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May 5. 1703.

At a MEETING
OF THE
ROYAL SOCIETY,

Sir JOHN HOSKYNS, V. P. *in the Chair,*

Mr. LOWTHROP *Preſented a* Propoſal *for Printing an* Abridgment *of the* Philoſophical Tranſactions. *This* Deſign *was Approv'd by the* Society, *and He was Deſir'd to* proceed *therein.*

May 12. 1705.

Impimatur,

Iſ. Newton, R. S. Pr.

London, Eng.—Royal Society.

THE PHILOSOPHICAL TRANSACTIONS AND COLLECTIONS,

To the End of the Year 1700.

ABRIDG'D

AND

Diſpos'd under GENERAL HEADS

vol. I

In Three Volumes.

By *JOHN LOWTHORP*, M.A.
and F. R. S.

The Third Edition.

LONDON:
Printed for J. KNAPTON, R. KNAPLOCK, R. WILKIN, J. and B. SPRINT, D. MIDWINTER, W. TAYLOR, W. and J. INNYS, R. ROBBINSON, and J. OSBORN.
M.D.CC.XXII.

LSoc 1816.~~11.5~~
10.3 (1)

Dr W. G. Howard
Cambridge

TO HIS
Royal Highneſs
THE
PRINCE,
LORD High ADMIRAL
OF
ENGLAND, *&c.*

SIR,

YOUR Royal Highneſs's *Great Condeſcenſion to Subſcribe the Statudes of the* Royal Society, *as one of their* Fellows, *Commands ſome Tribute from every Mem-*

Member. This, SIR, *I humbly hope, will in ſome meaſure excuſe the Preſumption of laying theſe* Papers *at Your* Royal Highneſs's *Feet. They have unavoidably, in this Abridgment, loſt much of their Original Beauty: But the* Order *wherein the Remaining Subſtance is Diſpoſed, will give Your* Royal Highneſs *a Nearer Proſpect of the Courſe of thoſe Studies you have the Goodneſs to* Protect.

It was a Noble Deſign, Worthy of their Royal Founder, *by* Incorporating *this* Society, *to Perpetuate a* Succeſſion *of* Uſeful Inventions: *But the Diſcouraging Neglect of the Great, the Impetuous Contradictions of the Ignorant, and the Reproaches of the Unreaſonable, have unhappily Retarded them in their Purſuit of thoſe Great Ends. To* Reſtore *them therefore to their firſt Vigour, is a Glory reſerv'd for Your* Royal Highneſs; *And already*

ready, Sir, *we feel the Chearful Influence of a Returning Spring.*

The Commands Your Royal Highneſs *has given, for Publiſhing, at Your own Expence, a moſt Magnificent* Uranigtaphy, (*far exceeding that of all the* Arabian Princes, *the Noble* Tycho Brahe, *and the Induſtrious* Hevelius) *cannot fail of ſurprizing Effects. All* Art *and* Nature *will exert their Powers upon this Occaſion, to keep pace with* Aſtronomy; *particularly* Navigation (*being under Your* Royal Highneſs's *Immediate* Care) *will Induſtriouſly apply thoſe* Accurate Obſervations *to all* Nautical *Purpoſes, and by ſome Familiar Method, Deliver the Anxious* Seamen, *from the* Fatal Accidents *that frequently attend their* Miſtaken Longitude.

Thus, Sir, *the* Munificence *of the* Prince, *and the* Vigilence *of the* Lord High Admiral,

ral, *will be equally a Bleſſing to the Preſent, and to all Future Ages,*

May Your Royal Highneſs *ever meet with with Returns of Gratitude, Suitable to the Univerſal* Beneficence. *So Wiſhes with great Sincerity,*

May it pleaſe Your ROYAL HIGHNESS,

Your ROYAL HIGHNESS's

Moſt Faithful, *and*

Moſt Obedient Servant,

John Lowthorp.

THE
PHILOSOPHICAL
TRANSACTIONS
AND
COLLECTIONS,

To the End of the Year 1700.

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ROBINSON, and J. OSBORN.

M. D. CC. XXII.

To the HONOURABLE
Sir *ISAAC NEWTON*, Kt.
PRESIDENT,
And to the
COUNCIL and FELLOWS
OF THE
Royal Society
OF
LONDON,
FOR THE
Advancement of *Natural Knowledge*,
THESE
Mathematical Papers,
ABRIDG'D
AND
Diſpos'd under GENERAL HEADS,
Are Moſt Humbly Dedicated
By *John Lowthorp*.

THE PREFACE.

THE Philoſophical Tranſactions *having met with* General Applauſe *and* Encouragement *for many Years, it would be a* Needleſs Trouble *to give any* Hiſtory *of them: 'Tis enough to ſay, That many of the* Diſcourſes *were* Compos'd, *and All of them* Collected *and* Publiſhed, *by* Particular Members *of the* Royal Society. *I ſhall therefore employ theſe very few Pages, only to acquaint the* Reader *with my own* Conduct *in this* Abridgment *of them.*

When I firſt reſolv'd *upon this* Undertaking, *I had Two Sorts of* Readers *in View, whom I was* deſirous *to ſerve*; Thoſe who make uſe of Books for their private Inſtruction or Entertainment, and thoſe who conſult them in order to publiſh ſomething of their own. *To a* Reader *of the Former Claſs, I thought it ſufficient to give him the Subſtance of ſo many* curious Papers, *in ſuch* Order *as would beſt ſuit with the* Courſe *of thoſe* Studies *that might Denominate him a* General Scholar. *But for the Sake of the Latter, I have, in the Margin, given the* Title *and* Author *of each Paper, and Directed to the* Number *and* Page *of the* Tranſactions *or* Collections, *where he may meet with the* Original *it ſelf. To the former I* deſign'd *this* Abridgment *to be as Uſeful as the* Volumes *at large, and to ſerve the Latter, inſtead of a not* inconvenient Repertorium: *And in the Proſecution of this Deſign, I have generally* confined *my ſelf to theſe Rules.*

I. I *have not only* Retain'd *the Eſſential Parts of the* Diſcourſes, *but I have kept in many Places to the very* Words *of their own* Authors, (*except where I was* Forc'd *to Vary them a little, to preſerve the* Connection:) *For, I thought it very unwarrantable to* Obtrude *any thing of mine under the Name of another Perſon*

II. *But to* Shorten *the* whole Work, *wherever I found any* Perſonal Addreſſes, *Long and Unneceſſary* Excurſions, *or* pompous Citations *of* Books, *I have taken the Liberty to* Suppreſs *them*, *yet, I hope, without Injuring the* Force *of the* Author's Reaſoning.

III. *I have* Omitted *all* Accounts *and* Extracts *of* Books, *which now, after ſo many Years* Publication, *ſeem almoſt Uſeleſs*: *Yet, to put the* Readers in Mind *of them, eſpecially ſuch as are about to Furniſh or Enlarge their* Libraries, *I have* added *a* Catalogue *at the* End *of each* Chapter, *to which they chiefly belong*; *and I have alſo* directed *them to ſuch* Additions, Emendations, *or* Refutations, *as ought to be* conſulted, *when thoſe* Books *fall under their* Examination.

IV. *I have alſo* Omitted *all* Heads *of* Enquiries *and* Experiments *ſimply propos'd, without further* Proſecution; *Believing that the* Anſwers *already given to many of them, and other* Diſcourſes *upon the ſame, or the like* Subjects, *will ſufficiently direct the* Notice *of an* Inquiſitive Reader.

V. *The previous* Calculations *of* Eclipſes, Lunar Appulſes, *and* Satellite Eclipſes *and* Occultations: *alſo* Tide-Tables, *and many other curious* Papers *of that kind*; *have long ago Out-liv'd the Reaſon of their* Publication.

VI. *All Simple* Catalogues *of* Natural Curioſities *(as of* Shells, Minerals, Plants, Animals, *&c.) without particular* Deſcriptions *of them, are little* Inſtructive : *And chiefly ſerve to* enlarge *the* Hiſtory *of the* Muſæum, *where they are* Depoſited ; *which is no Part of the* Deſign *of theſe* Volumes.

VII. *I have commonly* omitted *ſuch Papers as have been* collected *into Juſt* Volumes *by their own* Authors. *For this Reaſon, I have* omitted *ſome of thoſe ſurprizing* Microſcopial Diſcoveries *by the* Famous M. Leeuwenhoeck : *But I further confeſs,* I *was alſo leſs inclin'd to inſert them here, becauſe moſt of them Treat of* Subjects *not at all convenient* (in my Opinion) *for* Common Readers.

VIII. *But, to do all the* Right I *could to the* Ingenious Authors *of thoſe* Papers, *which the* Limits *of this* Abridgment *obliged me to* Omit, *I have, at the* End *of each* Chapter, *Annexed their* Titles, *and ſometimes a ſhort* Account *of them.*

Theſe are the Rules *I have carefully obſerved through the whole Conduct of this* Tedious Work : *Wherein I have faithfully aim'd at the* General Good *of all ſorts of* Readers ; *if* I *have fail'd in the* Performance, *'tis for Want of* Judgment *to do it* Better : *But* I *am bold to ſay, That if a* Kind Reception *of this ſhall encourage a like* Abridgment *of the* Foreign Philoſophical Journals, *in the* Same *or a* Better Order, *it will much Facilitate the many* Diſcoveries *ſtill ready to* Reward *the* Labours *and* Expences *of all* Induſtrious Promoters *of* Natural Knowledge.

A D-

ADVERTISEMENT

Of the BOOKSELLERS

To this Third EDITION.

THE *Philoſophical Tranſactions*, ſince the Year 1700. having been abridg'd and diſpos'd under General Heads, (after the ſame Method with this Work) by Mr. *MOTTE*, and by him publiſhed in Two Volumes: We thought it would tend to make the whole complete, if the ſeveral *Indices* of the Five Volumes were joined together; by which means, the Reader will have but one Trouble in ſeeking any Particular he has Occaſion for. To this Purpoſe we have taken Care, that in the *Contents*, the ſeveral Tracts ſhould be inſerted and refer'd to under their proper Chapters; and that in the *Index* of Authors Names, and the General one of Matters, the whole ſhould be plac'd in Alphabetical Order. At the Head of each *Index*, Directions are given for diſtinguiſhing the Matters occurring in theſe Three Volumes, from thoſe in the Continuation.

N. B. *In the enſuing* Contents, *thoſe Tracts whoſe Titles are printed in* Roman, *and the Authors in the* Italick *Character, are to be found in the Continuation.*

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THE
Mathematical Papers,
PUBLISH'D and DISPERS'D
IN THE
Philoſophical Tranſactions
AND
COLLECTIONS
ABRIDG'D;
AND
Diſpos'd under GENERAL HEADS.

CHAP. I.

Geometry, *Algebra*, *Arithmetick*, *Logarithmotechny*.

I. 1. QUÆ antehac de Augmentis Artium Mathematicarum executus ſum, eorum Summa huc ſere redit; Quamdiu homines Voluntate, Ingenio, Adminiculis, & Otio ad hæc Studia requiſitis deſtituantur, mirum non eſſe, ſi non majores in iis progreſſus faciant: Mihi itaque videri, ſequentium mediorum ope, Remedium huic Malo ſatis commode afferri poſſe. Nimirum ſi,

An Idea of Mathematicks; by Dr. J. Pell, An. 1638. *Ph. Col.* N. 5. *p.* 127.

§. 1. Conſcribatur Conſiliarius Mathematicus (ita appellare libet) qui ad tres haſce Quæſtiones reſpondeat. 1. Qui, qualeſq; Fructus ex ſtudiis Mathematicis ſperandi? 2. Quænam aſſequendæ tam fructuoſæ ſcientiæ Auxilia jam extent? 3. Qui Ordo in utendis iſtis Adminiculis obſervandus? Itaque continebit.

1. Facilem & perſpicuum diſcurſum, de limitibus ſeu ambitu Artium Mathematicarum, deque ingentibus utilitatibus, primo ad ipſarum ſtudioſos, tum ad gentem complurium in iis peritorum feracem inde redundaturis.

2. Catalogum Mathematicorum & ab iis editorum Operum, qui exhibeat, 1. Synopſin omnis generis Librorum Mathematicorum vel jam Publicatorum, vel Manuſcriptorum in publicis Bibliothecis deliteſcentium, peculiaribus, cuivis Generi, numeris appoſitis. 2. Catalogum Chronologicum omnium Mathematicorum celebrium ſecundum ordinem Annorum quibus ſinguli vixere, additis ubique annis quibus Opera ipſorum primo excuſa fuere. 3. Catalogum ipſorummet Operum ſecundum ordinem Annorum, quibus ulla Lingua impreſſa fuere. In quo concinnando ita procederem, ut notato Anno Domini ſubjungerem, more uſitatorum Catalogorum, Nomina omnia Mathematicorum Librorum in ulla Gente aut Lingua iſto Anno editorum, 1. Indicando tamen in ſingulis, rationem Voluminis, hoc eſt, non tantum magnitudinem paginarum (*e. g. 4to. 8vo. &c.*) ſed earundem ſummam, ut moles totius Operis ſic facile cuivis pateat. 2. Ante ipſummet Titulum exprimendo Annum, ad quem retroſpicere oporteat cogniturum quando opus conſcriptum, quando hac illave Lingua ultimo editum fuerit. 3. Notando in Margine poſt Titulum, 1. Annum quo opus aliquod proxime Typis exſcriptum ſit. 2. Numerum, qui remittat Lectorem ad Synopſin prima ſtatim Catalogi pagina delineatam. Quorum Numerorum beneficio quilibet è Veſtigio omnes unius argumenti libros Mathematicos percurrere queat.

3. Conſilium, inſtruens ſtudioſum, de optimis in quolibet genere libris: Quo ordine & quomodo legendi, quid obſervandum, quid evitandum in lectione quorundam Mathematicaſtrorum, quomodo procedendum, omniaque memoria tenenda.

4. Paræneſin, Primo ad omnes, opibus, otio, & ingenio horum Studiorum capaci inſtructos, ut, 1. Cum ingentium commodorum ad ipſummet eorum ſtudioſum & ad univerſum humanum genus inde redeuntium. 2. Tum etiam defæcatæ illius voluptatis, quæ ex ſcrutinio abſtruſarum Veritatum, lucta cum difficilibus Problematibus, & eorundem victoria emergit, reſpectu, ſerio ad ea excolenda incumbant, tanto quidem magis, quo, 3. Expeditiora adminicula reperta ſunt, quæ laborem, tempus, ſumptus, majoribus noſtris impendenda, nobis compendifaciant. Deinde ad omnes tum judicio ad æſtimandum horum ſtudiorum pretium, tum & opibus (æternæ noſtri memoriæ, ſi in prudentes ſui diſpenſatores inciderint, promicondis) pollentes ut majori curæ ſibi eſſe patiantur hoc ſtudioſorum genu, ampliſque propoſitis præmiis lectiſſimos eorum ſeponant perficiendis inventis, ad quæ ipſo genio ſuo ducuntur. Denique ad omnes Principes & Reſpublicas, qui majus ditionibus ſuis

ornamentum vix conſiliaverint, quam operam dando, 1. Ut harum Artium peritis abundent. 2. Via ad eas minus laborioſa ſumptuoſaq; reddatur. 3. Ipſaq; adeo Ingenia Mathematica magis innoteſcant, ſeq; dignis auxiliis gaudeant.

In quem finem utiliſſimum fuerit, ut,

§. 2. Conſtituatur Publica Bibliotheca, quæ & omnes ſupradictos Libros, & unum Inſtrumentorum unquam inventorum contineat, & præterea reditibus ſufficientibus inſtructa ſit. 1. Ad coemenda ſingulorum librorum Mathematicorum, quæ quotannis alibi locorum foras dantur, exemplaria. 2. Ad ſuſtentandum Bibliothecarium, cui incumbat.

1. Perlegere omnes hujus Claſſis libros in Regione iſthac publico committendos, 1. Suppreſſis quæ ab artis regulis exorbitant, ne Scriptorum errores Lectoribus fraudi ſint. 2. Monereq; Scriptores, ſi vetera & jam dicta tantum reponant.

2. Sub periculo exiſtimationis ſuæ approbare inventa inſignia, & ſincere commendare Inventores Mecænatibus.

3. Recipere, in Catalogum referre, & per Repoſitoria diſponere unum ita perlectorum librorum exemplar, donandum Bibliothecæ bene compactum, Authoris aut Bibliopolæ ſumptibus.

4. Promptum Reſponſum præbere cuivis ſtudioſo, circa Problema aliquod inquirenti, an ſolutum fuerit nec ne, ni illi neceſſe ſit actum agere, aut contra inventa ſua ſupprimere, metu, ne antiqua jam ſint, & forſan in aliquo Librorum Bibliothecæ jam elaborata.

5. Recipere, *&c.* omnia Manuſcripta, vel donatione, aut legato Bibliothecæ obvenientia.

6. Diligens cum ſui generis hominibus, extra patriam ſuam agentibus, Literarum commercium exercere, ne ipſum fugiat, quidnam ibi locorum prelo paratum ſit.

7. Obſervare in Conterraneis ſuis, qui docendis hiſce artibus apti ſint.

8. In numerato ſemper habere Artifices omnes conficiendorum Inſtrumentorum Mathematicorum & Repræſentationum, adeoque Operationum in Ligno, Magnete, Metallis, Vitro, *&c.* peritos.

9. Poſt legitimum Examen, omnis generis practicis, *e. g.* Gubernatoribus Navium, Gæodetis, Logiſtis, *&c.* Speculativæ Peritiæ & Practicæ Dexteritatis teſtimonium præbere, ne poſthac, quibus hoc hominum genere opus, imperiti magno ipſorum damno imponant.

Catalogus docuerit, in tanta multitudine librorum, mundum tantum non obruentium, quinam ad hanc tantum ſtudiorum partem pertineant. Bibliotheca, & ipſorummet Librorum copiam faciet, & ſimul cognoſcendi ubi locorum exemplaria plura venalia ſint: Prætereaq; promptuarii inſtar erit exteris & indigenis, unde extemplo diſcant, quænam ad hæc ſtudia adminicula Regio iſta ſit ſuppeditatura.

Atque hæc meo judicio expeditiſſima Methodus eſt, utendi illis auxiliis, quibus jam potimur. Si ampliora deſiderentur, conveniens fuerit, ut peritorum Artificum opera.

§. 3. Conficiantur & publicentur tres hi novi Tractatus,

1. Pandectæ Mathematicæ, complectentes, quam fieri poteſt, perſpicue, methodice, compendioſe, & ingenue quæcunque ex Mathematicis libris & inventis, quæ ante nos fuere, colligi aut veluti conſectaria inde deduci potuere, citatis ſub finem cujuſque periodi aut propoſitionis, authoribus, in quibus ea reperitur antiquiſſimis, ſequentibuſque adeo omnibus nota inuſta, ubi vel in furto deprehenſi fuerint, vel mutuati eſſe ſuppreſſo unde ſumpſerint, aut (quod peſſimum omnium) aliorum inventa audacter ſibi arrogaſſe. Hoc pacto magna illa Bibliotheca in multo auguſtius ſpatium contraheretur, majori omnibus poſtfuturis laboris, temporis & ſumptuum compendio, quam inpræſentiarum quis imaginetur. Sed cum & hoc opus portatu minus amplum futurum ſit : Elaboretur præterea.

2. Comes Mathematicus, libro manuali (adeoque quam licet breviſſime). continens utiliſſimas Tabulas una cum præceptis de uſu ipſarum in Problematibus ſolvendis, ſeu pure Mathematicis, ſeu ad diverſas materias, prout res requiſierit, applicatas.

Et denique, ne in hoc quoque Eruditionis genere libris amplius aſtricti ſimus : Excogitetur,

3. Mathematicus αὐτάρκης ſeu Inſtructio quomodo quilibet Mathematicus, laborem non exhorrens, eam peritiam adipiſci poſſit, ut ſane ulla Librorum aut Inſtrumentorum ope, cujuſvis tamen Problematis Mathematici reſolutio æque facilis ipſi ſit, ac alteri ſola librorum Volutatione nixo.

Atque hæc eſt Idea illa Matheſeos, quam, more meo, diu abhinc mihi effinxi, firmiter ſemper perſuaſus, tum demum feliciter magnis rebus vires intendi, ubi exactam prius earundem Ideam, ejuſque aſſequendæ media quam maxime oppoſita animus conceperit : Quam ſi exprimere re ipſa non detur, multum tamen eſſe ab ea quam proxime abſuiſſe. Hancce meam equidem adeo non ſupra vires humanas arbitror, ut vel viri unius, rei domeſticæ aliorumve negotiorum curis non diſtracti, diligentiam parem ei putem. Bibliothecam enim & Catalogum nummis præſentibus facile parari poſſe, quis non videt ? Et ſi mihi Pandectarum contexendarum (quales ſupra deſcripſi) Provincia demandaretur, longe duriores Leges, quam quas ibidem enumeravi, mihi impoſiturus forem. Delinearem, primo omnium, infallibilem Rationis humanæ in indagando, quodcunque objicitur, proceſſum, oſtenſo quomodo à primis principiis ſeu elementis nunquam interrupta Serie, ad ſublimiſſimas juxta & abjectiſſimas eorum applicationes conſcendendum ſit. Qua arte forſan haud diu carerent homines, ſi in poſterum ſollicite veſtigarent, quanam ratione illis quos mirantur tales obortæ eſſent cogitationes, quomodo tali fini, tam appoſita media reperta fuiſſent. Pandectas in Enchiridion, quotidianis uſibus opportunum, conſtringendi ratio forſan haud plures fugerit : At ita mentibus ſuis eas inſcribendi, ut ne Libris quidem amplius opus habeant (quod Mathematicus noſter αὐτάρκης flagitat) vires ingenii humani tranſcendere pleriſque haud dubie videbitur, cum nemo hucuſque, quod ſciam, tale quid vel animo concipere auſus fuerit. Nihilominus tamen remittent aliquid, credo, de incredulitate ſua homines, poſtquam quæ roborandæ imaginationi, juvandæ memoriæ, dirigendæ rationi artes excogitari, quam miri effectus conjunctione & jugi exercitio earum produci queant, paulo attentius ſecum conſideraverint.

Conſider'd, Oct. 1639. *by* Merſennus, *ibid.* p. 135.

2. Conſultius omnino mihi videtur ac præſtantius, ſi loco tanti apparatus quem Author Ideæ proponit de colligendis variis Mathematicorum ſcriptis, optimi ſolummodo & digniores ſeligerentur; Adductis primum Authoribus illis Antiquis quorum libri adhuc ſuperſunt, veluti Euclide, Apollonio, Archimede, Theodoſio, Pappo, Ptolemæo, cum reliquis ipſorum Fragmentis & Manuſcriptis quæ nondum viderunt lucem, ac quorum aliqua penes Golium Lugduni Batavorum, quædam vero Romæ aſſervantur. Ipſis deinde recentiores poſſent adjungi, veluti Vieta, Clavius, & noſter Herigonius. Similiter inter Opticos, quis ſeligeret Vitellionem, Keplerum, Aquilonium, & Dom. de Villes. Inter Arithmeticos poſt Diophantum, eminent Cardanus, Tartaglea & Veſter Nepperus. Pro Triangulis Sphæricis & Supputatione per Logarithmos extant Briggius, Gordanus, Pitiſcus, Snellius, & noſter Morinus. Pro Aſtronomicis, poſt Ptolemæum & aliquot Arabes, conducerent omnes illi qui Tabulas compoſuerunt, ſicut Alphonſus, Johannes Regiomontanus, Keplerus, & noſter Duretus. Et ut paucis dicam, pro Fortificatoriis & Muſicis octo vel decem Authores excerpendi eſſent, illi ſcilicet qui meliori inclaruere Praxi. Similiter pro Rebus Mechanicis, Viribus Motuum, Machinis, & Ductibus Aquarum, ita ut enucleatis decem vel duodecim Authoribus, deſiderium ſuum explere poſſet harum rerum cupidus. Et ſi duodecim Viri Intelligentes & Amici cordate in ſe ſuſciperent hoc negotium, ut quilibet unico Volumine convenienti & perſpicuo compingeret aliquam Scientiam, additis illis quæ deſunt in reliquis Authoribus, & ſublatis non neceſſariis, abſque dubio haberemus breviter & nervoſe, in duodecim Voluminibus totum id quod hac in re à quoquam poſſet expeti. Et mea quidem opinione, quicquid ſpectat ad Matheſin, tam Puram quam Mixtam, poterit comprehendi duodecim illis Voluminibus. Sic omnem elegantiorem Philoſophiam tribus, & omnes Artes Liberales & Mechanicas etiam Tribus Libris exponere liceret: Ideoque levi pretio ad Eruditionem quis poſſet pertingere. Quod autem Mathematica illa Inſtrumenta attinet, parum certe prodeſſet apparatus omnium illorum quæ hactenus inventa fuere. Præſtaret habere quatuor vel quinque, quæ nimirum optima ſunt in eo genere, & magis commoda præ reliquis judicantur.

Anſwer'd by Dr. Pell, *ib.* p. 137.

3. Tibi (Vir Clariſſime) ſi mentem tuam recte aſſequor, Omnia probantur, niſi quod majorem Apparatum, quam opus ſit, à me deſideratum cenſes: Selectiora tantum, non ut nos, Omnia Volumina atque Inſtrumenta Mathematica, colligi ſuades. Quod conſilium tuum neque ego improbarem, ſi & omnium calculo definitum eſſet quinam in tanta Scriptorum Turba reliquis omnibus Palmam præriperent, & id tantum à me ſpectaretur ut aliquid Laboris, Sumptuumque Compendium harum Artium ſtudioſis ſuppeditaretur. Nunc cum totius Matheſeos Perfectiſſimam Inſtaurationem optem, non aliud Conſilium à me proficiſci debet, quam quod tali ſcopo per omnia quadret. Hujus Conſilii fundamentum in univerſali ejuſmodi Bibliotheca haud immerito collo-

care mihi videor. Non audeo quenquam ſpernere, ſymbolum ſuum quocunq; modo huc conferentem, multo minus condemnare inauditum. Et ſi meo judicio locus ſit, vel Ineptiſſimum ſive Scriptum ſive Inſtrumentum Mathematicum dignum eſſe puto, cujus unum Exemplar, vel propter Errores, in amplæ cujuſcunque regionis loco certo & patente, ſervetur. Videmus enim non pauca ingenioſe inventa in ſuperiorum ſeculorum rudioribus Inſtrumentis, etiam nunc non obſervatu tantum digna, ſed & Imitatione, quemadmodum etiam Imperiti Scriptores non paucas optimas egregiorum inventorum ἀφορμὰς ſagacioribus ingeniis præbuere; digitum enim intendere poſſumus, quo pervenire non poſſumus. Videmus quamplurima Lemmata rectiſſime demonſtrata ab hujuſmodi Scriptoribus, & tamen propter unum Pſeudographema inter fundamenta collocatum, quicquid ſuperſtruxerant corruiſſe. Quod ſi non ſolum falſitate, ſed etiam verboſitate, ſtribligine, *&c.* offenſus multos rejiciendos putas, cogita quam varium & multiplex ſit hominum Palatum, neque omnes ex tui ingenii perſpicacitate judica. Sunt enim nonnulli qui nihil intelligunt niſi idem (& fere iiſdem verbis) dictum ſit centies; his credamus ταυτολογίας iſtas accommodatas eſſe. Et quoniam à Notioribus omnino incipiendum eſt, eadem vero non ſunt omnibus notiora, à diverſiſſimis initiis progrediuntur, ut vix reperias Diſcentis Ingenium, cujus Tyrocinio non ſit jamdudum accommodatum Rudius aliquod Scriptum vel Inſtrumentum quod neſcire nullo modo debet, qui ſe Conſiliarium Mathematicum profitetur. Itaque Librorum iſta Perfectior Collectio ſane neceſſaria mihi videtur.

Quo minus vero nobis placent iſti Mathematicaſtri, eo magis fuerit, cur Bibliothecam hujuſmodi optemus, quippe quæ ſola infinitam iſtam hoc in genere ſcriptitandi pruriginem facile coercuerit. Balbutientes enim iſti Magiſtelli qui Tyrocinia & Juventutis captum crepantes, nuſquam non pueriliter ineptiunt, videbunt ſat ſuperque eſſe hujus farinæ Elementarium jam pridem Compilatorum. Qui vero infinitis inventis Matheſin augere ſed fruſtra cupiunt incautiores, tot κενοδόξων παράδοξα publice damnata, atque ludibrio omnium expoſita videntes, malo cavebunt ſi ſapiunt. Plagiarii vero, publico odio digniſſimi, non audebunt poſthac veteres aliquos Libellos ſemel forſan typis impreſſos, aut aliquas tantum earum particulas, pro ſuis extrudere. Quin etiam illi, quibus alioqui nec Candor nec Ingenium deeſſet, ad cogitata ſua ſatis commode proferenda, quia tamen tam multos in ſingulis materiis ſe præceſſiſſe videbunt, ſatis monebuntur, ne alia in publicum edant, quam prioribus indicta. Qualia quæ ſint, ex inſpecta ampla ejuſmodi Bibliotheca, vel ſi hunc laborem gravantur, ex ipſomet, quem ei aſſigno Bibliothecario, facile cognoſcent. Atque hæ fere rationes ſunt, quæ conſilii iſtius de tam univerſali Bibliotheca pœnitere me adhuc non ſinunt.

To the Satisfaction of Merſennus, *Dec.* 10. 1639. *ibid. pag.* 143.

4. Statim atque tuas Literas perlegi (Vir Eruditiſſime) non ſolum totus fui tuus, tuæque ſubſcripſi ſententiæ, quam mihi egregie probatam teſtor; ſed & inſolitus Ardor Animi me propemodum abſtulit, nempe ut tuum illud opus quantumvis arduum Orbis Magnatibus proponendum curarem, ſi liber ad eos foret Aditus. Sed quis Regum incipiet? Hoc enim Opus Regium auſim appellare.

5. Ideam

The Judgment and Approbation of Des Cartes, *Feb.* 1640.
Ib. p. 144.

5. Ideam Mathematicam non niſi obiter inſpexi, jamq; tantum memini nihil me in illa reperiſſe à quo multum diſſentirem, & valde probaſſe quod primo loco omnis ſupellex Mathematica ibi enumeretur, & poſtea ipſe Mathematicus tanquam αὐτάρκης ex ſeipſo contentus deſcribatur. In eundem enim fere ſenſum duo ſoleo in Matheſi diſtinguere, Hiſtoriam ſcilicet & Scientiam: Per Hiſtoriam intelligo illud omne quod jam inventum eſt, atque in libris continetur: Per Scientiam vero, Peritiam Quæſtiones omnes reſolvendi, atque ideo inveniendi propria Induſtria illud omne quod ab humano ingenio in ea Scientia poteſt inveniri, quam qui habet, non ſane multum aliena deſiderat, atque adeo valde proprie αὐτάρκης appellatur. Valde autem optandum foret, ut illa Hiſtoria Mathematica, quæ in multis Voluminibus ſparſa, nondum integra & perfecta eſt, in unum Librum tota colligeretur: Neque ad hoc ulli ſumptus in perquirendis aut coemendis libris eſſent faciendi: Cum enim Authores alii ex aliis multa exſcripſerint, nihil ullibi extat, quod non in quavis mediocriter inſtructa Bibliotheca alicubi reperiatur; nec tam diligentia opus eſſet ad omnia colligenda, quam judicio ab ſuperflua rejicienda, & Scientia ad ea quæ nondum inventa ſunt ſupplenda. Atqui ſi talis Liber extaret, facile ex eo unuſquiſque omnem Hiſtoriam Mathematicam, atque etiam aliquam partem Scientiæ addiſceret. Si quis autem omnia quæ ad ejus Praxin pertinent, habere vellet, ut Inſtrumenta, Machinas, Automata, *&c.* næ ille ſi Rex eſſet, orbis Terrarum impenſis omnibus ad hoc neceſſariis ſufficere nunquam poſſet. Neque vere etiam illis opus habet, ſed ſatis eſt ſi omnium norit deſcriptionem, adeo ut ea, cum uſus exiget, vel ipſe facere, vel per Artifices fieri curare poſſit.

Some of Euclid's *Propoſitions, demonſtrated independently from the reſt, by* Mr. Aſh.
N. 162. *p.* 672.
32. I. E.

II. THE Propoſitions which I ſhall endeavour to demonſtrate independently from all others, ſhall be theſe; the 32d and 47th of the *Firſt Book*; moſt of the *Second* and *Fifth Books*; the 1ſt and 16th of the *Sixth*; with their Corollaries. In order to demonſtrate the 32d; I ſuppoſe it known what is meant by an Angle, Triangle, Circle, External Angle, Parallels, and that the Meaſure of an Angle is the Arch of a Circle intercepted between its Sides; that a Right Angle is meaſur'd by a Quadrant, and two Right Angles by a Semicircle. I ſay then, that in the Triangle A B C, the External Angle B C E is equal to the two oppoſite Internal ones A B C, B A C; for let a Circle be drawn, C being the Center, and B C the Radius; and let C D be drawn parallel to A B, thoſe two Lines being always equidiſtant, will both have the ſame Inclination to any third Line falling upon them; that is, (by the Definition of Angle) they will make Equal Angles with it: For if any Part of C D (for Inſtance) did incline more to B C than to A B, upon that very Account they would not be parallel; it follows therefore that the Angles A B C, B C D, are equal: Alſo $BAC = DCE$, becauſe A E falls upon two Parallels; but the External Angle $BCE = BCD + DCE$, which were before prov'd to be equal to A B C, B A C. *Q.E.D.* Hence may be inferr'd as a Corollary, That the three Angles of every Triangle are equal to two Right ones; for the Angles $ACB + BCE$, are meaſur'd by a Semi-

Fig. 1.

Semicircle, and therefore equal to two Right Angles; Corollaries alſo from
20, 22, 31. 3. E. hence are the 20th, 22d, and 31ſt of the *Third* Book, which contain the
Properties of Circles, whoſe Deduction from hence is moſt natural and
obvious.

47. 1. E. In order to demonſtrate the 47th, I ſuppoſe the Meaning of the Terms
made uſe of, to be known; and that two Angles or Superficies are equal,
when one being put on the other, it neither exceeds, nor is exceeded. This
being allow'd, I ſay, the Sides about the Right Angle are either equal or
Fig. 2. unequal; if equal, let all the Squares be deſcrib'd; the whole Figure exceeds
the Square of the Hypotenuſe B C, by the two Triangles M, V, and
exceeds alſo the Squares of the other two Sides, A B, A C, by the two
Triangles, A B C, and S; which Exceſſes are equal, for M is equal to
A B C, the two Sides about the Right Angle, being two Sides of a Square
upon A B, by Suppoſition equal to A C, and the third Side equal to B C;
therefore the whole Triangles are equal. After the ſame manner, S and V
are proved to be equal; therefore the Square of C B is equal to the Squares
of the two other Sides. *Q. E. D.*

Fig. 3. But if the Sides be unequal, let the Squares be deſcribed, and the Parallelogram
L Q compleated, the whole Figure exceeds the Square upon B C,
by three Triangles, X, R, Z, and exceeds alſo the Square L A, A D, by
the Triangle A B C, and the Parallelogram P Q: Which Exceſſes, I ſay, are
equal; for Z is equal to A B C, the Side $OC = BC$, $CD = AC$, the
Angle $D = A$, and $OCD = BCA$; which is manifeſt, by taking the
common Angle A C O, out of the two Right Angles B C O, A C D; therefore
by Superimpoſition the whole Triangles are equal. In like manner X
is proved equal to A B C, alſo R; and the Parallelogram P Q to be double
of the Triangle A B C: Thus the Exceſſes being proved equal, the Remainders
alſo will be equal; *viz.* the Square of B C to the Square of A B, A C.
35, 36. 3. E. *Q. E. D.* Manifeſt Corollaries from hence are, the 35th and 36th of the
12, 13. 2. E. *Third* Book; alſo the 12th and 13th of the *Second*.

1, 2, &c. 2. E. The firſt Ten Propoſitions of the *Second* Book are evidently demonſtrated,
only by ſubſtituting Species or Letters inſtead of Lines, and multiplying
them according to the Tenor of the Propoſition: Thus, to inſtance in one or
Fig. 4. two, Call the whole Line A, and its Parts B and C, therefore $A = B + C$,
and conſequently $AA = BB + CC + 2BC$, which is the very Senſe of
4. 2. E. the *Fourth* of the *Second* Book. Thus alſo, Let a Line be cut into equal
Fig. 5. Parts F, F, and let another Line, S, be added thereto; 'tis manifeſt, that
$4FF + 4SF + 2SS = 2FF + 2FF + 2SS + 4SF$, which is the
10. 2. E. *Tenth* Propoſition of the ſame Book.

Almoſt the whole Doctrine of Proportionals, *viz.* Permutation, Inverſion,
Converſion, Compoſition, Diviſion of Ratio's, and Proportion *ex æquo*; and
conſequently the moſt uſeful Propoſitions of the *Fifth* Book are clearly demonſtrated
by one Definition, and that is of Similar or Like Parts, which are
ſaid to be ſuch as are after the ſame manner, or equally contain'd in their
Fig. 6. Wholes: Thus the Antecedents A and C, are either equal to their Conſequents,
or greater, or leſs; if equal, the thing is manifeſt; if leſs, then

(by the Definition of Proportionals) A and C are like Parts of B and E; therefore what Proportions the whole B and E have to one another, the ſame will A and C have, which is Permutation; likewiſe $E : C :: B : A$, which is Inverſion; ſo alſo if from Proportionals you take like Parts, the Remainders will be Proportional; whence Converſion and Diviſion are Demonſtrated: And if to Proportionals you add Like Parts, the Wholes will ſtill be Proportional, which is Compoſition, *&c.* If the Antecedents be greater than the Conſequents, the Conſequents will be like Parts of them, and the Demonſtration exactly the ſame with the former.

1. 6. E.
Fig. 7.
Fig. 7.

The firſt of the *Sixth* Book is prov'd, by conſidering the Generations of the Parallelograms, which are produced by drawing or multiplying the Perpendicular upon the Baſis; that is, taking it ſo often as there be Parts and Diviſions in the Baſe: Therefore the ſame Proportion that R X ſingle, hath to N X ſingle, the ſame hath R X multiplied by X Z, that is, repeated a certain number of times, to N X multiplied by X Z, that is, repeated the ſame Number of times; which is as much as to ſay, $RX : NX ::$ Paral. RZ : Paral. NZ. Now that this Propoſition alſo is true in Oblique angled Parallelograms, is proved, becauſe they are equal to the Rectangled ones upon the ſame Baſis, and between the ſame Parallels, as does thus independently appear; the Triangles R Q X, and M P Z, are equal, for $RX = MZ$, $QX = PZ$, $RM = QP$; therefore adding to both M Q, $RQ = MP$; if therefore from theſe equal Triangles you take what is Common, *viz.* M L Q, the Remainders will be equal, $RXLM = QLZP$; to both which add X L Z, and the whole Parallelograms will be Equal, $RZ = QZ$. *Q. E. D.* That Triangles alſo having a Common Baſis, are in the Proportion of their Altitudes, does hence follow, becauſe they are the halves of Parallelograms upon the ſame Baſis. This alſo is true, and the Demonſtration exactly the ſame in Priſms, Pyramids, Cylinders, and Cones, having the ſame Baſis.

16. 6. E.
Fig. 6.

To prove the 16th of the *Sixth*, I ſuppoſe the 4 Lines, A, B, C, E, to be Proportional, that is, granting A and C to be the leſſer Terms; the ſame way that A is contained in B, ſo is C in E, and that D is the Denominator of the Ratio; 'twill follow then, that B is made up of A, multiplied by D, and E of C, multiplied by D; ſo that $AD = B$, and $CD = E$; draw therefore the Extremes upon one another, that is, A upon C D, and the Means, that is, C upon A D, the Factors being the ſame; I ſay, the Products A C D, and C A D, are the ſame, and conſequently Equal. *Q. E. D.*

The Problem compos'd by M. *Comier*, (with Oſtentation enough) as if it contain'd ſomething New, tho' in reality it be nothing but the Old Buſineſs of doubling the Cube a little diſguis'd, is eaſily ſolv'd Algebraically, as follows,

Fig. 8.

8. 6. E.	1	$a:2x::x:\frac{2x^2}{a}=p$
47. 1. E.	2	$aa+2ax-\frac{2x^3}{a}=\frac{4x^4}{aa}$
2 × by aa	3	$a^4+2a^3x=4x^4+2x^3a$
3 ÷ $-a+2x$	4	$a^3=2x^3$; that is, The Cube upon x, is half the Cube upon a.

The Squaring of the Hyperbola by the L. Viſcount Brounker. N. 34. *p.* 645. *Fig.* 9.

III. Let A B be one Aſymptote of the Hyperbola E d C; and let A E, and B C, be Parallel to the other: Let alſo A E, be to B C, as 2 to 1; and let the Parallelogram A B D E, be equal to 1.

Suppoſing the Reader knows, that E A, $\alpha\zeta$, K H, $\beta\eta$, $d\theta$, $\gamma\kappa$, $\delta\lambda$, $\epsilon\mu$, C B, &c. are in an Harmonick Series, or a Series *Reciproca-Primanorum, ſeu Arithmetice Proportionalium,* (otherwiſe he is referr'd for Satisfaction to *Arithm. Infinitor. Walliſii,* Prop. 87, 88, 89, &c.) I ſay,

$$ABCdEA=\frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+\frac{1}{7\times8}+\frac{1}{9\times10}, \text{ \&c.}$$

$$EdCDE=\frac{1}{2\times3}+\frac{1}{4\times5}+\frac{1}{6\times7}+\frac{1}{8\times9}+\frac{1}{10\times11}, \text{ \&c.}$$

$$EdCyB=\frac{1}{2\times3\times4}+\frac{1}{4\times5\times6}+\frac{1}{6\times7\times8}+\frac{1}{8\times9\times10}, \text{ \&c.}$$

For (in *Fig.* 10. and 11.) the *Parallelogram*, and (in *Fig.* 12.) the *Triangle*, *Fig.* 10, 11, 12.

	$CA = \frac{1}{1 \times 2}$	$EdC = \frac{1}{2 \times 3 \times 4} = \frac{\square dD - \square Fd}{2}$
$dD = \frac{1}{2 \times 3}$	$dF = \frac{1}{3 \times 4}$	$Ebd = \frac{1}{4 \times 5 \times 6} = \frac{\square br - \square bn}{2}$
$br = \frac{1}{4 \times 5}$	$bn = \frac{1}{5 \times 6}$	$dfC = \frac{1}{6 \times 7 \times 8} = \frac{\square fG - \square fk}{2}$
$fG = \frac{1}{6 \times 7}$	$fk = \frac{1}{7 \times 8}$	$Eab = \frac{1}{8 \times 9 \times 10} = \frac{\square aq - \square ap}{2}$
$aq = \frac{1}{8 \times 9}$	$ap = \frac{1}{9 \times 10}$	$bcd = \frac{1}{10 \times 11 \times 12} = \frac{\square cs - \square cm}{2}$
$cs = \frac{1}{10 \times 11}$	$cm = \frac{1}{11 \times 12}$	$def = \frac{1}{12 \times 13 \times 14} = \frac{\square et - \square el}{2}$
$et = \frac{1}{12 \times 13}$	$el = \frac{1}{13 \times 14}$	$fgC = \frac{1}{14 \times 15 \times 16} = \frac{\square gu - \square gh}{2}$
$gu = \frac{1}{14 \times 15}$	$gh = \frac{1}{15 \times 16}$	&c.
&c.	&c.	

Note, $\frac{1}{2}\, CA = dD + dF$
$\frac{1}{2}\, dD = br + bn$
$\frac{1}{2}\, dF = fG + fk$
$\frac{1}{2}\, br = aq + ap$
$\frac{1}{2}\, bn = cs + cm$
$\frac{1}{2}\, fG = et + el$
$\frac{1}{2}\, fk = gu + gh$ &c.

And that therefore in the firſt Series half the firſt Term is greater than the Summ of the two next, and half this Summ of the Second and Third greater than the Summ of the four next; and half the Summ of thoſe Four, greater than the Summ of the next Eight, *&c. in infinitum.* For $\frac{1}{2} dD = br + bn$; but $bn > fG$, therefore $\frac{1}{2} dD > br + fG$, *&c.* And in the ſecond Series, half the ſecond Term is leſs than the Summ of the two next, and half this Summ leſs than the Summ of the four next, *&c. in infinitum.*

That the firſt Series are the even Terms, *viz.* the 2d, 4th, 6th, 8th, 10th, *&c.* and the ſecond the odd, *viz.* the 1ſt, 3d, 5th, 7th, 9th, *&c.* of the following Series, *&c.*

$$\frac{1}{1\times 2},\ \frac{1}{2\times 3},\ \frac{1}{3\times 4},\ \frac{1}{4\times 5},\ \frac{1}{5\times 6},\ \frac{1}{6\times 7},\ \text{\&c. } \textit{infinitum} = 0,$$

Whereof a, being put for the Number of Terms taken at Pleaſure, $\frac{1}{a^{2}+a}$ is the laſt, $\frac{a}{a+1}$ is the Summ of all thoſe Terms from the Beginning, and $\frac{1}{a+1}$ the Summ of the reſt to the End.

That $\frac{1}{4}$ of the firſt Term in the third Series, is leſs than the Summ of the two next, and $\frac{1}{4}$ of this Summ leſs than the Summ of the four next, and $\frac{1}{4}$ of this laſt Summ leſs than the next eight, I thus Demonſtrate.

Let a, be equal to the Third or Laſt Number of any Term of the firſt Column, *viz.* of Diviſors;

$$\frac{1}{a\times\overline{a-1}\times\overline{a-2}} = \frac{1}{a^{3}-3a^{2}-+2a} = \frac{16a^{3}-48a^{2}+56a-24}{16a^{6}-96a^{5}+232a^{4}-288a^{3}+184a^{2}-48a} = \text{A}.$$

$$\left.\begin{array}{l} \dfrac{1}{2a\times\overline{2a-1}\times\overline{2a-2}} = \dfrac{1}{8a^{3}-12a^{2}+4a} \\[2ex] \dfrac{1}{\overline{2a-2}\times\overline{2a-3}\times\overline{2a-4}} = \dfrac{1}{8a^{3}-36a^{2}+52a-24} \end{array}\right\} =$$

$$\frac{16a^{3}-48a^{2}+56a-24}{64a^{6}-384a^{5}+880a^{4}-960a^{3}+496a^{2}-96a} = \text{B}.$$

$$\frac{64a^{6}-384a^{5}+928a^{4}-1152a^{3}+736a^{2}-192a}{64a^{6}-384a^{5}+880a^{4}-960a^{3}+496a^{2}-96a}\times\tfrac{1}{4}\text{A} = \text{B}.$$

And $48a^{4}-192a^{3}+240a^{2}-96a =$ Exceſs of the Numerator above the Denominator.

$$\left.\begin{array}{rcl} \text{But the Affirmat.} & > & \text{the Negat.} \\ \text{That is, } 48a^{4}+240a^{2} & > & 192a^{3}+96a \\ \text{Becauſe } a^{4}+5a^{2} & > & 4a^{3}+2a \\ a^{3}+5a & > & 4a^{2}+2 \end{array}\right\}\ \text{if } a>2.$$

$$\text{Therefore}\quad \text{B} > \tfrac{1}{4}\text{A}.$$

Therefore one Fourth of any Number of A, or Terms, is leſs than their ſo many reſpective B; that is, than twice ſo many of the next Terms. *Q. E. D.*

By any one of which three Series it is not hard to calculate, as near as you pleaſe, theſe and the other like Hyperbolic Spaces, whatever be the rational Proportion of A E to B C. As for Example; When A E is to B C, as 5 is to 4. (Whereof the Calculation follows, after that where the Proportion is, as to 2 to 1; and both by the *Third* Series.)

Firſt then, when (in *Fig.* 9.) A E : B C : : 2 : 1. *Fig.* 9.

2 × 3 × 4) 1.	(0. 0416666666	0. 0416666666
4 × 5 × 6) 1.	(0. 0083333333	0. 0113095237
6 × 7 × 8) 1.	(0. 0029761904	
8 × 9 × 10) 1.	(0. 0013888888	0. 0029019589
10 × 11 × 12) 1.	(0. 0007575757	
12 × 13 × 14) 1.	(0. 0004578754	
14 × 15 × 16) 1.	(0. 0002976190	
16 × 17 × 18) 1.	(0. 0002042484	0. 0007306482
18 × 19 × 20) 1.	(0. 0001461988	
20 × 21 × 22) 1.	(0. 0001082251	
22 × 23 × 24) 1.	(0. 0000823452	
24 × 25 × 26) 1.	(0. 0000641026	
26 × 27 × 28) 1.	(0. 0000508751	
28 × 29 × 30) 1.	(0. 0000410509	
30 × 31 × 32) 1.	(0. 0000336021	
32 × 33 × 34) 1.	(0. 0000278520	0. 0001829939
34 × 35 × 36) 1.	(0. 0000233426	
36 × 37 × 38) 1.	(0. 0000197566	
38 × 39 × 40) 1.	(0. 0000168691	
40 × 41 × 42) 1.	(0. 0000145180	
42 × 43 × 44) 1.	(0. 0000125843	
44 × 45 × 46) 1.	(0. 0000109793	
46 × 47 × 48) 1.	(0. 0000096361	
48 × 49 × 50) 1.	(0. 0000085034	
50 × 51 × 52) 1.	(0. 0000075415	
52 × 53 × 54) 1.	(0. 0000067193	
54 × 55 × 56) 1.	(0. 0000060125	
56 × 57 × 58) 1.	(0. 0000054014	
58 × 59 × 60) 1.	(0. 0000048704	
60 × 61 × 62) 1.	(0. 0000044068	
62 × 63 × 64) 1.	(0. 0000040002	

o. 0416666666
o. 0113095237
o. 0029019589
o. 0007306482
3) o. 0001829939 (o. 0000609980

o. 05679179
+ o. 00006100

o. 05685279 < E*d*C*y*

But o. 0007306482 }
o. 0001829939 } ∺
o. 0000458315 }

Therefore o. 05679179
+ o. 00004583
+ o. 00001528

o. 05685290 > E*d*C*y*.

For it has been demonſtrated. That $\frac{1}{4}$ of any Term in the laſt Column is leſs than the Term next after it ; and therefore that $\frac{1}{3}$ of the laſt Term, at which you ſtop, is leſs than the remaining Terms ; and that the Total of theſe is leſs than $\frac{1}{3}$ of a Third proportional to the two laſt.

And therefore A B C *y* E being = o. 75 ——— ——— ——— o. 57
and E *d* C *y* > o. 05685279 —— and < o. 05685290

And A B C *d* E is < o. 69314720 —- and > o. 69314709

Fig. 9. But when A E : B C : : 5 : 4, or as E A to K H ; then will the Space A B C E, or now, the Space A H K E (A H = $\frac{1}{4}$ A B) be found as follows.

8 × 9 × 10) 1. (o. 0013888888] o. 0013888888

16 × 17 × 18) 1. (o. 0002042484 }
18 × 19 × 20) 1. (o. 0001461988 } o. 0003504472

32 × 33 × 34) 1. (o. 0000278520 }
34 × 35 × 36) 1. (o. 0000233426 }
36 × 37 × 38) 1. (o. 0000197566 } o. 0000878204
38 × 39 × 40) 1. (o. 0000168691 }

o. 0013888888
o. 0003504472
3) o. 0000878204 (o. 0000292735

o. 0018271564
+ o. 0000292735

o. 0018564299 < E*ab*

But o. 0003504472
o. 0000878204
o. 00002200737 } ÷

Therefore o. 0018271564
+ o. 0000220074
+ o. 0000073358

o. 0018564996 > E*ab*.

Therefore E M *b* (*Fig*. 12.) being = o. 025 ——— o. 025
E*ab* > o. 0018564292 & < o. 0018564996.

E M*ba*(*Fig*. 12.) or EKM(*Fig*.9.) > o. 02685643 < o. 02685650
A H K E < o. 22314356 > o. 22314349

Therefore 3 A B C *d*E = 2. 07944154
and A H K E = o. 2231435

ABC*d*E(when A E : B C : : 10: 1.) = 2. 3025850

Therefore,
The Logar. of 10,
is to the Log, of 2,
As 2. 302585,
to o. 693147,

IV. The Quadrature of the Circle, or the turning it into an equal Square, or any other Right-lin'd Figure, (which depends upon the Ratio of the Circle to the Square of its Diameter, or of the Circumference to its Diameter) may be underſtood to be Fourfold, to wit, either by Calculation, or by Linear Conſtruction, and each of them again may be either perfectly axact, or elſe almoſt, or pretty near. The Quadrature by accurate Calculation, I call the Analytical; That which is done by accurate Calculation, I call the Geometrical: That which is done by Calculation pretty near, I call the Approach; That which is by Conſtruction pretty near, I call Mechanical.

The Quadrature of a Circle by M. Leibnitz, *Phil. Coll.* N. 7. *p.* 204.

The Approaches have been furtheſt carry'd on by *Ludolph van Ceulen*; *Vieta*, *Hugenius*, and others, have given ſeveral Mechanical. The Accurate Geometrical Conſtruction may be had, by which not only an entire Circle may be meaſur'd, but any Section or Arch of it alſo, which is by an exact and ordinate Motion, but ſuch notwithſtanding as ſuits with Tranſcendental Curves, which erroneouſly are accounted Mechanical, tho' in truth they are as Geometrical as thoſe which are commonly ſo eſteem'd, tho' they are not Algebraical, nor can be reduced to Equations Algebraical or of certain Degrees, they having Degrees proper to themſelves, which tho' they be not Algebraical, are yet nevertheleſs Analytical.

The Analytical Quadrature, or that which is made by Accurate Calculation, may be again ſubdivided into three Kinds; namely, into the Analytical Tranſcendent, the Algebraical, and the Arithmetical. The Analytical Tranſcendent is to be obtain'd, amongſt others, by Equations of Degrees indefinite, hitherto conſider'd by none. As if $X^x + X$, be equal to 30, and X be ſought, it will be found to be 3; becauſe $3^3 + 3$, is $27 + 3$, or 30.

The Algebraical is done by Vulgar Numbers, tho' irrationally, Vulgar, or by the Roots of common Equations, which for the General Quadrature of the Circle, or its Sectors, is indeed impoſſible. Now there remains the Arithmetical Quadrature, which is performed by certain Series exhibiting the Quantity of the Circle exact by a Progreſſion of Terms (firſt) Rational, ſuch as I ſhall here propound.

I have found therefore, that if the Square of the Diameter be put 1, the Area of the Circle will be $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17}$, *&c.* To wit, the entire Square of the Diameter being diminiſhed (that it may not be too big) by a third part; and again (becauſe hereby too much is taken away) being augmented by one fifth; and again (becauſe by this laſt too much is added) diminiſhed by one ſeventh; and ſo onward continually.

And the firſt Quantity will be too great, *viz.* 1. but the Error will be leſs than ——— $\frac{1}{3}$

The next too little, *viz.* $1 - \frac{1}{3}$, but the Error will be leſs than $\frac{\ }{5}$

The 3d too much, *viz.* $1 - \frac{1}{3} + \frac{1}{5}$, but, *&c.* ——— $\frac{1}{7}$

The 4th too little, *viz.* $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$, but, *&c.* ——— $\frac{1}{9}$ *&c.*
&c.

The whole Series contains all the Approaches together, or the Values both greater than they ought to be, and leſs than they ought to be.

So that by continuing the Series, the Errors may be made leſs than the Fraction given, and conſequently leſs than any Aſſignable Quantity. Whence it follows, that the whole Series muſt give the true Value. And tho' the Summ of the whole Series cannot be expreſs'd by one Number, and that the Series be infinitely continued: yet becauſe it conſiſteth of one regular Method of Progreſſion, the whole may ſufficiently enough be conceived by the Mind. And if *Van Ceulen* could have given a Rule by which his Numbers 314159, &c. could have been continued *in infinitum*, he would have given us the Arithmetical Quadrature exact in whole Numbers, which we have here done in Fractions.

There are ſeveral things relating to this Quadrature, which might be taken Notice of, eſpecially one, *viz.* That the Terms of this our Series $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, *&c.* are of Harmonical Progreſſion, or in a continued Harmonical Progreſſion, as will be evident to any one that ſhall examine them. And a Series made by skipping, as $\frac{1}{1}$, $\frac{1}{5}$, $\frac{1}{9}$, $\frac{1}{13}$, $\frac{1}{17}$, *&c.* is alſo of Harmonical Progreſſion. And $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{15}$, $\frac{1}{19}$, *&c.* is alſo a Series of Harmonical Proportionals. Wherefore ſince the Circle $\frac{1}{1}+\frac{1}{5}+\frac{1}{9}+\frac{1}{13}+\frac{1}{17}$, *&c.* $-\frac{1}{3}-\frac{1}{7}-\frac{1}{11}-\frac{1}{15}-\frac{1}{19}$, *&c.* by ſubſtracting the latter partial Series from the former partial Series, the Circle will be the Difference of two Series in Harmonical Progreſſion. And becauſe the Sum of any Number of Terms in Harmonical Progreſſion, how many ſoever, may by ſome Compendium be obtained; hence Compendious Approaches (if after *Van Ceulen* there be any need of them) may be deduc'd, if one would in this our Series take out the Terms affected with the Sign −, by adding the two next into one, $+\frac{1}{1}-\frac{1}{3}$, and $+\frac{1}{5}-\frac{1}{7}$, and $+\frac{1}{9}-\frac{1}{11}$, and $+\frac{1}{13}-\frac{1}{15}$, and $+\frac{1}{17}-\frac{1}{19}$, and ſo onward, he will have a new Series for the Magnitude of the Circle, namely $\frac{2}{3}$) that is $\frac{1}{1}-\frac{1}{3}$) $+\frac{2}{35}$ (that is $\frac{1}{5}-\frac{1}{7}$) $+\frac{2}{99}$ (that is $\frac{1}{9}-\frac{1}{11}$ (*&c.* Wherefore

The Square inſcribed being $\frac{1}{4}$

The Area of the Circle ſhall be $\frac{1}{3}+\frac{1}{35}+\frac{1}{99}+\frac{1}{195}+\frac{1}{323}$, *&c.*

But the Numbers 3, 35, 99, 195, 323, *&c.* by skipping are taken out of the Series of Square Numbers, 4, 9, 16, 25, *&c.* diminiſhed by an Unite, and ſo made the Series 3, 8, 15, 24, *&c.* out of the Members of which Series every fourth after the firſt, is a Number of this our Series. But I have found (which is worth nothing) the Summ of this infinite Series $\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+$ *&c.* to be $\frac{3}{4}$. Nay, and by culling out by ſingle skipping, as

$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}$, &c. the Summ of this infinite Series maketh $\frac{2}{4}$ or $\frac{1}{2}$. But if out of this again another Progreſſion be culled out by ſingle Skipping, as $\frac{1}{3}+\frac{1}{35}+\frac{1}{99}$, &c. the Summ of that infinite Series ſhall be the Semicircle, the Square of the Diameter being 1. Now becauſe by the ſame Means the Arithmetical Quadrature of the *Hyperbola* is obtain'd, I thought it not amiſs to repreſent to View the whole Harmony.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	&c.		
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	&c.		
0	3	8	15	24	35	48	63	80	99	120	143	168	195	224	255	288	323	360	399	&c.		
.	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{1}{15}$	$\frac{1}{24}$	$\frac{1}{35}$	$\frac{1}{48}$	$\frac{1}{63}$	$\frac{1}{80}$	$\frac{1}{99}$	$\frac{1}{120}$	$\frac{1}{143}$	$\frac{1}{168}$	$\frac{1}{195}$	$\frac{1}{224}$	$\frac{1}{255}$	$\frac{1}{288}$	$\frac{1}{323}$	$\frac{1}{360}$	$\frac{1}{399}$	&c.	=	$\frac{3}{4}$
.	$\frac{1}{3}$	.	$\frac{1}{15}$	.	$\frac{1}{35}$	.	$\frac{1}{63}$	.	$\frac{1}{99}$	.	$\frac{1}{143}$	.	$\frac{1}{195}$	.	$\frac{1}{255}$	.	$\frac{1}{323}$	.	$\frac{1}{399}$	&c.	=	$\frac{2}{4}$
.	.	$\frac{1}{8}$	.	$\frac{1}{24}$	.	$\frac{1}{48}$	.	$\frac{1}{86}$	.	$\frac{1}{120}$	.	$\frac{1}{168}$	.	$\frac{1}{224}$	.	$\frac{1}{288}$	.	$\frac{1}{360}$	.	&c.	=	$\frac{1}{4}$
.	$\frac{1}{3}$	.	.	.	$\frac{1}{35}$	.	.	.	$\frac{1}{99}$	.	.	.	$\frac{1}{195}$	.	.	.	$\frac{1}{323}$	.	.	&c.	≠	∞
.	.	$\frac{1}{8}$	.	.	.	$\frac{1}{48}$	.	.	.	$\frac{1}{120}$	.	.	.	$\frac{1}{224}$	.	.	.	$\frac{1}{360}$	.	&c.	=	∞

the Circle ABCD, whoſe Inſcribed Square is
the Hyperbola CBEHC, whoſe Power ABCD, is $\Big\} \frac{1}{4}$.

Fig. 13. To the Aſymptotes AF, AE, at Right Angles to each other, let there be deſcribed the Curve Line of an Hyperbola GCH, whoſe Vertex is C; and ABCD, the Power or Square to which every Rectangle made of the Ordinate, as EH, and the intercepted part AE, is always equal. About this Square let a Circle be drawn, and let the Hyperbola be continued from C to H, ſo that AE be double to AB. Then putting AE to be 1, AB ſhall be $\frac{1}{2}$ and its Square ABCD, ſhall be $\frac{1}{4}$, and the Circle (whoſe Power ABCD is inſcribed) ſhall be $\frac{1}{3}+\frac{1}{35}+\frac{1}{99}$, &c. but the Portion of the Hyperbola CBEHC (whoſe Power inſcribed is the ſame Square $\frac{1}{4}$) which repreſents the Logarithm of the Ratio of AE to AB, (or of 2 to 1) ſhall be $\frac{1}{8}+\frac{1}{48}+\frac{1}{120}$, &c.

Tangents to all Geometrical Curves; by Renatus Fran. Sluſius, n. 90. p. 513. Jan. An. 1673. *Fig.* 14.

V. 1. Data ſit quælibet Curva DQ, cujus puncta omnia referantur ad Rectam quamlibet datam EAB, per Rectam DA; ſive EAB ſit Diameter ſeu alia quælibet, ſive etiam aliæ ſimul lineæ datæ ſint, quæ, vel quarum poteſtates, Æquationem ingrediantur; parum id refert.

'In Æquatione Analytica, facilioris explicationis cauſa, DA perpetuo dicatur v, & BA, y; EB vero & aliæ quantitates datæ, Conſonantibus exprimantur.

Tum ſupponantur ducta DC, tangens Curvam in D, & occurrens EB, productæ, ſi opus ſit, in puncto C; & CA perpetuo quoque dicatur a. Ad inveniendam AC, vel a, hæc erit Regula Generalis;

1. Rejectis ab Æquatione partibus in quibus y vel v non invenitur; ſtatuantur ab uno latere omnes in quibus eſt y, & ab altero illæ in quibus habetur v, cum ſuis ſignis $+$ vel $-$. Hoc dextrum, illud ſiniſtrum latus facilitatis cauſa, vocabimus.

2. In latere dextro, præfigatur ſingulis partibus Exponens poteſtatis quam in illis obtinet v; ſeu, quod idem eſt, in illum ducantur partes.

3. Fiat idem in latere ſiniſtro, præponendo ſcil. unicuique illius parti Exponentem poreſtatis quam in illa habet y. Sed & hoc amplius: unum y in ſingulis partibus vertatur in a.

Aio, Æquationem ſic reformatam modum oſtendere ducendæ Tangentis ad punctum D datum. Cum enim eo dato, pariter datæ ſunt y & v, & cæteræ quantitates, quæ Conſonantibus exprimuntur, a non poterit ignorari.

Si quid forte ſit obſcuritatis in Regula, aliquot Exemplis illuſtrabitur: Data ſit hæc Æquatio, $by - yy = vv$; in qua E B ſit b; B A, y; D A, v; & quæratur a, ſive A C, talis ut juncta D C, tangat Curvam D Q in D. Ex Regula, nihil rejiciendum eſt ab hac Æquatione, cum in ſingulis ejus partibus reperiatur y vel v. Ita quoque diſpoſita eſt, ut ab uno latere ſint omnes illius partes in quibus y; ab altero, omnes in quibus v. Singulis itaque tantum præfigendus eſt Exponens poteſtatis, quam in illis habet y vel v; in latere ſiniſtro unum y vertendum in a, ut fiat $ba - 2ya = 2vv$. Aio nunc, hanc Æquationem oſtendere modum ducendæ Tangentis ad punctum D, ſive $a = \frac{2vv}{b - 2y} =$ A C.

Sic ſi data fuiſſet Æquatio, $qq + by - yy = vv$; eadem plane fieret cum priori Æquatio pro Tangente, abjecto ſcil. qq, ut Regula præſcribit.

Sic ex $2byy - y^3 = v^3$ fit $4bya - 3yya = 3v^3$, ſive $a = \frac{3v^3}{4by - 3yy}$: Ex $bby + zyy + y^3 = qvv$, fit $bba + 2zya + 3yya = 2qvv$, & $a = \frac{2qvv}{bb + 2zy + 3yy}$: Ex $b^4 + by^3 - y^4 = qqvv + zv^3$, fit $3byya - 4y^3a = 2qqvv + 3zv^3$, & $a = \frac{2qqvv + 3zv^3}{3byy - 4y^3}$.

Verum in ſimilibus Æquationibus nullam arbitror accidere poſſe difficultatem. Aliqua fortaſſe in illis occurrit, quarum partes quædam conſtant ex productis y in v: Ut yv, yyv, y^3vv, *&c.* Sed hæc quoque levis eſt, ut Exemplis patebit. Detur enim $y^3 = bvv - yvv$. Nihil ab illa rejiciendum erit, cum in ſingulis ejus partibus reperiatur y vel v.

Sed ut ex Regulæ præſcripto diſponatur, bis ſumendum erit yvv, & ſtatuendum tam in latere dextro, in quo ſunt partes quæ habent v, quam in ſiniſtro, cujus partes habent y; quandoquidem yvv, tam y quam v contineat. Faciendum igitur erit $y^3 + vvy = bvv - yvv$. Tum mutata, ut prius, hac Æquatione in aliam $3yya + vva = 2bvv - 2yvv$, dabitur $a = \frac{2bvv - 2yvv}{3yy + vv}$.

Ita enim intelligenda eſt Regula, ut nempe in latere ſiniſtro non conſideretur poteſtas ipſius v, ideoque ipſi yvv, Exponens vv, præfigi non debeat, ſed tantum ipſius y: Sicut contra ab alio latere, in yvv conſiderari non debet poteſtas ipſius y, ſed v tantum, eique ſuus Exponens præponi. Sic ſi foret $y^5 + by^4 = 2qqv^3 - yyv^3$, faciendum eſſet $y^5 + by^4 + v^3yy = 2qqv^3 - yyv$; & haberetur Æquatio pro Tangente $5y^4a + 4by^3a + 2v^3ya = 6qqv^3 - 3yyv^3$ & $a = \frac{6qqy^3 - 3yyv^3}{5y^4 + 4by^3 + 2v^3y}$.

Atque his Exemplis arbitror, me omnem, quæ dari poſſet, Caſuum varietatem complexum eſſe. Cæterum non erit fortaſſe inutile, ſi ea quæ generatim expoſui, ad lineam aliquem ſingularem applicem. Data ſit igitur Curva B D, cujus ea ſit proprietas, ut ſumpto in illa quolibet puncto D, ſi jungatur
Fig. 15. B D, & erigatur ad illam normalis D E, occurrens rectæ B E in E, recta D E ſit ſemper æqualis datæ rectæ B F. Ut habeatur Æquatio in terminis Analyticis, ſit $DA = v$, $BA = y$, BF vel $DE = q$. Erit itaq; $EA = \frac{vv}{y}$. Et cum quadratum D E æquale ſit duobus D A, A E; erit Æquatio $qq = \frac{v^4}{yy} + vv$; ſive $qqyy = v^4 + yyvv$; quæ pro Tangente ex Regulæ præſcripto, ſic reformanda erit, $qqyy - vvyy = v^4 + yyvv$, & $2qqya - 2vvya = 4v^4 + 2yyvv$, & $a = \frac{4v^4 + 2yyvv}{2qqy - 2vvy}$.

Quomodo autem Æquationes hujuſmodi ad faciliores terminos pro conſtructione reduci debeant, id ſane ſolertem Geometram minime latebit. Ut ecce in hoc exemplo, quoniam Rectangulum B A E ſupponitur æquale Quadrato A D, ſi E A dicatur e, erit $vv = ye$, & $v^4 = yyee$, & $qq = ye + ee$. Itaq; pro illis, poſito in Æquatione eorum valore, fit $a = \frac{4yyee + 2y^3e}{2eey + 2eyy - 2eyy}$, ſive $a = \frac{2ey + yy}{\;}$, hoc eſt, $ae = 2ey + yy$, & addito ee utrinque $ae + ee = ee + 2ey + yy$. Erant itaque tres e, $e + y$, $e + a$, ſive E A, E B, E C, in continua analogia, & facillima evadet conſtructio.

Cæterum quoniam hactenus ſuppoſuiſſe videmur, Tangentem verſus partes B ducendum eſſe, cum tamen ex datis accidere poſſit, ut vel parallela ſit ipſi A B; vel etiam ducenda ad partes contrarias; definiendum nunc ſupereſt, quomodo hæc caſuum diverſitas in Æquationibus diſtinguatur, facta igitur Fractione pro a, ut in Exemplis ſupra adductis, conſiderandæ ſunt partes tam Numeratoris quam Denominatoris, & earum Signa.

1. Nam ſi in utroque, partes vel habeant omnes Signum $+$, vel ſaltem Affirmatæ prævaleant Negatis, ducenda erit Tangens verſus B.

2. Si Affirmatæ prævaleant Negatis in Numeratore, ſed æquales ſint in Denominatore, recta per D ducta parallela A B, tanget Curvam in D: hoc enim in caſu, a eſt infinitæ longitudinis.

3. Si tam in Denominatore, quam Numeratore, partes Affirmatæ minores ſunt Negatis; mutatis omnibus Signis, ducenda erit rurſus Tangens verſus B: hic enim Caſus cum primo in idem recidit.

4. Si in Denominatore prævaleant, in Numeratore minores ſint, vel contra; mutatis Signis illius in quo ſunt minores, ducenda erit Tangens verſus partes contrarias, *h. e.* A C, ſumenda erit verſus E.

5. Ac tandem ſi in Numeratore partes Affirmatæ ſint æquales Negatis, quomodocunque ſe habeant in Denominatore, *a* abibit in nihilum Itaque vel ipſa A D erit Tangens, vel ipſa E A, aut ei Parallela; quod ex datis facile dignoſcitur. Horum autem Caſuum Varietas explicari poteſt per Æquationes ad Circulum.

Sit enim Semicirculus, cujus Diameter E B, & in eo punctum D datum, ex quo cadat Normalis D A $= v$. Sit B A $= y$, B E $= b$; erit Æquatio $by - yy = vv$, & ducta Tangente D C, erit A C, ſive $a \; \frac{2vv}{b - 2y}$. Nunc ſi b major ſit $2y$, ducenda eſt Tangens verſus B; ſi æqualis, fit Parallela E B; ſin autem minor, ducenda eſt verſus E; ut *n.* 1, 2, & 4. diximus. Fig. 16.

Detur rurſus alius Semicirculus inverſus, cujus puncta referri intelligantur ad rectam Diametro parallelam, & eidem æqualem, ut in Schemate. Denominatis, ut prius, partibus, & N B $= d$, fit Æquatio $by - yy = dd + vv - 2dv$. Igitur A C, ſive $a = -\frac{2vv - 2dv}{b - 2y}$. Cum vero Exemplo ſuppoſuerimus v ſemper eſſe minorem d; ſi b ſit major $2y$, ducenda erit Tangens verſus E; ſi æqualis, erit parallela; ſin minor, mutatis omnibus Signis, ducenda erit verſus B; ut *n.* 4, 5, & 3. Nulla autem ducenda eſſet Tangens, ſeu Tangens foret ipſa E B, ſi ſuppoſuiſſemus N B æqualem Semidiametro, ſive $2d = b$; ut *n.* 5. Fig. 17.

Sit tandem alius Semicirculus, cujus Diameter N B normalis ſit ad Rectam B E, ad quam ejus puncta referri intelligantur. N B dicatur b, & aliæ partes denominentur ut ſupra; fiet Æquatio $yy = bv - vv$; & $a = \frac{bv - 2vv}{2y}$. Jam ſi b ſit major $2v$, Tangens ducenda erit verſus B; ſi minor, verſus E; ſi autem æqualis, ipſa D A erit Tangens; ut *n.* 1, 4, & 5. Fig. 18.

Et hæc eſt, ni fallor, Caſuum omnium Varietas, quæ ex Æquationum conſideratione deprehendi poteſt.

The Lemmata, *whereby the preceding Method is demonſtrated, by* M. Sluſius. n. 95. *p.* 6059. n. 97. *p.* 6126. *June*, An. 1673.

2. (1.) Differentia duarum dignitatum ejuſdem gradus applicata ad differentiam laterum, dat partes ſingulares gradus inferioris ex binomio laterum; ut $\frac{y^3 - x^3}{y - x} = yy + yx + xx$. Quod facile oſtenditur.

(2.) Tot ſunt partes ſingulares ex binomio in gradu quolibet, quot unitates habet Exponens dignitatis immediate ſuperioris; tres nimirum in Quadrato, quatuor in Cubo, *&c.* Et hoc vulgo notum.

(3) Si quantitas eadem applicetur ad duas alias, quarum ratio data ſit. Quotientes erunt reciproce in eadem ratione data.

His Lemmatibus Methodus mea facile demonſtratur, cum eo ordine diſpoſita ſint, qui ad illam quaſi manu ducit.

The Teſtudo Veliformis Quadrabilis *Ænigmatically propos'd by* V. V. n. 196. *p.* 584. Jan. An. 1693.

VI. 1. *ÆNIGMA Geometricum, De miro Opificio Teſtudinis Quadrabilis Hemiſphæricæ, à D.* Pio Liſci Puſillo Geometra *propoſitum.*

Inter Venerabilia Eruditæ olim Græciæ Monumenta extat adhuc, perpetuo equidem duraturum, Templum Auguſtiſſimum Ichnographia Circulari, *Almæ Geometriæ* dicatum, quod, à Teſtudine intus perfecte Hemiſphærica, operitur: ſed in hac, Feneſtrarum quatuor æquales Areæ (circum, & ſupra Baſin Hemiſphæræ ipſius diſpoſitarum) tali Configu:atione, Amplitudine, tantaque Induſtria, ac Ingenii acumine ſunt extructæ, ut, his detractis, ſuperſtes Curva Teſtudinis Superficies, pretioſo Opere Muſivo ornata, Tetragoniſmi vere Geometrici eſt capax.

Quæritur modo, Quæ ſit, Qua Methodo, Quave Arte, Pars illa Hemiſphæricæ Superficiei Curvæ Quadrabilis, Tenſi ad inſtar Carbaſi, vel Turgidi Veli-Nautici, ab Architecto illo Geometra fuerit aſſecuta? & cui demum Plano Geometrici Quadrabili ſit æqualis?

Solv'd by Dr. Wallis, *ibid. p.* 587.

2. Accepi, V. C. nudius tertius (noctu decubiturus) Literas tuas quibus heri non vacabat, alias occupato, reſpondere, eiſque incluſam Chartulam, Typis impreſſam, quam ais Florentia te accepiſſe mihi mittendam.

Continet ea Chartula Ænigma Geometricum, quod (verborum involucris exemptum) hoc innuere judico Problema; Ab Hemiſphærii curva Superficie, Segmenta quatuor inter ſe æqualia ſic amputare, ut reliquum ſit Tetragoniſmi capax.

Simulque videtur innuere, in veteris Græciæ Monumentis etiamnum extare quidpiam quo illud fiat.

Hoc eſſe eſtimo Hippocratis Chii Quadraturam Lunulæ.

Quippe cum Archimedes demonſtravit, Curvam Hemiſphærii Superficiem æqualem duobus Circulis ejuſdem Sphæræ maximis, (id eſt quatuor Semicirculis;) docuitque Hippocrates Chius Lunulam quadrare quandam: Si ſingulis Hemiſphærici hujuſce Fornicis Quadrantibus tantundem eximatur, quanto deficit à Semicirculo ea Lunula, Reliquum æquabitur Quadrato, quod Circulo Sphæræ maximo (cui hic inſiſtit Fornix Hemiſphæricus) Inſcribatur.

Si tamen præter Ænigmaticam Problematis Involutionem, ſubſit aliquid (de Templo) Hiſtoricum; putaverim ego S. Sophiæ (quod eſt Conſtantinòpoli) Templum hic inſinuatum.

Fig. 19.

SCHOLIUM] Per Hippocratis Chii Quadraturam Lunulæ (primo Phyſicorum Ariſtotelis & Simplicii in eum locum Commentariis, indicatam,) Si Semicirculo A B D, in duos Quadrantes A C D, B C D diviſo, aptetur A D Subtenſa Quadrantalis Arcus, Radio C E biſecta in H: & Centro H ſcribatur Semicirculus A D F: Erit (propter Quadratum Rectæ A D Subduplum Quadrati Rectæ A B) Semicirculus A D F Subduplus Semicirculi A B D;

adeoque Quadranti A C D æqualis. Et (dempto utrinque communi Segmento A D E) reſidua Lunula A E D F reſiduo Triangulo A D C æqualis. Taleſque quatuor Lunulæ, talibus quatuor Triangulis; hoc eſt, Quadrato toti Circulo inſcripto A D B G.

Porro, per Archimedis demonſtrata, æquatur Sphæræ Superficies, quatuor Circulis in ea Sphæra Maximis; Adeoque Hemiſphærii Superficies Curva, talibus quatuor Semicirculis; taliſque Superficiei Hemiſphæricæ Quadrans, uni Semicirculo.

Circulus A D B G eſto jam Baſis Hemiſphæricæ Superficiei Curvæ; cujus Polus P, Axis C P, plano Baſis perpendicularis, ejuſq; Quadrans unus D P A; Fig. 20.
qui plano E P C per Axem tranſeunte, biſecetur.

Ponantur item (ob commodiorem Calculum) Circuli Radius R, Diameter D = 2 R, Peripheria P, Expoſitus Arcus *a*.

Poſitóque Quadrantali Arcu D E A = *a* = $\frac{1}{4}$ P; eſt Semicirculus A B D = *a* R = $\frac{1}{4}$ R P: Triangulum A D C = $\frac{1}{2}$ R^2 = $\frac{1}{4}$ R D; reliquumq; Semicirculo (dempto hoc Triangulo) $\frac{1}{4}$ R P − R D; cui æquale auferendum eſt ex D P A (Quadrante Superficiei Hemiſphæricæ Curvæ, æquali Semicirculo A B D) quo Reſiduum æquetur Expoſito Triangulo A D C.

Quod cum variis modis fieri poſſit; per ea quæ nos dudum docuimus *Anno* 1659. (ad Calcem Tractatus *de Cycloide*, tum editi, *p.* 122. inſerenda ad §. 68.) iterumq; *Anno* 1670. (in Tractatus *de Motu*, Cap. V. Prop. 24.) de Figura Plana, æquali cuivis in Superficie Sphæricæ Figuræ, Circulis quibuſvis (ſive Maximis, ſive Minoribus) terminatæ. Sic fiat ſimpliciſſime;

Cum Superficiei Sphæricæ Segmenta, Parallelis Planis Abſciſſa, ſint Axis Fig. 21.
Segmentis proportionalia (quod de expoſita quadrantalis Cunei Superficie D P A pariter valet:) Si ſumatur, in Axe C P, ut Semicirculus $\frac{1}{4}$ R P, ad Semicirculum dempto Triangulo $\frac{1}{4}$ R P − $\frac{1}{4}$ R D; hoc eſt, ut P ad P − D; ſic C P ad C Y: (ſive, quod tantundem eſt, ut P ad D; ſic C P ad P Y:) Planum per Y Z Baſi parallelum, abſcindet hujus Superficiei Curvæ portionem Polo adjacentem, æqualem Triangulo A D C. Quod cum in reliquis Superficiei Curvæ Quadrantibus, pariter fiat; æquabitur totum Abſciſſum (Polo adjacens) toti Quadrato Baſi inſcripto: Et ſic Tenſum ut oportuit. *Q. E. F.*

Vel ſic brevius. Eſt Hemiſphærii Superficies Curva (utpote duobus Circulis Maximis æqualis) = R P; Quadratum Circulo Maximo inſcriptum = 2 R R = R D; Illudque ad hoc, ut P ad D. Adeoque (propter Segmenta Superficiei Parallelis Planis abſciſſa, Segmentis Axis proportionalia) ſumptis C P ad P Y, ut P ad D, erit tum tota Superficies = R P, tum Portio ad Polum, plano Z Y abſciſſa, = R D, Quadrato Baſi inſcripto. *Q. E. F.*

Sic dicatur; Proceſſum hic eſſe ex præſumpti Circuli Quadratura, aut ratione quam habet Circuli Perimeter ad Diametrum: Id omnino verum eſt. Sed non eſt objiciendum. Quia non poſtulat Ænigma propoſitum ut Hemiſphæricæ Superficiei Portiones abſciſſæ, (quas Feneſtras vocat) ſed ut Portio Superſtes, ſi Tetragoniſmi capax. Et quidem ſi utrumq; poſtularet, poſtularet Circuli Quadraturam abſolute Geometricam: quod haberi non poſſe ſatis conſtat.

Opificium quod ſpectat; ſuper Baſem Planam, extra Baſem Hemiſphærii Fig. 21.
poſitam, ſed ipſi contiguam, cujus duo Latera in Angulum coeant ad A, intra

protractæ

protractas D A, G A. Rectas, (quo Feneſtrarum quas vocat utrinque adjacentium liber proſpectus pateat, non impeditus) extruatur Moles ſatis firma; ita quidem ut, aſſurgente Structura, promineat ejus acies, angulo ſuffulta, Fig. 21. Circuli Arcum efficiens qualis eſt D Z, ad altitudinem Y aſſurgens. Et ſimiliter ad reliquos Angulos D, B, G. Atque his demum Structuris (quaſi totidem Columnis) ad eam Altitudinem provectis, imponatur Teſtudo, ſic intus excavata, ut poſſit Hemiſphæricæ Superficies; Adeoque totum opus imperatum abſolvitur.

Aliter. Idem fiet ſi, pro Quadrato Baſi inſcripto, exponatur Quadratum quodvis QQ, (quod minus ſit quam Hemiſphærica Superficies curva). Quippe ſi ſumatur, ut R P (Hemiſphærica Superficies curva) ad Q Q (expoſitum Quadratum, ſic) C P (Axis Hemiſphærii) ad P Y (Axis Portionem Polo adjacentem: Planum Z Y (Baſi parallelum) abſcindet portionem Superficiei Sphæricæ Tetragoniſmi capacem: Utpote æqualem expoſito Quadrato QQ.

Idem ſic aliter abſolvi poteſt: ſed majore ſolicitudine.

Fig. 20. Cum ſit (ut jam oſtenſum eſt) Hemiſphæricæ Superficiei curvæ Quadrans D P A æqualis Semicirculo A B D; ejuſque Segmenta Planis Baſi parallelis abſciſſa, Segmentis Axis proportionalia: Sumatur in D P Quadrantali arcu, arcus P Q graduum 60; (quod mihi Caſwellus ſuggerit) Polo P deſcriptus Circulus Q T S biſecabit Axem (propter Sinum Verſum grad. 60 = $\frac{1}{2}$ R:) adeoque Quadrantem Hemiſphæricæ Superficiei curvæ D P A dirimet in duo Segmenta inter ſe æqualia. Quorum alterum, D Q T S A Quadrilineum, æquat Quadrantem circularem B C D; reliquumq; Trilineum P Q T S æquat Quadrantem A C D. Unde ſi porro auferatur Q R S T Bilineum, æquale ſegmento Circuli A D E: reliquum Trilineum P Q R S, æquabit A D C Triangulum. Taliaq; quatuor, in quatuor Quadrantibus Hemiſphærii, æquabunt Quadratum Baſi inſcriptum. Habebitur autem illud Bilineum per ea quæ nos dudum docuimus locis modo citatis.

Idem univerſalius ſic fiet.

Sumpto Q ubivis in Arcu D Z, (ne major ſit D Q quam D Z;) & quanto deficit Quadrilineum D Q T S A à toto auferendo, tantundem ſuppleat Bilineum Q R S T: Reliquum æquabit A D C Triangulum.

Et quidem, ſi ſumatur Q in D (quo evaneſcat Quadrilineum) ſumendum erit Bilineum æquale toti auferendo. Sin ſumatur Q in Z (ut Bilineo non ſit opus) æquabitur Quadrilineum toti auferendo.

Eademque omnia (de Quadrilineo & Bilineo quæ ſimul compleant totum auferendum) pariter accommodanda erunt (mutatis mutandis) ſi, pro Quadrato Baſi inſcripto, ſubſtituatur Q Q Quadratum quodvis; quod tota Superficie curva Hemiſphærica non ſit majus.

Poſtquam hæc ſcripta fuerant, erantque ſub prelo, reſcivi tandem huic eidem Problemati reſponſum dediſſe Cl. V. D. Leibnitz, illudque in Actis Lipſicis comparere pro Menſe Junio, 1692. Quod fecit ut Prelum ſufflaminandum curaverim per aliquot ſeptimanas donec illud conſpicerem; quod ægre tandem obtinui. Videoque Cl. Virum juxta mecum ſentire, non eſſe Problema Determinatum, ſed mille modis (nedum infinitis) ſolubile.

3. Ænigmatis hujus Author Problematis tandem Conſtructionem ingenioſe admodum & expedite dedit in Tractatu Italico, *de Formatione & Menſura Teſtudinum omnium*, ad Sereniſ. *Etruriæ* Principem, ubi & Nomen ſuum profiteri dignatus eſt, nempe à *V. V. Poſtremo Galilei Diſcipulo*, cum antea diſpoſitis horum verborum ut in Anagrammate elementis, ſub ficto nomine *D. Pio Liſci Puſillo Geometra* tectus latuiſſet.

The Propoſer's Solution Demonſtrated, by Dr D. Gregory. N. 207 p. 25. Jan. An. 1694.

Ænigma verum in ſequens Problema ab Authore convertitur: *Super Hemiſphærii ſuperficiem, aſſignare portionem dato quadrato æqualem:* Quod ſic conſtruit.

Fig. 22.

Sphæra cujus Axis æqualis lateri dati Quadrati exponatur per Circulum A C B D in propoſita Sphæra verticalem, cujus Diameter Horizontalis eſt A B, centrum E. Perforetur Sphæra duobus Cylindris rectis quorum communes Sectiones cum plano A C B D ſunt Circuli B L E G, A H E I, diametris E B, E A deſcripti. Dico factum; hoc eſt à quolibet Hemiſphærio *v. g.* ſuperiori A C B ablatas eſſe per Cylindros perforantes quatuor Figuras bilineares, duas ſcil. in parte antica & duas in poſtica æquales ſimiles & ſimiliter poſitas, ita ut reſidua ſuperficies Hemiſphærica ſit æqualis Quadrato rectæ A B. Et quoniam Hemiſphærica Superficies, demptis ſpatiis quatuor Bilinearibus prædictis, refert Velum vento Inflatum & Tenſum, Teſtudinemve Hemiſphæricam quatuor Feneſtris interruptam, quæ Circulari baſi A E B impoſita, ipſi ad puncta, A, E, E, B, innititur, hanc pro jure ſuo appellat *Teſtudinem Veliformem Florentinam Quadrabilem.*

Auctor deinceps in memorato Tractatu plurima ad Praxin attinentia profert, ut ope Torni & Terebeæ Cylindricæ tam hujus quam reliquarum quinque Teſtudinum fiant exemplaria: Atque in hanc rem alia quædam Problemata ſubtilia conſtruit quorum omnium demonſtrationes ab Auctore conſulto omiſſæ facillime ex nunc proferendis conſequentur.

Quod quatuor Feneſtræ in Hemiſphærio, ut dictum eſt, extructæ ſint Figuræ æquales ſimiles & ſimiliter poſitæ, ſatis liquet, reliquum eſt ut oſtendamus reliquam ſuperficiem Hemiſphæricam Tetragoniſmi vere Geometrici eſſe capacem.

Ad planum C A D B, in puncto E, erigi intelligatur Normalis recta æqualis E A, & ſuper Peripheriam A C B D ſuperficies Cylindrica recta ejuſdem Altitudinis. Vulgo notum eſt portionem Superficiei Sphæricæ inter quælibet duo plana Circulo A B C D Parallela comprehenſam, æqualem eſſe portioni Superficiei Cylindricæ inter eadem plana; & horum Annulorum ſimiles portiones reſectas à planis in erecta ex E, Normali ſe mutuo inter ſecantibus eſſe etiam æquales. Si jam ducendo innumera plana Baſi A C B D parallela dicto modo deſignari intelligantur in ſuperficie Cylindrica partes reſpondentibus Sphæricis æquales, quæ è regione Superficiei perforatione ablatæ deſignatur illi æqualis eſt. Quare patet reſiduam à perforatione Superficiem æqualem eſſe reſiduæ ſuperficiei Cylindricæ dempta illa quæ è Regione ablatæ per dicta innumera plana deſignatur. Ducatur Diameter quælibet P M, ſecans Peripheriam A H E utcunque in H. Jungatur H A, per H ducatur R T normalis ad A B, & parallela ad C D per E ductam, occurrens Peripheria

A C B D in R & T, & Peripheriæ A I E in I. Super R T Diametro fiat Semicirculus cujus Peripheriæ occurrant H S, I Q ad R T normales S & Q. Hujus Semicirculi Planum intelligatur normaliter erectum ad Circulum ABCD. Unde Peripheria R S Q T erit in Superficie Hemiſphærica, rectaque H S nunc ad Planum ACBD normalis, erit Altitudo Superficiei Cylindricæ perforantis ſupra Baſeos punctum H. Idemq; de quolibet puncto ſuperficiei Cylindricæ perforantis verum eſt, *ſcil.* ejus altitudinem uſque ad Superficiem Sphæræ ſupra quodvis in Baſi punctum H, eſſe rectam H S, ut dictum eſt, genitam, ſed H S æqualis eſt H A ſinui recto Arcus M A, quoniam tam hæc quam illa eſt Media Geometrica inter P H & H M, altera in Circulo M A P, altera in Circulo Sphæræ etiam Maximo per puncta M, S, & P, tranſeunte.

Si in erecta in E ad planum A C B D normali, ab E ſumatur recta æqualis H S aut H A, & ab extremo ejus puncto ducantur rectæ Parallelæ ad P M & V N, planum per illas extenſum erit ad Planum A C B D parallelum, & rectæ hæ per puncta S & Q tranſibunt, & productæ uſque ad Superficiem Cylindricam Hemiſphærio circumſcriptam abſcindent ex lateribus Cylindri rectas ipſis H S vel H A itidem æquales; comprehendentque Arcus æquales & reſpondentes Arcubus M N & V P. Quod ſi alterum planum huic ad minimam diſtantiam parallelum ſimiliter ductum intelligatur, hæc duo per ſupra oſtenſa deſignabunt in Superficie Cylindrica annuli portionem æqualem portioni inter eadem plana à Superficie Hemiſphærica perforatione ablatæ. Quod ſi ſimilis Conſtructio fieri ſupponatur ad quodlibet in Peripheria AHE, punctum portiones omnes in Superficie Cylindrica Hemiſphæricæ circumſcripta dicto modo genitæ & deſignatæ erunt æquales Superficiei Sphæricæ perforatione ablatæ. Quare reſidua Superficies Hemiſphærica æqualis erit reliquæ ſuperficiei Cylindricæ conflatæ ex rectis omnibus H A ad reſpectiva puncta M, N, V, & P erectis, ſeu figuræ ſinuum rectorum Semiperipheriarum A C B, A D B, hoc eſt, per dudum à Geometris cognita, quadruplo Quadrato Radii A E, ſive denique quadrato Diametri A B. Cumque duæ integræ Figuræ comprehenſæ à communi ſectione prædictæ Superficiei Cylindricæ perforantis cum Superficie Sphærica, æquales ſint quatuor ſemiſſibus earundem, patet reſiduam ſuperficiem Hemiſphæricam A C B, ablatis quatuor ſpatiis Bilinearibus (ut ſupra in Conſtructione) æqualem eſſe quadrato Diametri A B, *Q. E. D.*

Si Semiperipheria A H E, ita inflectatur ut congruat cum æquali quadrante Peripheriæ A R C; punctum H incident in punctum M ob æquales Arcus A H, A M, & H S altitudo ad H ſuperficiei Cylindricæ ſuper A H E inſiſtentis congruet cum æquali H A altitudine ad M Figuræ Sinuum rectorum ſuper A M C erectæ; idemque in reliquis punctis fiet. Unde curva quæ eſt communis interſectio Superficiei Sphæricæ cum Superficie Cylindrica ſuper Baſi A H E, quamvis non jaceat in eodem Plano inflexa, ut dictum eſt, congruet, & proinde æqualis eſt curvæ terminanti figuram Sinuum rectorum; hoc eſt communi Sectioni Superficiei Cylindricæ ſupra Quadrantalem Arcum A R C erectæ cum Plano ſecante Planum Baſeos in recta B A ad Angulos ſemirectos; ſive quadranti curvæ Ellipſeos cujus minor Axis eſt A B, major vero poteſt hujus duplum. Adeoque Perimeter Veli Quadrabilis Florentini ex hujuſmodi quatuor conſtans æqualis eſt Perimetro dictæ Ellipſeos.

Sed & hoc amplius adnotare non pigebit, Superficies Cylindorum duorum perforantium intra Sphæram, æquales eſſe Superficiei Sphæræ poſt perforationem relictæ, ſive duplici Velo Florentino, hoc eſt duplo quadrato Diametri. Atque hoc exinde patet quod Velum Florentinum æquale ſit Figuris quatuor ſinuum rectorum Quadrantis & Superficies perforans iiſdem etiam ſit æqualis, quoniam illis congruit ſi inflectio fiat ut ſupra.

Hoc tantum addam, Conſiderationem Figuræ Sinuum rectorum (cujus etiam partes in Quadrata facile mutantur) ſufficere ad Demonſtrationem eorum omnium quæ de aliis ſolidis Torno elaboratis vel Cylindro perforatis, eorumque Superficiebus ab Acutiſſimo Geometra V. V. (Vicentio Viviani, ni fallor) Digniſſimo Galilæi Diſcipulo proferuntur; dum Fabricam & Menſuram Teſtudinum docet. Speciatim Superficies Teſtudinis Scaphoidis Romanæ [*Volta a Schifo alla Romana*] ex octo Figuris Sinuum rectorum Arcus Quadrantalis conſtat, ac proinde Teſtudini Veliformi Florentinæ æqualis eſt. Unde patet quomodo æqualibus quadratis ſuperimponi poſſunt duæ Teſtudines, quarum altera eſt undique clauſa, altera quatuor Feneſtris interrupta, utraque Quadrati Baſeos dupla.

VII. 1. *Drawing the ſtreight Lines E A, and E B (cutting the Arc AB in G,) and on A G, a Perpendicular E F, (which will therefore paſs to the Center C, becauſe biſecting A G at Right Angles;) the Right-lined Triangle A F E, is equal to A D E, the propoſed Portion of the Lunula.*

The Quadrature of the Parts of the Lunula, *by* Mr. J. Perks, *a little vary'd by* Dr. J. Wallis, N. 259. *p.* 411. Dec. An. 1699. *Fig.* 23.

The Demonſtration is to this purpoſe; *viz.* ADB being a Quadrantal Arc; the Angle A G B will be three Halves of a Right Angle; (and its conjunct Angle E G A, half a Right Angle) and that Angle (being external to the Triangle A G E, is equal to the two oppoſite Intervals G E A + E A G. Whereof G E A (becauſe an Angle in the Semicircle A E B) is a Right Angle, and therefore E A G is half a Right Angle, (as are alſo F E G, and F E A) and the three Triangles A F E, G F E and G E A, each of them half a Square. And A G to A E, as √2 to 1, (proportional to the reſpective Radii of the two Circles.) And the like Segments A D G, A E, in their reſpective Circles (as the Squares of their reſpective Radii) as 2 to 1. And therefore the Semiſegment A F D, equal to the Segment A E. And conſequently (one taking from the Triangle as much as the other adds to it) the Portion of the Lunula A D E, equal to the Triangle A F E. *Q. E. D.*

If the Point E chance to be in K (the middle of the Arc A E B) there will be no Interſection at G (the Points G, B, being then coincident, but without any Diſturbance to the Demonſtration): If it happen beyond it, toward B; then G will be on the other ſide; and what is here ſaid of E G B, muſt be accommodated to E G A.

The Ground of the whole Proceſs is plainly this; The Angle A C E, being an Angle at the Center of the greater Circle; but at the Circumference of the leſſer, the Line C D E (as it paſſeth from C A to C B) doth in the ſame Proportion, divide the Quadrantal Arc A D B, and the Semicircular A E B: whence all the reſt doth naturally follow.

Improved by Dr. Gregory, *ibid.* p. 414. Dec. An. 1695. *Fig.* 24.

2. If you compleat the two Circles, whoſe Arcs contain the *Lunula* of *Hipocrates*; the ſame is true as well of the Points in the other Semicircle A C B, as of thoſe in the Semicircle A E B; and for the ſame Reaſons. As appears in the Scheme annex'd, wherein I have mark'd the Points in the Semicircle A C B, (correſpondent to thoſe of Mr. *Perks* in A E B,) with the correſpondent ſmall Letters of the *Roman* and *Greek* Alphabets.

If Mr. *Perks* had made his Conſtruction univerſal by making both E A and E B, meet with the greater Circle, (which he might have done by protracting theſe Lines and the greater Circle till they meet;) he might have found that the Portions of the Spaces A ε C M, B H C N, (ſuppoſing M C N Parallel to A B) are Quadrable as well as thoſe of *Hippocrates*'s *Lunula*: And that E A γ being a ſtreight Line, the Portion A E D of *Hippocrates*'s *Lunula*, is to A ε δ (the Correſpondent of A ε C M) in duplicate Proportion of C ε to A ε. For E R ε (at R the Center of the leſſer Circle) is in this Caſe, a Right Angle.

Moreover, If you take any Point ε, in the Semicicle A C B, and proceed according to Mr. *Perks*'s Conſtruction Univerſaliz'd as above ſaid; you will find on the one ſide, the Trilineum A ε δ (contain'd by the Arcs A ε, A δ, and the ſtreight Line ε δ) equal to the Rectilineal Triangle A ε φ. And on the other ſide, the Trilineum contained by the Arc B ε (the Complement of ε A to the Semicircumference,) and the Arc B *d* (the Complement of A δ to the fourth Part of the Circumference,) and the ſtreight Line ε *d*, (that is the Trilineum B H C *d* diminiſhed by the Segment C ε;) to be equal to the Rectilineal Triangle B ε *f*. And that thoſe two Spaces A ε δ, and the difference of B H C *d*, from the Segment C ε (Parts of the *Lunula* A C B *g* γ A) taken together, are equal to the Triangle A C B, as well as to the two Spaces A E D and B E D, Parts of the *Lunula* of *Hippocrates*.

So that upon the whole it appears, that the two Circles (containing the *Lunula* of *Hippocrates*) being compleated, this *Lunula* A E B G A, and the other A C B *g* γ A, make up one Syſtem, and are conjugate Figures.

For, drawing a ſtreight Line C D E, or C ε δ, or ε C *d*, at Pleaſure, thro' C the Center of the greater Circle, and cutting thoſe two Circles, the Space contained within two Arcs of theſe two Circles, and part of the ſaid ſtreight Line (as A E D, or A ε δ, or B H ε *d*) is equal to the rectilineal Triangle A E F, or A ε φ, or B ε *f*, reſpectively.

And it ſo happens, that, if this Line going out from C, be on the ſame ſide of the Diameter M N, with the *Lunula* of *Hippocrates*; the aforeſaid Space (which receives a perfect Quadrature) is ſolitary; ſuch as are the Parts of *Hippocrates*'s *Lunula*, and of the two Spaces A ε C M, B H C N, which therefore are Parts of the *Lunula* more nearly relating to one another).

But if that Line going out from C, be on the other ſide of M N; then the Space which is equal to the Rectilineal Triangle, is the difference of two Mixtilineal Figures (the one a Trilineum, the other a Segment of the leſſer Circle) as is aboveſaid; neither of which can be ſquared ſeverally.

All theſe Particulars are plain from Mr. *Perks*'s Demonſtration; which with a little Variation (ſuch as is uſual in the different Caſes of the ſame Theorem) is applicable to them all; tho' perhaps he was not aware of it.

The like was done (without any Demonſtration) by M. *Tſchirnhauſe* in the *Acta Lipſiæ* 1687, to this purpoſe; If from any Point E, in the Circumference of the leſſer Circle, we let fall on AB, a Perpendicular Line cutting it in L, and draw the Line CL; the Triangle CAL, is equal to the Portion of the *Lunula* AED. (And conſequently the Triangle CBL, equal to the Portion BED.) Which I ſhall demonſtrate, ſo as the Demonſtration may alſo reach the Portions of the conjugate Space ACB$g\gamma$A.

For the Triangles ACB, AEF, are like Triangles, each being the half of a Square: and therefore by 19 *El.* 6. the Triangle ACB is to the Triangle AEF, in the duplicate Proportion of BA to AE, that is, by 8 *El.* 6, as BA is to AL. But, by 1 *El.* 6, the Triangle ACB is to the Triangle ACL, as BA is to AL. Therefore by 9 *El.* 5, the Triangles ACL and AEF are equal. But the Triangle AEF is (by Mr. *Perks*) proved equal to the Portion AED. And therefore the Portion AED is alſo equal to the Triangle ACL.

By Mr. Caſwell *ib. p.* 417.

3. On the Center B, Mr. *Caſwell* draws by A, a third Circle, which forms another *Lunula* than that of *Hippocrates*: and he doth (very dexterouſly) ſquare the Portions of this *Lunula*. And doth thereby let us in, to a new Syſtem, which may be purſued in like manner, as Dr. *Gregory* hath done that of *Hippocrates*.

By Dr. Wallis *ibid.* *Fig.* 25.

4. M. *Tſchirnhauſe*, letting fall, from E, (on AB) a Perpendicular EL, determines the Angle ALC, equal to the Portion ADE. Which being admitted, we may thus divide the *Lunula* in any given Proportion; if we divide AB, at L, in ſuch given Proportion; CL will, in the ſame Proportion (becauſe of the common Altitude) divide the Triangle ACB (which is equal to the whole *Lunula*.) And LE (erected at Right Angles on ALB) will determine the Point E; from whence if we draw to C, the ſtreight Line EC, this will, at DE, divide the *Lunula* in the ſame Proportion.

Fig. 26.

Mr. *Perks*, on EDC, drawing the Perpendicular AF, determines the Semiquadrate AFE, equal to the propoſed Portion ADE. Which Semiquadrate, is a like Figure, and alike ſituate to AE, as is ACB to AB.

And therefore (becauſe like Figures are in duplicate Proportion of their reſpective ſides) if we ſo inſcribe AE, as that the Square of AE be to the Square of AB in ſuch given Proportion, the *Lunula* will at DE be ſo divided as is required.

And this will hold (if duly apply'd, according as the different Caſes may require) tho' E be taken (in the Continuation of the Semicircle) beyond A. For, ſtill like Figures will be in duplicate Proportion of their reſpective ſides; and CE=CD±DE. And the ſame is yet improveable much further.

The Dimenſion of Solids generated by the Converſion of Hippocrates *Lunula by M.* Ab. de Moivre, N. 265. *p.* 624. Jul. An. 1700.

VIII. If upon BC you take any two Points D, E, and draw the Perpendiculars DH, EM, meeting BA in I and L, and cutting a Portion FGMH, of the *Lunula*; the Solid generated by the Converſion of this Portion about the Axis BC, is equal to a Priſm, whoſe Baſe is ILMH, and Height the Circumference of a Circle, whoſe Diameter is BC; and the Solid generated by the

Fig. 27. the Semicircle B K A, is equal to a Priſm or Semicylinder, whoſe Baſe is the Semicircle B K A, and Height, the Circumference of a Circle whoſe Diameter is B C.

Having biſected B A in R, and B C in P, the Surface generated by the Converſion of the Arc H M about the Axis B C, is equal to $\frac{c}{r} \times \overline{BP \times HM + BR \times DE}$ (ſuppoſing the Ratio of the Radius to the Circumference to be as r to c) and the Surface generated by the Semicircumference B K A is equal to a Rectangle, whoſe Baſe is the Summ of that Semicircumference and Diameter B A, and Height, the Circumference of a Circle, whoſe Diameter is B C. As for the Surface generated by the Arc G F, 'tis well known that it is equal to a Rectangle, whoſe Baſe is the Circumference of a Circle whoſe Radius is B C, and Height D E; therefore the Surface generated by the Converſion of the Portion M H F G, is known.

Fig. 28. If upon B A, you take any two Points I, L, and draw I N, L V, perpendicular to it, cutting the Quadrant in O and T, and the Circumference in N and V, the Solid generated by the Converſion of the Portion O N V T about the Axis B A, is equal to a Priſm, whoſe Baſe is I O T L, and Height the Circumference of a Circle whoſe Diameter is B A.

Having biſected B A in R, and drawn C R, meeting the Quadrant in G, the Surface generated by the Converſion of the Arc O T about BA, is equal to $\frac{c}{r} \times \overline{CG \times IL - CR \times OT}$.

Fig. 27. Biſect D E in 1, thro' the Center draw S Q, parallel to B C, meeting the Circumference B K A in S, B K parallel to A C in V, and the Lines D H, EM, in N and O; the Solid generated by the Converſion of the Portion FGMN about the Axis A C, is $\frac{c}{r} \times \overline{\tfrac{1}{3}MO^3 - \tfrac{1}{3}NH^3 + PC \times NOMH + CY \times DNOE - \tfrac{1}{3}EG^3 + \tfrac{1}{3}DF^3}$, and the Solid generated by the Segment KBS is $\frac{c}{r} \times \overline{\tfrac{2}{3}VK^3 + PC \times BVKS}$. Therefore the Solid generated by the Semicircle BKA about AC is $\frac{c}{r} \times \overline{PC \times VQAK + PC \times BCQV - \tfrac{1}{3}AC^3 + \tfrac{2}{3}VK^3 + PC \times BVKS}$, which by due Reduction will be found equal to the Solid generated by the Converſion of the ſame Semicircle about the Axis B C.

Fig. 28. The Solid generated by the Portion O N V T, about the Axis C P, is equal to $\frac{c}{r} \times \overline{\tfrac{1}{3}LV^3 - \tfrac{1}{3}IN^3 - \tfrac{1}{3}QT^3 + \tfrac{1}{3}PO^3 + CS \times PQIL}$.

Fig. 27 From the Points M, H, drop the two Perpendiculars M Z, H W, upon C A prolong'd, if need be; the Surface generated by the Converſion of the Arc HM, about the Axis CA is equal to $\frac{c}{r}\times\overline{PC}\times\overline{HM}$ $-\overline{RA\times WZ}$, when the Point Z is next to C, or $\frac{c}{r}\times\overline{PC\times HM+RA\times WZ}$, when the Point W is next to it.

Thoſe that will think it worth their while to beſtow ſome little Pains to find the Demonſtration of this, may ſolve the following Problem.

Any two Conic Sections, being given, forming a Lunula by their Interſection, and a Right Line being given by Poſition, about which, as an Axis, this Lunula is imagined to turn, to find the Solids generated by the Converſion of any of its Parts, cut off by Lines Perpendicular to that Axis, or Parallel to it, or making any given Angle with it, as alſo the Surfaces made by that Converſion.

IX. Suppoſe DPV, to be half of an exterior Epicycloid, VB its Axis, V the Vertex, VLB half the generant Circle, E its Center; DB the Baſe, C its Center: Biſect the Arc of the Semicircle VB in L, and on the Center C thro' L, braw a Circle cutting the Epicycloid in P: Then, I ſay, the Curvilinear Triangle VLP will be $=BEq$ in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle, will be to the Curvilinear Triangle VLP, as CB the Semidiameter of the Baſe to CE: Which CE in the Exterior Epicycloid is the Summ of the Semidiameters of the Baſe and Generant, but in the Interior Epicycloid D*pu*, 'tis the difference of the ſaid Semidiameters.

The Quadrature of a Portion of the Epicycloid, by Mr. Caſwell. N. 217, *p.* 113. Oct. An. 1695. *Fig* 29.

COROL. I.] In the Interior Epicycloid, if CE is $\frac{1}{2}$ CB, the Epicycloid then degenerating into a Right Line, the Quadrature of the Triangle *lpu*, will be in effect the ſame with the Quadrature of *Hyppocrates Chius*.

COROL. II.] If the Semidiameter of the Baſe is ſuppoſed infinite, the Epicycloid then being the common Cycloid, the Area of the ſaid Triangle will be equal to the Square of the Radius of the Generant; and ſo it falls in with that *Theorem* which *Lalovera* found and calls *Mirabile*.

The general Propoſition from whence I deduced the aboveſaid Quadrature, is this; *viz.* the Segments of the generant Circle are to the Correſpondent Segments of the Epicycloid, as CB, to 2 CE+CB. For Example. Suppoſe F*m*G, the Poſition of part of the Generant, when the Point F of the exterior Epicycloid, was deſigned, then the Segment F*m*G*n* is to the Segment DF*n*G, as CB to 2 CE + CB. And conſequently the whole Epicycloid to the whole Generant in the ſame Proportion; which is the only Caſe demonſtrated by M. *de la Hire*.

It follows alſo, that in the vulgar Cycloid, its Segments are triple of the correſpondent Sectors of the Generant, which was firſt ſhewn by Dr. *Wallis*.

A general Propoſition for meaſuring all Cycloids and Epicycloids, by Mr. Edm. Halley, N. 218. *p.* 125.

X. *Area Cycloidis vel Epicycloidis, ſive Primariæ, ſive Contractæ vel Prolatæ, eſt ad Aream Circuli Generantis; atque etiam Area partium genitarum in iiſdem Curvis, ad Areas analogorum ſegmentorum Circuli, ut ſumma duplæ Velocitatis centri ac Velocitatis motus Circularis, ad Velocitatem motus Circularis.*

Fig. 30.

Demonſtratio.] Deſcribatur Epicyclois quævis Y P S Q V B, revolutione circuli V L B, ſuper Baſi circulari Y M N B; ponatur centrum circuli generantis in *c*, ductaque *c* M K, inſiſtat circulus Baſi in puncto M; ſitque punctum lineans S. Jam diviſis motibus transferatur primum motu circulari punctum S in R, ut augeatur arcus S M particula indiviſibili R S; deinde progrediatur centrum *c* in C; hoc motu, traducto ſegmento R S M in ſitum Q T N, punctum Q tanget Curvam. Patet triangulum R S M eſſe momentum ſive fluxionem Areæ ſegmenti Circuli: Trapezium vero Q S M N eſſe Fluctionem Areæ ſpatii curvilinei ſimulgeniti. Jam cum S M, R M, Q M, non niſi puncto inter ſe deferre intelligantur, concipe Areolam Q S M N conſtare ex tribus ſectoribus R M S, R M Q, M Q N; adeoque Areolam R M S, eſſe ad Areolam Q S M N, ut eſt angulus R M S ad ſummam trium angulorum R M S + R M Q + M Q N. At anguli R M Q + M Q N, æquantur angulis M C N + M K N, ſive angulo *c* M C; propter lineas R M, Q N invicem inclinatis ſub angulo ipſi M K N æquali ac propter angulum M Q N ipſius M C N dimidium (per *Eucl.* 3. 20.) Proinde angulus R M S eſt ad angulos R M S + *c* M C, hoc eſt (per eadem 3. 20.) arcus $\frac{1}{2}$ R S, ad duos arcus C *c* + $\frac{1}{2}$ R S, ſive R S ad 2 C *c* + R S, ut areola R S M ad areolam Q S M N: ſive ut momentum ſegmenti circularis Q T N, ad momentum ſegmenti in Epicycloide ſimul geniti Q S Y M N. Cumque hæc momenta ſemper ſint in eadem illa ratione, ubicunque aſſumpſeris punctum Q, conſtat Areas ipſas Q T N, Q S Y M N his momentis genitas, eandem habere conſtantem rationem, nempe velocitatis motus circularis R S, ad duplam velocitatem centri addito motu circulari, ſive 2 C *c* + R S. Sicut etiam Aream V B Z ad Aream Q V B N, ac proinde ſemicirculum V L B ad ſpatium Curvilineam V Q Y N B. Ergo conſtat propoſitio. Nulla autem alia eſt differentia in modo demonſtrandi, ſi Circulus genitor ſuper Arcu Baſis Concavæ moveatur, niſi quod angulus *c* M C, hoc in caſu, ſit differentia angulorum M C N, M K N. Si vero Baſis ſit linea recta, evaneſcente M K N, ac ob R M, Q N parallelas, etiam facilior erit probatio. In omnibus autem hujuſmodi Curvis portiones analogæ portionibus illis quas in Cycloide primaria perfectæ Quadraturæ capaces invenit *Cl. Walliſius*, ſunt æque quadrabiles, quod quidem facile conſequitur ex præmiſſis.

Centro K, per punctum Q duc circularem arcum Q Z, ac age Z B abſcindens ſegmentum Z L B, æquale ſegmento Q T N, dein biſeca ſemicirculum V B in L, ac per punctum L, centro etiam K, deſcribe arcum P L, ſecantem Epicycloidem in P, circulum Genitorem in T, ac Chordas Q N, Z B in y & X. Jam ſit Arcus V Z $= a$, ejuſque ſinus $= s$, Radius Genitoris $= r$, Radius vero Baſis $=$ R; ſitque arcus C E, ſive motus centri $= m$, Patet Sectorem CKE eam rationem habere ad ſpatium XyNB, quam habet quadratum ex KE, ad differentiam quadratorum ex KL & KB; ſive ut $RR + 2Rr + rr$ ad $2Rr + 2rr$; hoc eſt ut $R + r$ ad $2r$, vel KE ad BV; ac proinde rectangulum BE in CE ſive rm æquari ſpatio XyNB. Spatium vero VZB æquale eſt rectang. $\frac{1}{2}ar + \frac{1}{2}sr$; adeoque juxta noſtram propoſitionem, erit ut a ad $2m$, ita $\frac{1}{2}ar + \frac{1}{2}sr$ ad $\frac{mar + msr}{a}$, æquale ſpatio Curvilineo QVZLBNQ: Ex hoc ſubduc ſpatium XyNB $= rm$, & remanebit ſpatium QVZX$y = \frac{mrs}{a}$: Cumque ſpatia ZXL, QyT æquentur inter ſe, ſpatium QVLTQ etiam æquabitur ipſi $\frac{mrs}{a}$: Quoties itaque a ad m, ſive motus circularis ad progreſſivum Centri, fuerit in data ratione, dabitur etiam perfecta Quadratura ſpatiorum Curvilineorum QVLTQ: Totumque ſpatium VPL, ad Quadratum Radii BE, erit in eadem ratione motuum m ad a, hoc eſt, in omni Epicycloide primaria, in ratione radiorum KE ad KB, quæ eſt ipſa *D. Caſwelli* Propoſitio. Spatia autem minora QVLTQ erunt inter ſe ut Sinus arcuum VZ; ac ſpatia Triangularia QTP, eodem argumento erunt ut Sinus Verſi arcuum QT vel ZL: ac proinde etiam quadrantur. Pari modo probabuntur ſpatia PΛϒ, pLn, pλϒ, ſemper eſſe ad Radii BE quadratum (in omnibus his Figuris) in prædicta ratione m ad a; eorumque portiones pqt, ut Sinus Verſi arcuum interceptorum qt. Reſidua autem ſegmenta, ut qtϒΛ, qtϒΛ, &c. erunt ut ſinus recti complimentorum eorundem arcuum qt.

Componitur autem ratio velocitatum m ad a, ex ratione radiorum KE, BE, ac ratione angulorum ſimul æquabiliter deſcriptorum CKE, VEZ: ac proinde data etiam illa angulorum ratione, etiam Quadrabuntur ſpatia omnia Epicycloidalia prædicta.

XI. 1. *Quæritur Curva ejus Proprietatis, ut duo Segmenta* (lineæ rectæ à dato puncto per curvam dictæ) *ad quamcunque potentiam datam elevata & ſimul ſumpta faciant ubique unam eandemque ſummam.* Generalem ſolutionem, Analyſtis eruendam relinquimus.

A Problem propos'd by M. J. Bernouilli. N. 224. *p.* 387. Jan. An. 1697.

2. Problema (ſi recte intellexi) ſic proponi poteſt. Quæritur Curva KIL ea lege ut ſit recta PKL à dato quodam puncto P, ceu Polo utcunque ducatur, & eidem Curvæ in punctis duobus K & L occurrat, poteſtates duorum ejus ſegmentorum PK & PL, à dato illo puncto P ad occurſus illos ductorum ſi ſint æque altæ (id eſt vel quadrata, vel cubi vel quadrato-quadrata, &c.) datam ſummam $PKq + PLq$, vel $PK^{cub.} + PL^{cub.}$ &c. (in omni rectæ illius poſitione) conficiant.

Solv'd by I. N. *ibid. p.* 389. *Fig.* 31.

SOLUTIO.] Per datum quodvis punctum A, ducatur recta quævis infinita poſitione data ADB, recte mobili PKL occurrens in D, & nominentur AD, x, & PR, vel PL, y, ſintque Q & R quantitates ex quantitatibus quibuſcunque datis & quantitate x quomodocunque conſtantes, & relatio inter x & y definiatur per hanc æquationem $YY + QY + R = 0$. Et ſi R ſit quantitas data, Rectangulum ſub ſegmentis PK & PL dabitur. Si Q ſit quantitas data, ſumma ſegmentorum illorum (ſub ſignis propriis conjunctorum) dabitur. Si $QQ - 2R$ datur, ſumma quadratorum ($PKq + PLq$) dabitur. Si $Q^3 - 3QR$ data ſit quantitas, ſumma cuborum ($PK^{cub.} + PL^{cub}$) dabitur. Si $Q^4 - 4QQR + 2RR$ data ſit quantitas, ſumma quadrato-quadratorum ($PKqq + PLqq$) dabitur. Et ſic deinceps in infinitum. Efficiatur itaque ut R, Q, $QQ - 2R$, $Q^3 - 3QR$, &c. datæ ſint quantitates, & Problema ſolvetur. *Q. E. F.*

Ad eundem modum Curvæ inveniri poſſunt quæ tria vel plura abſcindent ſegmenta ſimiles proprietates habentia. Sit æquatio $Y^3 + Qyy + Ry + S = 0$; ubi Q, R & S, quantitates ſignificant ex quantitatibus quibuſcunque datis, & quantitate x utcunque conſtantes; & Curva abſcindet ſegmenta tria. Et ſi S data ſit quantitas, contentum ſolidum illorum trium dabitur. Si Q ſit quantitas data, ſumma trium illorum dabitur. Si $QQ - 2R$ ſit data quantitas, ſumma quadratorum ex tribus illis dabitur.

The Uſe of Fluxions in the Solution of Geometrick Problems, by Mr. Abr. de Moivre. N.216. p. 52. March, Anno 1695.

XII. Habes hic Methodum de Figurarum Curvilinearum Quadraturis; de Solidorum a Rotatione Plani genitorum eorumque ſuperficierum dimenſione; de Rectificatione Curvarum, deque Centri Gravitatis Calculo. Priuſquam, verum, ulterius progrediar, hoc te monitum velim me uſurpare illa, quæ demonſtravit Clariſſimus *Newtonus,* in *pag.* 251, 252 & 253. *Prin. Phil.* circa Momentanea Incrementa vel Decrementa Quantitatum quæ Fluxu continuo creſcunt vel decreſcunt, præſertim quod dignitatis cujuſcunque $A^{\frac{n}{m}}$, Momentum ſit $\frac{n}{m} a A^{\frac{n}{m} - 1}$.

Porro data Fluxione $\frac{n}{m} a A^{\frac{n}{m} - 1}$ viciſſim reperiri poteſt Quantitas Fluens $A^{\frac{n}{m}}$, 1° tollendo a de Fluxione, 2° Fluxionis Indicem Unitate augendo, 3° denique Fluxionem dividendo per Indicem ſic Unitate auctum.

Curvæ abſciſſa deſignabitur deinceps per x, ejus Fluxio per $\dot{x}$, ordinatim applicata per y, ejuſque Fluxio per $\dot{y}$.

His poſitis ut ad Quadraturas deveniamus. 1° aſſumatur valor ordinatim applicatæ ope Æquationis naturam Curvæ exprimentis. 2° Multiplicetur hic valor per Fluxionem abſciſſæ; rectangulum hinc ortum erit Fluxio Areæ. 3. Data Fluxione Areæ reperiatur quantitas Fluens, habebitur Area quæſita.

Proponatur æquatio $x^m = y^n$ cujuſvis Paraboloidos naturam exprimens, valor ordinatim applicatæ y eſt $x^{\frac{m}{n}}$, qui ſi multiplicetur per $\dot{x}$ Rectangulum

$x^{\frac{m}{n}}\dot{x}$ erit Fluxio Areæ, proindeque Area quæſita erit $\frac{n}{m+n}x^{\frac{m}{n}+1}$, ſeu (poſito y pro $x^{\frac{m}{n}}$) $\frac{n}{m+n}xy$.

Rurſum proponatur Curva cujus Æquatio ſit, $x^4+aaxx=yy$ (illa ſcilicet quæ inter Exempla *Cl. Craigi* extat prima) aſſumpto $x\sqrt{xx+aa}=y$, Fluxio Areæ erit $x\dot{x}\sqrt{xx+aa}$; cum autem ſub Radicalitate involvatur, ſupponatur $\sqrt{xx+aa}=z$, hinc $xx+aa=z^2$, ideoque $x\dot{x}=z\dot{z}$; poſitiſque $z\dot{z}$ & z pro $x\dot{x}$ & $\sqrt{xx+aa}$, Fluxio à Surdis liberata erit $z^2\dot{z}$, quam ſi ad Originem ſuam $\frac{1}{3}z^3$ revocaverimus, repoſitoque $\sqrt{xx+aa}$ pro z, habebitur $\frac{1}{3}\overline{xx+aa}\sqrt{xx+aa}$ pro Area quæſita.

Sed quo magis conſtet quam facili negotio conficiantur hujuſmodi Quadraturæ, unum amplius Exemplum proferre viſum eſt; Æquatio Curvæ talis ſit $\frac{x^2}{x+a}=y^2$, igitur $y=\frac{x}{\sqrt{x+a}}$, ideoque $\frac{x\dot{x}}{\sqrt{x+a}}$ eſt Fluxio Areæ: ſupponatur $\sqrt{x+a}=z$, hinc $x=zz-a$, & $\dot{x}=2z\dot{z}$, itaque $\frac{x\dot{x}}{\sqrt{x+a}}=2z^2\dot{z}-2a\dot{z}$, ac proinde $\frac{2}{3}z^3-2az$ ſeu $\overline{\frac{2}{3}x-\frac{4}{3}a}\sqrt{\overline{+a}}$ erit Area quæſita.

Verum ſæpe accidit ut quædam Curvæ, quales Circulus, & Hyperbola, ejus naturæ ſint, ut fruſtra tentaveris earum Fluxiones Surdis immunes facere; tunc valore ordinatæ in Seriem infinitam conjecto ſinguliſque hujus Seriei Terminis Fluxionem abſciſſæ, ut ſupra, multiplicatis, reperiatur ſingulorum Terminorum Quantitas Fluens, orietur nova Series quæ Quadraturam Curvæ exhibebit.

Methodus hæc eadem facilitate ad Dimenſionem Solidorum à Plani circumvolutione genitorum accommodatur, nempe aſſumendo pro eorum Fluxione productum ex Fluxione abſciſſæ per Circulum Baſis; Ratio Quadrati ad Circulum ſibi inſcriptum vocetur $\frac{n}{1}$, Æquatio Circulo competens eſt $yy=dx-xx$, igitur $4\frac{\overline{dx\dot{x}-x^2\dot{x}}}{n}$ eſt Fluxio Portionis Sphæræ, igitur $4\frac{\overline{\frac{1}{2}xdx-\frac{1}{3}x^3}}{n}$ eſt Portio ipſa, huic circumſcriptus Cylindrus eſt $4\frac{\overline{dxx-x^3}}{n}$, ideoq; Ratio Portionis Sphæræ ad circumſcriptum Cylindrum eſt ut $\frac{1}{2}d-\frac{1}{2}x$ ad $d-x$.

Rectificatio Curvarum obtinebitur ſi Hypothenuſa Trianguli Rectanguli cujus latera ſunt Fluxiones abſciſſæ & ordinatæ tanquam Curvæ Fluxio conſideretur, ſed curandum eſt ut in expreſſione iſtius Hypothenuſæ, alterutra Fluxionum ſolummodo ſuperſit, ac una tantum Indeterminatarum, illa ſcilicet cujus Fluxio retinetur. Res Exemplis clarior fiet.

Fig. 32. Ex dato ſinu recto C B, arcum A C invenire, poſitis A B $= x$, C B $= y$, O A $= r$; ſit C E Fluxio abſciſſæ, E D Fluxio ordinatim applicatæ, C D Fluxio Arcus C A; ex Circuli proprietate $2rx - xx = yy$, unde $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, ideoque $\dot{x} = \frac{y\dot{y}}{r-x}$, ſed C D $q = \dot{y}\dot{y} + \dot{x}\dot{x} = \dot{y}\dot{y} + \frac{y^2\dot{y}\dot{y}}{rr - 2rx + xx} = \dot{y}\dot{y} + \frac{y^2\dot{y}\dot{y}}{rr - yy} = \frac{rr\dot{y}\dot{y}}{rr - yy}$, igitur C D $= \frac{r\dot{y}}{\sqrt{rr-yy}}$, ſed $\frac{r\dot{y}}{\sqrt{rr-yy}}$ factum eſt ex $\frac{1}{\sqrt{rr-yy}}$ ſeu $\overline{rr-yy}|^{-\frac{1}{2}}$ in $r\dot{y}$; proindeq; ſi $\overline{rr-yy}|^{-\frac{1}{2}}$ conjiciatur in Seriem infinitam cujus ſingula membra per $r\dot{y}$ multiplicentur, & ex unoquoque producto ad Quantitatem Fluentem fiat retrogreſſus, habebitur Longitudo arcus A C.

Non abſimili modo ex dato Sinu Verſo reperietur idem arcus; Reſumatur æquatio ſupra inventa $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, fit $\dot{y} = \frac{r\dot{x} - x\dot{x}}{2y}$, ſed C D $q = \dot{x}\dot{x} + \dot{y}\dot{y} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + x^2\dot{x}\dot{x}}{yy} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + x^2\dot{x}\dot{x}}{2rx - xx}$, ſeu (omnibus ſub eodem denominatore reductis, expunctiſque iis quæ ſub diverſis ſignis continentur) $= \frac{rr\dot{x}\dot{x}}{2rx - xx}$, unde C D $= \frac{r\dot{x}}{\sqrt{2rx - xx}}$, ideoque Longitudo arcus A C, per ea quæ jam dicta ſunt facile obtinebitur.

Fluxio curvæ facilius interdum reperitur per comparationem inter Triangula ſimilia C E D, C B O, inſtitui enim poteſt hæc proportio C B : C O :: C E : C D, hoc eſt, pro Circulo $\sqrt{2rx - xx} : r :: \dot{x} : \frac{r\dot{x}}{\sqrt{2rx - xx}}$.

Fig. 33. Curva Cycloidis eadem opera cognoſci poterit. Sit ALK ſemicyclois, cujus circulus genitor ADL. Aſſumpto in diametro AL quovis puncto B, ducatur BI parallela Baſi LK, peripheriæ circuli in puncto D occurrens; compleatur rectangulum A E I B, ducaturque F H erectæ E I parallela, idemque infinite vicina, B I productam ſecans in G, curvamque A K in H; ponatur $AL = d$, $AB = EI = x$, $GH = \dot{x}$; Notum eſt rectam B G eſſe ubique aggregatum arcus A D & ſinus recti B D, hinc manifeſtum eſt Fluxionem I G eſſe aggregatum Fluxionum arcus A D & ſinus recti B D. Porro Fluxio arcus A D reperta eſt $= \frac{\frac{1}{2} d\dot{x}}{\sqrt{dx - xx}}$, Fluxio autem Sinus Recti B D reperietur $= \frac{d\dot{x} - 2x\dot{x}}{2\sqrt{dx - xx}}$, igitur $IG = \frac{d\dot{x} - x\dot{x}}{\sqrt{dx - xx}}$, ideoque $IHq = IGq + GHq = \frac{dd\dot{x}\dot{x} - dx\dot{x}\dot{x}}{dx - xx}$; Quamobrem $IH = \frac{\dot{x}\sqrt{dd - dx}}{\sqrt{dx - xx}} = \frac{\dot{x}\sqrt{d}}{\sqrt{x}} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$, proindeque $AI = 2 d^{\frac{1}{2}} x^{\frac{1}{2}} = 2\sqrt{dx} = 2\,AD$.

Hæc concluſio minimo cum labore deduci poteſt ex nota proprietate Tangentis, cum enim illius portiuncula I H ſemper ſit parallela chordæ A D, fit ut Triangula I G H, A B D ſint ſimilia, unde $AB : AD :: GH : IH$, hoc eſt $x : \sqrt{dx} :: \dot{x} : \frac{\dot{x}\sqrt{dx}}{x}$, igitur $IH = \frac{\dot{x}\sqrt{dx}}{x} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$.

Sed nihil vetat quo minus adhibito Fluxionis I H auxilio, ipſam Cycloidis aream inveſtigemus. Fluxio Areæ A E I eſt rectangulum E I G $= \frac{dx\dot{x} - x^2\dot{x}}{\sqrt{dx - xx}} = \dot{x}\sqrt{dx - xx}$, ſed Fluxio portionis A B D non alia eſt ab illa: Itaque Area A E I, correſpondenſque circuli portio A B D, ſemper ſunt æquales.

Eſto A B curva Parabolæ cujus Axis A F, Parameter a; Ponatur $AE = x$, $EB = y$, $AB = z$, $BD = \dot{x}$, $DC = \dot{y}$, $BC = \dot{z}$, aſſumpta æquatione Parabolæ naturam conſtituente, videlicet $ax = yy$, fit $a\dot{x} = 2y\dot{y}$, unde $\dot{x} = \frac{2y\dot{y}}{a}$, ſed $BCq = BDq + CDq$, hoc eſt $\dot{z}\dot{z} = \dot{x}\dot{x} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y}}{aa} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y} + aa\dot{y}\dot{y}}{aa}$, ideoque $\dot{z} = \dot{y}\frac{\sqrt{4y^2 + aa}}{a}$ vel, quod idem eſt, $\dot{z} = \dot{y}\frac{\sqrt{y^2 + \frac{1}{4}aa}}{\frac{1}{2}a}$: ſi ergo $\dot{y}\frac{\sqrt{y^2 + \frac{1}{4}a^2}}{\frac{1}{2}a}$, in ſeriem infinitam transformetur, Curva A B haud difficulter innoteſcet. Fig. 34.

Inſuper

Fig. 35. Inſuper ſtatim apparet dato Hyperbolico ſpatio curvam hanc dari, & viciſſim. Nam $\frac{1}{2} a \dot{z} = \dot{y} \sqrt{y^2 + \frac{1}{4} aa}$, ac proinde $\frac{1}{2} a z =$ ſpatio cujus Fluxio eſt $\dot{y} \sqrt{y^2 + \frac{1}{4} aa}$, ſed hujuſmodi ſpatium nihil aliud eſt quam Hyperbola æquilatera exterior A B E G, cujus ſemiaxis A B $= \frac{1}{2} a$, abſciſſa A E $= y$, ordinatim applicata E G $= x$.

Ad dimetiendam ſuperficiem converſione curvæ circa ſuum Axem deſcriptam, aſſumi debet pro ejus Fluxione Cylindrica ſuperficies cujus altitudo eſt ipſa curvæ Fluxio, cujuſque diſtantia ab Axe eſt ordinatim applicata huic Fluxioni conveniens.

Fig. 12. Sit *ex. gr.* A C Circuli Arcus qui circa Axem A D revolvendo ſuperficiem Sphæricam generet, quamque dimetiri ſtatuamus; D C arcus Fluxio jam reperta eſt $\frac{r \dot{x}}{\sqrt{2rx - xx}}$, hanc ſi multiplicemus per Circumferentiam ad Radium B C pertinentem, hoc eſt $\frac{c}{r} \sqrt{2rx - xx}$ (poſita ratione Circumferentiæ ad Radium $= \frac{c}{r}$) habebimus Fluxionem ſuperficiei Sphæricæ $= c \dot{x}$; adeoque ſuperficies ipſa eſt $c x$.

Ad centra gravitatis quod attinet, reperta Superficiei Solidive Fluxione, hacque ducta in ſuam à Vertice diſtantiam, ad quantitatem Fluentem recurrendum eſt: qua diviſa per Superficiem ipſam Solidumve ipſum, prodibit diſtantia Centri Gravitatis à Vertice.

Inveniendum ſit Centrum Gravitatis omnium Paraboloidum horum Fluxio ſic generaliter exprimitur $x^{\frac{m}{n}} \dot{x}$, hanc multiplica per x, fit $x^{\frac{m}{n}+1} \dot{x}$ cujus quantitatem Fluentem $\frac{n}{m + 2n} x^{\frac{m}{n}+2}$, divide per Paraboloidos Aream, puta $\frac{n}{m+n} x^{\frac{m}{n}+1}$; prodibit $\frac{m+n}{m+2n} x$, diſtantia Centri Gravitatis à Vertice.

Centrum Gravitatis in Portione Sphæræ eodem modo colligitur, namque illius Fluxione $4 \frac{dx\dot{x} - x^2\dot{x}}{n}$ in x ducta, fit $4 \frac{dx^2\dot{x} - x^3\dot{x}}{n}$, cujus quantitas Fluens $4 \frac{\frac{1}{3} dx^3 - \frac{1}{4} x^4}{n}$ per Portionis Soliditatem diviſa, puta $4 \frac{\frac{1}{2} dxx - \frac{1}{3} x^3}{n}$, exhibet $\frac{\frac{1}{3} d - \frac{1}{4} x}{\frac{1}{2} d - \frac{1}{3} x} x$, ſeu $\frac{4d - 3x}{6d - 4x} x$ Diſtantiam Centri Gravitatis à Vertice.

XIII. 1. Prop.

XIII. 1. Prop. I. Probl.] *Invenire relationem inter Fluxionem Axeos & Fluxionem Ordinatæ in Curva Catenaria.*

The Catena; by Dr. Dav. Gregory. n. 231. *p.* 637. Aug. An. 1697. *Fig.* 56.

Sit Catena F A D ab extremitatibus F & D dependens, cujus punctum imum (ſeu Curvæ vertex) A, Axis A B ad horizontem erectus, eique applica B D horizonti parallela. Invenienda eſt relatio inter B *b*, ſeu D δ & *d* δ; poſito *b* puncto ipſi B proximo, & *b d* ad B D, item D δ ad B A parallela.

Ex Mechanicis conſtat potentias tres in æquilibrio poſitas eandem habere rationem cum rectis tribus ad ipſarum directionis parallelis, vel in dato angulo inclinatis, à mutuo occurſu terminatis; adeoque ſi D *d* exponat gravitatem abſolutam particulæ D *d* (ut in Catena æqualiter craſſa rite ſit) *d* δ repræſentabit gravitatis partem eam quæ normaliter in D *d* agit, quaque ſit ut *d* D (ob Catenæ flexilitatem circa *d* mobilis) in ſitum verticalem ſe componere conatur. Adeoque ſi δ *d* (ſive fluxio ordinatæ B D) conſtans ſit; gravitatis actio in partes correſpondentes Catenæ D *d* normaliter exerta etiam conſtans erit ſive ubique eadem. Exponatur hæc per Rectam *a*. Porro ex ſupracitato Lemmate Mechanico, D δ ſive fluxio axeos A B, exponet vim ſecundum directionem ipſius *d* D exerendam, quæ priori conatui Lineæ gravis *d* D ad componendam ſe in ſitum verticalem æquipolleat, eumque impedire poſſit. Hæc vero vis oritur à linea gravi D A ſecundum directionem *d* D trahente; eſtque proinde (cæteris manentibus) lineæ D A proportionalis. Eſt igitur δ *d* fluxio ordinatæ ad δ D fluxionem abſciſſæ, ſicut conſtans recta *a* ad D A curvam. *Q. E. F.*

COROL.] Si recta T D tangat Catenariam, & axi B A producto occurrat in T, erit B D : B T : : *(d* δ : D : :*) a* : D A Curvam.

Prop. II, Theorem.] *Si ad perpendiculum A B tanquam axem, vertice A, deſcribatur hyperbola æquilatera A H, cujus ſemiaxis A C æqualis* a; *& ad eundem axem & verticem, parabola A P cujus parameter æqualis quadruplo axi hyperbolæ, & producatur ſemper hyperbolæ ordinata H B, donec H F æqualis Curvæ A P: Dico Curvam F A D, in quo puncta F & D verſantur (poſitis B D, B F æqualibus) eſſe Catenariam.*

Vocetur A B, x, erit B$b = \dot{x}$, & B H $= \sqrt{2ax + x^2}$. Unde ex Methodo Fluxionum, Fluxio ipſius B H (ſive *m b*) $= \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$, rurſus quia parabolæ A P parameter $= 8a$, erit B P $= \sqrt{8ax}$. Unde np (hoc eſt Fluxio ipſius B P) æqualis $\frac{2a\dot{x}}{\sqrt{2ax}}$. Quare Fluxio Curvæ A P (= P p $= \sqrt{npq + Pnq}$) $= \sqrt{\frac{4a^2\dot{x}^2}{2ax} + \dot{x}^2} = \frac{\sqrt{2a\dot{x}^2 + x\dot{x}^2}}{x}$, quæ ducendo do tam numeratorem quam denominatorem in $\sqrt{2a+x}$, $= \frac{2a\dot{x}+x\dot{x}}{\sqrt{2ax+x^2}}$. Et cum HE ſit ubique = AP, erit Fluxio HF rectæ, hoc eſt $mh + sf$ $= \frac{2a\dot{x}+x\dot{x}}{\sqrt{2ax+x^2}}$. Sed hactenus inventa eſt $mh = \frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+x^2}}$. Unde sf, ſive Fluxio ipſius BF ordinatæ ad axem Catenariæ, eſt æqualis $\frac{a\dot{x}}{\sqrt{2ax+x^2}}$. Et igitur Fluxio Curvæ AF (ſive ipſa $Ff = \sqrt{sf\text{q} + Fs\text{q}}$ $= \sqrt{\frac{a^2\dot{x}^2}{2ax+x^2} + \dot{x}^2}) = \frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+x^2}}$, cujus fluens modo oſtenſa eſt $\sqrt{2ax+x^2}$. Et igitur $AF = \sqrt{2ax+x^2}$. Patetque fluxionem ordinatæ BF ſive $\frac{a\dot{x}}{\sqrt{2\,[illegible]+{}^2}}$ eſſe ad $\dot{x}$ fluxionem abſciſſæ AB, ſicut data a ad Curvam AF, quæ eſt ſuperius inventa Catenariæ proprietas. Igitur Catenariæ puncta recte determinantur, per præcedentem conſtructionem. *Q. E. D.*

COROL. 1.] Ex Conſtructione patet BF ordinatam Catenariæ æquari Curvæ parabolicæ AP, dempta BH correſpondente ordinata hyperbolæ conterminatæ AH.

2. Ex demonſtratione conſtat Catenariam Curvam AF æquari BH correſpondenti ordinatæ conterminæ Hyperbolæ æquilateræ. Cum enim harum linearum Fluxiones æquentur & ſimul naſcantur ipſæ lineæ, patet illas ubique eſſe æquales. Unde data Catena, dabitur AC ſive a, quippe æqualis ſemiaxi Hyperbolæ æquilateræ cujus vertex A, & ordinata ad abſciſſam AB catenæ AD eſt æqualis.

3. Catenariæ omnes ſunt inter ſe ſimiles, cum ex ſimili ſimilium, & ſimiliter poſitarum figurarum conſtructione generentur. Undæ duæ rectæ ad Horizontem ſimiliter inclinatæ per Catenarum vertices ductæ abſcindent figuras ſimilis & Catenarum portiones abſcindentibus rectis proportionales.

4. Si Catena QAD ſuſpendatur à punctis Q & D inæqualiter altis, Curvæ pars FAD eadem manet, ac ſi ex punctis æquialtis F & D eſſet ſuſpenſa, quoniam nihil refert utrum punctum F affixum ſit vel non affixum ad planum verticale.

5. Si Catenæ vis trahens ſecundum directionem, dD exponatur per Dd, dividetur, ut vulgo notum in vim ut $d\delta$ ſecundum directionem horizontalem, & vim ut δD, ſecundum directionem verticalem: Igitur vis in Catenæ extre-

mo directe accedendi ad axem, eſt ad vim in eodem deſcendendi ſecundum perpendiculum; ſive vis ſuſtinentis pars ſecundum directionem B D agens, eſt ad ejuſdem partem ſecundum directionem D ꝺ agentem, ut ſemi-axis Hyperbolæ conterminæ A H ad D A, longitudinem Catenæ uſque ad verticem Curvæ: Unde data Catena ratio hæc datur. Et in eadem Catena nunc magis nunc minus laxe ſuſpenſa, vis iſta Horizontalis eſt ut Hyperbolæ conterminæ axis, cum D A eadem maneat ſi extrema æquialta ſint.

6. Catena in Plano verticali, ſed ſitu inverſo, figuram ſervat nec decidit, adeoque arcum ſeu fornicem facit tenuiſſimum: Hoc eſt Sphæræ minimæ rigidæ & lubricæ in inverſa Curva Catenaria diſpoſitæ, arcum conſtituunt, cujus nulla pars ab aliis extrorſum vel introrſum propellitur; ſed manentibus infimis punctis immotis, virtute ſuæ figuræ ſuſtinetur. Cum enim punctorum Curvæ Catenariæ ſitus, partiumque inclinatio ad Horizontem eadem ſit, ſive in ſitu F A D, ſive in ſitu inverſo, dummodo Curva ſit in plano ad Horizontem recto, patet illam æque ſervare figuram immutatam in uno ſitu ac in altero. Et è converſo ſolæ Catenariæ ſunt fornices ſive arcus legitimi: Et cujuſcunque alterius figuræ arcus ideo ſuſtinetur, quod in illius craſſitie quædam Catenaria incluſa ſit: Neque, ſi tenuiſſimus eſſet, parteſque haberet lubricas ſuſtineretur. Ex præcedente *Corol.* 5. colligitur, quali vi arcus muros quibus inſiſtit extra propellit; nempe hæc eadem eſt cum parte vis Catenam ſuſtinentis, quæ ſecundum directionem Horizonta[illegible] trahit. Quæ enim in Catena introrſum trahit vis, in arcu Catènæ æquali, extrorſum propellit. Alia omnia de murorum quibus fornices imponuntur firmitate requiſita, ex hac theoria Geometrice determinantur, quæ in ædificiorum extructione præcipua ſunt.

7. Si loco gravitatis alia quælibet vis ſimiliter agens in lineam flexilem vires ſuas exerat, eadem producetur linea, *v. g.* Si ventus æquabilis ſupponatur, & ſecundum rectas datæ poſitione rectæ parallelas ſpirans, linea vento inflata eadem erit cum Catenaria. Nam cum omnia quæ in gravitate conſideravimus, in altera hac vi obtineant, patet eandem Curvam productum iri.

Prop. 3. Theor.] *Si manente prædicta Hyperbola A H, per A ducatur recta G A L Axi A B normalis, & deſcribatur Curva K R ejus natura, ut B K ſit tertia proportionalis rectis B H & A C, & ad A C applicetur rectangulum A V æquale ſpatio interminato A B K R L A, erit F concurſus rectarum H B, V G ad Catenariam.* Fig. 37.

Nam ex conſtructione eſt $\mathrm{B\,K} = \frac{a^2}{\sqrt{2ax + x^2}}$, quare fluxio ſpatii

$\mathrm{A\,B\,K\,R\,L\,A} = (\mathrm{B\,K}\,kb = \mathrm{B\,K} \times \mathrm{B}\,b =) \frac{a^2 \dot{x}}{\sqrt{2ax + x^2}}$. Cumque

B F = ſpatio $\frac{ABKRLA}{AC}$, & A C detur, erit fluxio ipſius B F = fluxioni ſpatii $\frac{ABKRLA}{AC} = \frac{a\dot{x}}{\sqrt{2ax+x^2}}$. Sed in præcedentis *Prop.* conſtructione, fluxio ordinatæ B F = $\frac{a\dot{x}}{\sqrt{2ax+x^2}}$. Quare hæc conſtructio eodem redit cum conſtructione *Prop.* præcedentis, & conſequenter punctum F eſt ad Catenariam. *Q. E. D.*

COROL.] Sicut in *Prop.* præced. Catenaria deſcribitur ex data longitudine Curvæ Parabolicæ, ita in hac, illius deſcriptio pendet à quadratura ſpatii in quo $x^2y^2 = a^4 - 2axy^2$. Nam y (ſive B K) = $\frac{a^2}{\sqrt{2ax+x^2}}$

Fig. 36. Prop. 4. Theor.] *Spatium AGF ſub Catenaria AF & Rectis FG, AG ad AB, BF parallelis comprehenſum, æquale eſt rectangulo ſub ſemi-axe AC, & DH intervallo applicatarum in Hyperbola & Catenaria.*

Nam $\dot{D}H = (\dot{B}H - \dot{B}D =$, ex *Prop.* 2. hujus, $\frac{a\dot{x} + x\dot{x}}{\sqrt{2ax+x^2}} - \frac{a\dot{x}}{\sqrt{2ax+x^2}} =) \frac{x\dot{x}}{\sqrt{2ax+x^2}}$. Quare fluxio rectanguli ſub data A C & H D = $(\frac{ax\dot{x}}{\sqrt{2ax+x^2}} = x \times \frac{a\dot{x}}{\sqrt{2ax+x^2}} =$ *fs* × F G =) fluxioni ſpatii A G F. Cumque figuræ hæ ſimul naſcantur, ſequitur rectangulum ſub A C & D H æquari ſpatio A G F. *Q. E. D.*

COROL.] Hinc ſequitur, ſpatium F A D, ſub Catena F A D, & recta Horizontali F D comprehenſum, æquari rectangulo ſub F D & B A, dempto rectangulo ſub Hyperbolæ A H axe alterutro, & D H exceſſu rectæ B H, vel Curvæ A D, ſupra ordinatam B D.

Fig. 36. Prop. 5. Theor.] *Si ad rectam A L, applicetur Rectangulum L E æquale ſpatio Hyperbolico A L H, erit E centrum Æquilibrii Curvæ Catenariæ A F D.*

Concipiatur curva gravis F A librari ſuper axe G L. Ex Centro barycis conſtat momentum gravis F A exponi per ſuperficiem Cylindrici recti ſuper F A erecti, & reſecti plano per G L tranſeunte, cum plano Curvæ angulum ſemi-rectum faciente. Et hujus ſuperficiei fluxio, ſive $\dot{F}A \times FG$, æqualis eſt fluxioni ſpatii A L H ſive $\dot{B}H \times HL$; quia $\dot{F}A$, $\dot{B}H$, item F G & H L æquantur. Ac propterea (cum ſimul naſcantur) dicta ſuperficies Cylindrici recti æqualis eſt ſpatio Hyperbolico A L H. Hoc proinde applicatum ad ipſum grave A F, vel illi æqualem rectam A L, facit latitudinem A E æqualem diſtantiæ centri gravitatis ab axe librationis G L. Unde Curvæ F A D, æqualiter ad utramque Axeos A B partem jacentis, centrum æquilibrii eſt E. *Q. E. D.*

COROL. 1.] Spatia A B H L, B A H, & A G F ſunt Arithmetice proportionalia. Nam fluxio ſpatii A L H $= \left(\frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+x^2}} \times x = \frac{\overline{ax+x^2} \times \dot{x}}{\sqrt{2ax+x^2}} = \frac{\overline{2ax+x^2-ax} \times \dot{x}}{\sqrt{2ax+x^2}} = \dot{x}\sqrt{2ax+x^2} - \frac{ax\dot{x}}{\sqrt{2ax+x^2}} =\right)$ fluxioni ſpatii B A H, multatæ Fluxione ſpatii A G F, per *Prop.* 4. hujus. Cumque hæ tres figuræ ſimul naſcantur, erit B A H − A G F = (A L H =) B L − B A H. Quare 2 B A H = B L + A G F. Unde ſequitur ſpatia B L, B A H & A G F eſſe in proportione Arithmetica.

2. Catenæ centrum gravitatis eſt omnium linearum ejuſdem Longitudinis, eoſdemque terminos habentium, infimum. Nam tantum deſcendet grave quantum poteſt. Cumque tantum deſcendat figura, quantum ejus centrum gravitatis deſcendit, ſe ſic diſponet linea gravis flexilis, ut ejus centrum gravitatis ſit inferius quam ſi aliam quamcunque figuram indueret. Atque ex hoc Symptomate lineæ gravis flexilis, reliqua omnia facile deduci poſſent.

3. Si ſuper quaſcunque Curvas eandem longitudinem eoſdem terminos D & F cum Catenaria F A D habentes, erecti Cylindrici recti ſecentur plano per D F tranſeunte; ſuperficierum Cylindricarum ſic reſectarum maxima eſt quæ ſuper Catenariam inſiſtit. Hæ enim ſuperficies (ſi angulus ſub planis fuerit ſemirectus) ad ipſas Curvas (quæ ſunt in caſu præſenti longitudinis ejuſdem) applicatæ, latitudines faciunt æquales diſtantiis centrorum gravitatis Curvarum à D F recta. Cum diſtantia hæc ſit in Catenaria maxima (ob maximum deſcenſum centri gravitatis) erit Cylindrica ſuperficies applicanda etiam maxima. Et quoniam ſuperficierum Cylindricarum reſectarum plano, cum plano baſeos angulum quemvis continente, eadem eſt ratio, atque cum dictus angulus eſt ſemirectus, patet propoſitum univerſaliter.

LEMMA.] *Si in cujuſvis Curvæ A F Q, deſcripta evolutione alterius Curvæ K V, ordinatam quamvis F B ad axem A B normalem, à correſpondente in K V puncto V demittatur normalis V R ordinatæ occurrens in R: erunt, manente fluxione axeos A B eadem, fluxio fluxionis ordinatæ B F, fluxio Curvæ A F, & recta F R, continuæ proportionales.* Fig. 36.

Producatur rectula F f, donec proximæ ordinatæ W φ occurrat in o. Et quoniam ex hypotheſi F s − f W, erit of = F f, adeoque $o\varphi$ erit fluxio ipſius fs, hoc eſt fluxio fluxionis ordinatæ. Porro triangula $o\varphi f$, f F R ſunt æquiangula, quia $o\varphi f$ = alterno f F R, & $fo\varphi$ = (F fr =) F f R, quia illorum intervallum R fr alterutrius reſpectu evaneſcit, cum R r præ fr nulla ſit. Et igitur $o\varphi : \varphi f :: f$F : F R, ſed of, fF æquales ſunt, cum fluxione utriuſvis, tantum differant. Quare $o\varphi : f$F $:: f$F : F R. *Q. E. D.*

Fig. 16. Prop. 6. Probl.] *Invenire Curvam K V cujus evolutione Catenaria A F Q deſcribitur.*

Vocetur ut prius A B, x, item B F, y. Eſt, ex *Prop.* 2. hujus, $\dot{y} = \dfrac{a\dot{x}}{\sqrt{2ax+x^{2}}}$, ſive $2ax\dot{y}^{2} + x^{2}\dot{y}^{2} = a^{2}\dot{x}^{2}$. Quare, per ſatis nunc uſurpatam *Newtoni* methodum, $2a\dot{x}\dot{y}^{2} + 4ax\dot{y}.\ddot{y} + 2x\dot{x}\dot{y}^{2} + 2x^{2}\dot{y}\ddot{y}$ (= $2a^{2}\dot{x}\ddot{x}$ quæ, propter $\ddot{x} = 0$, cum conſtans $\dot{x}$ non fluat) = 0. Quare $\ddot{y} = \left(\dfrac{-a\dot{x}\dot{y} - x\dot{x}\dot{y}}{2ax+x^{2}} =\right) \dfrac{\overline{a+x} \times a\dot{x}^{2}}{\overline{2ax+x^{2}} \times \sqrt{2ax+x^{2}}}$, ponendo loco $\dot{y}$ ejus valorem $\dfrac{a\dot{x}}{\sqrt{2ax+x^{2}}}$. (Nam ſignum − quantitati $\ddot{y}$ præfixum, tantum denotat locum puncti R ex F ſpectati, oppoſitum eſſe loco puncti F ex B ſpectati, cum Curva A F Q eſt cava verſus axem A B) & F f, per *Prop.* 2. hujus, $= \dfrac{\overline{a+x} \times \dot{x}}{\sqrt{2ax+x^{2}}}$: Quare per præcedens Lemma,

$$\text{F R} = \left(\frac{\text{F}f\,q}{\ddot{y}} = \frac{\overline{a+x}^{2} \times x^{2}}{2ax+x^{2}} \times \frac{\overline{2ax+x^{2}} \times \sqrt{2ax+x^{2}}}{\overline{a+x} \times a\dot{x}^{2}} =\right) \frac{\overline{a+x} \times \sqrt{2ax+x^{2}}}{a}.$$

Rurſus ob triangula rectangula F sf, F R V, habentia angulos f F s, V F R æquales, quia V F s eſt utriuſque complementum ad rectum, eſt F s : sf :: F R : V R, ſive $\dot{x} : \dfrac{a\dot{x}}{\sqrt{2ax+x^{2}}} :: \dfrac{\overline{a+x} \times \sqrt{2ax+x^{2}}}{a}$: V R, quæ proinde æqualis $a + x$. Hæc igitur eſt natura Curvæ K V, ut ſi A B vocetur x, erit F R $= \dfrac{\overline{a+x} \times \sqrt{2ax+x^{2}}}{a}$. & V R $= a + x$. *Q. E. D.*

COROL. 1.] AC : CB :: BH : FR. Hæc enim eſt proprietas rectæ FR ſuperius inventa.

2. Recta CB æqualis eſt rectæ BI, ſive VR. Utraque enim eſt æqualis $a+x$.

3. Recta evolvens VF, eſt tertia proportionalis ipſis AC, CB. Nam ob æquiangula triangula fFs, VFR, eſt sF : Ff :: FR : VF. Sive

$x : \frac{ax+xx}{\sqrt{2ax+x^2}} :: \frac{\overline{a+x}\times\sqrt{2ax+x^2}}{a} :$ VF, quæ proinde $= \frac{\overline{a+x}|^2}{a}$. Unde $a : a+x :: a+x :$ VF, quæ præterea eſt radius circuli Catenæ in F æquicurvi.

4. Cum punctum F eſt in A, ſive cum vertex evolutione deſcribitur, id eſt cum $x = 0$, valor evolventis rectæ VF, quæ in hoc caſu eſt KA, nempe $\frac{\overline{a+x}|^2}{a}$ fiet a: hoc eſt punctum K, ubi Curva VK occurrit axi, tantum extat ſupra Catenæ verticem A, quantum C deprimitur infra eundem. Unde diameter circuli, Catenæ ad verticem æquicurvi, æqualis eſt axi conterminæ Hyperbolæ AH. Adeoque Catenæ AD & Hyperbolæ AH eadem eſt curvatura in vertice A: Nam vulgo notum eſt circulum prædictum, Hyperbolæ æquilateræ AH in vertice A æquicurvum eſſe. Sed & hoc aliunde, ex ipſa Catenæ natura *Prop.* 2. hujus demonſtrata, conſtat. Nam naſcens FH ſive (AP = naſcenti BP =) $\sqrt{8ax}$ dupla eſt naſcentis BH ſive ($\sqrt{2ax+x^2}$, hoc eſt, evaneſcente x^2, cum x minima ſit) $\sqrt{2ax}$; Et igitur idem punctum eſt tam in naſcente Hyperbola quam naſcente Catenaria; hoc eſt naſcens Hyperbola AH cum naſcente Catenaria AD coincidit, & proinde æquicurvæ ſunt hæ lineæ ad verticem A.

5. Curva KV tertia proportionalis ad rectam AC & curvam AF ſive rectam AL. Ex natura enim evolutionis, KV = (VKA − KA = VF − KA $= \frac{\overline{a+x}|^2}{a} - a = \frac{a^2+2ax+x^2}{a} - a =$) $\frac{2ax+x^2}{a}$. Et igitur $a : \sqrt{2ax+x^2} :: \sqrt{2ax+x^2} :$ KV. Sed $\sqrt{2ax+x^2}$, ex *Corol.* 2. *Prop.* 2. = AF. Unde AC : AF :: AF : KV.

6. Recta KI dupla eſt ipſius AB. Cum enim BI = (BC =) CA + AB, erit AI = CA + 2AB; At AK = AC, per *Corol.* 4. hujus; igitur KI = 2AB.

7. Rectangulum ſub AC & BR eſt æquale duplo ſpatio hyperbolico BAH. Nam FR × AC = ($\frac{\overline{a+x}\times\sqrt{2ax+x^2}}{a}\times a = \overline{a+a}\times$

$\sqrt{2ax+x^2} = x \times \sqrt{2ax+x^2} + a \times \sqrt{2ax+x^2} = AB \times BH + AC \times BH =)$ $AB \times BH + AC \times BD + AC \times DH$. Quare $FR \times AC - BD \times AC$, hoc eſt $BR \times AC = AB \times BH + AC \times DH$. Sed per *Prop.* 4. hujus, $AC \times DH = AGF$ ſpatio. Et igitur $BR \times AC = (ABHL + AGF =$ per *Corol.* 1. *Prop.* 5.) $2\,BAH$.

Fig. 38. Prop. 7. Theor.] *Si in Curva Logarithmica LAG cujus data ſubtangens HS æqualis recta* a, Corol. 2. Prop. 2. hujus definitæ, *ſumatur punctum A cujus diſtantia ab HP aſymptoto, nempe AC, æqualis ſit ſubtangenti HS, & ex punctis H & P utcunque in aſymptoto ſumptis à puncto C æqualiter diſtantibus, erigantur HL, PG ordinatæ ad Logarithmicam, quarum ſemiſumma ponatur æqualis HD vel PF, erunt D & F ad Catenariam rectæ AC correſpondentem.*

Vocetur AB, x, adeoque CB vel DH ſemiſumma ordinatarum HL, PG erit $a+x$; ſemidifferentia earundem vocetur y. Unde $HL = a+x+y$, & $PG = a+x-y$. Cumque ex natura Logarithmicæ, CA ſit inter has media proportionalis, erit $a^2 + 2ax + x^2 - y^2 = a^2$. Unde $y = \sqrt{2ax+x^2}$. Adeoque $HL = a+x+\sqrt{2ax+x^2}$, & $PG = a+x-\sqrt{2ax+x^2}$. Quare fluxio ipſius HL, ſive ipſa lm eſt $\frac{a\dot{x} + x\dot{x} + \dot{x}\sqrt{2ax+x^2}}{\sqrt{2ax+x^2}}$. Et ob æquiangula triangula lm L, LHS, eſt $LH : HS :: lm : m$L, unde mL ſive *dſ* fluxio ipſius $BD = \frac{a\dot{x}}{\sqrt{2ax+x^2}}$. Hoc eſt Curva AD ex Logarithmica ſupradicto modo genita, ejus eſt naturæ, ut ſi axis vocetur x, ejuſque fluxio $\dot{x}$, fluxio ordinatæ BD ſit $\frac{a\dot{x}}{\sqrt{2ax+x^2}}$. Sed hæc ipſa eſt proprietas Catenariæ ad quam a pertinet, *Prop.* 1. hujus demonſtrata. Ergo Curva FAD ſuperius deſcripta eſt hæc ipſa Catenaria. *Q. E. D.*

COROL. 1.] Sicut ope Logarithmorum Catenaria deſcribitur, vice verſa ope Catenariæ per ipſam rerum naturam productæ, numeri dati vel potius rationis datæ Logarithmus invenitur. Ut ſi poſita CA unitate, cujus Logarithmus eſt nihilo æqualis, quæratur Logarithmus Numeri CQ ſive rationis inter CA & CQ; Rectis CQ & CA tertia proportionalis ſit CV, ipſarumque CQ, CV ſemiſumma CB; ex B ordinata ad Catenariam, nempe BD eſt Logarithmus quæſitus. Ratio ex propoſitione manifeſta eſt.

2. Viciſſim ſi dato Logarithmo CH vel CP, quæratur correſpondens numerus HL vel PG, ſeu ratio HL ad CA, ſive PG ad CA. Ex H vel P erigatur perpendiculum Catenæ occurrens in D vel F, ipſique HD vel PF hoc eſt CB, fiat æqualis CR ad horizontalem AR terminata; eritque

AR ſemidifferentia quæſitarum LH, GP, ſicut ex ſupra demonſtrata Catenæ natura HD vel CR eſt earundem ſemiſumma: (Nam in tribus quantitatibus Geometrice proportionalibus quales ſunt HL, CA, PG, quadratum ſemiſummæ extremarum multatum quadrato mediæ, æquatur quadrato ſemidifferentiæ extremarum) adeoque CR + AR, & CR − AR ſunt numeri HL vel GP, dato Logarithmo CH vel CP congrui.

3. Ex demonſtratione patet quod ſicut HD ſemiſumma Logarithmicæ ordinatarum HL, PG, ad CH normaliter applicata in H, eſt ordinata Catenariæ, ſic ſemidifferentia earundem HL, PG, ad CA normaliter applicata in B eſt ordinata Hyperbolæ æquilateræ centro C vertice A deſcriptæ: ac proinde, per *Corol.* 2. *Prop.* 2. hujus, æqualis Catenæ AD. Nam $y = \sqrt{2ax + x^2}$. Cumque *Corol. præced.* oſtenſum ſit, AR eſſe etiam ſemidifferentiam rectarum HL, PG, patet AR eſſe æqualem Catenariæ portioni AD. Unde obiter eluceſcit modus, data Catena AD, inveniendi C centrum Hyperbolæ conterminæ, vel punctum id aſymptoto Logarithmicæ GL. Nam ſi ſumatur AR æqualis Catenæ AD, & ex junctæ rectæ BR puncto medio erigatur ad ipſam BR normalis, hæc occurret BA Axi Catenæ in quæſito puncto C, uti patet. Nam ſic erit CR = CB.

4. Hinc etiam ſequitur ſi BDT, angulus fiat æqualis ACR, rectam DT tangere Catenariam in D. Nam ſic fiet in triangulis æquiangulis DBT, CAR; DB : BT :: CA : AR ſive huic æqualem AD curvam. Et igitur per *Corol. Prop.* 1. hujus, DT tangit Catenariam.

5. Sequitur etiam ſpatium ACHD æquari rectangulo ſub CA & AR. Nam quoniam AYD eſt, per *Prop.* 4. æquale triangulo ſub CA & (AD − BD =, per *Corol.* 3. *hujus Prop.* AR − AY =) YR, patet propoſitum. Et quoniam CA datur, conſtat ſpatium ACHD eſſe ſicut AD curva, illiuſque fluxionem H*d* ſicut D*d* fluxio hujus.

6. Si per punctum K, ubi CR ſecat HD, ducatur KZ parallela PH, rectæ AC occurrens in Z, ſumaturque CE æqualis ſemiſummæ ipſarum BC, CZ, erit E centrum Æquilibrii Curvæ FAD.

Intelligatur ſuper FAD erecta ſuperficies Cylindrici recti reſecti plano per PH ad angulos ſemirectos cum plano Curvæ FAD; exponet hæc ſuperficies momentum Curvæ FAD ſuper axe PH libratæ, ejuſque fluxio eſt

$$DH \times Dd + PF \times Ff = 2BC \times AD = 2 \times \overline{a + x} \times \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + x^2}}$$

$$= \frac{2a^2\dot{x} + 4ax\dot{x} + 2x^2\dot{x}}{\sqrt{2ax + x^2}} = \frac{a^2\dot{x}}{\sqrt{2ax + x^2}} + \frac{a^2\dot{x} + ax\dot{x}}{\sqrt{2ax + x^2}}$$

$$+ \frac{3ax\dot{x} + 2x^2\dot{x}}{\sqrt{2ax + x^2}},$$ cujus fluens $a \times BD + a\sqrt{2ax + x^2} + x\sqrt{2ax + x^2} = CA \times BD + CB \times AD$. Quare $CA \times BD + CB \times AD =$ (quoniam ſimul naſcitur, dictæ ſuperficiei Cylindricæ =) momento Curvæ FAD ſuper axe PH libratæ. Unde diſtantia centri gravi-

tatis Curvæ F A D à puncto C eſt $\frac{CA \times BD + CB \times AD}{2\,AD}$ ſive $\frac{1}{2}\frac{CA \times BD}{AD} + \frac{1}{2}CB$. Porro ob Z K parallelam A R, eſt $AD : BD :: (AR : ZK ::) \; CA : CZ$, unde $CZ = \frac{CA \times BD}{AD}$, & igitur C E quæ per conſtructionem eſt $= \frac{1}{2}BC + \frac{1}{2}CZ$, erit $= \frac{1}{2}\frac{CA \times BD}{AD} + \frac{1}{2}BC$: hoc eſt Curvæ F A D centrum gravitatis, & E punctum ex conſtructione definitum æqualiter diſtant à C; ſed & in eadem recta & verſus eaſdem partes ſita ſunt, ergo coincidunt illa.

Poteſt & coincidentia puncti E, ut ſupra determinati, cum centro æquilibrii, *Prop.* 5. hujus definito, Synthetice ſic oſtendi. Per *Corol.* 1. *Prop.* 5. $2\,BAX = AYD + BA \times AR$. Unde $AH + 2\,BAX = (ACHD + BA \times AR =$ per præced. *Corol.*) $AR \times CA + BA \times AR$: hoc eſt $BD \times AC + 2\,BAX = AR \times CB$; ſive $BD \times AC = AR \times CB - 2\,BAX$. Unde $BD \times AC + AD \times BC = (AD \times BC + AR \times CB - 2\,BAX = 2\,AD \times BC - 2\,BAX =) \; 2\,AD \times AC + 2\,AD \times AB - 2\,BAX$. Et applicando ad 2 A D, erit $\frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC = (AC + \frac{AB \times AD - BAX}{AD} =) \; CA + \frac{ARX}{AR}$. Sed $\frac{ARX}{AR}$ eſt diſtantia centri æquilibrii Catenæ à vertice A, per *Prop.* 5. hujus determinata, ac proinde, ſecundum dictam *Prop.* 5. $CA + \frac{ARX}{AR}$ eſt diſtantia puncti E à C, & $\frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC$, eſt ejuſdem E diſtantia ab eodem C, ſecundum hoc *Corol.* 6. Unde patet duas iſtas determinationes puncti E eodem recidere, quoniam $CA + \frac{ARX}{AR} = \frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC$.

7. Spatii P F A D H centrum gravitatis eſt in I, medio puncto rectæ C E. Cum centrum gravitatis fluxionis ipſius A D, ſive Dd & Ff, duplo magis diſtet à P H quam centrum gravitatis fluxionis ipſius A C H D ſive $DHhd$ & $FPpf$, & $\overline{Dd + Ff} \times AC$ datam, æquale $DdhH + FfpP$ patet & fluentis F A D centrum gravitatis E duplo magis diſtare à P H, quam fluentis P F A D H centrum I. Sed libet propoſitum aliter & ad modum ſuperiorum oſtendere.

Intelligatur ſupra figura P F A D H erectus Cylindricus rectus & reſectus plano per P H tranſeunte, cum plano baſeos angulum ſemirectum comprehendente; exponet iſtud ſolidum momentum figuræ P F A D H ſuper axe PH libratæ: hujuſque ſolidi ſive prædicti momenti fluxio (ſolida nempe erecta ſuper PF *fp* & HD *dh*) producitur, ſi momentum fluxionis, ſive fluxio momenti ipſius AD, ducatur in $\frac{1}{2}$ AC datam. Nam per *Corol.* 5. hujus *Prop.* HD *dh* = D*d* × AC: Quare ipſum momentum fluens producitur ducendo momentum Curvæ FAD reſpectu axis PH, ſuperiore *Corol.* determinatum, nempe CA × BD + CB × AD, in $\frac{1}{2}$ AC; eritque proinde $\frac{1}{2}$ AC × AC × BD + $\frac{1}{2}$ AC × CB × AD. Adeoque ſi hoc applicetur ad figuram libratam PFADH ſive 2 CA × AD, per hujus *Prop. Corol.* 5. fiet diſtantia centri gravitatis figuræ PFADH ab axe PH = $\left(\frac{1}{4}\frac{CA \times BD}{AD} + \frac{1}{4} CB =\right)$ dimidiæ rectæ CE ſuperius determinatæ.

8. Si per N punctum ubi DT tangens Catenariam in D, ſecat AR, ducatur recta parallela ipſi BC, Occurrens rectæ per E ad AR parallelæ in O; erit O centrum gravitatis Curvæ AD. Nam per *Corol.* 6. centrum gravitatis curvæ AD eſt in recta EO, ſed demonſtrabitur illud eſſe in NO recta, & proinde erit ipſum O punctum. Intelligatur DA librari circa HL axem; hujus momentum eſt curva DA, ducta in diſtantiam centri gravitatis ab HL: & ejus proinde fluxio = DA × H*h* (H*h* eſt fluxio diſtantiæ axis librationis à gravitatis centro) $= \sqrt{2ax + x^2} \times \frac{a\dot{x}}{\sqrt{2ax + x^2}} = a\dot{x}$. Ac proinde ipſum momentum Curvæ gravis DA circa axem HL libratæ $= ax$. Et igitur diſtantia centri gravitatis ab eodem axe eſt ax applicata ad AD, ſive $\frac{AC \times DY}{AR}$. Sed quia DT tangit Catenariam, per *Corol.* 4. hujus *Prop.* angulus BDT ſive DNY = ACR, & anguli ad A & Y ſunt recti, quare in triangulis æquiangulis RAC, DYN; RA : AC :: DY : YN. Unde $YN = \frac{AC \times DY}{RA}$, hoc eſt YN eſt diſtantia centri gravitatis Catenæ AD ab axe HL, ſive centrum prædictum eſt in recta NO.

9. Si ſuper I ducatur recta ad AR parallela, rectæ ON productæ occurrens in W, erit W centrum gravitatis ſpatii ACHD. Nam per *Corol.* 7. centrum gravitatis ſpatii ACHD eſt in recta IW, ſed ut mox oſtendetur, eſt in NW, & proinde eſt ipſum W punctum. Eodem enim modo, quo in *Corol.* præced. fluxio momenti ſpatii ACHD circa HL librati oſtenditur eſſe ACHD × H*h* = AC × AD × H*h* $= ax\sqrt{2ax + x^2} \times \frac{a\dot{x}}{\sqrt{2ax + xx}} = aa\dot{x}$. Ac proinde ipſum momentum ſpatii ACHD circa axem HL librati, æquale eſt fluenti cujus $a2\dot{x}$ eſt Fluxio, hoc eſt ipſi a^2x. Hoc igitur applicatum ad ipſum ſpatium ACHD ſive $a \times \sqrt{2ax + x^2}$

dat diſtantiam centri gravitatis ſpatii A C H D ab H L $= \frac{ax}{\sqrt{2ax + x^2}} = \frac{AC \times DY}{RA}$. Sed *Corol.* præced. oſtenſa eſt $YN = \frac{AC \times DY}{RA}$. Et igitur centrum gravitatis ſpatii ACHD eſt in N W. Atque ex duobus hiſce ultimis *Corol.* invenitur centrum gravitatis cujuſvis portionis Catenæ etiam ad verticem A non pertingentis: vel cujuſvis ſpatii Catenariæ portione quavis, & aliis rectis præter prædictas comprehenſi.

10. Hinc menſurantur ſuperficies & ſolida genita rotatione Catenæ aut ſpatii ſub illa & rectis comprehenſi, circa axes datos. Nam figura rotatione genita æquatur, uti vulgo notum, figuræ rotatæ ductæ in peripheriam à centro gravitatis inter rotandum percurſam, etiam datam cum detur illius radius ſive diſtantia centri gravitatis ab axe dato. Sic ſi Catena A D rotetur circa axem A B, $\frac{\pi}{\rho}$ A N eſt peripheria à centro gravitatis O percurſa, ($\frac{\pi}{\rho}$ denotat rationem peripheriæ circuli ad ſemidiametrum) adeoque ſuperficies rotatione Catenæ A D genita $= (\frac{\pi}{\rho} \times AN \times AD =) \frac{\pi}{\rho} \times AN \times AR$. Hoc eſt circulus cujus radius poteſt duplum rectangulum R A N, æquabitur ſuperficiei à Catenæ A D rotatione circa axem A B genitæ. Pari modo ſolidum genitum rotatione ſpatii ACHD circa AC, æquale oſtendetur Cylindro cujus baſis eſt prædictus circulus, altitudo vero æqualis A C. Similiterq; ſuperficies & ſolida, ex rotatione harum figurarum circa alios quoſvis datos axes facta menſurantur. Nam dato centro gravitatis hæc non latebunt.

The Animadverſions of ...... Anſwered by Dr. D. Gregory. N. 259. p. 419. Dec. An. 1699.

2. Quæ in Animadverſione ad noſtras de Catenaria Demonſtrationes objicit Anonymus, *Act. Lipſ. M. Feb. A.* 1699. ſunt hæc; Quod rem ab aliis jam ante ſeptennium inventam & publice expoſitam demonſtrare aggreſſus ſim, modo quodam meo. Ita quidem eſt; Quid vero hic redarguendum ſit non capio. Celeberrimi viri *Hugenius*, *Leibnitius*, & *Bernoullius*, plurimas Catenariæ proprietates detexerunt, & ediderunt, at non demonſtrarunt: Ego quod ſuſcepi, demonſtrationes pertexui.

Sed an res hæc (nempe Catenariæ Natura & proprietates primariæ) ab aliis inventa & publice expoſita fuit? Certe iſta Catenariæ proprietas, *Corol.* 6. *Prop.* 2. aliis indicta eſt penitus ante editas haſce demonſtrationes: Cum tamen ſit ni fallor inter primarias illius proprietates, & omnium longe utiliſſima, & ad vitæ communis uſus facillime reducenda. Ab omni ævo, in ædificiis publicis Fornices arcuſque tam ad firmitatem quam pulchritudinem adhibuerunt Architecti: Qualis tamen ſit Fornicis figura legitima aduſque editas noſtras demonſtrationes ignoratum eſt.

Primum autem quod reprehendat invenit, quod quædam *ex Mechanicis conſtare* dixerim, quæ diſtinctius enunciare atque etiam applicare operæ pretium fuiſſe ait. Ego qui Geometris demonſtranda Theoremata quædam ſuſceperam, omnia minutim exequenda non credebam.

Verum ut *Animadverſori* gratum faciam, *Lemma* iſtud (*Prop.* 1.) demonſtrabo, cum diſtinctius enuntiare nequeam, quam eſt hactenus factum in hæc verba.

Potentiæ tres in æquilibrio poſitæ eandem habent rationem cum rectis tribus ad ipſarum directiones parallelis, vel in dato Angulo inclinatis, à mutuo occurſu terminatis.

Puta ſi potentiæ tres trahentes, impellentes vel utcunque agentes, ſecundum rectas P A, P B, P C ſint in Æquilibrio; & inclinentur ad has directiones tres rectæ E F, F D, D E, in angulo quovis dato, hoc eſt ſi anguli EAP, FBP, DCP, fuerint æquales, dico potentias A, B, & C, eſſe inter ſe ut rectæ F E, F D & D E. *Fig.* 39.

Producantur rectæ A P, B P, C P in G, H & K.

In Quadrilatero F A B P, cum Angulus externus E A P ſit, ex Hypotheſi, æqualis interno & oppoſito P B F, erunt interni duo oppoſiti F A P, & F B P æquales duobus rectis; cumque omnes quatuor interni quatuor rectis æquentur, erunt reliqui duo F & A P B in eodem quadrilatero oppoſiti, duobus rectis etiam æquales. Sed A P B & B P G efficiunt duos rectos, & igitur angulus F eſt æqualis angulo B P G. Similiter oſtendentur D & B P K æquales, item E & A P K.

Quoniam tres Potentiæ ſunt in Æquilibrio, ſunt immotæ, & igitur earum quælibet pro Hypomochlio haberi poteſt reliquarum duarum reſpectu quæ in æquilibrio manent. Si B habeatur pro Hypomochlio, per *Mechanicæ notiſſimum Theorema*, Potentia A eſt ad Potentiam C, ſicut ſinus Anguli B P K, ad ſinum Anguli BPG, hoc eſt ſinus Anguli D ad ſinum Anguli F, hoc eſt recta F E ad rectam D E. Rurſus poſito C Hypomochlio, potentia A eſt ad potentiam B, ut ſinus Anguli CPH ad ſinum Anguli CPG, ſive ſinus Anguli B P K ad ſinum Anguli A P K, hoc eſt ſinus Anguli D ad ſinum Anguli E, hoc eſt ut recta F E ad rectam F D. Tres igitur Potentiæ A, B, & C, ſunt ut rectæ F E, F D, & D E. *Q. E. D.*

De Applicatione hujus *Lemmatis Mechanici* nunc agendum. Si concipiatur *Fig.* 40. (ut ſupra-dictum *Prop.* 1.) lineolæ *d* D gravitas abſoluta per *d* D expoſita, in ejus centro gravitatis M collecta, & grave hoc ſecundum directionem M F ad *d* D normalem vi gravitatis ſuæ deſcendere: Potentia ſecundum M D trahens quæ in æquilibrio eſt cum prædicto gravi, per præmiſſum *Lemma*, eſt ad ejus momentum ſive potentiam trahentem ſecundum M F, ſicut δ D ad δ *d*. Nam Angulus δ *D d*, quo D δ inclinatur ad M D, æqualis eſt angulo *d* ε F, quo *d* δ inclinatur ad M F; *viz.* uterque complementum anguli *d* ad rectum. Atque hoc etiam obtinet, agnoſcente *Animadverſore*, ſi ut in vulgari *Mechanica*, prædictum grave, plano M F incumbens, interpoſita trochlea ad M, trahatur ab alio grave ipſi M D incumbente: Erit hoc ad illud ſicut D δ ad *d* δ.

Quod ſi, reliquis manentibus, modus applicationis harum potentiarum mutetur, ita ut ad flexilis lineæ *d* D, cujus extremum *d* immotum, punctum medium M applicetur pondus ſecundum M F vires exerens, quippe arcum centro *d*, radio *d* M, in deſcenſu deſcripturum: Erit ponderis hujus vis, ad flexilem lineam rectam ad M incurvandam, infinita reſpectu vis ſuæ gravitatis abſolutæ; & vis ſecundum M D trahens ad modo deſcriptam incurvationem impediendam requiſita, etiam infinita reſpectu ejus quæ prius requirebatur ad pondus M in plano M F ſuſtinendum. Adeo ut Potentiæ quæ in priore applicationis modo exponebantur per $d\delta$, δ D, nunc exponendæ veniant per infinite majores prioribus proportionales: Nam ut prius pondus M trahit ſecundum directionem M F, & Potentia illud ſuſtinens ſecundum M D; & hæc duo eſſe in æquilibrio ex partium Catenæ quiete conſtat. Eadem igitur manebit harum ratio quæ prius fuerat. Sed cauſa quæ lineam flexilem d D (cujus extremum d immotum, cujuſque medio puncto M applicatur grave infinite quidem parvum, ſed cujus vires per hunc applicationis modum infinite majores redduntur, & proinde in *Animadverſoris* Phraſi aſſignabiles fiunt) in rectam extendit, eſt Catenæ D A gravitas quæ eſt ipſius longitudini proportionalis. Hæc ergo eſt ad conſtantem & aſſignabilem a (conſtanti ſed inaſſignabili $d\delta$ proportionalem) ut D δ, ad δd. Atque ſic *Animadverſori* patere credo veram concluſionem abſque aſſumptis erroneis fuiſſe probatam.

The Quadrature of Figures Geometrically Irrational, by Mr. J. Craig. N. 2.2. p. 708. Sept. An. 1697. Fig. 41.

XIV. Sit A C F Semicirculus, cujus Diameter eſt A F, A D E Curva Geometrice irrationalis, cujus ordinatim applicata B D, ſecat Semicirculum in C. Quantitates vero ſic deſignentur; Diameter A F $= 2a$, abſciſſa A B $= y$, Arcus A C $= v$, ordinata B D $= z$: ſitque $z = rvy$ æquatio generalis exprimens naturas Curvarum Geometrice irrationalium A D E, in qua r denotat quantitatem quamlibet datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y. Dico Aream

$$\text{ABD} = \frac{rvy^{n+1}}{n+1} - qv + v\sqrt{2ry - yy} \times \frac{ra}{\overline{n+1}|}y^{n} + \frac{2nra^{2} + ra^{2}}{n \times \overline{n+1}|^{2}}y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2}y^{n-3}$$

$$+ \frac{aC \times \overline{2n-5}}{n-3}y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4}y^{n-5} + \frac{aE \times \overline{2n-9}}{n-5}y^{-6}$$

$+$ &c.

De hac Serie Infinita hæc ſunt notanda: (1.) Quod Literæ majuſculæ A, B, C, D, E, &c. deſignent coefficientes Terminorum ipſis immediate præcedentium, ſcil. $\text{A} = \frac{2nra^{2} + ra^{2}}{n \times \overline{n+1} \times \overline{n+1}}$, $\text{B} = \frac{a\text{A} \times \overline{2n-1}}{n-1}$, $\text{C} = \frac{a\text{B} \times \overline{2n-3}}{n-2}$, & ſic porro. (2.) Quod ſi exponens n ſit numerus integer & poſitivus, aut nihilo æqualis, vel etiam ſi $2n$ ſit numerus impar, tum Quadratura ſpatii ABD exhibeatur per Quantitatem finitam; ſerie in his caſibus abrumpente. (3.) Quod q deſignet Terminum ultimo abrumpentem. (4.) Quod omnes illæ Figuræ in quibus Series abrumpitur, habeant unam portionem Geometrice Quadrabilem, ex ipſa Serie facillime aſſignabilem: Nimirum ſi capiatur abſciſſa $y = \overline{r}|^{\frac{1}{n+1}} \times \overline{nq+q}|^{\frac{1}{n+1}}$; erit huic abſciſſæ competens Area Geometrice Quadrabilis. (5.) Quod ſolus Terminus Irrationalis $\sqrt{2ay-yy}$ in Terminos ipſum ſequentes ſit multiplicandus.

Exemplum 1.] Sit $z=v$, quia in hoc caſu $r=1$, $n=0$, ideo $\frac{ra}{\overline{n+1}|^{2}}y^{n}$ eſt Terminus ultimo abrumpens, quare $q=a$, unde $ABD=vy-av+a\sqrt{2ay-y^{2}}$: & proinde ſi (per *Not.* 4.) capiatur Abſciſſa $y=a$, id eſt, ſi ordinata tranſeat per Circuli Centrum, erit portio huic competens Geometrice Quadrabilis, ſcil. Area $=aa$, id eſt Radii, Quadrato.

Exemplum 2.] Sit $z=\frac{vy}{a}$, quia in hoc caſu $r=\frac{1}{a}$, $n=1$, ideo $\frac{anraa+raa}{n\times\overline{n+1}|^{2}}y^{n-1}$ eſt terminus ultimo abrumpens, quare $q=\frac{3a}{4}$; unde $ABD=\frac{vy^{2}}{2a}-\frac{3av}{4}+\frac{\overline{y+3a}}{4}\sqrt{2ay-y^{2}}$, & proinde ſi (per *Not.* 4.) capiatur $y=\sqrt{\frac{3aa}{2}}$, erit huic abſciſſæ competens Area Geometrice Quadrabilis, ſcil. Area $=\sqrt{:\sqrt{6a^{4}}-\frac{3a^{2}}{2}}\times\sqrt{\frac{3a^{2}}{32}}+\frac{3a}{4}$.

Exemplum 3.] Sit $z=\frac{vy^{2}}{aa}$; in hoc caſu $r=\frac{1}{aa}$, $n=2$, ideo $\frac{aA\times\overline{2n-1}}{n-1}y^{n-2}$ eſt Terminus ultimo abrumpens, ergo $q=\frac{5a}{6}$; unde per ſeriem infinitam erit $ABD=\frac{6vy^{3}-15a^{2}v+\overline{2ay^{2}+5a^{2}y+15a^{3}}}{18a}\sqrt{2ay-yy}$, & proinde ſi (per *Not.* 4.) capiatur $y=\sqrt[3]{\frac{5a^{3}}{2}}$ erit abſciſſæ competens Area Geometrice Quadrabilis: ſcil. Area $\frac{2ay^{2}+5a^{2}y+15a^{2}}{18a}\times\sqrt{2ay-yy}$.

Secundo.] Sit ACF Parabola, cujus Axis AE, vertex A, & latus rectum BA. Sitque ADG Curva Geometrice irrationalis, cujus ordinatim applicata BD ſecat Parabolam in C. Et vocetur abſciſſa AB $=y$, Ordinata BD $=z$, Arcus Parabolicus AC $=v$. Sitque æquatio generalis, exprimens Naturas Infinitarum Curvarum irrationalium, hæc, $Z=rvy^{n}$, in qua r denotat Quantitatem datam & determinatam, & n exponentem indefinitum Quantitatis indeterminatæ y. Dico Aream Fig. 42.

ABD

$$\mathrm{ABD} = \frac{ry^{\overline{n+1}} \times v}{n+1} - qv + \sqrt{2ay+yy} - \frac{r}{\overline{n+2} \times \overline{n+1}} y^{n+1}$$

$$- \frac{ra}{\overline{n+2} \times \overline{n+1}|^2} y^{n} + \frac{raa \times \overline{2n+1}}{n \times \overline{n+2} \times \overline{n+1}|^2} y^{n-1} -$$

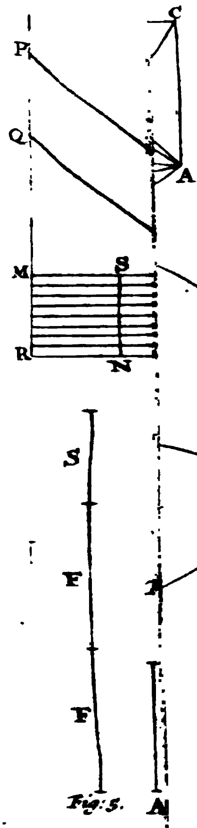
$$\frac{a\mathrm{A} \times \overline{2n-1}}{n-1} y^{n-2} + \frac{a\mathrm{B} \times \overline{2n-3}}{n-2} y^{n-3} - \frac{a\mathrm{C} \times \overline{2n-5}}{n-3} y^{n-4}$$

$+$ &c.

De hac Serie hæc ſunt notanda: (1.) Quod Literæ majuſculæ, A, B, C, &c. denotent coefficientes terminorum ipſis præcedentium. (2) Quod ſi exponens n ſit integer poſitivus aut nihilo æqualis, aut etiamſi $2n$ ſit numeris impar, tum Quadratura exhibeatur per numerum terminorum finitum; Serie in his caſibus abrumpente. (3.) Quod $+q$ ſit æqualis ultimo termino abrumpenti. (4.) Quod ex terminis Quantitatem $\sqrt{2ay+yy}$ multiplicantibus ultimo abrumpens ſit duplicandus. (5.) Quod omnes illæ Figuræ, in quibus n eſt numerus integer poſitivus & impar; vel generaliùs, omnes illæ figuræ, in quibus ultimus terminus abrumpens habet ſignum affirmativum ſeu $+$, habeant unam portionem Geometricè Quadrabilem, & ex ipſa ſerie facile aſſignabilem, ſumendo abſciſſam, ut in *Not.* 4. præcedentis ſeriei.

Exemplum 1.] Sit $z = v$, quia in hoc caſu $r = 1$, $n = 0$, ideo terminus ultimo abrumpens eſt $- \frac{ra}{\overline{n+2} \times \overline{n+1}|^2} y^{n}$, unde $+q = -\frac{a}{2}$ (per *Not.* 3.) & quia in hoc caſu $-\frac{a}{2}$ eſt terminus ultimo abrumpens, ideo $-a$ eſt ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus, (per *Not.* 4.) Adeoque $\mathrm{ABD} = vy + \frac{av}{2} + \sqrt{2ay+y^2} \times \overline{-\frac{1}{2}y - a}$.

Exemplum 2.] Sit $x = \frac{vy}{a}$ quia in hoc caſu $r = \frac{1}{a}$, $n = 1$, ideo terminus ultimo abrumpens eſt $\frac{raa \times \overline{2n+1}}{n \times \overline{n+2} \times \overline{n+1}|^2} y^{n-1} = \frac{a}{4}$; unde $q = \frac{a}{4}$, & $\frac{a}{2}$ ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus; adeoque $\mathrm{ABD} = \frac{vyy}{2a} - \frac{av}{4} + \sqrt{2ay+y^2} \times -\frac{y^2}{6a} - \frac{y}{12} + \frac{a}{2}$; & ſi capiatur



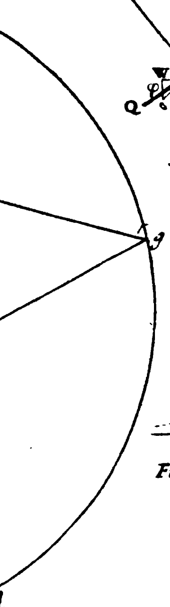


Fig. 5.

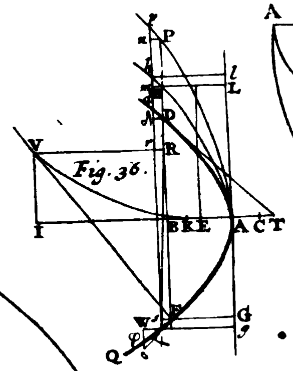


Fig. 36.

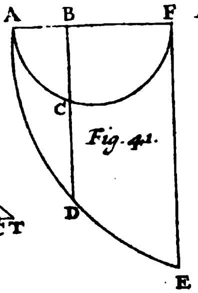


Fig. 41.

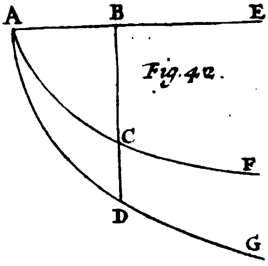


Fig. 42.

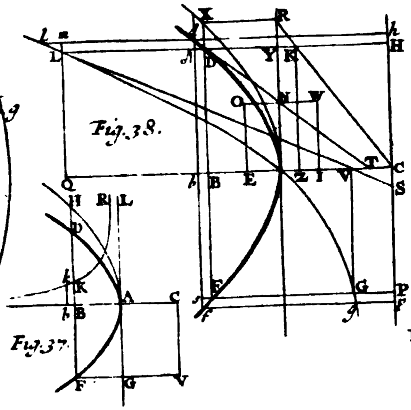


Fig. 38.

Fig. 37.

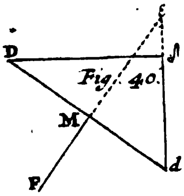


Fig. 40.

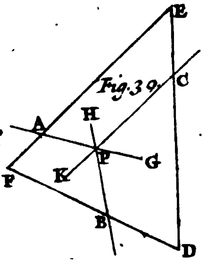


Fig. 39.

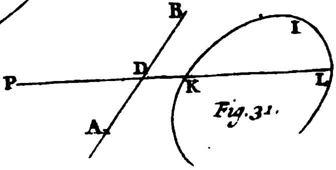


Fig. 31.

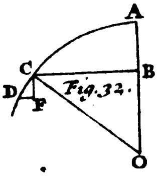


Fig. 32.

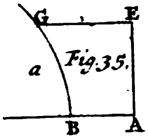


Fig. 35.

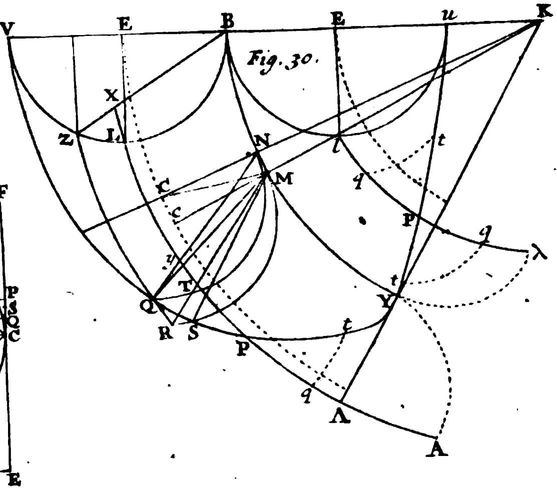


Fig. 30.

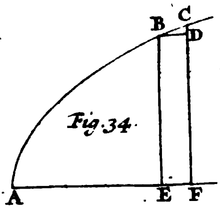


Fig. 34.

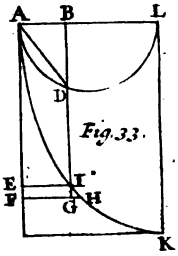


Fig. 33.

piatur $y = \sqrt{\frac{aa}{2}}$, erit Area competens hinc Abſciſſæ Geometrice Quadrabilis, ſcil. Area $= \frac{1}{12}\sqrt{:\sqrt{2a^4}+\frac{a^2}{2}} \times 5a - \sqrt{\frac{a^2}{2}}$.

Tertio.] Sit ACF ſemicirculus, ADE Curva Geometrice irrationalis, cujus ordinatim applicata BD ſecat ſemicirculum in C. Quantitates vero deſignentur ut prius, ſcil. Diameter $AF = 2a$. Abſciſſa $AB = y$, Arcus $AC = v$, Ordinata $BD = z$; ſitque $z = rv^2y^n$ Æquatio exprimens Naturas Curvarum ADE, in qua r denotat quantitatem quamlibet datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y. Dico Aream N. 235. p. 784. Fig. 41.

$$ABD = rv^2y^{n+1} - qv^2 + v\sqrt{2ay-y^2} \times \frac{2ra}{\overline{n+1}|^2}y^n + \frac{2ra^2 \times \overline{2n+1}}{n \times \overline{n+1}|^2}y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2}y^{n-3}$$

$$+ \frac{aC \times \overline{2n-5}}{n-3}y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4}y^{n-5} + \frac{aE \times \overline{2n-9}}{n-5}y^{n-6}$$

$$\&c. - \frac{2ra^2}{\overline{n+1}|^3}y^{n+1} - \frac{2ra^3 \times \overline{2n+1}}{n^2 \times \overline{n+1}|^2}y^n - \frac{a^2A \times \overline{2n-1}}{\overline{n-1}|^2}y^{n-1}$$

$$- \frac{a^2B \times \overline{2n-3}}{\overline{n-2}|^2}y^{n-2} - \frac{a^2C \times \overline{2n-5}}{\overline{n-1}|^2}y^{n-3}, \&c.$$

De hoc Theoremate hæc ſunt notanda; (1.) Quod componatur ex duabus ſeriebus infinitis, quarum prior (ſigno + connexa) multiplicatur in $v\sqrt{2ay-y^2}$; termini autem poſterioris (ſigno − affecti) ſunt abſoluti. (2.) Quod in priori ſerie literæ majuſculæ, A, B, C, D, E, &c. deſignent coefficientes terminorum ipſis reſpective præcedentium; nec non in poſteriori eoſdem obtineant valores, quos in priori. (3.) Quod Quadratura exhibeatur per quantitatem finitam, quando n eſt numerus integer poſitivus, aut nihilo æqualis; vel etiam ſi $2n$ ſit numerus impar: nam in his caſibus utraque Series abrumpitur. (4.) Quod $2q$ ſit æqualis ultimo termino abrumpenti prioris Seriei.

Exemplum 1.) Sit $z = \frac{v^2}{a}$. Quia in hoc caſu $n = 0$, $r = \frac{1}{a}$, ideo erit Area $ABD = \frac{yv^2}{a} - v^2 + 2v\sqrt{2ay-y^2} - 2ay$. *Corol.* Integra figura AFE eſt æqualis duplo Quadrato, cujus Latus eſt ACF, dempto diametri Quadrato.

Exemplum 2.] Sit $z = \frac{yv^2}{a^2}$, quia hoc in caſu $n = 1$, $r = \frac{1}{a}$, ideo erit Area ABD $= \frac{y^2v^2}{2a^2} - \frac{3}{4}v^2 + v\sqrt{2ay - y^2} \times \overline{\frac{v}{2a} + \frac{3}{2}} - \frac{1}{4}y^2 - \frac{3ay}{2}$.

Exempl. 3.] Sit $z = \frac{y^2v^2}{a^3}$, quoniam in hoc caſu $n = 2$, $r = \frac{1}{a^3}$, ideo erit Area ABD $= \frac{y^3v^2}{3a^3} - \frac{5}{6}v^2 + v\sqrt{2ay - y^2} \times \overline{\frac{2y^2}{9aa} + \frac{5y}{9a} + \frac{5}{3}} - \frac{2y^3}{27a} - \frac{5y^2}{18} - \frac{5ay}{3}$.

The Quadrature of the Logarithmick Curve, by Mr. J. Craig. N. 245. *p.* 73. Oct. An. 1698. *Fig.* 43.

XV. Eſto ONF Curva Logarithmica, cujus Aſymptotos AR, in qua tale ſumatur punctum A, ut ejus prima ordinata AO ſit ſubtangenti ſeu unitati æqualis: Quæritur ſpatium Curvilineum AONM à duabus ordinatis AO, MN, Abſciſſa AM, & Curva Logarithmica ON comprehenſum.

Ex O ducatur OE ad AM parallela, & ſecans MN in E; dico quod rectangulum ex ſegmentis ME, EN ſit æquale ſpatio quæſito.

Demonſtratio.] Vocetur Ordinata MN, z; ſubtangens AO ſeu ME, s: & ad axem AR conſtruatur alia Curva HGE, cujus æquatio $2sz = x^2$, ubi ejus ordinata GM $= x$; dico quod ſit quadratrix Logarithmicæ juxta Methodi meæ fundamentum; ſcil. ejus ſubnormalis eſt reſpectivæ hujus Ordinatæ æqualis: ut ex calculo iſtius Methodi patebit: Ergo (juxta alibi à me expoſita) ſi ad G ducatur GC perpendicularis & æqualis lineæ GM, nec non HD parallela ad GC, & lineis GM, CM occurrens in B & D; erit trapezium GBDC = AONM. Sed GBDC = GMC − BMD = $\frac{1}{2}x^2 - \frac{1}{2}BMq = SZ - \frac{1}{2}HAq$; ſed HA $= \sqrt{2AOq}$ ex natura Curvæ HGQ, ergo GBDC $= SZ - AOq = AO \times \overline{MN - AO}q = AO \times \overline{MN - AO} = ME \times \overline{MN - ME} = ME \times EN$; Ergo etiam AONM $= ME \times EN$. *Q. E. D.*

A Quadratrix to the Circle; being the Curve deſcrib'd by its Equable Evolution, by .... N. 260. *p.* 445. Jan. An. 1700. *Fig.* 44.

XVI. 1. By the Equable Evolution of a Circle, I mean ſuch a gradual approach of its Periphery to Rectitude, as that all its Parts do together, and equally, evolve or unbend; or ſo that the ſame Line becomes ſucceſſively a leſs and leſs Arc of a reciprocally greater Circle.

2. Let AHKA be the Periphery of a Circle, AE a Tangent to the Point A. Let this Circular Line be ſuppos'd, cut or divided at A, and then to unbend (like a Spring), its upper end remaining fixt to its Tangent AE, whilſt the

other parts do equally evolve or extend themſelves thro' all the degrees of leſs Curvature (as in ABD, AMC, &c.) till they become ſtreight in coincidence with the Tangent AE.

3. Let AMC be the Evolving Curve in any middle poſition between its firſt and laſt. Join the fixt end A, and the moving end C, by the Chord-line AC, interſecting the firſt Circle at H. I ſay, that AMC is a like Segment to A*n*H, cut off in the firſt Circle by the Chord AH. For, by the Suppoſition, AMC is the Arc of a Circle, having AE a Tangent common both to it and A*n*H, and both Arcs are terminated in the ſame Right-Line AC.

4. Hence the Curve ADCE (deſcrib'd by the moving end of the Periphery in its Evolution) may be thus conſtructed. Let the Circle AHKA be by Biſections divided into any number of equal Parts. Let H be one of the Points of ſuch diviſion. Then ſay, as the Number of equal Parts in the Arc A*n*H, is to the Number of Parts in the whole Periphery AHKA; ſo is the Chord AH, to a fourth Line, which let be AC in AH produc'd. So is C a point in the Curve ADCE.

5. *Dem.* Upon AC deſcribe AMC, an Arc like to the Arc A*n*H. Whence AH : AC : : A*n*H : AMC. But by conſtruction, AH : AC : : A*n*H : Periph. AHKA, therefore is the Arc AMC equal to the whole Periphery AHKA, and like to the Arc A*n*H. Conſequently AMC repreſents the Evolving Periphery, in a Poſition like to the Arc A*n*H, and C is the deſcribing Point.

6. After the ſame manner may be found other Points, thro' which the Curve may be drawn. But here (as in the old *Quadratrix* of *Dinoſtratus*) the Point E cannot be preciſely determined, but the Curve may be brought ſo near it, that its flexure or tendency will ſo lead to the Point E, that AE ſhall be near enough to the truth for common Uſes.

7. Suppoſing the Point E found, a Tangent to any point of the Curve may be drawn: and ſuppoſing a Tangent drawn, the point E may be determined; the property of the Tangent being this, that ſuppoſing RT a Tangent to the Point C, and CA, CE, drawn from C to each end of the rectify'd Circle, the Angle ACT (the leſſer Angle that AC makes with the Tangent) is equal to ACE the Angle made by the two Lines drawn from C.

8. *Let c be* a point in the Quadratrix indefinitely near to C; and draw A*c* interſecting AHKA in *h*, and AMC in *o*. To A*c* as a Chord, draw the Arc A*mc*, like unto the Arc A*nh*. To the point C of the Arc AMC draw the Tangent CL=AE, and joyn LA: ſo is *o*C an indefinitely little particle of the Arc coincident with its Tangent.

9. Becauſe of the like Segments A*nh*A, AM*o*A, A*mc*A, as Chord A*c*, to Chord A*o*; ſo is Arc A*mc* (=AMC), to Arc AM*o*. Or, A*c* : A*o* : : A*mc* (=AMC) : AM*o*. And dividing, A*c*−A*o* (=*co*) : A*o* : : A*mc*−AM*o* (=C*o*) : AM*o*: That is, *co* : A*o* : : C*o* : AM*o*, and alternately, *co* : C*o* : : A*o* : AM*o*. Put AC for A*o*, and AMC for AM*o* as differing infinitely little) and then 'tis *co* : C*o* : : AC : AMC. But by *conſtruction* CL=AE=AMC, whence *co* : C*o* : :

A C : C L, and the Angle L C A = C *o c*, (*o c* being infinitely near to A C, is therefore parallel to it.) And therefore C *o c*, A C L, are like Triangles.

10. Becauſe of C L = A E, Angle E A C = L C A, (C L and E A being Tangents to the two ends of the ſame Circular Arch A M C, make equal Angles with its Chord A C) and A C common to both, the Triangles EAC, and ACL, are like and equal: therefore are all three C *o c*, ACL, EAC, like Triangles. Whence it follows, That the Angle A C E (in the Triangle EAC) is equal to the Angle *o c* C in the Triangle *c o* C) but *o c* C = A C T becauſe *o c* and A C are parallel; therefore the Angle A C E = A C T. *Q. E. D.*

The Dimenſions of a Sphere and Cylinder compar'd, by D. Wallis N. 263. *p.* 547. Mar. An. 1700.

XVII. In Epiſtola quadam mea (*Oper. Mathem.* Vol. III.) inter alias meas Methodos (quibus in Tetragoniſmis utor) occurrunt hæ duæ; quarum alteram appello Methodum *Convolutionis* & *Evolutionis*; alteram, Methodum *Complicationis* & *Explicationis*. Quarum ope oſtendo (tum aliarum Figurarum, tum ſpeciatim) Cycloidis dimetiendæ quis ſit modus omnium ſimpliciſſimus.

Simili Artificio colligetur, tota Sphæræ cum Cylindro collatio: Quod ſibi Monumentum fecit Archimedes.

Fig. 45. Quippe ſi ad Baſin P (Peripheriæ Circuli æqualem) ſumatur altitudo R (æqualis Radio) fiet Parallelogrammum Rectangulum = R P. Quod ex minutis Parallelogrammis æque altis, numero infinitis, (juxta receptam Methodum Indiviſibilium) conflatum intelligatur. Quorum ſi omnes vertices intel-

Fig. 46. ligantur in unicum punctum contrahi, quo ex illis minutis Parallelogrammis totidem fiant Triangula ſuper eiſdem Baſibus æque alta; ſingula ſingulorum, adeoque omnia omnium, dimidia; (curvata Baſi in Circuli Peripheriam) fiet Circulus (Centro C, Radio R,) Parallelogrammi dimidius = $\frac{1}{2}$ R P.

Quæ eſt ipſa Archimedis Dimenſio Circuli: æqualis utique Triangulo Rectangulo, cujus laterum (circa Angulum Rectum) æquatur alterum Peripheriæ, alterum Radio expoſiti Circuli. Quippe $\frac{1}{2}$ R (ſemi-altitudo Trianguli) in P (Baſin) ducta, exhibet Magnitudinem iſtius Trianguli = $\frac{1}{2}$ R P, circulo æqualem. Idemque accommodabitur Sectori Circulari, ſumpto arcu A pro P Peripheria.

Fig. 47. Porro; ſi ad illud Parallelogrammum = R P (ut Baſim) ſumatur itidem (in ordine ad Hemiſphærium) Altitudo R; fiet Parallelepipedum = R R P. Quod pariter, ex minutis Parallelepipedis æque altis, numero infinitis, conflatum intelligatur (minutis areolis iſtius Plani inſiſtentibus; quorum omnium communis altitudo ſit R; & Baſium Aggregatum = R P. Quod ſi Parallelogrammum hoc (manente magnitudine = R P) intelligatur in Curvam Superficiem Cylindricam curvari (cujus Baſis ſit P, jam in Peripheriam circuli convoluta, Altitudo R) quo minuta illa Parallelepipeda in totidem Cuneos, ſeu Priſmata baſium triangularium, (Parallelepipedorum ſingula ſingulorum, adeoque omnia omnium, ſub-dupla) redigantur; Acies ſeu Vertices habentia totidem C puncta (ſeu lineolas minutas) in Axe Cylindri conſtituta, eumque complentia, fiet Cylindrus (Parallelepipedi Dimidius) = $\frac{1}{2}$ R R P.

Ve

Vel (in ordine ad Sphæram integram) ſi ſumatur utrinque Altitudo R, (ut ſit tota Altitudo D = 2 R;) fiet convolutione pariter facta) Cylindrus (ut prius) ex Cuneis ſeu Priſmatibus numero infinitis (Vertices ſeu Acies habentibus in Axe Cylindri) $= RRP = \frac{1}{2} RP \times 2R$, æqualis facto ex $\frac{1}{2} RP$ (circulari Baſe) in altitudinem 2 R: ſeu (quod tantundem eſt) $= \frac{1}{2} R \times 2RP$, æqualis facto ex $\frac{1}{2}$ R (ſemiſſe communis Altitudinis Cuneorum) in (Baſium aggregatum) 2 R P.

Quod quidem Baſium Aggregatum eſt ipſa Cylindrica Superficies curva $= P \times 2R$) æqualis Facto ex Baſis Circularis Peripheria P in Altitudinem 2 R ducta:) ſeu $\frac{1}{2} RP \times 4$, (æqualis quatuor Circulis in Sphæra maximis:) Quibus ſi accenſeantur, oppoſitæ duæ Baſes circulares; fiet Cylindri (Sphæræ circumſcripti) tota ſuperficies, æqualis ſex Circulis maximis, $\frac{1}{2} RP \times 6 = 3RP$. Et Cylindri Magnitudo, $= RRP = \frac{1}{2} RP \times 2R$, æqualis Facto ex Baſe Circulari $\frac{1}{2}$ R P in altitudinem 2 R ducta: ut prius.

Quod ſi porro, Cuneorum horum omnium Vertices (Cylindri Axem Complentes) intelligantur in unum punctum contrahi: quo Cunei illi, ſeu Priſmata, jam fiant totidem Pyramides, ſuper iiſdem Baſibus æque altæ; ſingulæ ſingularum, adeoque omnes omnium, ſubſeſqui-tertiæ, ſeu ut $\frac{1}{3}$ ad $\frac{1}{2}$; & Superficies, prius Curva Cylindrica, jam fiat Sphærica propter ejus omnia puncta æqualiter à Centro remota; manente, quod prius erat, Baſium aggregato $= 2RP$, (quatuor Circulis Maximis æquali,) habebitur, tum tota Sphæræ Superficies $= 2RP = \frac{1}{2} RP \times 4$ (æqualis quatuor Circulis maximis; & quidem toti Curvæ Cylindricæ æqualis & partes partibus reſpective æquales, eaſdem Axis partes reſpicientibus;) tum Sphæræ magnitudo $= \frac{2}{3} RRP = \frac{1}{3} RP \times 2R$; æquales Facto ex $\frac{1}{3}$ R (triente communis Altitudinis Pyramidum omnium) in 2 R P (Baſium Aggregatum, jam factam ſuperficiem Sphæricam) ducto.

Eſt itaque Cylindri Sphæræ circumſcripti tum Superficies tum Magnitudo, ad Superficiem & Magnitudinem inſcriptæ Sphæræ, ſeſqui-altera, ſeu ut 3 ad 2. (Illic quidem, ut ſex Circuli Maximi $= 3RP$, ad quatuor Circulos maximos $= 2RP$: Hic vero ut R R P ad $\frac{2}{3}$ R R P.) quod eſt illud ipſum Archimedis Inventum celebre.

Idem paulo brevius haberetur; ſi, in Parallelepipedo illo (ſuper plana Baſe 2 R P (cum Altitudine R) ex minutis Parallelepipedis conflato; Horum omnium Vertices immediate cenſeantur in unicum C (punctum) comprimi. Quo, manente ut prius Baſium Aggregato $= 2RP$, Parallelepipeda illa in totidem Pyramides redigantur; Vertices habentes ad Sphæræ Centrum coeuntes; cujus Radius R, (communis Pyramidum omnium altitudo;) & Sphærica Superficies, Baſium omnium Aggregatum. Quippe $\frac{1}{3}$ R (triens communis Altitudinis) in 2 R P (Baſium Aggregatum) exhibet Sphæræ magnitudinem (ut prius) $\frac{2}{3}$ R R P; & Sphæræ Superficiem $= 2RP$.

Poteſtque hoc itidem Sectori Sphærico accommodari. Ducto $\frac{1}{3}$ R (triente communis Altitudinis Pyramidum inibi omnium) in portionem Sphæricæ Superficiei plano abſciſſam: Quæ eſt ad totum Superficiem Sphæricam, ut eſt Diametri (ſeu Axis) pars abſciſſa ad totum Diametrum; ut ſupra oſtenſum

Cujus quidem proceſſus totius Ratio his Principiis nititur; nempe, quod figura ex Triangulis eſt dimidia figuræ ex Parallelogrammis, ſuper eiſdem Baſibus, æque-altis: (Illam ego appello Figuram *Convolutam*; hanc *Evolutam*:) Et figura ex Pyramidibus, eſt, triens figuræ ex Parallelepipedis, ſuper eiſdem Baſibus æque-altis: (Illam ego appello Figuram *Complicatam*; hanc *Explicatam*.) Quæ poſſunt mille modis accommodari Figuris Curvilineis (tum Superficialibus tum Solidis) mirum in modum perplexis.

Improvements in England *in the Reſolution of Equations in Numbers, by Mr.* J. Collins. N. 46. *p.* 929. Apr. An. 1669.

XVIII. 1. It hath been obſerved by divers of this Nation, that in any Equation, howſoever affected, if you give a Root, and find the abſolute Number or Reſolvend, (which *Vieta* calls *Homogeneum Comparationis*;) and again give Roots and find more Reſolvends; that if theſe Roots, or rather rank of Roots, be aſſumed in Arithmetical Progreſſion, the Reſolvends, as to their firſt, ſecond, or third Differences, *&c.* imitate the Laws of the pure Powers of an Arithmetical Progreſſion of the ſame degree, that the higheſt Power, or firſt Term of the Equation is of. *E. g.* In this Equation $aaa - 3aa + 4a = N$.

If a be =	Then N or the Abſolutes or Reſolvends will be found to be	1 diff.	2 diff.	3 diff.
10	740	218		
9	522	170	48	
8	352	128	42	6
7	224	92	36	6
6	132			

To wit, the 3d differences of thoſe Abſolutes are equal, as in the Cubes of an Arithmetical Progreſſion.

2. To find what habitude thoſe differences have to the Coefficient of the Equation, 'tis beſt to begin from an Unite.

3. In any Arithmetical Progreſſion, if you multiply Numbers by Pairs, you ſhall create a rank of Numbers whoſe ſecond differences are equal; and if by Ternaries, then the 3d differences of thoſe Products ſhall be equal. And how to find the greateſt Product of an Arithmetical Progreſſion of any Number of Terms having any common difference aſſign'd, contain'd in any Number propos'd, is ſhewed by *Paſcal* in his Tract *du Triangle Arithmetique*, where he applies it to the Extraction of the Roots of Simple Powers.

4. It appears, how this Rank may be carried eaſily by Addition, till you have a Reſolvend either equal or greater or leſs than that propoſed.

5. When you have a *Majus* and *Minus*, you may interpoſe as many more Terms in the Arithmetical Progreſſion as you will, that is to ſay, Subdivide the common difference in the Arithmetical Progreſſion, and render it leſs; and then renew, and find the Reſolvends, which are eaſily obtained out of the Powers and their Coefficients, which are ſuppoſed known, and may be readily raiſed from a Table of Squares and Cubes, *&c.* with which kind the Reader may be furniſhed in *Guldini Centrobaryca*, and *Babington*'s *Fireworks*. By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more eaſily, and is ſo far improved, that after any Figure is placed in the Root, moſt certain Characters

are given to know by Aid of the Subſequent Dividend and Diviſor, whether the Figure before aſſumed be too great or too ſmall: or laſtly it may be well concluded, that, as in Logarithms, when you propoſe ſuch an one as is not abſolutely given in the Canon, you do by proportional Work, uſing the Aid of their firſt Differences (when their abſolute Numbers differ by Unite) find the abſolute Number true to 5 or 6 Places farther than the Canon gives it (the reaſon whereof is, that the firſt Differences do likewiſe agree to about the ſame number of Places;) that I ſay the like may be done in Equations, after divers of the firſt Figures of the Roots are found; provided there be the like Agreement in the firſt Differences of the Interpoled Reſolvends.

Moreover we ought here to take notice of a more ſubtile kind of Interpolation, common to all Gradual Ranks or Progreſſion of Numbers, wherein Differences happen to be equal: Of which kind the Reader may find Examples in *Briggii Arithmetica, Logarithmica & Trigonometria Britannica*, relating to Logarithms, Sines, and the Powers of an Arithmetical Progreſſion: But the Method there delivered may be rendred more eaſy and general, *viz.* by Aid of a Table of Figurate Numbers, by deriving Generating Differences ſought, from thoſe given; a Doctrine that eaſily flows from *Mercator*'s *Logarithmotechnia*, and of uſe in the Caſe in hand, ſhould we ſuppoſe theſe Powers and their Coefficients unknown, or a Table of Squares and Cubes wanting, and give nothing more than a few Reſolvends belonging to equal Moments or Spaces. And this may likewiſe be of good Uſe in *Gauging*, when having the Contents of a Solid, for every three Inches more or leſs given, without knowing the Dimenſions of the Figure, and even in moſt Caſes, when the Differences are Progreſſive of one kind, without knowing the Figure it ſelf, having nothing given but its Contents at ſeveral equal parallel Diſtances, each ſuch diſtance may be ſubdivided, and made as many as you pleaſe, and the reſpective Contents found by this general Method of Interpolation.

After one Root is obtained, the Methods of *Huddenius* and others will depreſs the Equations ſo as to obtain more, and conſequently all of them.

6. It is eaſy by a Table of Figurate Numbers to give the Sum of any ſuch Rank, or any Term in it relating to a known part of the Series of Equals or Roots; but *è converſo*, giving the Reſolvend to find the Root, comes to an Equation as difficult as that propos'd; as in Dr. *Wallis*'s Chapter of *Figurate Numbers*.

7. Some affirm, they can give good Approaches for the obtaining a Root of any pure Power, affected Equation, or for the finding any of the mean Proportionals in any Rank between the two Extreams given.

8. Others pretend to have found out the Method (incited thereto by an Example in *Albert Gerard*'s *Invention Nouvelle en Algebre, a Amſterdam.* 1629) ſo much, by comparing of Equations, to increaſe or diminiſh the unknown Root of Equation as to render it a whole Number (or leſs differing therefrom than any Error aſſign'd,) and by *Albert Gerard*'s Method of Aliquot Parts to find the ſame, and thereby the Root ſought, altho' it be a mixt Number, Fraction, or Surd.

Probably this may Sympathize with what is promiſed by the Learned *Huddenius in Annexis Geometriæ Carteſianæ*, where he ſaith he intended not then to publiſh certain Rules he had ready; whereof one was to find out all the Irrational Roots both of Literal and Numeral Equations. This muſt be underſtood, when ſuch Roots are poſſible; for 'tis certain there are Infinite Equations, whoſe Roots are no ways explicable, either in whole or mixt Numbers, Fractions, or Surds, and can be no otherwiſe explain'd, but by a *quamproxime*.

Ibid. p. 932. 9. The *Author of this Narrative* conſidering, that the Conick Sections may be projected from leſſer Circles placed on the Sphere, and thence eaſily (otherwiſe than hitherto hath been handled) deſcribed by Points, and that by their Interſections, ſome Spherick Problem is determined; accordingly he found that this following Problem, according to the various Situation of the Eye and of the projecting Plan, would take in all Caſes.

The Diſtances of an unknown Star are given from two Stars of known Declination and Right Aſcenſion; the Declination and Right Aſcenſion of the unknown Star is required.

And ſaith, He hath obſerved, that, admitting the Mechaniſm of dividing the Periphery of a Circle into any Number of equal Parts, or (which is equivalent, the uſe of a Line of Chords, that this Problem, wherever the Eye be placed, may be reſolved by plain *Geometry*, and yet the Eye ſhall be ſo placed, as to determine it by the Interſection of the Conic Sections; conſequently thoſe Points of Interſection (the Species and Poſition of the Figures being given) may be found without deſcribing any more Points than thoſe ſought; and the Lenghts of Ordinates falling from thence on the Axes of either Figure Calculated by mixt *Trigonometry*, and hence likewiſe the Roots of all *Cubick* and *Biquadratick Equations* found by *Trigonometry*.

For giving, from the *Meſolabe* of *Sluſius*, the Scheme that finds theſe Roots, it will then be required to fit thoſe Sections into Cones, which have their Vertex either in the Center, or an aſſigned Point in the Surface of the Sphere, to which they relate as Projected, and proceed to the Reſolution of the Problem propoſed: And how to fit in thoſe Sections, ſee the 7 *Books* of *Apollonius*, *Mydorgius*, the 3d Volume of *des Cartes*'s Letters; *Leotandi Geometria Practica*, *Anderſonii Exercitat. Geometricæ*.

As to the Problem it ſelf, it is determined on the Sphere by the Interſections of the two leſſer Circles of Diſtance, whoſe Poles are the known Stars. And this Problem hath divers Geometrick ways of Reſolution.

1. By plain Geometry (in the Senſe before mentioned; ſuppoſing a Plain to touch the Sphere or the North Pole: if the Eye be at the South Pole, projecting thoſe Circles into the ſaid Plain, they are ſtill Circles (by reaſon of the ſub-contrary Sections of the viſual Cones) whoſe Centers fall in the ſides of the Right-lined Angle, made by the Projected Meridians, that paſs thro' the known Stars; and thus the Problem is eaſily ſolved in this manner.

2. If it be required to be performed by Conick Geometry; in one Caſe it may be done, by placing the Eye at the Center of the Sphere, and projecting

as before; to wit, when the longer Axes of the Figures being produced, concur above the Vertex: Here the Problem is determined by the Interſections of two Conick Sections (whereof a Circle cannot be one, unleſs its Center be in the Axis of the other Figure:) And in this ſecond Caſe, theſe points of Interſection fall in the ſame Right-line or projected Meridian, they did before, but at a more remote diſtance from the Pole Point, to wit, in the former Suppoſition, the Polar diſtance was meaſured by a Right Line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident, how one Projection may beget another, yea, infinite others, altering the Scale: and how the leſſer Circles in the *Stereographick* Projection help to deſcribe the Conick Sections in the *Gnomonick* Projection: But (to reduce the matter to the common Radius) if we ſuppoſe two Spheres equal, and ſo placed about the ſame Axis, that the Pole point of the one ſhall paſs thro' the Center of the other, and the Touch-Plain to paſs thro' the ſaid Center or Pole-Point; and that a leſſer Circle hath the ſame Poſition in the one as in the other; then, if the Eye be at the South Pole of the one, it is at the Center of the other; and any Projected Meridian drawn from the Projected Pole point to paſs thro' both the Projections of theſe leſſer Circles, the diſtances of the Points of Interſection are the Tangents of the half and the whole Arch of the Meridian ſo interſected. But as to the Points of Interſection, which determine the Problem propos'd, they may be found without the aid of the former way, from a *Gnomonick* and *Stereographick* Method of meaſuring and ſetting off the Sides and Angles of Spherical Triangles in thoſe Projections, which is neceſſary in what follows.

3. If the Problem is to be performed by mixt Geometry, as by a Circle, and either a Parabola, Hyperbola, or Ellipſis, the Circle may be conceived to be the ſub-contrary Section of a Cone projected by the Eye at the South Pole, and any of the reſt of the Sections by the Eye at the Center of the Sphere.

4. If by any of the Conick Sections however poſited, the Projecting Plain may remain the ſame; but the Eye muſt be in ſome other Part of the Surface of the Sphere, and not in the Axis.

The Conſtruction of Cubick and Biquadratic Æquations, by a Parabola and a Circle. By M. Edmund Halley. N. 188. p. 335. Jul. An. 1687.

XIX. 1. Conſtructio quam tradit *Carteſius*, quæque facillime radices Æquationum omnium Cubicarum vel Biquadraticarum, ubi deficit ſecundus terminus, eruit, ut nota ſupponi poteſt; attamen cum cardo ſit a quo ſubſequentia pendent, ex illius Geometria deſumptam placuit Regulam adjungere, pauculis nonnullis in melius (uti reor) tranſpoſitis.

Deficiente ſecundo termino omnes æquationes Cubicæ reducuntur ad hanc formam $z^3 . * . apz . aaq . = 0$, ac Biquadraticæ ad hanc $z^4 . * . apzz . aaqz . a^3 r = 0$, (ubi a deſignat latus rectum Parabolæ cujuſvis datæ, quam in Conſtructione adhibere licet,) vel ſumendo a pro unitate, ad hanc $z^3 . * . pz . q = 0$, vel ad hanc $z^4 . * . pzz . qz . r = 0$.

Jam data Parabola F A G, cujus Axis ſit A C D K L ac latus rectum a vel 1, fiat A C ejus dimidium ac collocetur ſemper a vertice A verſus interiora figuræ; dein ſumatur $CD = \frac{1}{2} p$ in linea illa A C continuata verſus C ſi in æquatione fuerit $- p$, vel verſus alteram partem ſi habeatur $+ p$. Porro è Fig. 48.

puncto D, aut ex puncto C ſi non habeatur quantitas p, erigenda eſt ad axem perpendicularis DE æqualis $\frac{1}{2}q$, dextrorſum quidem ſi fuerit $-q$, ad alterum vero axis latus ſi fuerit $+q$; ac Circulus, Centro E, Radio AE deſcriptus, ſi æquatio fuerit tantum Cubica, Parabolam tot punctis F & G interſecabit quot veras habet Radices, quarum quidem affirmativæ, ut GK, erunt ad dextram Axis partem, Negativæ, ut FL, ad ſiniſtram.

Aſt ſi Æquatio Biquadratica fuerit, augeri vel minui debet Circuli Radius AE, addendo ſi fuerit $-r$, vel ſubducendo ſi ſit $+r$, ex ejus quadrato rectangulum ar, ſeu contentum ſub Latere Recto & quantitate data r; id quod nullo fere negotio efficitur Geometrice. Hujus vero Circuli interſectiones cum Parabola omnes veras Biquadraticæ Æquationis Radicis, dimiſſis ad axem perpendiculis, exhibebunt; affirmativas quidem ad dextram Axis, Negativas vero ad ſiniſtram. Totius demonſtrationem *Carteſio*, ejus Inventori, relinquo.

Notandum hic me operam dare ut ſemper habeantur Radices affirmativæ ad dextrum Axis latus, ut evitetur confuſio, à pluribus cautionibus, quarum cauſa minime evidens eſt, neceſſario oritura.

His præmiſſis, ut aditus pateat ad conſtructionem etiam earum æquationum ubi reperitur terminus ſecundus, conſideranda venit Regula pro tollendo termino ſecundo, ac reducenda æquatione ad aliam quæ methodo præcedente conſtrui poſſit. Omnes vero hujus claſſis æquationes cubicæ ad hanc formam, $z^3.\,bzz.\,apz.\,aaq = 0$, vel ad hanc, $z^3.\,bzz.\,*.\,aaq = 0$; Biquadraticæ vero ad hanc, $z^4.\,bz^3.\,apzz.\,aaqz.\,a^3r = 0$, vel ad hanc, $z^4.\,bz^3.\,*.\,aaqz.\,a^3r = 0$, vel $z^4.\,bz^3.\,apzz.\,*.\,a^3r = 0$, vel denique ad hanc, $z^4.\,bz^3.\,*.\,*\,a^3r = 0$, reduci poſſunt: è quibus omnibus, prout ſignis $+$ & $-$ diverſimode connectuntur, ingens oritur varietas; unde Regula generalis omnibus inſerviens obſcura ac maxime difficilis redditur, niſi methodo quam ſubjungimus illuſtrata nodiſque extricata tractetur.

Tollitur in Biquadraticis ſecundus terminus ponendo $x = z + \frac{1}{4}b$, ſi fuerit $+b$ in æquatione, vel $z = z - \frac{1}{4}b$, ſi fuerit $-b$: hinc $x - \frac{1}{4}b$ in primo caſu, & $+\frac{1}{4}b$ in altero æquatur z; & in Æquatione quavis propoſita, ſubſtituta loco z quantitate æquali, prodibit nova æquatio termino ſecundo carens, cujus radices omnes x data differentia $\frac{1}{4}b$ vel excedunt vel deficiunt à radice quæſita z.

Exemp. 1.] $z^4 + bz^3 - apzz - aaqz + aaar = 0.$
ſit $x - \frac{1}{4}b = z$ Et erit

$$xx - \tfrac{1}{2}bx + \frac{1}{16}bb = zz$$

$$xxx - \frac{3}{4}xxb + \frac{3}{16}xbb - \frac{1}{64}bbb = z^3$$

$$\&\ x^4 - bx^3 + \frac{3}{8}bbxx - \frac{1}{16}b^3x + \frac{1}{256}b^4 = z^4.$$

Hinc
$$\begin{array}{rrrrrl}
x^4 - bx^3 & + \frac{3}{8}bbxx & - \frac{1}{16}bbbx & + \frac{1}{256}b^4 & = & +z^4 \\
+bx^3 & - \frac{3}{4}bbxx & + \frac{3}{16}bbbx & - \frac{1}{64}b^4 & = & +bz^3 \\
 & -apxx & + \frac{1}{2}apbx & - \frac{1}{16}apbb & = & -apzz \\
 & & -aaqx & + \frac{1}{4}aaqb & = & -apqz \\
 & & & +aaar & = & +aaar
\end{array}$$

Harum omnium ſumma fit æquatio nova ſecundo termino carens, quæque proinde juxta Regulam *Carteſianam* conſtrui poſſit, ſecundo loco ½ p dimidium coefficientis termini tertii per a ſive Latus rectum diviſi, hoc eſt $-\frac{3}{16}\frac{bb}{a} - \frac{1}{2}p$; ac Loco $\frac{1}{2}q$ dimidium coefficientis termini quarti per aa diviſi, ſive $+\frac{1}{16}\frac{bbb}{aa} + \frac{1}{4}\frac{pb}{a} - \frac{1}{2}q$. Cujus partes ſigno + notatæ ſiniſtrorſum ab Axe, ſigno − notatæ dextrorſum collocandæ ſunt, ut habeatur centrum Circuli ad conſtructionem requiſiti, ac cujus interſectiones cum Parabola, dimiſſis in axem perpendiculis, radices omnes veras x deſignet, affirmativas quidem ad dextram axis, negativas vero ad ſiniſtram. Cum vero $x - \frac{1}{4}b = z$, ducendo lineam axi Parallelam, ad dextrum ejus latus & ad diſtantiam $\frac{1}{4}b$, perpendicula illa ad hanc Parallelam terminata deſignabunt omnes radices quæſitas z, affirmativas ad dextram, negativas vero ad ſiniſtram. Radium circuli quod attinet, habetur ille addendo partes negativas ac auferendo partes affirmativas termini quinti aa diviſi, e quadrato lineæ A E, à centro invento E ad verticem Parabolæ A ductæ: id quod maxima ex parte efficitur capiendo loco lineæ A E lineam E O, quæ ad O interſectionem Parabolæ ac parallelæ prædictæ terminatur; ejus enim quadratum omnes termini quinti partes ex ablatione termini ſecundi æquationi novæ ingeſtas complectitur (uti facile probabitur): ac reſtat ſolummodo ut ipſius E O quadratum augeatur, ſi in æquatione habeatur $-r$, vel minuatur, ſi ſit $+r$; additione vel ſubductione rectanguli ar, unde conflatur quadratum Radii Circuli quæſiti.

Hæc eſt Methodus inveſtigandi regulam centralem *D. Bakeri* omnibus cautionibus libera, ac ſatis facilis; ac ſola differentia ex eo provenit, quod ego juxta Axem, ille vero juxta Axi parallelam circuli ejuſdem centrum determinat: quodque ego ſemper radices affirmativas ex Axis dextro latere invenio, quas ille nunc dextro nunc ſiniſtro conſtituit.

Æquationes cubicas quod attinet, eæ reduci debent ad Biquadraticas, antequam eadem regula generali conſtrui poſſint; id quod fit ducendo æquationem propoſitam in radicem ſuam z, unde provenit æquatio Biquadratica in qua deficit terminus ultimus ſive r: quapropter ſublato ſecundo termino & invento centro E, linea E O eſt radius Circuli; cum ſcilicet ar ſit $= 0$, & in nova æquatione totus terminus quintus ex ipſa ablatione termini ſecundi oriatur.

Exemp. 2. $z^3 - bzz + apz + aaq = 0$: Quæ ducta in z fit
$z^4 - bz^3 + apzz + aaqz = 0$

Ad tollendum ſecundum terminum ponatur $x + \frac{1}{4}b = z$, & fiet

$$\begin{array}{rrrrrr} x^4 + bx^3 & + \frac{3}{8}bbxx & + \frac{1}{16}b^3x & + \frac{1}{256}b^4 & = & + z^4 \\ - bx^3 & - \frac{3}{4}bbxx & - \frac{3}{16}b^3x & - \frac{1}{64}b^4 & = & - bz^3 \\ & + apxx & + \frac{1}{2}abpx & + \frac{1}{16}apbb & = & + apzz \\ & & + aaqx & + \frac{1}{4}aaqb & = & + aaqz \end{array}$$

In hac nova æquatione, tertii termini ſemi-coefficiens per a diviſa, *viz.* $-\frac{3}{16}\frac{bb}{a} + \frac{1}{2}p$, loco $\frac{1}{2}p$ uſurpanda eſt; ac coefficientis termini quarti dimidium, diviſum per aa Lateris recti quadratum, *viz.* $-\frac{1}{16}\frac{bbb}{aa} + \frac{1}{4}\frac{pb}{a} + \frac{1}{2}q$, vicem ipſius $\frac{1}{2}q$ in conſtructione *Carteſii* ſubit: unde centrum E determinatur. Deinde ducta Axi Parallela ad diſtantiam $\frac{1}{4}b$ ad ſiniſtram ejus latus (ob $x + \frac{1}{4}b = z$) cujus interſectio cum Parabola ſit O; circulus centro E, Radio E O deſcriptus Parabolam ſecet vel tanget in tot punctis quot æquatio veras habet radices: quæ quidem radices ſeu z ſunt perpendicula de punctis illis in Axi parallelam demiſſa; ad dextram quidem Affirmativæ, Negativæ ad ſiniſtram.

Si in æquatione defuerit terminus tertius vel quartus vel uterque, inveſtiganda regula centrali nulla omnino obſervanda eſt methodus differentia, ſed deficiente quantitate p vel q, deerunt partes illæ linearum C D ac D E ex quantitate illa aliquo modo deductæ, ac producendum eſt cum reliquis coefficientibus termini tertii & quarti in æquatione nova, ſicut in præmiſſis exemplis præſcriptum eſt.

Hactenus *Cl. Bakeri* methodum generalem pertractavimus, qua quidem nulla alia facilior ac paratior expectanda eſt, aſſumpta ad conſtructionem ſive Parabola, ſive alia quævis linea curva, cum ſcilicet æquatio ad *Biquadraticam* aſcendit. Etenim dum hæc ſcribo mihi occurrit regulæ Centralis Effectio Geometrica præter omnem ſpem expedita, ac harum rerum Curioſis abunde ſatisfactura.

Deſcripta Parabola N A M, cujus vertex A, Axis ABC, ac latus rectum a, reducatur æquatio ad hanc formam $z^4 . bz^3 . apzz . aaqz . a^3r . = 0$, vel ad hanc $z^3 . bzz . apz . aaq = 0$, ſi cubica tantum fuerit: dein ad diſtantiam Fig. 49.
B D $= \frac{1}{4}b$ ducatur linea D H Axi parallela, ad ſiniſtram quidem ſi fuerit $-b$, ad dextram ſi $+b$, Parabolæ occurrens in puncto D; de quo demittatur perpendiculum in axem B D. In linea A B continuata verſus B fiat B K $= \frac{1}{2}a$, & ducatur linea D K utrinque interminata. Porro ſit K C $=$ A B in Axe ſemper ultra K continuato; ac ſic habeatur quantitas p ſigno $-$ affecta, verſus eaſdem partes etiam ſumatur C E $= \frac{1}{2}p$, vel in contrarias ſi habeatur $+ p$, ac è puncto E erigatur Axi perpendiculum

diculum EF (vel è puncto C ſi defuerit quantitas p) lineæ DK, ſi opus eſt continuatæ, occurrens in puncto F; quod quidem Circuli requiſiti centrum eſt, ſi defuerit quantitas q; aſt ſi habeatur q, ſumenda eſt in FE, ſi opus eſt continuata, linea $FG = \frac{1}{2} q$, ſiniſtrorſum quidem ſi fuerit $+ q$, dextrorſum ſi $- q$ collocanda: Et punctum G erit centrum Circuli ad conſtructionem propoſitam idonei; ejuſque Radius, ſi defuerit quantitas r, hoc eſt, ſi tantum Cubica fuerit, erit linea GD; cujus quadratum in Biquadraticis augendum eſt, ſi fuerit $- r$, vel minuendum, ſi $+ r$, additione vel ſubductione rectanguli ſub r & latere recto. Deſcripto ſic Circulo, ab interſectionibus ejus cum Parabola demiſſis in lineam DH perpendiculis, quæ ad ſiniſtram ſunt, ut NO, radices æquationis negativas ſemper deſignant, quæ ad dextram, ut *ML*, affirmativas.

Aliter ac paulo ſimplicius Æquationes Cubicæ juxta *Schooteni* Regulam conſtruuntur, quæque etiam radices ad Axem referuntur: quoniam vero ipſe inventor nec modum inveniendi, nec demonſtrationem inventi exponit, non abs re erit ejuſdem fundamentum hic adjicere, ſimul atque Effectionem Geometricam concinniorem reddere, atque cautionibus quibus implicatur extricare.

Hæc Regula derivatur ex eo quod omnis æquatio Cubica reduci poſſit ad Biquadraticam, in qua deficiet terminus ſecundus: Hoc fit ducendo æquationem propoſitam in $z - b = 0$, ſi fuerit $+ b$ in æquatione, vel in $z + b = 0$, ſi fuerit $- b$; & æquatio nova producta eaſdem habebit radices cum Cubica, atque inſuper alteram ipſi $- b$ æqualem, ſi fuerit $- b$ in æquatione; vel contra.

Proponatur conſtruenda $z^3 - z^2 b + apz + aaq = 0$.
Hæc ducta in $z + b$ fit $z^4 - z^3 b + apz^2 + aaqz$
$\qquad + z^3 b - bbzz + abpz + aaqb$.

Hic deficit ſecundus terminus, ac coefficiens tertii $- bb + ap$ dat $- \frac{1}{2} \frac{bb}{a} + \frac{1}{2} p$ loco $\frac{1}{2} p$ vel CD in Conſtructione *Carteſii*, & ex dimidio coefficientis termini quarti fit $+ \frac{1}{2} q + \frac{1}{2} \frac{bp}{a}$ loco $\frac{1}{2} q$ vel DE uſurpanda; adeoque determinatur centrum Circuli quæſiti: atque ob datam unam ex radicibus æquationis novæ, viz. $-$ vel $+ b$, dabitur etiam punctum in circumferentia, id eſt Radius ejus. Denique deſcripto Circulo, ab interſectionibus ejus cum Parabola demiſſa in axem perpendicula æquationis radices exhibebunt, affirmativas & negativas, eadem lege ac ſupra.

Inveſtigatur autem centrum Circuli conſtructione perquam facili, cæteriſque omnibus in Cubicis præferenda. Deſcriptæ Parabolæ AMD ſit vertex A, atque Axis AF: ad diſtantiam ipſi b æqualem ducatur Axi parallela DK, ad dextram ſi fuerit $+ b$ in æquatione, ad ſiniſtram ſi $- b$, quæ Parabolæ occurrat in puncto D. Centris D & A deſcribantur Radiis æqualibus arcus occulti utrinque ſeſe interſecantes, ac per ſectionum puncta ducatur linea interminata BC, quæ medio lineæ ſubtenſæ AD perpendiculariter inſiſtat,

Fig. 50.

ſtat, & Axi occurrat in puncto E. Ab E, inferne quidem ſi in æquatione habeatur $-p$, vel ſuperne verſus A ſi fuerit $+p$, ponatur $EF = \frac{1}{2}p$; & ex F (vel ex E ſi defuerit p) educatur perpendiculum F G, linea B C occurrens in puncto G; & in G F producta fiat $GH = \frac{1}{2}q$, dextrorſum quidem ſi in æquatione habeatur $-q$, aliter ſiniſtrorſum, applicanda: ac punctum H erit Centrum quæſitum, H D vero Circuli Radius, qui demiſſis in Axem Perpendiculis ab interſectionibus ſuis cum Parabola, ut L M, Radices omnes, ut prius, commonſtrabit.

The Number of Roots in ſuch Æquations, with their Limits and Signs; by Mr. E. Halley N. 190. p. 589. Nov. An. 1687.

2. Ex *Carteſio* & ex ſupradictis conſtat, tam in Cubicis quam in Biquadraticis æquationibus, Radices exponi poſſe demittendo perpendicula in Axem, datamve diametrum Parabolæ datæ, ab interſectionibus Curvæ illius cum Circulo. Cum Circulus Parabolam ſecans, vel in quatuor vel in duobus punctis eam interſecare neceſſe eſt, conſtat in Biquadraticis vel duas vel quatuor radices veras, Affirmativas vel Negativas, ſemper haberi; uti etiam, ſi forte Circulus illam tangat, quo in Caſu æqualitas duarum Radicum ejuſdem ſigni concluditur. In Cubicis autem, quoniam una ex interſectionibus ad Conſtructionem requiritur, non niſi una vel tres reliquæ Radices deſignant unam vel tres; uti in Caſu Contactus, unde conſtat duas æquales reperiri Radices, Problemaque unde reſultat æquatio revera Planum eſſe.

Cubicæ itaque omnes quomodocunque affectæ una vel triplici Radice explicabiles ſunt, utique ſemper poſſibiles, nempe ſi Radices Negativas pro veris admiſeris: ſic Biquadraticæ, quarum terminus ultimus r ſigno $-$ affecta eſt, duabus vel quatuor. Aſt ſi habeatur $+r$ in æquatione, eaque tanta ſit, ut

Fig. 49.

$\sqrt{GDq - ar}$, minor ſit quam ut Circulus, eo radio ac centro G deſcriptus, Parabolam contingere in aliquo puncto poſſit, æquatio data omnino impoſſibilis eſt, nec ulla Radice Negativa vel Affirmativa explicabilis: Sed de his plura in ſequentibus.

Quoniam vero tanta intercedit differentia inter caſus Cubicarum & Biquadraticarum, ut ſimul comprehendi nequeant; primum Cubicas deinde alteras tractabimus. Cubicæ vero infinitis Circulis in data Parabola conſtruuntur, Biquadraticæ autem unico tantum (ſaltem his methodis). id adeo quia ponendo $z - e$ ſive indeterminata aliqua, æqualem nihilo, æquatio Cubica reducitur ad Biquadraticam eaſdem Radices cum Cubica habentem, atque inſuper aliam ipſi e æqualem; unde fit ut tot Circulis diverſis conſtrui poſſit Cubica, quot imaginari velis quantitates e, id eſt infinitis. Inter has vero Conſtructiones, illa quam ſuperius (§. *ult.*) dedi longe facillima eſt. Huic tamen non multum cedit alia, quæ ad enucleationem Numeri Radicum, earumque Limitum magis accommodata videtur, quæque ortum trahit ex ablatione ſecundi termini; ponendo modo vulgari $x = z +$ vel $-$ tertia parte Coefficientis termini ſecundi. Hæc autem eſt: Data Parabola A B Y ejuſque vertice A, Axe A E &

Fig. 51.

Latere recto a, reducatur æquatio ad formam conſuetam, *viz.* $z^3 \cdot bz^2 \cdot apz \cdot aaq = 0$. Deinde ad diſtantiam $\frac{1}{3}b$ ducatur Axi parallela B K, dextrorſum quidem ſi fuerit $+b$, aliter ſiniſtrorſum, Parabolæ occurrens in B; ac lineæ ſuppoſitæ A B erigatur perpendicularis utrinque interminata D P, Axi occurrens in puncto G. De B in Axem demitte perpendiculum B C, & ipſi A C fiat G E ſemper æqualis, ac verſus inferiora ponatur. Ab E fiat

E H = $\frac{1}{2}p$, ſurſum quidem, ſi in æquatione fuerit $+p$, deorſum vero, ſi $-p$, ac è puncto H (vel ex E ſi defuerit quantitas p) educatur perpendiculum H Q interminatæ D P occurrens in puncto O. Denique in linea H Q interminata, fiat O R = $\frac{1}{2}q$, ab O dextrorſuuı ſi fuerit $-q$, Siniſtrorſum ſi $+q$, collocanda: ac Circulus centro R, radio R A deſcriptus, tot punctis ſecabit Parabolam, quot æquatio propoſita veras habet radices; eæque erunt perpendicula Z Y à punctis interſectionem Y in Axi parallelam B K demiſſa; quarum quæ ad dextram lineæ B K Affirmativæ ſunt, ad Siniſtram Negativæ.

Hujus Conſtructionis commoditas in eo conſiſtit, quod Circulo per verticem tranſeunte peragitur, perinde ac ſi defuiſſet ſecundus terminus; ideoque ad Radicum Numerum determinandum, ſufficit Loci ſive Lineæ Curvæ proprietates perſpectas habere, quæ ſpatia diſcriminat, ubi ſi ponatur Centrum Circuli qui per Parabolæ Verticem tranſeat, circumferentia ejus vel uno vel tribus aliis punctis eam ſecabit; hoc eſt Lineæ curvæ, in quam incidunt centra omnium Circulorum per verticem tranſeuntium ac deinde Parabolam tangentium, naturam definire.

Locus autem ille eſt Parabolois, quam cum *Cl. Walliſio* ſemicubicalem appellare licet, ſive in qua Cubi applicatarum ad Axem ſunt inter ſe ut Quadrata portionum Axis. Cujus Latus rectum eſt $\frac{27}{8}$ Lateris Recti datæ Parabolæ, Vertex vero punctum V exiſtente A V dimidium Lateris recti ejuſdem Parabolæ. Hoc eſt, ſi ponatur Unitas pro Latere Recto datæ Parabolæ, $\frac{8}{27}$ cui ordinatim applicatæ æquabuntur Quadrato partis diametri, ſive cubus ex $\frac{2}{3}$ V H quadrato ex H R, ſi ſcilicet R ſit Centrum Circuli qui per verticem Parabolæ tranſeat, eamque deinde contingat; Hæc eſt Curva illa quam primus mortalium *Nelius* Noſtras rectæ datæ æqualem demonſtravit, eaque occaſione apud Principes Geometras dudum celebris; ejuſque proprietates *Cl. Walliſius* ſub finem Libri *de Ciſſoide*, & *Hugenius Prop.* 8 & 9, *de Linearum Curvarum Evolutione*, aliique acri ingenio diſquiſivere, quorum ſcripta conſultat Lector. Hæc Curva utrinque ab Axe Parabolæ deſcripta, *viz.* V N L, V P X, ſpatium complectitur, in quo ſi ponatur centrum Circuli, qui per verticem A tranſeat, interſecabit ille Parabolam in tribus aliis punctis; ſpatia vero ab Axe remotiora Centra præbent Circulis non niſi uno præter verticem puncto Parabolam ſecantibus.

His probe intellectis jam ad determinandum Radicum Numerum accingimur: Ac primum deficiat ſecundus terminus; ſitque Latus Rectum 1, vel A V = $\frac{1}{2}$; in conſtructione V H eſt $\frac{1}{2}p$, H R vero $\frac{1}{2}q$; cumque ſi fuerit $+p$, ab V verſus ſuperiora ponenda ſit $\frac{1}{2}p$, Centrum Circuli extra ſpatium L V X ſemper conſtituitur; ideoque una tantum Radice explicabilis eſt, Affirmativa ſi $-q$, Negativa ſi $+q$: que quidem Radices *Cardani* Regulis inveſtigantur. Si vero fuerit $-p$, V H = $\frac{1}{2}p$ inferne ponitur, ac fieri poteſt ut H R cadat inter Axem & Curvam V X vel V L, ſi ſcilicet Cubus ex $\frac{2}{3}$ V H, ſive ex $\frac{1}{3}p$, major ſit quam quadratum ex $\frac{1}{2}q$, ſive $\frac{1}{27}p^3$ major quam $\frac{1}{4}qq$, quo in caſu tres dantur Radices, duæ Negativæ, ſi fuerit $-q$, ac una Affirmativa earum ſummæ æqualis; vel ſi $+q$, duæ Affirmativæ unaque Negativa. Quod ſi $\frac{1}{27}p^3$ minor ſit quam $\frac{1}{4}qq$ una tantum reperitur Radix, Affirmativa ſi $-q$, Negativa ſi $+q$. Atque hæc paſſim docentur ab iis qui hanc Geometriæ partem tractarunt.

Jam adſint omnes termini, ac primum proponatur, *e. g.* æquatio hæc $z^3 - z^2 b + zp - q = 0$; cui etiam Figuram (51) adaptavimus. In hujus conſtructione $BC = \frac{1}{3} b$, $VG = \frac{1}{2} AC = \frac{1}{18} bb$, $VE \frac{1}{6} bb$, $VH \frac{1}{6} bb - \frac{1}{2} p$, $GH \frac{1}{9} bb - \frac{1}{2} p$ vel $\frac{1}{2} p - \frac{1}{9} bb$, hinc $HO = \frac{1}{27} b^3 - \frac{1}{6} bp$, vel $\frac{1}{6} bp - \frac{1}{27} b^3$, atque HR, ſive diſtantia Centri Circuli R ab Axe, eſt ſemper differentia inter $\frac{1}{6} bp$ & $\frac{1}{27} b^3 + \frac{1}{2} q$; quæ ſi æquantur, Centrum cadit in Axe; ſi $\frac{1}{6} bp$ major ſit quam $\frac{1}{27} b^3 + \frac{1}{2} q$ ad Siniſtram Axis, ſin minor ad Dextram. Si itaque Cubi ex $\frac{2}{3}$ VH, (hoc eſt ex $\frac{1}{9} bb - \frac{1}{3} p$ quam nominemus d) Latus Quadratum ſive $\sqrt{ddd}$, majus ſit quam HR, ſive differentia inter $\frac{1}{27} b^3 + \frac{1}{2} q$ & $\frac{1}{6} bp$; reperitur Centrum R intra ſpatium NPV, Paraboloidibus VPX, VNL, ac recta interminata DNP, circumſcriptum: ac proinde Circulus Parabolam ſecabit in tribus punctis Y, Y, Y, ad dextram lineæ BK ſitis, atque adeo æquatio tres habet Radices Affirmativas. Centro vero extra hoc ſpatium NVP conſtituto, non niſi una Radice Affirmativa explicari poteſt. Hic obiter notandum Rectam DP Paraboloidem VPX tangente in puncto P, exiſtente $EP = \frac{1}{27} b^3$; alteram vero VNL ſecare in puncto N, ita ut demiſſo in Axem Perpendiculo NF, VF ſit pars quarta ipſius EV, ſive $\frac{1}{24} bb$, NF vero $\frac{1}{108} b^3$. VW autem, quæ è puncto V Axi perpendiculariter erecta lineæ DP occurrit in W, æqualis eſt $\frac{1}{54} b^3$ ſive $\frac{1}{2}$ EP.

Hinc tuto concluditur, ſi in æquatione vel p major ſit quam $\frac{1}{3} bb$, vel q major quam $\frac{1}{27} b^3$, non niſi unam eamque Affirmativam Radicem reperiri; Fallit itaque Regula *Carteſii* (Edit. *Amſt.* 1659, *pag.* 70.) ubi tot veras dari Radices quot ſunt in æquatione mutationes ſignorum + & − pronunciat, fruſtra etiam in Commentariis ſuis Sphalma hoc excuſante *Schootenio*; Pingi enim poſſunt infinite plures æquationes præcedentis formulæ tres ſignorum mutationes habentis, quæ unam tantum quam quæ tres habeant Radices. *Propoſitio* etiam *quinta Sectionis quintæ Artis Analyticæ Harriotti Noſtri,* uti *Prob.* 18. *Numeroſæ Poteſt. Reſol. Vietæ,* vix ſatis firma eſt, cum ex Limitationibus quas ibi poſuerunt, toti Parallelogrammo PIVW id conveniat, quod ſoli ſpatio NVP jam competere probavimus, hoc eſt ut Centrum præbeat Circulo tribus aliis punctis præter Verticem Parabolam ſecante.

Quantitas autem q, ſive terminus ult., datis, b & p ea lege ut p minor ſit quam $\frac{1}{3}bb$, accurate limitatur ex præcedente æquatione $\sqrt{ddd} = \frac{1}{27}b^3 + \frac{1}{2}q \backsim \frac{1}{6}bp$; cum ſcilicet Circulus Parabolam contingat. Itaque $\frac{1}{2}q$ minor eſſe debet quam $\frac{1}{6}bp - \frac{1}{27}b^3 + \sqrt{ddd}$; at ſi p major fuerit quam $\frac{1}{4}bb$, majorem etiam eſſe oportet $\frac{1}{2}q$ quam $\frac{1}{6}bp - \frac{1}{27}b^3 - \sqrt{d^3}$, ne cadat centrum in ſpatiolo N V W. Atque his conditionibus æquatio ſemper triplici Radice explicabilis erit, aliter non niſi una. Semper vero, ſive tres ſive una, Affirmativæ ſunt, ob poſitionem centri R ad dextram lineæ D P.

Atque hic eſt caſus maxime difficilis, ita ut quicunque præmiſſa bene calleat ſequentia facili negotio intelliget. Detur jam æquatio $z^3 - bz^2 + pz + q = 0$. Hic ut tres habeantur Radices, oportet Centrum Circuli alicubi intra ſpatium P N △, rectis P N, P △, & curva Paraboloidis N △, definitum, reperiri; quapropter cum E F ſit $= \frac{1}{8}bb$, p minor eſſe debet quam $\frac{1}{4}bb$: jam ad determinationem quantitatis q, exiſtente $d = \frac{1}{9}bb - \frac{1}{3}p$, ut antea, $\sqrt{ddd} + \frac{1}{7}bbb - \frac{1}{6}bp$ ſemper major eſſe debet quam $\frac{1}{2}q$, ut conſtituatur Centrum Circuli in ſpatio prædicto P N △; quod cum ſit æquatio talis duas habet radices Affirmativas ac unam Negativam. Si vero p major eſt quam $\frac{1}{3}bb$, vel $\frac{1}{2}q$ major quam $\sqrt{ddd} + \frac{1}{27}bbb - \frac{1}{6}bp$, non niſi una eaque Negativa Radice explicabilis eſt.

Proponatur jam æquatio $z^3 - bz^2 - pz - q = 0$. Ut hæc æquatio tres habeat Radices, oportet Centrum Circuli alicubi inveniri in ſpatio indefinito, inter rectam D P D & curvam Paraboloidis P X; hinc quantitas p non eſt obnoxia limitationibus, $\frac{1}{2}q$ vero ſemper minor eſſe debet quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, poſito $d = \frac{1}{9}bb + \frac{1}{3}p$: Hoc pacto duæ dantur Radices Negativæ, ac una Affirmativa; aliter vero ſi $\frac{1}{2}q$ major ſit quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, unica tantum Affirmativa exponi poteſt. Quarto loco ſit æquatio $z^3 - bz^2 - pz + q = 0$, quæ duas Affirmativas habet Radices ac unam Negativam ſi Centrum Circuli reperiatur in ſpatio indefinito inter rectas P △, P D, ac curvam Paraboloidis △ L; hoc eſt, (poſito $d = \frac{1}{9}bb + \frac{1}{3}p$) ſi $\frac{1}{2}q$ minor ſit quam $\sqrt{ddd} + \frac{1}{27}bbb + \frac{1}{6}bp$; ſi vero $\frac{1}{2}q$ major hac quantitate fuerit, una tantum Negativa ineſt Radix.

Quatuor autem æquationes reliquæ, in quibus habeatur $+b$, quoad limitationem Numeri Radicum non differunt à prædictis, ſi ſignum termini ultimi mutetur, ſervato ſigno termini tertii; quæ vero Affirmativæ erunt Radices in illis hic fiunt Negativæ, & vice verſa. Sic in æquatione, $z^3 - bz^2 + pz - q = 0$, una vel tres erant Affirmativæ Radices; in hac vero, $z^3 + bz^2 + pz + q = 0$, vel una vel tres Negativæ ſunt, ſub iiſdem conditionibus; nulla vero omnino Affirmativa. Sic in $z^3 + bz^2 + pz - q = 0$, duæ ſunt Negativæ & una Affirmativa, ſi p minor ſit quam $\frac{1}{3}bb$, ac $\frac{1}{2}q$ minor quam $\sqrt{d^3} + \frac{1}{27}b^3 - \frac{1}{6}bp$, quemadmodum in $z^3 - bz^2 + pz + q = 0$, duæ erant Affirmativæ & una Negativa; excedentibus autem leges præſcriptas p vel q, una tantum hic eſt Radix Affirmativa, quæ ibi Negativa erit. Pari modo in $z^3 + bz^2 - pz + q = 0$, vel duæ ſunt Affirmativæ ac una Negativa, vel una Negativa tantum; denique iiſdem de cauſis in æquatione, $z^3 + bz^2 - pz - q$ duæ ſunt Negativæ & una Affirmativa, vel una Affirmativa tantum, quibus in æquatione, $z^3 - bz^2 - pz + q$, duæ erant Affirmativæ & una Negativa, vel una Negativa tantum, nempe prout $\frac{1}{2}q$ major vel minor fuerit quam $\sqrt{d^3} + \frac{1}{27}b^3 + \frac{1}{6}bp$.

Si defuerit terminus tertius, ſive pz, Centrum R ſemper cadit in linea I P E $\triangle$, quocirca ſi fuerit $z^3 - bz^2 . * . - q$, vel $z^3 + bz^2 . * . + q$, una tantum eſſe poteſt Radix, ſi $-b$ Affirmativa, ſi $+b$ Negativa. At ſi fuerit $z^3 - bz^2 . * . + q$, vel $z^3 + bz^2 . * . - q$, duæ poſſunt eſſe Affirmativæ ac una Negativa in priore, vel una Affirmativa & duæ Negativæ in poſteriore, cadente Centro in linea P $\triangle$ inter P ac $\triangle$, hoc eſt ſi $\frac{1}{4}q$ minor ſit quam $\frac{1}{27}b^3$; ſin major fuerit, una tantum Negativa in priore, vel una Affirmativa in poſteriore, dare poteſt.

Hactenus Numerum Radicum in Cubicis æquationibus plenius aſſecuti ſumus, reſtat ut nonnulla adjiciam de Quantitate Radicum. Hic primum Notandum quod omnis æquatio tres habens Radices ope Tabulæ Sinuum, Triſectione ſcilicet Anguli, ſatis expedite reſolvi poſſit; Ponendo ſcil. $\sqrt{\frac{4}{9}bb - \frac{4}{3}p}$, vel $\sqrt{4d}$, ſi fuerit $+p$ in æquatione, vel $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$, ſi $-p$, pro Radio Circuli; Angulum vero triſecandum qui Sinum habeat in Tabula Sinuum $\frac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3}}$: Invento hoc angulo, Sinus tertiæ partis ejus, ut & Sinus tertiæ partis compl. ad Semicirculum, eorumque ſumma, ex Tabula Sinuum dabuntur. Hi vero Sinus in Radium $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$ ducendi ſunt, & habebuntur Quantitates (y &, y &, y &, in *Fig.*) quarum & $\frac{1}{3}b$ vel

Summa vel differentia, prout caſus poſtulat, veras Radices Æquationis exhibebunt. Hæc omnia ex inventis *Carteſii* derivantur: Ut vero caſus omnes, quantum fieri poſſit, breviter complectar, dico quod Centro R, in prima æquationum formula, cadente in ſpatio V G P, Sectiones duæ Y, Y, cadunt inter A & B, ac proinde utraque ex Minoribus Radicibus Minor eſt quam $\frac{1}{3}b$, tertia autem & Major ſemper ſuperat $\frac{1}{3}b$, ſuperatur vero à b. Quod ſi cadat in ſpatio G N V, duæ majores ſunt quam $\frac{1}{3}b$, minores vero quam $\frac{2}{3}b$, tertia vero eſt b − duabus alteris, ac proinde minor quam $\frac{1}{3}b$: Sed adhibita Limitatione Quantitatis p, arctioribus terminis Radices includuntur. Maxima enim Radix minor eſt quam $\sqrt{\frac{4}{9}bb-\frac{4}{3}p}+\frac{1}{3}b$, major vero quam $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$; at cum $\frac{1}{4}bb$ minor eſt quam p, limes ille fit $\sqrt{\frac{1}{9}bb-\frac{1}{3}p}+\frac{1}{3}b$; Radix media ſemper minor eſt quam $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$; major vero quam $\frac{1}{5}b-\sqrt{\frac{1}{9}bb-\frac{1}{3}p}$; hunc vero limitem nunquam excedit Radix Minima, ſed cum Quantitate q evaneſcit.

In ſecunda formula præſcriptis legibus duæ ſunt Affirmativæ ac una Negativa, ac cadente Centro in ſpatio G P E, altera ex Affirmativis major eſt, altera minor quam $\frac{1}{3}b$, Major vero non excedit b, Negativa autem Major non eſſe poteſt quam $\sqrt{\frac{1}{3}bb}-\frac{1}{3}b$, eſt autem differentia ipſius b & ſummæ Affirmativarum. Centro autem in ſpatio E N G △ poſito, utraque Affirmativa major eſt quam $\frac{1}{3}b$, minor vero quam $\sqrt{\frac{1}{3}bb}+\frac{1}{3}b$, Negativa vero ſemper minor eſt quam $\frac{1}{3}b$. Limites autem propiores ex data p evadunt Radicis quidem maximæ Affirmativæ $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$, qua ſemper minor eſt, ut & major quam $\sqrt{\frac{1}{9}bb-\frac{1}{3}p}+\frac{1}{3}b$; hoc tamen limite minor eſt altera Affirmativa, quæ cum quantitate q minuitur. Negativa vero ſemper minor eſt quam $\sqrt{\frac{4}{9}bb-\frac{4}{3}p}-\frac{1}{3}b$, ac deficiente quantitate q evaneſcit.

In tertia formula duæ Negativæ ſunt ac una Affirmativa: in hac, ut & in quarta, Radices non limitantur à quantitate b. Affirmativa vero ſemper minor eſt quam $\sqrt{\frac{4}{9}bb+\frac{4}{3}p}+\frac{1}{3}b$, major tamen quam $\sqrt{p+\frac{1}{4}bb}$

$\sqrt{p + \frac{1}{4}bb} + \frac{1}{2}b$: Maxima vero ex Negativis ſemper major eſt quam $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} - \frac{1}{3}b$, minor vero quam $\sqrt{p + \frac{1}{4}bb} - \frac{1}{2}b$. Minor autem ex Negativis ſemper minuitur cum minuta quantitate q.

In quarta formula, cadente Centro intra ſpatium L △ P D; ſi duæ ſint Affirmativæ ac una Negativa, Maxima ex Affirmativis major eſſe nequit quam $\sqrt{p + \frac{1}{4}bb} + \frac{1}{2}b$, nec minor quam $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} + \frac{1}{3}b$; Minor vero Radix ab hoc Limite minuitur, minuta quantitate q. Negativa autem minor eſt quam $\sqrt{\frac{4}{9}bb + \frac{4}{3}p} - \frac{1}{3}b$; major vero quam $\sqrt{p + \frac{1}{4}bb} - \frac{1}{2}b$.

Notandum vero hic Radices Negativas ubique ſigno Affirmativo notari, quia hæ ſunt Radices Affirmativæ quatuor æquationum illarum, in quibus habetur $+ b$, ac q ſigno contrario notatur; ut ſupra monui. Horum omnium Demonſtratio ex eo conſequitur, quod ubicunque Centrum Circuli R incidit in Lineas curvas V P X, vel V △ L, circumferentia ejus Parabolam tangit in puncto, cujus diſtantia ab axe eſt $\sqrt{\frac{2}{3}}$ V H; eamque ſecat ex altera Axis parte, ad diſtantiam $2\sqrt{\frac{2}{3}}$ VH; cum vero Centrum cadit in Lineam DPD, altera ex Radicibus fit $= 0$, ac proinde Cubica reducitur ad Quadraticam, ſive ad $z^2 - bz + p = 0$, cujus Radices Limites deſignant ubi evaneſcit quantitas q: ac quo minor eſt q, eo propius ad hos limites accedunt Radices. Quadratica eſt etiam cum Centrum cadit in Axe; hoc eſt, cum $\frac{1}{2}q = \frac{1}{6}bp - \frac{1}{27}b^3$, in prima formula; vel $\frac{1}{2}q = \frac{1}{27}b^3 - \frac{1}{6}bp$, in ſecunda; in tertia impoſſibile eſt; at in quarta cum $\frac{1}{2}q = \frac{1}{27}b^3 + \frac{1}{6}bp$, quo in caſu Minor ex Radicibus Affirmativis eſt $\frac{1}{3}b$, Major $\sqrt{\frac{1}{3}bb + p} + \frac{1}{3}b$; Negativa vero $\sqrt{\frac{1}{3}bb + p} - \frac{1}{3}b$. In prima, Radices ſunt $\frac{1}{3}b$, & $\frac{1}{3}b \pm \sqrt{\frac{1}{3}bb - p}$. In ſecunda vero formula, $\frac{1}{3}b$, & $\sqrt{\frac{1}{3}bb - p} + \frac{1}{3}b$, ſunt Affirmativæ: Negativa autem $\sqrt{\frac{1}{3}bb - p} - \frac{1}{3}b$.

Atque hæc in Cubicis ſufficere poſſe videntur; ob eximium vero Uſum Methodi, qua ope Tabulæ Sinuum Radices harum æquationum inveniuntur, placuit unum vel alterum Exemplum adjungere, ut Praxis illius compendium inde innoteſcat. Proponatur Æquatio $z^3 - 39z^2$ $+ 472z$

$+ 479x - 1881 = 0$; quæruntur Radices x. $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} = \sqrt{9\frac{1}{3}} = \sqrt{d}$, cujus duplum $\sqrt{37\frac{1}{3}}$ Radius eſt Circuli; & $\frac{\frac{1}{27}b^3 + \frac{1}{3}q - \frac{1}{6}bp}{\sqrt{d^3}} = \frac{2197 + 940\frac{1}{2} - 3113\frac{1}{2}}{9\frac{1}{3}\sqrt{9\frac{1}{3}}}$, ſive $\frac{24}{9\frac{1}{3}\sqrt{9\frac{1}{3}}}$ eſt Sinus Tabularis Anguli, hoc eſt, facta diviſione ope Logarithmorum, Log. 9. 9251560, cui reſpondet Angulus 57 *gr.* 19 *m.* 11 $\frac{1}{3}$ *s.* Hujus tertia pars 19 *gr.* 6 *m.* 24 *s.* & complementi 40 *gr.* 53 *m.* 36 *s.* Sinus dant Log. 9. 514983, & 9. 816011, qui ducti in Rad. $\sqrt{37\frac{1}{3}}$ producunt Y &, et Y &, Log. 0. 301030. = 2, et Log. 0. 601059 = 4, tertio vero Y &, æqualis eſt eorum ſummæ ſive 6. Ideoque Radices ſunt $13 - 4 = 9$, $13 - 2 = 11$, & $13 + 6 = 19$, ex quibus ſingulis conflatur prædicta æquatio. Ubi Notandum duas Minores Radices non excedere $\frac{1}{6}b$ vel 13, quia Centrum R in conſtructione cadit ad dextram Axis; id eſt $\frac{1}{6}bp$ minor eſt quam $\frac{1}{27}b^3 + \frac{1}{2}q$.

Exemplum alterum ſit $x^3 - 15x^2 - 229x - 525 = 0$, & quærantur Radices $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} = \sqrt{101\frac{1}{3}} = \sqrt{d}$, & Radius Circuli $\sqrt{405\frac{1}{3}}$. $\frac{\frac{1}{27}b + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3}} = \frac{125 + 572\frac{1}{2} + 262\frac{1}{2}}{101\frac{1}{3}\sqrt{101\frac{1}{3}}} = \frac{960}{101\frac{1}{3}\sqrt{101\frac{1}{3}}}$ = Sinui Tabulari Arcus, cujus Log. 9. 9736426, & Arcus ipſe 70 *gr.* 14 *m.* 22 *s.* hujus pars tertia eſt 23 *gr.* 24 *m.* 47 $\frac{1}{2}$ *s.* & Complementi 36 *gr.* 35 *m.* 12 $\frac{1}{2}$ *s*; quorum Sinus Log. ſunt 9. 599183, & 9. 775275, quibus addito Log. $\sqrt{405\frac{1}{3}}$ fiunt Log. 0. 903089 = 8, & Log. 1. 079181 = 12, & eorum ſumma = 20. Hinc concluditur $20 \pm \frac{1}{3}b$, vel 25, æquari Radici Affirmativæ, & 8 & $12 - \frac{1}{3}b$, ſive 3 & 7, Negativis. Quod ſi æquatio fuiſſet $x^3 + 15x^2 - 229x + 525 = 0$, 3 & 7 fuiſſent Affirmativæ; 25 vero Negativa. Cæteræ autem Cubicæ unica tantum Radice explicabiles juxta Regulas *Cardani* reſolvendæ ſunt, poſtquam demptus fuerit ſecundus terminus; nec video quo pacto minori calculo hoc negotium peragi poſſit. At ſi deſideretur Radix hæc in Quantitatibus, b, p, q, expreſſa, dico eam eſſe in prima formula, $\frac{1}{3}b$ + vel − ſumma vel differentia Radicum Cubicarum ex $\sqrt{\frac{1}{4}qq - \frac{1}{108}p^2b^2 + \frac{1}{27}b^3q - \frac{1}{6}bpq + \frac{1}{27}p^3} \pm \frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp$: *viz.* +, ſi $\frac{1}{27}b^3 + \frac{1}{2}q$ major ſit quam $\frac{1}{6}bp$, aliter −; Summa vero quoties $\frac{1}{3}bb$ major eſt quam p; ſin minor fuerit $\frac{1}{3}bb$, differentia. Inque cæteris formulis Radix ſemper conflatur ex iiſdem elementis, variatis tamen ſignis + & −, ut facile percipiet qui velit experiri.

Ope vero Tabulæ Logarithmicæ Sinuum Verſorum Radices hæ ſatis prompte inveniuntur: nempe ſi coefficientis Numeri ſint ſurdi vel fracti, ac Radices Numeris ineffabiles; ut plerumque fit. Hæc autem eſt Regula: in prima ac ſecunda formula, ſi $\frac{1}{3}bb$ minor ſit quam p; ſit $\frac{1}{3}p - \frac{1}{9}bb = d$, & poſita differentia inter $\frac{1}{6}bp$ & $\frac{1}{27}b^3 + \frac{1}{2}q$, hoc eſt HR, in prima, ac inter $\frac{1}{6}bp + \frac{1}{2}q$ & $\frac{1}{27}b^3$, in ſecunda, pro Radio; inveniatur Angulus cujus Tangens eſt $d\sqrt{d}$. Deinde ut Co-ſinus hujus Anguli, ad ejuſdem Sinum Verſum: Ita differentia pro Radio habita, ad quartum; cujus Latus cubicum triſecando Logarithmum habebitur: ac diviſo $\frac{1}{3}p - \frac{1}{9}bb$ per hoc Latus Cub. è Quoto ſubducatur Diviſor, Reſiduum erit quantitas Y & : Hujus Reſidui ac $\frac{1}{3}b$ ſumma, ſi centrum cadit ad dextram Axis, aliter differentia earundem, Radix erit quæſita. Quod ſi $\frac{1}{3}bb$ major ſit quam p, poſito HR pro Radio, ſit $d\sqrt{d}$, ſive diſtantia Paraboloidis ab Axe, Sinus Arcus cujuſdam; Hujus Sinus verſus ducatur in Radium, ſive $\frac{1}{6}bp - \frac{1}{27}b^3 \pm \frac{1}{2}q$, ac triſecto producti Logarithmo, habebitur ejus Latus Cubicum, per quod dividatur $\frac{1}{9}bb - \frac{1}{3}p$. Dico Quoti ac Diviſoris ſummam eadem lege additam vel ablatam ex $\frac{1}{3}b$, Radicem quæſitam exhibere. Ac par eſt Ratio in tertia ac quarta formulis, niſi quod $\frac{1}{27}b^3 + \frac{1}{6}bp \pm \frac{1}{2}q$ pro Radio aſſumenda eſt, ac $\frac{1}{9}bb + \frac{1}{3}p$ in $\sqrt{\frac{1}{9}bb + \frac{1}{3}p}$ ſive $d\sqrt{d}$ pro Sinu: Sed hæc præcepta exemplis fortaſſe melius percipientur.

Sit æquatio Cubica, $zzz - 17zz + 54z - 350$, ac quæratur Radix z: Hic $\frac{1}{3}bb$ major eſt quam p, ſed q major eſt quam Cubus ex $\frac{1}{3}b$, ideoque una tantum Affirmativa Radice explicabilis eſt. Jam $\frac{289}{9} - \frac{54}{3}$ eſt d, ac $\frac{127}{9}\sqrt{\frac{127}{9}}$ pro Sinu habenda eſt, ad Radium $\frac{4913}{27} + 175 - 153$, hoc eſt $\frac{5507}{27}$: Arcus vero competens fit 15 *gr.* 3. *m.* 49 *s.* Hujus Sinus Verſi Log. 8.5362376, additus Log. Radii 2.3095913, dat 0.8457889, cujus tertia pars 0.2819276 eſt Log. Radicis Cubicæ 1.91394, quo Diviſore diviſo, $\frac{127}{9}$ ſive d, fit Quotus 7.37281; Quoti ac Diviſoris ſumma, aucta editione $\frac{1}{3}b$, fit Radix quæſita, nempe 14,9534, &c.

Exactis Cubicis Biquadraticas jam aggrediamur. Hæ ſemper vel nullam, vel duas, vel quatuor Radices veras habent, quarum determinatio, partim à Coefficientibus, partim à ſigno & magnitudine Numeri abſoluti dati, pendet. In Conſtructione æquationis $z^4 - bz^3 + pzz - qz + r = 0$, fit $BD = \frac{1}{4}b$, $AB = \frac{1}{16}bb$, $BK = \frac{1}{2}$, ſive dimidio Lateris recti, $KC = 2AB = \frac{1}{8}bb$, $KE = \frac{1}{8}bb - \frac{1}{2}p$, $AE - \frac{1}{2} = \frac{3}{16}bb - \frac{1}{2}p$, $FE = \frac{1}{16}b^3 - \frac{1}{4}bp$, ac $EG = \frac{1}{16}b^3 - \frac{1}{4}bp + \frac{1}{2}q$; quo facto Circulus, Centro G, Radio $\sqrt{GD^2 - r}$, interſecabit Parabolam vel nullo, duobus, aut quatuor punctis, quæ perpendiculis in lineam HD Radices omnes z exhibent. Ut autem quatuor ſint, evidens eſt Centrum Circuli alicubi conſtitui debere intra ſpatium, de cujus puncto quovis tria perpendicula in Curvam Parabolæ demitti poſſint; atque ſimul Radium minorem eſſe maximo ex illis perpendiculis majorem vero medio. Quod ſi Centrum conſtituatur extra hoc ſpatium, ut non niſi una perpendicularis in Parabolam demitti poſſit, qua major ſit Radius; vel ſi minor ſit media ex tribus perpendicularibus, major vero quam minima ex illis, duæ tantum poſſunt eſſe Radices; nulla vero omnino datur, quoties Radius $\sqrt{GD^2 - r}$, minor eſt minima ex tribus, vel una illa, quoties una tantum eſt. Jam quale ſpatium hoc ſit, quibuſque limitibus diſcernitur, ac quibus conditionibus Radius Circuli minor vel major ſit prædictis perpendicularibus, nobis reſtat inquirendum; ac primum quo pacto perpendicularis in Parabolam demitti poſſit oſtendendum eſt. Fig. 49.

Sit ABC Parabola, AE Axis ejus, AV Semi-Latus Rectum, G punctum de quo demittenda eſt perpendicularis: Ducatur Axi perpendicularis GE, ac biſecetur VE in F, & erecta perpendiculari FH ad idem Axis latus, fiat $FH = \frac{1}{4}GE$; dico quod Circulus, Centro H, Radio HA deſcriptus, Para- Fig. 53.

bolam interſecabit in punctis tribus, vel uno, Z; ad quæ ductæ rectæ G Z Curvæ Parabolicæ perpendiculariter inſiſtunt.

Ut autem tres ſint hujuſmodi interſectiones, oportet Centrum Circuli H ita Fig. 51. collocari, ut ſit intra ſpatium Paraboloidibus incluſum; hoc eſt ut F H minor ſit quam $\surd \frac{8}{27} VF^3$, ſive FH^2 minus quam cubus ex $\frac{2}{3} VF$: atque adeo $GE = 4 FH$, minor erit quam $\surd \frac{8}{27} VF^3$, ſive $4 \surd \frac{1}{27} VE^3$, hoc eſt Quadratum ex G E minus erit quam $\frac{16}{27} VE^3$. Coincidunt itaque hi Limites cum Paraboloidibus duabus ejuſdem generis cum iis quibus in Cubicis uſi ſumus, ſed quarum Latus Rectum duplo minor eſt; *viz.* $\frac{27}{16}$ Lateris Recti Parabolæ, hoc eſt $\frac{27}{8}$ ipſius A V: ideoque ea ipſa eſt linea Curva cujus Evolutione generatur Parabola, ſic demonſtrante *Hugenio*; quamque ſemper con- Fig. 52. tingit linea D F, quæ Parabolæ perpendiculariter inſiſtit in puncto D. Punctum autem P, ſive in quo contingit recta D F Paraboloidem, Centrum eſt Circuli, qui Radio D P deſcriptus cum Parabola in puncto D coincidit, ſive ejuſdem Curvitatis eſt; ut per ſe ſatis conſtat.

Deſcriptis itaque hujuſmodi Paraboloidibus V X P, V N Δ, utrinque ab Axe; Fig. 52. perſpicuum eſt quod, niſi Centrum Circuli conſtituatur intra hos limites, non poſſit ille pluribus quam duobus in punctis Parabolam interſecare: unde determinare licet quibus ſub conditionibus Coefficientes terminorum intermediorum coercentur, in æquationibus Biquadraticis, ut habeantur quatuor Radices. Ac prima fronte clarum eſt p majorem eſſe non poſſe quam $\frac{3}{8} bb$, (*ſcil.* in formulis ubi habetur $+ p$) nec q quam $\frac{1}{16} b^3$. Generaliter vero $\frac{1}{16} b^3 \mp \frac{1}{4} pb \mp \frac{1}{2} q$, id eſt diſtantia Centri ab Axe E G, minor eſſe debet quam $EH = 4 \surd \frac{1}{27} VE^3$, hoc eſt (ob $VE = \frac{3}{16} bb \mp \frac{1}{2} p$) quam $\frac{1}{4} bb \mp \frac{2}{3} p \surd \overline{\frac{1}{16} bb \mp \frac{1}{6} p}$; ſignis + & − in dubio relictis, ut ſecundum æquationis cujuſvis naturam variari poſſint; quemadmodum in Cubicis ſuperius oſtenſum eſt.

Termini autem ultimi r limitatio eadem facilitate inveniri nequit; id adeo, quia Problema ſit Solidum in Curvam Parabolæ demittere perpendicularem, quodque non ſine ſolutione æquationis Cubicæ reſolvi poſſit. Itaque primo loco deficiat ſecundus terminus, vel ſi adfuerit tollatur, ut æquatio habeat formulam, $z^4 * . p z^2 . q z . r . = 0$. Ac ſi fuerit $- r$, ſemper duabus vel quatuor Radicibus explicari poteſt; ut autem quatuor ſint, oportet Centrum Circuli infra Paraboloides prædictas conſtitui, ſive ut ſit $- p$, ac qq minus quam $\frac{8}{27}p^3$, ſive cubo ex $\frac{2}{3}p$. Deinde habeantur Radices æquationis hujus $y^3 . * . \frac{1}{2}py . \frac{1}{4}q = 0$, quantitatibus p & q iiſdem ſignis annexis quibus in Biquadratica. Hæ autem Radices auxilio Tabulæ Sinuum ſatis expedite inveniuntur. Inventis autem tribus illis y, (quæ ſunt ordinatim applicatæ ad Axem Parabolæ, de punctis ubi incidunt perpendicula in Curvam ejus *ſcil.* ZY) $pyy - 3y^4$ ex minore y, quantitatem maximam r deſignabit, ſi fuerit $-r$; qua ſi minor fuerit r, æquatio quatuor habebit Radices, aliter duas. Aſt ſi fuerit $+r$, oportebit eam minorem eſſe quam $3y^4 - pyy$ ex media y, nam ſi major ſit, non niſi duas habere poteſt Radices, ſaltem ſi minor ſit r, quam $3y^4 - pyy$ ex maxima y. Hac vero ſi major ſit, nulla omnino Radice vera explicabilis eſt æquatio. Hi vero iidem Limites aliter deſignantur ex quantitate q, *ſcil.* $\frac{1}{2}qy - y^4$ in primo caſu; $y^4 - \frac{1}{2}qy$ in ſecundo, ac $y^4 + \frac{1}{2}qy$ in tertio. Fig. 53.

Fieri autem poteſt ut duæ minores quantitates y non longe diſtant ab invicem, unde evenit quod utraque ex perpendicularibus major ſit quam recta GA, *ſcil.* cum qq majus ſit quam $\frac{4}{27}p^3$, minus vero quam $\frac{8}{27}p^3$; cadente centro intra ſpatium Paraboloidibus (utriuſque Figuræ 51 & 52) interjectum. Hoc in caſu, ſi fuerit $+r$, non niſi duæ poſſunt eſſe Radices, exiſtente $y^4 + \frac{1}{2}qy$ ex maxima y, major quam r; aliter nulla. At ſi $\frac{1}{2}qy - y^4$ ex minima y, major fuerit quam r ſigno $-$ notata, r vero major quam $\frac{1}{2}qy - y^4$ ex media y, tunc habentur quatuor radices; at duæ tantum, ſi vel major priore vel minor poſteriore inventa ſit r.

Si vero in æquatione fuerit $+p$, vel ſi ſit $-p$ & qq majus fuerit quam $\frac{8}{27}p^3$, æquatio $y^3 * . \frac{1}{2}py . \frac{1}{4}q$, unica tantum explicatur Radice y; hoc eſt, una tantum perpendicularis de Centro Circuli demitti poteſt: unde certe concluditur duas tantum Radices haberi poſſe in æquatione data, quarum Summa, ſi fuerit $-r$, cum quantitate r augetur; at ſi habeatur $+r$, obtenta quantitate y, quantitas illa r minor eſſe debet quam $y^4 + \frac{1}{2}qy$; nam ſi ea major ſit, æquatio propoſita abſurda & impoſſibilis eſt.

Longum & ſuperfluum eſſet omnes hujus ſenſus æquationes percurrere, cum ex jam dictis attendenti ſatis evidens ſit, quæ Negativæ quæ Affirmativæ ſint; atque quod Radicum harum Limites ex quantitatibus inventis y petantur. In exemplum vero, quod cuivis in cæteris imitari licet, proponantur indagandi Limites ſive Conditiones ſub quibus in Æquatione Biquadratica 4 Radices Affirmativæ dari poſſint. Hoc autem fit quoties Centrum Circuli G, ponitur Fig. 52.

in ſpatio V P K, ac ſimul habetur $+ r$, ſive Circuli Radius minor quam G D: Unde patet, æquationem de qua agitur hujus eſſe formulæ, $z^4 - bz^3 + pz^2 - qz + r = 0$; p vero majorem eſſe non poſſe quam $\frac{3}{8}bb$, nec $\frac{1}{4}pb$, hoc in caſu, quam $\frac{1}{16}b^3 + \frac{1}{2}q$; deinde opus eſt ut $\frac{1}{4}bb - \frac{2}{3}p$ in $\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$ major ſit quam $\frac{1}{16}b^3 + \frac{1}{2}q - \frac{1}{4}pb$; & ex his Limitibus certo conſtabit Centrum intra ſpatium V P K inveniri. Ut vero definiatur quantitas r, ſolvenda primum eſt Cubica, $y^3.\ *.\ - \overline{\frac{3}{16}b^2 - \frac{1}{2}p}\,y = \frac{1}{32}b^3 + \frac{1}{4}q - \frac{1}{8}pb$; & habebuntur puncta, in quæ perpendiculares de Centro in Curvam Parabolæ cadunt.

Inventis autem tribus valoribus hujus y, r minor eſſe debet quam $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}b^2yy + pyy$ ex media y, major vero quam $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}b^2yy + pyy$ ex minima y. Hos vero Limites ſi excedat r, non niſi duæ Radices haberi poſſunt. Deniq; ſi $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^4 - \frac{3}{8}bbyy + pyy$ ex maxima y, minor fuerit quam r, æquatio propoſita impoſſibilis eſt.

Accidit etiam ut quatuor ſint Affirmativæ, cum Centrum G conſtituitur in ſpatiolo V T S, ducta *ſcil.* R T S perpendiculari in medium ſuppoſitæ lineæ A D: hoc autem fit cum p major eſt quam $\frac{5}{16}bb$, ac $\frac{1}{4}bb - \frac{2}{3}p\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$ major quam $\frac{1}{8}pb - \frac{5}{128}bbb - \frac{1}{2}q$. Quo in caſu ſemper duæ, aliquando tres, ex Radicibus fiunt majores quam $\frac{1}{4}b$.

Fig. 52. Notandum vero hic limitem illum ex minima y productum, aliquando negativum fieri, ſive minorem nihilo; quoties *ſcil.* maxima ex tribus perpendicularibus major eſt quam G D. Hoc, ſi acciderit quantitas $+ r$, à Limite præſcripto ex media y, in nihilum minui poteſt. Defectus vero Limitis ex minima y monſtrat quanta poſſit eſſe $- r$ in æquatione, ſi habeantur tres Radices Affirmativæ ac una Negativa; quam ſi excedat, non niſi duæ, altera Affirmativa, altera Negativa, dari poſſunt. Hæc autem omnia demonſtrantur ex eo quod prædicti Limites quantitatis r, ſint differentiæ Quadratorum lineæ G D & perpendicularium in Curvam Parabolæ.

Ob perplexas vero cautiones, quas parit in æquationibus hiſce ſignorum diverſitas, præſtat ſemper ſecundum terminum tollere, ac deinde juxta præcepta jam tradita Radicum numerum ac ſigna inquirere; præſertim ſi quantitates illæ y non ab invicem. Ex quatuor autem hiſce Radicibus Affirmativis, duæ ſemper ſunt minores quam $\frac{1}{4}b$, duæ vero majores; nempe ſi D G, minor ſit quam A G, ſive $\frac{1}{4}pb$ quam $\frac{3}{64}b^3+q$. Tres autem minores ſunt quam $\frac{1}{4}b$ quoties perpendicularis media, ſive ex media y inventa, major eſt quam A G, ſive $\frac{3}{8}b, b, y$ major quam $3y^3-pyy$ ex eadem media y; Quarta vero & maxima Radix major eſt quam maxima $y+\frac{1}{4}b$; æquatur autem differentiæ ipſius b & ſummæ cæterarum trium Radicum, ideoque minor eſt b. Sed jam manum de Tabula; Fortaſſis illi qui naturam Parabolæ penitius perſpectam habent, majori compendio hæc omnia peragere valebunt; at ſi quantitates hæ omnes $b.\ p.\ q$ & r, abſque reſolutione Cubicæ æquationis rite determinari poſſint, non ſine cauſa ambigitur; quæcunque enim æquationibus planis hac in re fiunt, non veros Limites, ſed Approximationes tantum exhibent.

The Extraction of all Roots without any previous Reduction; *by* Mr. Ed. Halley. N. 210. *p.* 136. May. An. 1694.

XX. Regulas binas compendioſas admodum pro Approximatione Radicis Cubicæ nuper protulit *D. de Lagney*, alteram rationalem, alteram irrationalem; nempe Cubi $aaa+b$ latus eſſe inter $a+\frac{ab}{3a^3+b}$ ac $\sqrt{\frac{1}{4}aa+\frac{b}{3a}}+\frac{1}{2}a$. Radicem autem poteſtatis Quintæ a^5+b ſic exprimit $=\frac{1}{2}a+\sqrt{\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}-\frac{1}{4}aa}$ (non $\frac{1}{2}aa$ ut perperam legitur in libro Gallico impreſſo). Demonſtrantur autem Regulæ prædictæ ex Geneſi Cubi & Poteſtatis quintæ. Poſito enim Latere Cubi cujuſque $a+e$, Cubus inde conflatus fit $aaa+3aae+3aee+eee$, adeoque ſi ſupponatur aaa numerus Cubus proxime minor dato quovis non Cubo, eee minor erit Unitate, ac reſiduum ſive b æquabitur reliquis Cubi membris $3aae+3aee+eee$: rejectoque eee ob parvitatem, $b=3aae+3aee$. Cumque aae multo majus ſit quam aee, $\frac{b}{3aa}$ non multum excedet ipſam e, poſitoque $e=\frac{b}{3aa}$, $\frac{b}{3aa+3ae}$, cui proxime æquatur quantitas e, invenietur $=\frac{b}{3aa+\frac{3ab}{3aa}}$ ſive $\frac{b}{3aa+\frac{b}{a}}$: hoc eſt $\frac{ab}{3aaa+b}=e$, adeoque latus Cubi $aaa+b$ habebitur $a+\frac{ab}{3aaa+b}$, quæ eſt ipſa formula rationalis *D. de Lagney*. Quod ſi aaa fuerit Numerus Cubus proxime major dato, Latus Cubi $aaa - b$, pari ratiocinio invenietur $a - \frac{ab}{3aaa - b}$; atque hæc Radicis Cubicæ approximatio ſatis expedita ac facilis parum admodum fallit in defectu, cum ſcilicet e reſiduum Radicis hoc pacto inventum paulo minus juſto ſit. Irrationalis vero formula etiam ex eodem fonte derivatur, *viz.* $b = 3aae + 3aee$, ſive $\frac{b}{3a} = ae + ee$; adeoque $\sqrt{\frac{1}{4}aa + \frac{b}{3a}} = \frac{1}{2}a + e$, atque $\sqrt{\frac{1}{4}aa + \frac{b}{3a}} + \frac{1}{2}a = a + e$, ſive Radici quæſitæ. Latus vero Cubi $aaa - b$ eodem modo habebitur $\frac{1}{2}a + \sqrt{\frac{1}{4}aa - \frac{b}{3a}}$. Atque hæc quidem formula aliquanto propius ad ſcopum collimat, in exceſſu peccans ſicut altera in defectu, ac ad praxim magis commoda videtur, cum reſtitutio Calculi nihil aliud ſit quam continua additio vel ſubductio ipſius $\frac{eee}{3a}$, ſecundum ac quantitas e innoteſcat; ita ut potius ſcribendum ſit $\sqrt{\frac{1}{4}aa + \frac{b - eee}{3a}} + \frac{1}{2}a$ in priori caſu, ac in poſteriori $\frac{1}{2}a + \sqrt{\frac{1}{4}aa + \frac{eee - b}{3a}}$. Utraque autem formula Ciphræ jam cognitæ in Radice extrahenda ad minimum triplicantur, quod quidem Arithmeticæ ſtudioſis omnibus gratum fore confido, atque ipſe Inventori abunde gratulor. Ut autem harum Regularum utilitas melius ſentiatur, exemplum unum vel alterum adjungere placuit.

Exemp. I. Quæratur Latus Cubi dupli, ſive $aaa + b = 2$. Hic $a = 1$ atque $\frac{b}{3a} = \frac{1}{3}$, adeoque $\frac{1}{2} + \sqrt{\frac{7}{12}}$ ſive 1, 26 invenietur Latus prope verum. Cubus autem ex 1, 26 eſt 2, 000376, adeoque $0,63 + \sqrt{0,3969 - \frac{0,000376}{3,78}}$ ſive $0,63 + \sqrt{0,3968005291005291} = 1,259921049895 -$; quod quidem tredecim figuris Latus Cubi dupli exhibet, nullo fere negotio, *viz.* una Diviſione & Lateris Quadrati extractione, ubi vulgari operandi modo quantum deſudaſſet Arithmeticus norunt experti. Hunc etiam calculum quouſque velis continuari licet, augendo quadratum additione $\frac{eee}{3a}$. Quæ quidem correctio hoc in caſu non niſi unitatis in Radicis figura decima-quarta augmentum affert.

Exemplum II. Quæratur Latus Cubi æqualis menſuræ Anglice *Gallon* dictæ, uncias ſolidas 231 continentis. Cubus proxime minor eſt 216 cujus Latus $6 = a$, ac reſiduum $15 = b$; adeoque pro prima approximatione provenit $3 + \sqrt{9 + \frac{5}{6}} =$ Radici. Cumque $\sqrt{9,\ 8333\ldots}$ ſit 3, 1358... patet $6,\ 1358 = a + e$. Supponatur jam $6,\ 1358 = a$, & habebimus Cubum ejus 231, 00085 38947 12, ac juxta regulam $3,\ 0679 + \sqrt{9,\ 41201041 - \frac{0,\ 00085 38947 12}{18,\ 4074}}$ æquatur accuratiſſime Lateri Cubi dati, id quod intra horæ ſpatium calculo obtinui 6, 135792439661 95897, in octodecima figura juſtum, at deficiens in decima-nona. Hæc vero formula merito præferenda eſt rationali, ob ingentem diviſorem, non ſine magno labore tractandum; cum Lateris Quadrati extractio multo facilius procedat, ut experientia multiplex me docuit.

Regula autem pro Radice Surſolidi puri, ſive Poteſtatis quintæ, paulo altioris indaginis eſt, atque etiam adhuc multo perfectius rem præſtat: datas enim in Radice ciphras ad minimum quintuplicat, neque etiam multi nec operoſi eſt calculi. Author autem nullibi inveniendi methodum ejuſve demonſtrationem concedit, etiamſi maxime deſiderari videatur: præſertim cum in Libro impreſſo non recte ſe habeat; id quod imperitos facile illudere poſſit. Poteſtas autem quinta Lateris $a + e$ conficitur ex his membris $a^5 + 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4 + e^5 = a^5 + b$, unde $b = 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4$, rejectio e^5 ob parvitatem ſuam: quo-circa $\frac{b}{5a} = a^3e = 2a^2e^2 + 2ae^3 + e^4$, atque utrinque addendo $\frac{1}{4}a^4$ habebimus $\sqrt{\frac{1}{4}aaaa + \frac{b}{5a}} = \sqrt{\frac{1}{4}a^4 + a^3e + 2a^2e^2 + 2ae^3 + e^4} = \frac{1}{2}aa + ae + ee$. Dein utrinque ſubducendo, $\frac{1}{4}aa$, $\frac{1}{2}a + e$ æquabitur $\sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa}$, cui ſi addatur $\frac{1}{2}a$, erit $a + e = \frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa} =$ Radici poteſtatis $a^5 + b$. Quod ſi fuiſſet $a^5 - b$, (aſſumpta a juſto majore) regula ſic ſe haberet, $\frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a - \frac{b}{5a}} - \frac{1}{4}aa}$.

Atque hæc regula mirum in modum approximat, ut vix reſtitutione opus ſit; at dum hæc mecum penſitavi, incidi in formularum methodum quandam generalem pro quavis poteſtate ſatis concinnam, quamque celare nequeo; cum etiam in ſuperioribus poteſtatibus datas radices figuras triplicare valeant.

Hæ autem formulæ ita ſe habent tam rationales quam irrationales.

$$\sqrt{aa+b} = \sqrt{aa+b}, \text{ vel } a + \frac{ab}{2aa+\frac{1}{2}b}$$

$$\sqrt[3]{a^3+b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa + \frac{b}{3a}}, \text{ vel } a + \frac{ab}{3a^3+b}$$

$$\sqrt[4]{a^4+b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa + \frac{b}{6aa}}, \text{ vel } a + \frac{ab}{4a^4+\frac{3}{2}b}$$

$$\sqrt[5]{a^5+b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa + \frac{b}{10a^3}}, \text{ vel } a + \frac{ab}{5a^5+2b}$$

$$\sqrt[6]{a^6+b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa + \frac{b}{15a^4}}, \text{ vel } a + \frac{ab}{6a^6+\frac{5}{2}b}$$

$$\sqrt[7]{a^7+b} = \tfrac{5}{6}a + \sqrt{\tfrac{1}{36}aa + \frac{b}{21a^5}}, \text{ vel } a + \frac{ab}{7a^7+3b}$$

Et ſic de cæteris etiam adhuc ſuperioribus. Quod ſi aſſumeretur a radice quæſita major, (quod cum fructu ſit quoties Poteſtas reſolvenda multo propior ſit poteſtati Numeri integri proxime majoris quam proxime minoris) mutatis mutandis eædem radicum expreſſiones proveniunt.

$$\sqrt{aa-b} = \sqrt{aa-b}, \text{ vel } a - \frac{ab}{2aa-\frac{1}{2}b}$$

$$\sqrt[3]{a^3-b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa - \frac{b}{3a}}, \text{ vel } a - \frac{ab}{3a^3-b}$$

$$\sqrt[4]{a^4-b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa - \frac{b}{6aa}}, \text{ vel } a - \frac{ab}{4a^4-\frac{3}{2}b}$$

$$\sqrt[5]{a^5-b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa - \frac{b}{10a^3}}, \text{ vel } a - \frac{ab}{5a^5-2b}$$

$$\sqrt[6]{a^6-b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa - \frac{b}{15a^4}}, \text{ vel } a - \frac{ab}{6a^6-\frac{5}{2}b}$$

$$\sqrt[7]{a^7-b} = \tfrac{5}{6}a + \sqrt{\tfrac{1}{36}aa - \frac{b}{21a^5}}, \text{ vel } a - \frac{ab}{7a^7-3b}$$

Atque inter hos duos terminos ſemper conſiſtit vera radix, aliquanto propior irrationali quam rationali; *e* vero juxta formulam irrationalem inventa, ſemper peccat in exceſſu, ſicut in defectu à rationali formula reſultans.

tans Quotus; adeoque ſi fuerit $+ b$, Irrationalis majorem juſto exhibet Radicem, rationalis minorem; è contrario vero ſi fuerit $- b$. Atque hæc de eliciendis Radicibus è Poteſtatibus puris dicta ſunto, quæ quidem ad uſus ordinarios ſufficientes multo facilius habentur ope Logarithmorum: quoties vero ultra Tabularum Logarithmicarum vires accuratiſſime definienda eſt radix, ad hujuſmodi methodos neceſſario recurrendum eſt. Præterea cum ex harum formularum inventione ac contemplatione, Univerſalis Regula pro Æquationibus Affectis (quam non ſine fructu Geometriæ ac Algebræ ſtudioſis omnibus uſurpandam confido) mihi ipſi oblata ſit, volui ipſius inventi primordia qua poſſim claritate aperire.

Æquationum quidem Affectarum Quadrato-quadratum non excedentium Conſtructionem generalem concinnam admodum ac facilem, *An. circiter* 1687. jam tum inventam, publici juris feci: ex quo ingens cupido animum inceſſit, idem Numeris efficiendi. At brevi poſt D. *Raphſon* magna ex parte voto ſatisfeciſſe viſus eſt, uſque dum *D. de Lagney* etiam adhuc compendioſius rem peragi poſſe hoc ſuo libello mihi ſuggeſſit. Methodus autem noſtra hæc eſt. Vid. ſup. Sect. XIX. 1.

Supponatur Radix cujuſvis æquationis z compoſita ex partibus $a +$ vel $- e$, quarum a ex hypotheſi aſſumatur ipſi z quantum fieri poſſit propinqua (quod tamen commodum eſt, non neceſſarium) & ex quantitate $a +$ vel $- e$ formentur Poteſtates omnes ipſius z in Æquatione inventas, iiſque affigantur Numeri Coefficientes reſpective: deinde Poteſtas Reſolvenda ſubducatur è ſumma partium datarum in prima columna, ubi e non reperitur, quam Homogeneum Comparationis vocant, ſitque differentia $+ b$. Dein habeatur ſumma omnium Coefficientium ipſius lateris e in ſecunda Columna, quæ ſit s; denique in tertia addantur omnes coefficientes quadrati ee, quarum ſummam vocemus t, ac Radix quæſita z, formula rationali habebitur $= a +$ vel $- \dfrac{sb}{ss + \text{vel} - tb}$; irrationali vero fiet $z = a \dfrac{\overline{+} \frac{1}{2}s \overline{+} \sqrt{\frac{1}{4}ss \overline{+} bt}}{t}$, id quod exemplis illuſtrare fortaſſe operæ pretium erit. Inſtrumenti vero loco ad ſit Tabella, Poteſtatum omnium ipſius $a +$ vel $- e$ Geneſin exhibens, quæ ſi opus fuerit continuari facile poſſit. A ſeptima vero incipiam cum pauca Problemata eo uſque aſſurgere deprehendantur. Hanc Tabellam jure optimo *Speculum Analyticum Generale* appellare licet. Poteſtates autem prædictæ ex continua multiplicatione per $a + e = z$ ortæ, ſic proveniunt, cum ſuis coefficientibus adjunctis.

$$\begin{array}{rcl@{\;}c@{\;}l@{\;}c@{\;}l@{\;}c@{\;}l@{\;}c@{\;}l}
 & & & & s & & t & & u & & w \\
lz^7 & = & la^7 & + & 7ta^6e & + & 21la^5ee & + & 35la^4e^3 & + & 35la^3e^4 \\
kz^6 & = & ka^6 & + & 6ka^5e & + & 15ka^4ee & + & 20ka^3e^3 & + & 15ka^2e^4 \\
bz^5 & = & ba^5 & + & 5ba^4e & + & 10ba^3ee & + & 10ba^2e^3 & + & 5bae^4 \\
gz^4 & = & ga^4 & + & 4ga^3e & + & 6ga^2ee & + & 4gae^3 & + & ge^4 \\
fz^3 & = & fa^3 & + & 3fa^2e & + & 3faee & + & fe^3 & & \\
dz^2 & = & da^2 & + & 2dae & + & dee & & e^3 & & \\
cz & = & ca & + & ce & + & & & & &
\end{array}$$

$$\begin{array}{c@{\;}l@{\;}c@{\;}l@{\;}c@{\;}l}
 & x & & y & & \\
+ & 21la^2e^5 & + & 7lae^6 & + & le^7 \\
+ & 6kae^5 & + & ke^6 & & \\
+ & be^5 & & & &
\end{array}$$

Quod ſi fuerit $a - e = z$, ex iiſdem membris conficitur Tabella, negatis ſolummodo imparibus Poteſtatibus ipſius e, ut ee^3, e^5, e^7: & affirmatis paribus e^2, e^4, e^6. Sitque ſumma Coefficientium lateris $e = s$: Summa Coefficientium Quadrati $ee = t$; Cubi $eee = u$; Biquadrati $e^4 = w$; Surſolidi $e^5 = x$; Summa vero Coefficientium Cubo-cubi $e^6 = y$; &c. Cum autem ſupponatur e exigua tantum pars Radicis inquirendæ, omnes poteſtates ipſius e multo minores evadunt ſimilibus ipſius a Poteſtatibus, adeoque pro prima hypotheſi rejiciantur ſuperiores (ut in poteſtatibus puris oſtenſum eſt) ac formata æquatione nova, ſubſtituendo $a \pm e = z$ habebimus ut diximus $\pm b = \pm se \pm tee$. Cujus rei cape exempla ſequentia, quo melius intelligatur.

Exemp. I. Proponatur æquatio $z^4 - 3zz + 75z = 10000$. Pro prima hypotheſi ponatur $a = 10$, ac conſequenter prodibit æquatio.

$$\begin{array}{rl}
z^4 = & + a^4 \pm 4a^3 + 6a^2ee \pm 4ae^3 + e^4 \\
- dz^2 = & - da^2 \pm dae - dee \\
+ cz = & + ca \pm ce \\
= & + 10000 \pm 4000e + 600ee \pm 40e^3 + e^4 \\
 & - 300 \pm 60e - 3ee \\
 & + 750 \pm 75e \\
 & - 10000 \\
\hline
 & + 450 \pm 4015e + 597ee + 40e^3 + e^4 = 0 \\
 & \qquad\quad s \qquad\quad t \qquad\quad u
\end{array}$$

Signis + ac − (reſpectu e & e^3) in dubio relictis, uſque dum ſciatur an e ſit Negativa vel Affirmativa; quod quidem aliquam pariet difficultatem, cum in æquationibus plures radices admittentibus, ſæpe augeantur Homogenia Comparationis, ut appellant, à minuta quantitate a, ac e contra ea aucta minuatur. Determinatur autem ſignum ipſius e ex ſigno quantitatis b; ſublata enim Reſolvenda ex Homogenio ab a formato, ſignum ipſius se, ac proinde partium in ejus compoſitione prævalentium, ſemper contrarium erit ſigno differentiæ b. Unde patebit an fuerit $-e$ vel $+e$, ſive an a major vel minor radice vera aſſumpta. Ipſa autem e ſemper æquatur $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, quoties b ac t eodem ſigno notantur; quoties vero diverſo ſigno connectuntur, eadem e fit $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{4}s}{t}$. Poſtquam vero compertum ſit fore $-e$, in affirmatis æquationis membris negentur e, e^3, e^4, &c. in negatis affirmentur; ſcribantur *ſcil.* ſigno contrario; ſi vero fuerit $+e$, affirmentur in affirmatis, negentur in negatis. Habemus autem in hoc noſtro exemplo 10450 loco Reſolvendæ 10000, ſive $b = +450$, unde conſtat a majorem juſto aſſumptam, ac proinde haberi $-e$: Hinc æquatio fit $10450 - 4015e + 597ee - 4e^3 + e^4 = 10000$. Hoc eſt $450 - 4015e + 597ee = 0$. Adeoque $450 = 4015e - 597ee$ ſive $b = se - tee$ cujus Radix e fit $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, vel ſi mavis $\frac{s}{2t} - \sqrt{\frac{ss}{4tt} - \frac{b}{t}}$, id eſt, in præſenti caſu, $e = \frac{2007\frac{1}{2} - \sqrt{3761406\frac{1}{4}}}{597}$, unde provenit radix quæſita prope verum 9, 886. Hoc vero pro ſecunda Hypotheſi ſubſtituto, emergit $a + e = z$ accuratiſſime 9,88626039364 95 . . . ., in ultima figura vix binario juſtum ſuperans; nempe cum $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$. Atque hoc etiam, ſi opus fuerit, multo ulterius verificari poſſit, ſubducendo $\frac{\frac{1}{2}ne^3 + \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss + tb}}$ ſi fuerit $+e$, vel addendo $\frac{\frac{1}{2}ne^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - tb}}$, radici prius inventæ, ſi ſit $-e$. Cujus compendium eo pluris æſtimandum quod quandoque, ex ſola prima ſuppoſitione, ſemper vero ex ſecunda, iiſdem conſervatis coefficientibus quouſque velis calculum continuare poſſis. Cæterum æquatio prædicta etiam negativam habet radicem, *viz.* $z = 10, 26$ . . . . . . quam cuilibet accuratius expiſcari licet.

Exemp. II. Sit $z^3 - 17zz + 54z = 350$; ac ponatur $a = 10$. Ex præſcripto Regulæ,

$$\begin{array}{rcl} zzz & = & aaa + 3aae + 3aee + eee \\ -dzz & = & daa - 2dae - dee \\ +cz & = & ca + ce \end{array}$$

$$\begin{array}{lllll} & & b & s & t \\ \text{Id eſt} & + & 1000 & + 300e & + 30ee + eee \\ & - & 1700 & - 340e & - 17ee \\ & + & 540 & + 54e & \\ & - & 350 & & \\ \hline \end{array}$$

$$\text{Sive } - 510 + 14e + 13ee + eee = 0.$$

Cum autem habere -510, conſtat a minorem juſto aſſumi, ac proinde e affirmativam eſſe, ac ex $510 = 14e + 13ee$ fit $\frac{\sqrt{bt + \frac{1}{4}ss} - \frac{1}{2}s}{t} = e = \frac{\sqrt{6679} - 7}{13}$, unde z fit $15,7\ldots$ quæ nimia quidem eſt ob late ſumptam a; ideo ſupponatur ſecundo $a = 15$, ac pari ratiocinio habebimus $e = \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - tb}}{t} = \frac{109\frac{1}{2} - \sqrt{11710\frac{1}{4}}}{28}$; ac proinde $z = 14,954068$. Quod ſi calculum adhuc tertio reſtaurare velis, uſque in vigeſimam quintam figuram vero conformem invenies radicem: Paucioribus vero contentus, ſcribendo $tb \pm teee$ loco tb, vel ſubtrahendo aut addendo radici prius inventæ $\frac{\frac{1}{2}eee}{\sqrt{\frac{1}{4}ss \mp tb}}$ ad ſcopum ſtatim perveniet. Æquatio vero propoſita nulla alia radice explicari poteſt, quia Poteſtas Reſolvenda 350 major eſt Cubo ex $\frac{17}{3}$ vel $\frac{1}{3}d$.

Exemp. III. Sit Æquatio illa quam in Reſolutione difficillimi Problematis Arithmetici adhibet *Clariſſimus Walliſius*, Cap. LXII. *Algebræ* ſuæ, quo radicem *Vietæ* Methodo accuratiſſime quidem aſſecutus eſt. Eandemque exemplum Methodi ſuæ affert laudatus D. *Raphſon*, pag. 25, 26. nempe $z^4 - 80z^3 + 1998z^2 - 14937z + 5000 = 0$. Hæc autem æquatio ejus formulæ eſt, ut plures habeat radices Affirmativas, ac quod difficultatem ejus augeat, prægrandes ſunt Coefficientes reſpectu Reſolvendæ datæ: Quo melius autem tractetur, dividatur, ac juxta notas punctationum regulas ponatur $-z^4 + 8z^3 - 20z^2 + 15z = 0,5$ (ubi z eſt $\frac{1}{10}z$ in æquatione propoſita) ac pro prima Hypotheſi habeamus $a = 1$. Proinde $+ 2 - 5e - 2ee + 4e^3 - e^4 - 0{,}5 = 0$.

Hoc eſt $1\frac{1}{2} = 5e + 2ee$; hinc $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$ fit $\frac{\sqrt{37} - 5}{4}$ adeoque $z = 1{,}\ 27$, unde conſtat $12{,}\ 7$ radicem eſſe æquationis propoſitæ vero vicinam. Secundo loco ſupponatur $z = 12{,}\ 7$, ac juxta præſcriptum Tabellæ Poteſtatum oritur.

$$\begin{array}{lllll}
b & s & t & u & \\
-\ 26014{,}\ 4641 & -\ 8193{,}\ 532\,e & -\ 967{,}\ 74\,ee & -\ 50{,}\ 8\,e^3 & -\ e^4 \\
+\ 163870{,}\ 640. & +\ 38709{,}\ 60.\,e & +\ 3048{,}..\,ee & +\ 80{,}\ .\,e^3 & \\
-\ 322257{,}\ 42.. & -\ 50749{,}\ 2..\,e & -\ 1998{,}..\,ee & & \\
+\ 189699{,}\ 9... & +\ 14937{,}...\,e & & & \\
-\ 5000{,}\ .... & & & & \\
\hline
+\ 298{,}\ 6559 & -\ 5296{,}\ 132\,e & +\ 82{,}\ 26\,ee & +\ 29{,}\ 2\,e^3 & -\ e^4 = 0
\end{array}$$

Adeoque $- 298{,}\ 6559 = - 5296{,}\ 132e + 82{,}\ 26ee$, cujus radix e juxta regulam $= \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$ fit $\frac{2648{,}\ 066 - \sqrt{6987686{,}\ 106022}}{82{,}\ 26} =$ $0{,}\ 05644080331\ .... = e$ minori vero: ut autem corrigatur, $\frac{\frac{1}{2}ue^3 - e^4}{\sqrt{\frac{1}{4}ss - bt}}$ ſive $\frac{0{,}\ 0026201\ .....}{2643{,}\ 423\ ......}$ fit $0{,}\ 00000099117$; ac proinde e correcta $=$ $0{,}05644179448$; Quod ſi adhuc plures radicis figuras deſideras, formetur ex e correcta $tue^3 - te^4 = 0{,}\ 4310560242 3\ ...$, ac $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt - tue^3 + te^4}}{t}$ ſive $\frac{2648{,}\ 066 - \sqrt{6987685{,}\ 6749659757 7\ .....}}{82{,}\ 26} =$ $0{,}\ 05644179448074402 = e$, unde $a + e = z$ radix accuratiſſima fit $12{,}\ 75644179448074402\ ....$ qualem invenit *Cl. Walliſius* in loco citato. Ubi obſervandum, redintegrationem calculi ſemper triplicare notas veras in aſſumpta a, quas prima correctio ſive $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - bt}}$ quintuplices reddit, quæque etiam commode per Logarithmos efficitur. Altera autem correctio poſt primam, etiam duplum Ciphrarum numerum adjungit, ut omnino aſſumptas ſeptuplicet; prima tamen plerumque uſibus Arithmetices abunde ſufficit. Quæ vero dicta ſunt de numero Ciphrarum in radice recte aſſumptarum, ita intelligi velim, ut cum a non niſi decima parte diſtet à vera radice, prima figura recte aſſumatur; ſi intra centeſimam partem, duæ primæ; ſi intra milleſimam, tres priores rite ſe habeant; quæ deinde juxta noſtram regulam tractatæ ſtatim novem evadunt.

Reſtat jam ut nonnulla adjiciam de noſtra formula rationali, *viz.* $e = \frac{sb}{ss \pm tb}$, quæ quidem ſatis expedita videbitur, nec multum cedet priori, cum etiam datas ciphras triplicare valeat. Formata autem æquatione ex $a \mp e = z$, ut prius, ſtatim patebit an a aſſumpta ſit major vel minor vero, cum *ſcil.* se ſigno ſemper notari debeat contrario ſigno differentiæ Reſolvendæ ac Homogenii ſui ex a producti. Deinde poſito quod $+ b \mp se +$ vel $- tee = 0$. Diviſor fit $ss - tb$, quoties b ac t iiſdem ſignis notantur; idem vero fit $ss + bt$, ſi ſigna diverſa ſint. Praxi autem magis accommodata videtur ſi ſcriberetur Theorema, $e = \frac{b}{s \pm \frac{tb}{s}}$ nempe cum una multiplicatione ac duabus diviſionibus res peragatur, quæ tres multiplicationes ac unam diviſionem alias requireret. Hujus etiam Methodi Exemplum capiamus à prædictæ Equationis radice 12, 7. . . : ubi

$$\underset{+b}{298,6559} - \underset{-s}{5296,132}\,e + \underset{+t}{82,26}\,ee + \underset{+u}{29,2}\,e^3 - e^4 = 0,$$

adeoque $\frac{b}{s - \frac{tb}{s}} = e$, hoc eſt, fiat ut s ad t ita b ad $\frac{tb}{s} = 5296,132)\ 298,6559$ in $82,26\ (4,63875\ldots$; quocirca diviſor fit, $s = \frac{tb}{s} = 5291,49325\ldots.)\ 298,6559\ (0,056441\ldots.. = e$, *viz.* quinque figuris veris adjectis radici aſſumptæ. Corrigi autem nequit hæc formula ſicut præcedens irrationalis; adeoque ſi plures deſiderentur radicis figuræ, præſtat aſſumpta nova Hypotheſi calculum de integro repetere: ac novus Quotus triplicando figuras in radice cognitas ſupputatori etiam maxime ſcrupuloſo abunde ſatisfaciet.

A Method of Raiſing an Infinite Multinomial, to any given Power; by M. de Moivre. N. 230. p. 619. July. An. 1697.

XXI. *Theorem.*] $\overline{az + bz^2 + cz^3 + dz^4 + ez^6 + fz^6 + gz^7 + hz^8 + iz^9\ \&c.}|^m = a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1}$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3}$$

$$+ \frac{m}{1} a^{m-1} c \qquad\qquad + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc$$

$$\times \frac{m}{1} a^{m-1} d$$

$$+ \frac{m}{1}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}a^{m-4}b^4z^{m+4}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2c$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bd$$

$$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}c^2$$

$$+\frac{m}{1}a^{m-1}e$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}a^{m-5}b^5z^{m+5}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3c$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2d$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2}a^{m-3}bc^2$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}be$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}cd$$

$$+\frac{m}{1}a^{m-1}f$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}a^{m-6}b^{6}z^{m+6}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{1}a^{m-5}b^{4}c$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^{3}d$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times\frac{m-3}{2}a^{m-4}b^{2}c^{2}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^{2}e$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}bf$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}c^{3}$$

$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$$

$$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^{2}$$

$$+\frac{m}{1}a^{m-1}g,\ \&c.$$

I ſuppoſe that the Infinite Number Multinomial is $az+bzz+cz^{3}+dz^{4}+ez^{5}$, *&c.* m is the Index of the Power, to which this Multinomial ought to be rais'd, or if you will, 'tis the Index of the Root, which is to be extracted: I ſay, That this Power or Root of the Multinomial is ſuch a Series as I have expreſs'd.

For the underſtanding of it, it is only neceſſary to conſider all the Terms by which the ſame Power of z is Multiplied; in order thereto, I diſtinguiſh two things in each of theſe Terms; 1°. The Product of certain Powers of the Quantities, a, b, c, d, *&c.* 2°. The Unciæ (as *Oughtred* calls 'em) prefixt to theſe Products. To find all the Products belonging to the ſame Power of z, to that Product, for Inſtance, whoſe Index is $m+r$ (where r may denote any Integer Number) I divide theſe Products into ſeveral Claſſes; thoſe which immediately after ſome certain Power of a (by which all theſe Products begin) have b, I call Products of the firſt Claſſis; for Example, $a^{m-4}b^{3}e$ is a Product of the firſt Claſſis, becauſe b immediately follows a^{m-4}; thoſe which immediately after ſome Power of a have c, I call Products of the ſecond Claſſis, ſo $a^{m-3}ccd$, is a Product of the ſecond Claſſis;

ſis; thoſe which immediately after ſome Power of a have d, I call Products of the third Claſſis, and ſo of the reſt.

This being done, I Multiply all the Products belonging to z^{m+r-1} (which preceeds immediately z^{m+r}) by b, and divide 'em all by a; 2°. I Multiply by c, and Divide by a, all the Products belonging to z^{m+r-2} except thoſe of the firſt Claſſis; 3°. I Multiply by d and Divide by a, all the Products belonging to z^{m+r-3}, except thoſe of the firſt and ſecond Claſſis: 4°. I Multiply by e and Divide by a, all the Terms belonging to z^{m+r-4}, except thoſe of the firſt, ſecond and third Claſſis, and ſo on, till I meet twice with the ſame Term. Laſtly, I add to all theſe Terms, the Product of a^{m-1} into the Letter, whoſe Exponent is $r+1$.

Here I muſt take notice, that by the Exponent of a Letter, I mean the Number which expreſſes what Place the Letter has in the Alphabet, ſo 3 is the Exponent of the Letter c, becauſe the Letter c is the 3d in the Alphabet.

It is evident by this Rule, you may eaſily find all the Products belonging to the ſeveral Powers of z, if you have but the Product belonging to z^m, *viz.* a^m.

To find the Unciæ which ought to be prefix'd to every Product, I conſider the Sum of Units contain'd in the Indices of the Letters which compoſe it (the Index of a excepted) I write as many Terms of the Series $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, *&c.* as there are Units in the Sum of theſe Indices, this Series is to be the Numerator of a Fraction, whoſe Denominator in the Product of the ſeveral Series $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. the firſt of which contains as many Terms as there are Units in the Index of b, the ſecond as many as there are Units in the Index of c, the third as many as there are Units in the Index of d, the fourth as many as there are Units in the Index of e, &c.

Demonſtration,] To raiſe the Series $az + bzz + cz^3 + dz^4$, &c. to any Power whatſoever, write ſo many Series equal to it, as there are Units in the Index of the Power demanded. Now it is evident, that when theſe Series are alſo Multiplied, there are ſeveral Products in which there is the ſame Power of z; thus if the Series $az + bzz + cz^3 + dz^4$, &c. is raiſed to its Cube, you have the Products b^3z^6, $abcz^6$, $aadz^6$, in which you find the ſame Power z^6. Therefore let us conſider what is the Condition that can make ſome Products to contain the ſame Power of z; the firſt thing that will appear in relation to it is, that in any Product whatſoever, the Index of z is the Sum of the particular Indices of z in the Multiplying Terms (this follows from the Nature of the Indiees): thus b^3z^6 is the Product of bz^2, bz^2, bz^2, and the Sum of the Indices in the Multiplying Terms, is $2+2+2=6$; $abcz^6$ is the Product of az, bzz, cz^3, and the Sum of the Indices of z in the Multiplying Terms is $1+2+3=6$; $aadz^6$ is the Product of az, az, dz^4, and the Sum of the Indices of z in the Multiplying Terms is $1+1+4=6$: the next thing that appears, is, that the Index of z in the Multiplying Terms is the ſame with the Exponent of the Letter to which z is joyn'd; from which two Conſiderations it follows, that, to have all the Products belonging to a certain Power of z, you muſt find all the Products where the Sum of the Exponents of the Letters which compoſe them, ſhall always be the ſame with the Index of that Power. Now this is the Method I uſe, to find eaſily all the Products belonging to the ſame Power of z, let $m+r$ be the Index of that Power, I conſider that the Sum of the Exponents of the Letters which compoſe the Products, muſt exceed by 1, thoſe which belong to m^{m+r-1}, now becauſe the exceſs of the Exponent of the Letter b, above the Exponent of the Letter a, is 1, it follows, that if each of the Products belonging to z^{m+r-1} is Multiply'd by b, and Divided by a, you will have Products, the Sum of whoſe Exponents will be $m+r$; Likewiſe the Sum of the Exponents of the Letters which compoſe the Products belonging to z^{m+r} exceeds by 2 the Sum of the Exponents of the Letters which compoſe the Products belonging to z^{m+r-1}: Now becauſe the Exponent of the Letter a is leſs by 2 than the Exponent of the Letter c, it follows, that if each Product belonging to z^{m+r-2} is Multiplyed by c, and Divided by a, you will have other Products, the Sum of whoſe Exponents is ſtill $m+r$; Now if all the Products belonging to z^{m+r-2} were Multiplied by c and Divided by a, you would have ſome Products that would be the ſame as ſome of them found before, therefore you muſt except out of 'em thoſe that I have called Products of the firſt Claſſis; what I have ſaid ſhows why all the Products belonging to z^{m+r-3}, except thoſe of the firſt and ſecond Claſſis muſt be Multiplied by d, and Divided by a: Laſtly, you ſee the Reaſon why to all theſe Products is added the Product of a^{m-1} by the Letter whoſe Exponent is $r+1$; 'Tis becauſe the Sum of the Exponent is ſtill $m+r$.

As for what relates to the Unciæ; obſerve that when you Multiply $az+bzz+cz^{3}+dz^{4}$, &c. by $az+bzz+cz^{3}+dz^{4}$, &c. each Letter, a, b, c, d, &c. of the ſecond Series, is Multiplied by each of the Letters a, b, c, d, &c. of the firſt Series. Thus the Letter a of the ſecond Series is Multiplied by the Letter b of the firſt Series, and the Letter b of the ſecond Series is Multiplied by the Letter a of the firſt; therefore you may have the two Planes, ab, ab, or $2ab$; for the ſame Reaſon you have $2ac$, $2ad$, &c. Therefore you muſt prefix to each Plane of thoſe that compoſe the Square of the Infinite Series $az+bzz+cz^{3}$, &c. the Number which expreſſes how many ways the Letters of each Plane may be changed; Likewiſe if you Multiply the Product of the two preceeding Series by $az+bzz+cz^{3}$, &c. each Letter a,b,c,d, of the third Series is Multiplied by each of the Planes form'd by the Product of the firſt and ſecond Series; Thus the Letter a is Multiplied by the Planes bc and cb; the Letter b is Multiplied by ac and ca; the Letter c is Multiplied by ab and ba; therefore you have the ſix Solids, abc, acb, bac, bca, cab, cba, or ſix abc; Therefore you muſt prefix to each Solid whereof the Cube of the Infinite Series is compos'd, the Number which expreſſes how many ways the Letters of each Solid may be changed. And generally, you muſt prefix to any Product, whereof any Power of the Infinite Series $az+bzz+cz^{3}$, &c. is compos'd, the Number which expreſſes how many ways the Letters of each Product may be changed.

Now to find how many ways the Letters of any Product, for inſtance, $a^{m-}\ b^{h} c^{p} d^{r}$ may be changed; this is the Rule which is commonly given: write as many Terms of the Series $1 \times 2 \times 3 \times 4 \times 5$, &c. as there are Units in the Sum of the Indices, *viz.* $m-n+h+p+r$; let this Series be the Numerator of a Fraction, whoſe Denominator ſhall be the Product of the Series, $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. whereof the firſt is to contain as many Terms as there are Units in the firſt Index $m-n$; the ſecond as many as there are Units in the ſecond Index h; the third as many as there are Units in the third Index p; the fourth as many as there are Units in the fourth Index r. But the Numerator and Denominator of this Fraction have a common Diviſor, *viz.* the Series of $1 \times 2 \times 3 \times 4 \times 5$, &c. continued to ſo many Terms as there are Units in the firſt Index $m-n$; therefore let both this Numerator and Denominator be divided by this common Diviſor, then this new Numerator will begin with $m-n+1$, whereas the other began with 1, and will contain ſo many Terms as there are Units in $h+p+r$, that is, ſo many as there are Units in the Sum of all the Indices excepting the firſt; as for the New Denominator, it will be the Product of three Series only; that is, of ſo many as there are Indices, excepting the firſt. But if it happens withall, that n be equal to $h+p+r$, as it always happens in our *Theorem*; then the Numerator beginning by $m-n+1$, and being continued to ſo many Terms as there are Units in $h+p+r$ or n, the laſt Term will be m neceſſarily, ſo if you invert the Series, and make that the firſt Term which was the laſt, the Numerator will be $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c. continued to ſo many Terms as there are Units in the Sum of the Indices of each Product, excepting the firſt Index. And whatever is here ſaid of Powers, whoſe Index is an Integer, may be adapted to Roots or Powers whoſe Index is a Fraction.

The Extraction of a Root of an Infinite Equation, by Mr Abr. de Moivre. N. 240. *p.* 190. May. An. 1698.

XXII. *Theorem.*] If $az + bzz + cz^{3} + dz^{4} + cz^{5} + fz^{6}$, &c. $= gy + hyy + iy^{3} + ky^{4} + ly^{5} + my^{6}$, &c. then will z be

$$= \frac{g}{a}y + \frac{h - bAA}{a}y^{2} + \frac{i - 2bAB - cA^{3}}{a}y^{3} + \frac{k - bBB - 2bAC - 3cAAB - dA^{4}}{a}y^{4} + \frac{l - 2bBC - 2bAD - 3cABB - 3cAAC - 4dA^{3}B - eA^{5}}{a}y^{5} + \frac{m - 2bBD - bCC - 2bAE - cB^{3} - 6cABC - 3cAAD - 6dAABB - 4dA^{3}C - 5eA^{4}B - fA^{6}}{a}y^{6}, \text{\&c.}$$

For the underſtanding of this Series, and in order to continue it as far as we pleaſe; it is to be obſerved, 1. That every Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coefficient $\frac{h - bAA}{a}$. 2. That the Denominator of each Coefficient is always a. 3. That the firſt Member of each Numerator, is always a Coefficient of the Series $gy + hyy + iy^3$, &c. *viz.* the firſt Numerator begins with the firſt Coefficient g, the ſecond Numerator with the ſecond Coefficient h, and ſo on. 4. That in every Member after the firſt, the Sum of the Exponents of the Capital Letters, is always equal to the Index of the Power to which this Member belongs: Thus conſidering the Coefficient $\frac{k - bBB - 2bAC - 3cAAB - dA^4}{a}$, which belongs to the Power y^4, we ſhall ſee that in every Member bBB, $2bAC$, $3cAAB$, dA^4, the Sum of the Exponents of the Capital Letters is 4, (where I muſt take notice, that by the Exponent of a Letter, I mean the Number which expreſſes what place it has in the Alphabet; thus 4 is the Exponent of the Letter D) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; Combine the Capital Letters as often as you can make the Sum of their Exponents equal to the Index of the Power to which they belong. 5. That the Exponents of the ſmall Letters, which are written before the Capitals, expreſs how many Capitals there is in each Member. 6. That the Numerical Figures or Unciæ that occurr in theſe Members, expreſs the Number of Permutations which the Capital Letters of every Member are capable of.

For the *Demonſtration* of this; ſuppoſe $z = Ay + Byy + Cy^3 + Dy^4$, &c. Subſtitute this Series in the room of z, and the Powers of this Series in the room of the Powers z, there will ariſe a new Series; then take the Coefficients which belong to the ſeveral Powers of y in this new Series, and make them equal to the correſponding Co-efficients of the Series $gy + hyy + iy^3$, &c. and the Coefficients A, B, C, D, &c. will be found ſuch as I have determined them.

But if any one deſires to be ſatisfied, that the Law by which the Coefficients are form'd, will always hold, I'll deſire 'em to have recourſe to the Sect. XXI. *Theorem* I have given for *Raiſing an Infinite Series to any Power*, or extracting any Root of the ſame; for if they make uſe of it, for taking ſucceſſively the Powers of $Ay + Byy + Cy^3$, &c. they will ſee that it muſt of neceſſity be ſo. I might have made the *Theorem* I give here, much more general than it is; for I might have ſuppos'd, $az^m + bz^{m+1} + cz^{m+2}$, &c, $= gy^m + hy^{m+1} + iy^{m+2}$, &c. then all the Powers of the Series $Ay + Byy + Cy^3$, &c. deſigned by the univerſal Indices, muſt have been taken ſucceſſively; but thoſe who will pleaſe to try this, may eaſily do it, by means of the *Theorem* for raiſing an Infinite Series to any Power, &c.

This *Theorem* may be applied to what is called the Reverſion of Series, ſuch as finding the Number from its Logarithm given; the Sine from the Arch; the Ordinate of an Ellipſe from an Area given to be cut from any point in the Axis: But to make a particular Application of it, I'll ſuppoſe we have this Problem to ſolve; *viz. The Chord of an Arc being given, to find the Chord of another Arc, that ſhall be to the firſt as* n *to* 1. Let y be the Chord given, z the Chord required; now the Arc belonging to the Chord y, is $y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^5}$ &c. And the Arc belonging to the Chord z is $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$ &c. The firſt of the Arcs is to the ſecond as 1 to n; therefore multiplying the Extreams and Means together, we ſhall have this Equation.

$$z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6} \text{ \&c.} = ny + \frac{ny^3}{6dd} + \frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^5} \text{ \&c.}$$

Compare theſe two Series with the two Series of the Theorem, and and you will find $a = 1$, $b = 0$, $c = \frac{1}{6dd}$, $d = 0$, $e = -\frac{3}{40d^4}$, $f = 0$, &c. $g = n$, $h = 0$, $i = \frac{n}{6dd}$, $k = 0$, $l = -\frac{3n}{40d^4}$, $m = 0$, &c. hence z will be $= ny + \frac{n-n}{6dd}y^3$, &c. or $ny + \frac{1-nn}{2 \times 3dd}yyA$, &c. Suppoſing A to denote the whole preceeding Term, which will be the ſame Series as Mr. *Newton* has firſt found.

By the ſame Method, this general *Problem* may be ſolved; *The Abſciſſa correſponding to a certain Area in any Curve being given, to find the Abſciſſa, whoſe correſponding Area ſhall be to the firſt in a given Ratio.*

The Logarithmic Series might alſo be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whoſe Logarithm we enquire, be $1 + z$, ſuppoſe its Log. to be $az + bzz + cz^3$, &c. Let there be another Number $1 + y$; therefore its Logarithm will be $ay + byy + cy^3$, &c. Now if $1 + z = \overline{1+y}|^n$, it follows that $az + bzz + cz^3$, &c. : $ay + byy + cy^3$, &c. : : n : 1. that is, $az + bzz + cz^3$, &c. $= nay + nbyy + ncy^3$, &c. therefore we may find a Value of z expreſt by the Powers of y; again, ſince $1 + z = \overline{1+y}|^n$, therefore $z = \overline{1+y}|^n - 1$, that is $z = ny + \frac{n}{1} \times \frac{n-1}{2}yy + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3}y^3$, &c. Therefore z is doubly expreſt by the Powers of y. Compare theſe two Values together, and the Coefficients $a, b, c,$ &c. will be determin'd, except the firſt a, which may be taken at Pleaſure, and gives accordingly all the different Species of Logarithms.

Doctrine of Exhauſtions, by Dr. Wallis. N. 255. *p.* 281. Aug. An. 1699.

XXIII. De Differentiis Infiniteſſimarum-infiniteſſimis explicandis, non eſt ut ſis porro ſolicitus. Nam, ut tu mihi facilis concedis, quod nihili quodvis multiplum ſit adhuc nihil; eadem ego facilitate tibi permitto, ut Differentias Infiniteſſimas in Infiniteſſimas ductas, tu merito negligas, poteſtque id tuto fieri, modo caute; Quippe, in quovis genere Quantitatum, quæ differunt dato minus, reputanda ſunt Æqualia. Quo nititur *Exhauſtionum* Doctrina tota, Veteribus pariter & Recentioribus neceſſaria.

The Approximations of the Ancients in the Extracting of Roots, Improv'd; by Dr. Wallis. N. 215. *p.* 2. Jun. An. 1695.

XXIV. It is agreed by all, that if a Number propoſed be not a true Square, it is in vain to hope for a juſt Quadratick Root thereof, explicable by rational Numbers, Integers or Fracted. And therefore in ſuch Caſes, we muſt content our ſelves with Approximations (ſomewhat near the truth) without pretending to Accuracy.

And ſo, for the Cubick Root, of what is not a perfect Cube. And the like for Superior Powers.

Now the Ancients (being aware of this) had their Methods of Approximation, which tho' ſcarce apply'd by them beyond the Quadratick, or perhaps the Cubick Root, yet are equally practicable (by due adjuſtments) to the Superior Powers alſo.

I ſhall begin with the Square Root: For which the Ancient Method is to this purpoſe.

From the propoſed Non-quadrate (ſuppoſe N) ſubſtract (in the uſual manner) the greateſt Square in Integers therein contained (ſuppoſe $A\,q$.) The remainder (ſuppoſe $B = 2\,A\,E + E\,q$) is to be the Numerator of a Fraction, for deſigning the near Value of E (the remaining part of the Root ſought $A + E = \surd N$,) whoſe Denominator or Diviſor is to be $2\,A$ (the double Root of the ſubſtracted Square) or $2\,A + 1$ (that double Root increaſed by 1) the true Value falling between theſe two; ſometime the one, ſometime the other, being neareſt to the true Value. But (for avoiding of Negative Numbers) the latter is commonly directed.

This Method Monſieur *De L'agny* affirms to be more than 200 Years old: And it is ſo; for I find it in *Lucas Pacciolus* (otherwiſe called *Lucas de Burgo*, or *de Burgo Sancti Sepulchri*) Printed at *Venice* in the Year 1494 (if not even ſooner than ſo, for I find there have been ſeveral Editions of it.) And how much older than ſo, I cannot tell: For he doth not deliver it as a New Invention of his own, but as a received Practice, and derived from the *Moors* or *Arabs*, from whom they had their *Algoriſm*, or Practice of Arithmetick by the 10 Numeral Figures now in uſe.

And it is continued down hitherto in Books of Practical Arithmetick in all Languages, which teach the Extraction of the Square Root, and (therein) this Method of Approximation, in caſe of a Non-quadrate.

The true ground of this Rule is this: $A\,q$ being (by Conſtruction) the greateſt Integer Square contained in N, 'tis evident that E muſt be leſs than 1; (otherwiſe not $A\,q$ but the Square of $A + 1$, or ſome greater than ſo, would be the greateſt Integer Square contained in N.) Now if the remainder $B = 2\,AE + E\,q$ be divided by $2\,A$, the reſult will be too great for E, (the Diviſor being too little; for it ſhould be $2\,A + E$, to make the Quotient E.) But if (to rectify this) we diminiſh the Quotient, by increaſing the Diviſor, adding 1 to it, it then becomes too little; becauſe the Diviſor is now too big. For (E being leſs than 1) $2A + 1$ is more than $2A + E$; and therefore too big.

As for Inſtance; If the Non-quadrate propoſed be $N = 5$, the greateſt Integer Square therein contained is $Aq = 4$ (the Square of $A = 2$:) which being ſubſtracted, leaves $N - Aq = 5 - 4 = 1 = B = 2AE + Eq$. Which divided by $2A = 4$ gives $\frac{1}{4}$: But divided by $2A + 1 = 4 + 1 = 5$, gives $\frac{1}{5}$. That too great, and this too little, for E. And therefore the true Root ($A + E = \surd N$) is leſs than $2\frac{1}{4} = 2.25$, but greater than $2\frac{1}{5} = 2.2$; And this was anciently thought an Approach near enough.

If this Approach be not now thought near enough, the ſame Proceſs may be again repeated; and that as oft as is thought neceſſary.

Take now for A, $2\frac{1}{5} = 2.2$, whoſe Square is $4.84 = Aq$, (now conſidered as an Integer in the ſecond place of Decimal Parts.) This ſubſtracted from 5.00, (or, which is the ſame, 0.84, the Exceſs of this Square above the former, from 1, which was then the Remainder) leaves a new Remainder $B = 0.16$: which divided by $2A = 4.4$, gives $\frac{0.16}{4.40} = \frac{2}{55} = 0.03636 +$, too much. But divided by $2A + 1 = 4.5$, it gives $\frac{0.16}{4.50} = \frac{8}{225} = 0.03555 +$, too little. The true Value (between theſe two) being 2.236 proxime, whoſe Square is 4.999696.

If this be not thought near enough, ſubſtract the Square from 5.000000: the Remainder $B = 0.000304$, divided by $2A = 4.472$, or by $2A + 1 = 4.473$, gives either way $0.000068 -$; which added to $A = 2.236$, makes $2.236068 -$, ſomewhat too big; but $2.236067 +$ would be much more too little.

Which gives us the Square Root of 5, adjuſted to the ſixth place of Decimal parts, at three Steps. And by the ſame Method, if it be thought needful, we may proceed further.

For the Cubick Root the Rule is this.

From the Non-Cubick propoſed, (ſuppoſe N,) ſubſtract the greateſt Cube in Integers therein contained, (ſuppoſe Ac:) the Remainder ſuppoſe $B = 3AqE + 3AEq + Ec$,) is to be the Numerator of a Fraction for deſigning the Value of E, (the remaining part of the Root ſought $A + E = \surd cN$.) To this Numerator, if (for the Denominator or Diviſor) we ſubjoyn $3Aq$, the Reſult will certainly be too great for E, becauſe the Diviſor is too little: (For it ſhould be $3Aq + 3AE + Eq$, to give the true Value of E. If for the Diviſor we take $3Aq + 3A + 1$, it will certainly be too little, becauſe the Diviſor is too great. (For E by conſtruction is leſs than 1.) It muſt therefore (between theſe Limits) be more than this latter. And therefore this latter Reſult being added to A, will give a Root whoſe Cube may be ſubſtracted from the Non-Cubick propoſed, in order to another Step.

But if, for the Diviſor, we take $3Aq + 3A$, (or even leſs than ſo) the Reſult may be too great; or in caſe B be ſmall) it may be too little, (and oft is ſo.)

Which comes to paſs from hence; becauſe E (by Conſtruction) is leſs than 1; and therefore $3AE$ leſs than $3A$; and perhaps ſo much as that the Addition of Eq will not redreſs it. And when it ſo happens $3Aq + 3A$ is a better Diviſor than $3Aq + 3A + 1$, (or even ſomewhat leſs than either.) But becauſe it doth not always ſo happen (though for the moſt part it doth) the Rule doth rather direct the other; as which doth certainly give a Root leſs than the true Value, whoſe Cube may always be ſubſtracted from the Non-Cubick propoſed. The Deſign being to have ſuch a Cube as (being ſubſtracted) may leave another B, to be ordered in like manner for a new Approach.

But, for the moſt part, $3Aq$ may be ſafely taken for the Diviſor. For tho' the Reſult will then be ſomewhat too big, yet the Exceſs may be ſo ſmall, as to be neglected; or, at leaſt, we may thence eaſily judge what Number (ſomewhat leſs than it) may be ſafely taken. And if we chance to take it ſomewhat too big, the Inconvenience will be but this, that B for the next Step will be a Negative. Of which Caſe we ſhall ſpeak anon.

Thus for Inſtance; if the Non-Cube propoſed be $9 = N$. The greateſt Integer Cube therein contained is $8 = Ac$, (whoſe Cubick Root is $A = 2$.) Which Cube ſubſtracted, leaves $9 - 8 = 1 = B = 3AqE + 3AEq + Ec$. This divided by $3Aq = 12$, gives $\frac{1}{12} = 0.08333+$, too big for E. But the ſame divided by $3Aq + 3A + 1 = 12 + 6 + 1 = 19$, gives $\frac{1}{19} = 0.05263+$, too little. Or if but by $3Aq + 3A = 12 + 6 = 18$, it gives $\frac{1}{18} = \frac{5}{90} = 0.05555+$, yet too little, for the Cube of $A + 0.06$, $= 2.06$, is but $8.742-$, which is ſhort of 9. And ſo much ſhort of it, that we may ſafely make 2.07 as not too big: Or perhaps 2.08, which upon tryal will be found not too big; for the Cube of 2.08, is but 8,998912.

If this firſt Step be not near enough, this Cube ſubſtracted from 9.000000, leaves a new $B = 0.001088$, which divided by $3Aq = 12.9796$, gives $0.000084-$; which will be ſomewhat too big, but not much. (For E is now ſo ſmall, as that $3AE$ may be ſafely neglected, and Eq much more.) So that if to 2.08, we add $0.000084-$, the Reſult 2.080084 will be too big; but 2.080083 will be more too little. (As will appear if we take the Cube of each.) So that neither of them, at the ſecond Step, gives the true Root within an Unite in the ſixth place of Decimal parts. But when I ſay, taking the Cube of each, (which I do that the thing may be more clearly apprehended) it is not neceſſary that we trouble our ſelves with the whole Cube. For, Ac being already ſubſtracted, for finding $B = 3AqE + 3AEq + Ec$, we have no more to try, but whether $3AqE + 3AEq + Ec$, be greater or leſs than B, according as we take 0.000084, or 0.000083, for E.

Which may conveniently be done in this manner: Take $3A + E$, and multiply this by E, (or E by it) ſo have we $3AE + Eq$. To this add $3Aq$, and multiply the whole by E, (ſo have we $3AqE + 3AEq + Ec$,) too ſee whether this be greater or leſſer than B.

That is, in the preſent caſe, if we take $E = 0.000084$, and add to this $3A = 6.24$, then is $6.240084 = 3A + E$. This multiplied by $E = 0.000084$, is $3AE + Eq = 0.000524+$. To which if we add $3Aq = 12.9792$, it is $3Aq + 3AE + Eq = 12.979724$.

Which multiplied again by $E = 0.000084$, is $0.0010902 + = 3\,A q E + 3\,A E q + E c$, which is more than $B = 0.001088$.

But if we take $E = 0.000083$, and proceed as before, we ſhall have $3\,A q E + 3\,A E q + E c = 0.001077 +$, which is leſs than $B = 0.001088$. And therefore (if we Subſtract that from this) the Remainder 0.000011, will be another B for the next Step, if we pleaſe to proceed further.

Hitherto I have purſued the Method moſt affected by the Ancients, in ſeeking a Square or Cube (and the like of other Powers) always leſs than the juſt Value, that it might be Subſtracted from the Number propoſed, leaving B a poſitive Remainder; thereby avoiding Negative Numbers.

But ſince the Arithmetick of Negatives is now ſo well underſtood, it may in this (and other Operations of like Nature) be adviſeable, to take the neareſt, whether it be greater or leſs than the juſt Value.

According to this Notion, for the Square Root of 5, I would ſay it is $(2 +)$ ſomewhat more than 2; and enquire how much more? But for the Square Root of 8, I would ſay, it is $(3 -)$ ſomewhat leſs than 3; and enquiring how much leſs? Taking (in both Caſes) that which is neareſt to the juſt Value.

Thus in the Cubick Root before us; I would take for E (in the laſt Enquiry $0.000084 -$ (where for the next Step we have $B = -0.000002$,) rather than $0.000083 +$ (where for the next Step we ſhould have $B = +0.000011$.) In the latter Caſe we are to divide $B = +0.000011$, by $3\,A q = 12.980236 -$, to find (by the Quotient) how much is to be added to 0.000083. In the other Caſe we are to divide $B = +0.000002$, by $3\,A q = 12.980248$, to find (by the Quotient) what is to be abated of 0.000084, ſo have we $\frac{0.000011}{12.980236} = 0.00000085 +$ to be added to 6.240083: Or $\frac{0.000002}{12.980248} = 0.00000015 +$ to be abated of 6.240084.

(Or it may ſuffice, in either to divide by $12.98 +$, or even by $13 -$, without being incumbred with a long Diviſor) either of which gives us, for the Root ſought, 2.08008385 proxime. True (at the third Step) to the eighth Place of Decimal Parts. And if this be not near enough, the Cube of this, compared with the Number propoſed, will give us another B for the next Step. And ſo onwards as far as we pleaſe.

Now, what is ſaid of the Cube, is eaſily applicable to the higher Powers.

I ſhall omit that of the Biquadrate; becauſe here perhaps it may be thought moſt adviſable, to Extract the Square Root of the Number propoſed; and then the Square Root of that Root. But if we would do it at once, we are from N (the Number propoſed, being not a Biquadrate) to Subſtract $A q q$ (the greateſt Biquadrate contained in it) to find the Remainder $B = 4\,A c E + 6\,A q E q + 4\,A E c + E q q$. Which Remainder, if we divide by $4\,A c$, the Quotient will certainly be too big for E, (though perhaps not much:) If by $4\,A E + 6\,A q + 4\,A + 1$, it will certainly be too little (for Reaſons before mentioned.) And we are to uſe our diſcretion in taking ſome intermediate Number. And if we chance not to hit on the neareſt, the Inconvenience will be but this, that our Leap will not be ſo great as otherwiſe it might be. Which will be rectify'd by another B at the next Step.

For the Surſolid (of five Dimenſions) we are, from N (the Number propoſed, being not a perfect Surſolid) to Subſtract Aqc (the greateſt Surſolid therein contain'd) to find the Remainder $B = 5AqqE + 10AcEq + 10AqEc + 5AEqq + Eqc$. Which (as before) if we divide by $5Aqq$, the Reſult will be ſomewhat too big (becauſe the Diviſor is too little:) If by $5Aqq + 10Ac + 10Aq + 5A + 1$, the Reſult will certainly be leſs than the true E. The juſt Value of E being ſomewhat between theſe two, where we are to uſe our Diſcretion, what Intermediate Number to take. Which, according as it proves too great or too little, is to be rectify'd at the next Step.

But for the moſt part it will be ſafe enough (and leaſt Trouble) to divide by $5Aqq$, which gives a Quotient ſomewhat too big; which we may either rectify at Diſcretion, by taking a Number ſomewhat leſs, or proceed to another B, (Affirmative or Negative, as the Caſe ſhall require) and ſo onward to what Exactneſs we pleaſe. Which is for Subſtance, in a manner co-incident with Mr. *Raphſon*'s Method, even for Affected Equations.

Thus, in the preſent Caſe; If the Number propoſed be $N = 33$, then is $Aqc = 32$, and $B = 33 - 32 = 1 = 5AqqE + 10AcEq + 10AqEc + 5AEqq + Eqc$. Which if we divide by $5Aqq = 5 \times 16 = 80$, the Reſult $\frac{1}{80} = 0.0125$, is ſomewhat too big for E, but not much. And if we examine it, by taking the Surſolid of 2.0152, or of $2\frac{1}{80}$, we ſhall find a Negative B (for the next Step,) but not very conſiderable. Or if we think it conſiderable, we may proceed further to another Step, or more than ſo.

The like Method may be apply'd (and with more Advantage) in the higher Powers, according as the Compoſition of each Power requires.

And the ſame Method may be of uſe (with good Advantage) in long Numbers (if duly applied) even before we come to the Place of Units; for the ſame will equally hold there alſo. Which the Reader may eaſily apprehend, without a long Diſcourſe upon it.

The Proportion of Infinite Quantities; by Mr. E. Halley. N. 195. p. 556. Oct. An. 1692.

XXV. The very Idea of Magnitudes Infinitely great, or ſuch as exceed any aſſignable Quantity, does include a Negation of Limits: yet if we nearly examine this Notion, we ſhall find that ſuch Magnitudes are not equal amongſt themſelves, but that there are really, beſides Infinite Length, and Infinite Area, three ſeveral ſorts of Infinite Solidity: all of which are *Quantitates ſui generis*; and that thoſe of each Species are in given Proportions.

Infinite Length, or a Line Infinitely long, is to be conſidered either as beginning at a Point, and ſo Infinitely extended one way, or elſe both ways from the ſame Point; in which Caſe the one, which is a beginning Infinity, in the one half of the whole, which is the Summ of the beginning and ceaſing Infinity; or as I may ſay, of Infinity *à parte ante* and *à parte poſt*, which is analogous to Eternity in Time or Duration, in which there is always as much to follow as is paſt, for any Point or Moment of Time: Nor doth the Addition or Subduction of Finite Length or ſpace of Time, alter the Caſe either in Infinity or Eternity, ſince both the one or the other cannot be any part of the whole.

As to Infinite Surface or Area, any Right Line, Infinitely extended both ways on an Infinite Plane, does divide that Infinite Plane into equal Parts, the one to the Right, and the other to the Left of the ſaid Line; but if from any Point in ſuch a Plane, two Right Lines be infinitely extended, ſo as to make an Angle, the Infinite Area, Intercepted between thoſe Infinite Right Lines, is to the whole Infinite Plane, as the Arch of a Circle on the Point of Concourſe of thoſe Lines as a Center, intercepted between the ſaid Lines, is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degree of a Circle. For Example, two right Lines meeting at a right Angle do include, on an Infinite Plane, a quarter part of the whole Infinite Area of ſuch a Plane.

But if ſo be two Parallel Infinite Lines be ſuppoſed drawn on ſuch an Infinite Plane, the Area intercepted between them will be likewiſe Infinite; but at the ſame time will be Infinitely leſs than that Space, which is intercepted between two Infinite Lines that are inclined, tho' with never ſo ſmall an Angle, for that in the one Caſe, the given finite diſtance of the Parallel Lines diminiſhes the Infinity in one Degree of Dimenſion; whereas in a Sector, there is Infinity in both Dimenſions: and conſequently the Quantities are the one Infinitely greater than the other, and there is no Proportion between them.

From the ſame Conſideration ariſe theſe three ſeveral Species of Infinite Space or Solidity; for a Parallelopipid, or a Cylinder Infinitely long, is greater than any Finite Magnitude how great ſoever; and all ſuch Solids ſuppos'd to be formed on given Baſes, are as thoſe Baſes, in proportion to one another. But if two of theſe three Dimenſions are wanting, as in the Space intercepted between two Parallel Planes Infinitely extended, and at a Finite Diſtance; or with Infinite Length and Breadth, with a Finite Thickneſs, all ſuch Solids ſhall be as the given Finite Diſtances one to another; but theſe Quantities, tho' Infinitely greater than the other, are yet Infinitely leſs than any of thoſe wherein all the three Dimenſions are Infinite. Such are the Spaces intercepted between two inclined Planes Infinitely extended; the Space intercepted by the Surface of a Cone, or the ſides of a Pyramid, likewiſe Infinitely continued, &c. of all which notwithſtanding the Proportions one to another, and to the τὸ πᾶν or vaſt Abyſs of Infinite Space (wherein is the *Locus* of all things that are or can be; or to the Solid of Infinite Length, Breadth and Thickneſs taken all manner of ways) are eaſily aſſignable. For the Space between two Planes is to the whole, as the Angle of thoſe Planes to the 360 Degrees of the Circle. As for Cones and Pyramids, they are as the Spherical Surface intercepted by them, is to the Surface of the Sphere, and therefore Cones are as the verted Sines of half their Angles, to the Diameter of the Circle: Theſe three ſorts of Infinite Quantity are Analogous to a Line, Surface, and Solid; and after the ſame manner cannot be compared, or have no Proportion the one to the other.

Infinitely-Infinite Fractions; *by* D. R. Wood. *Ph. Col.* N. 3. p. 45. Dec. An. 1681.

XXVI. Infinitely-Infinite Fractions, or all the Powers of all the Fractions whoſe Numerator is 1, are all of them together equal to (1) an Unit.

$$\begin{array}{lllll}
 & & & \text{P.} & \\
\text{A.} & \text{R.} & \text{q.} & \text{c.} & \text{qq.} \\
1=\frac{1}{1} & =\frac{1}{2}: & +\frac{1}{4} & +\frac{1}{8} & +\frac{1}{16}\ \&c. \\
\frac{1}{2} & =\frac{1}{3}: & +\frac{1}{9} & +\frac{1}{27} & +\frac{1}{81}\ \&c. \\
\frac{1}{3} & =\frac{1}{4}: & +\frac{1}{16} & +\frac{1}{64} & +\frac{1}{256}\ \&c. \\
\frac{1}{4} & =\frac{1}{5}: & +\frac{1}{25} & +\frac{1}{125} & +\frac{1}{625}\ \&c. \\
\frac{1}{5} & =\frac{1}{6}: & +\frac{1}{36} & +\frac{1}{216} & +\frac{1}{1296}\ \&c. \\
\&c. & \&c. & & &
\end{array}$$

Or thus

$$\begin{array}{l|l|l|}
 & {}^{1}\,\text{anor}\ +. & \\
\frac{1}{1\times 1}= & \frac{1}{2}+ & \frac{1}{1\times 2}= \\
\frac{1}{2\times 1}= & \frac{1}{3}+ & \frac{1}{2\times 3}= \\
\frac{1}{3\times 1}= & \frac{1}{4}+ & \frac{1}{3\times 4}= \\
\frac{1}{4\times 1}= & \frac{1}{5}+ & \frac{1}{4\times 5}= \\
\frac{1}{5\times 1}= & \frac{1}{6}+ & \frac{1}{5\times 6}=
\end{array}$$

$$\begin{array}{l|l|l|l|l}
{}^{2}\,\text{anor}\ +. & & {}^{3}\,\text{anor}\ +. & & {}^{4}\,\text{anor}\ +. \\
\frac{1}{2^{2}}+ & \frac{1}{1\times 2^{2}}= & \frac{1}{2^{3}}+ & \frac{1}{1\times 2^{3}}= & \frac{1}{2^{4}}+\ \&c. \\
\frac{1}{3^{2}}+ & \frac{1}{2\times 3^{2}}= & \frac{1}{3^{3}}+ & \frac{1}{2\times 3^{3}}= & \frac{1}{3^{4}}+\ \&c. \\
\frac{1}{4^{2}}+ & \frac{1}{3\times 4^{2}}= & \frac{1}{4^{3}}+ & \frac{1}{3\times 4^{3}}= & \frac{1}{4^{4}}+\ \&c. \\
\frac{1}{5^{2}}+ & \frac{1}{4\times 5^{2}}= & \frac{1}{5^{3}}+ & \frac{1}{4\times 5^{3}}= & \frac{1}{5^{4}}+\ \&c. \\
\frac{1}{6^{2}}+ & \frac{1}{5\times 6^{2}}= & \frac{1}{6^{3}}+ & \frac{1}{5\times 6^{3}}= & \frac{1}{6^{4}}+\ \&c.
\end{array}$$

A. Is a File or Row of abſolute Numbers, or rather of all the Fractions, whoſe Numerator is 1; which Row is ſuppoſed to be continued *in infinitum* (downwards).

R. Is another File or Row of all the Roots (whoſe Numerator is 1) of all the Powers of ſuch Fractions; ſuppoſed likewiſe to be continued *in infinitum* (downwards.)

P. Are all the reſpective Powers of ſuch Fractions, (*viz.* Squares, Cubes, *&c.* (or ſo many Ranks of Geometrical Proportionals; ſuppoſed to be continued *in infinitum*; both to the Right-hand, and alſo downwards.

Lemma.] *Each of the ſaid Ranks of Powers, together with their reſpective Roots, is equal to each of the ſeveral Numbers under A reſpectively.*

Demonſtration.] If from the Line *a b* you take (for inſtance) $\frac{1}{4}$ part towards *a*, ſuppoſe *a c*; and alſo from the other end of the ſame Line *a b*, you take two ſuch Parts (or $\frac{2}{4}$ parts) towards *b*, ſuppoſe *b d*, (*viz.* a Number of Parts leſs by two than the whole Line *a b*, was firſt ſuppoſed to be divided into) there will remain the Line $c\,d = a\,c = \frac{1}{4}$ of *a b*. Then again, if from *c d* you take $\frac{1}{4}$ part thereof towards *a*, ſuppoſe *c e*, and from the other end $\frac{2}{4}$ parts of the ſame *c d*, ſuppoſe *d f*, there will remain only $e\,f = c\,e = \frac{1}{4}$ of *c d*. And if you ſtill go on without ceaſing, to take on the Side towards *a*, $\frac{1}{4}$ part of what was taken laſt before, and twice as much on the other Side towards *b*, there ſhall be found between the two Lines laſt taken always remaining $\frac{1}{4}$ part of the Line from which they were taken. From which $\frac{1}{4}$ part there may ſtill after the ſame manner be ſuppoſed to be taken two other ſuch Lines. But if this be ſuppoſed to be done Infinite times actually, then there will nothing more remain (between), and ſo the continued Diviſion on either Side will come exactly to the Point *g*, ſuppoſing *a g* to be $\frac{1}{3}$ of *a b*, and $b\,g = 2\,a\,g$. For, becauſe that which was taken away towards *b*, was always twice as much as that which was taken away towards *a*, the total Summ of all the Lines taken away towards *b*, (which all together do make up the Line *b g*) muſt be twice as much as the Line *a g*, (which is the total Summ of all the Lines taken away towards *a*) *viz.* $b\,g = 2\,a\,g$; and conſequently $b\,g + a\,g$ (or the whole Line *a b*) is equal to $3\,a\,g$: And therefore $a\,g = \frac{1}{3}$ of *a b*. *Q. E. D.* Fig. 54.

The like *Conſtruction* and *Demonſtration (mutatis mutandis)* may be made uſe of in taking away any other part of any quantity, and the like part again of the firſt mentioned part, and ſo *in infinitum*. The total Sum of all the parts ſo taken, or ſuppoſed to be taken, ſhall be equal to any certain Quantity, or Part, or Fraction, whoſe Denominator ſhall be leſs by an Unite than the Denominator of the firſt mentioned part; as $\frac{1}{6} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343}$ &c. $\frac{1}{9} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ &c. And ſo, that which the incomparable *Archimedes* (in his Squaring the *Parabola*) has only Demonſtrated in one particular Caſe, *viz.* $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024}$ &c. and that too, not without an huge Apparatus of Preliminary Propoſitions, amounting to an whole Book, is here univerſally Demonſtrated in all Caſes (which are Infinite) and by a very ſimple and eaſy Method (in *Des Cartes*'s way) and on one ſingle Page.

Now if each of the ſaid Ranks of Powers, together with their reſpective Roots, be equal to the ſeveral Numbers of Fractions under A; (as is demonſtrated by the *Lemma)* then is A the Sum of them all, or equal to them all: that is to ſay, $R + P = A = R + 1$: For R is the ſame with A wanting $\frac{1}{1}$ or 1, (as appears upon View) or but $(\frac{1}{\infty})$ one infinite part bigger. Wherefore $P = 1$, *Q. E. D. viz. Infinities Infiniti numeri Fracti æquantur Unitati, ſc. minimæ Radici Integræ.*

Corollaria,] Hinc patet,

1°. *Dari progreſſum in infinitum.*

2°. *Et non modo in unum Infinitum, ſed etiam in plurima; ſeu potius infinita Infinita, vel infinities Infinita.*

3°. *Et hoc fieri poſſe, id eſt, hunc Calculum inſtitui, ab ingenio valde finito aut exiguo.*

4°. *Et totum hunc Progreſſum, vel progreſſus hujuſmodi infinitos, poſſe Numerari, ſive aggregari in unam ſummam.*

5°. *Et in ſummam, non modo non infinitam; ſed adeo tantillam, ut ſit minor omni numero.* Patet ulterius

Infinitorum alia eſſe æqualia, alia inæqualia.

Et unum Infinitum æquari duobus, tribus, pluribus, vel Finitis vel Infinitis.

For 1. The Infinite Powers of the firſt Rank are $= \frac{1}{2} = \frac{1}{1 \times 2}$, and alſo equal to all the Infinitely-infinite Powers of all the other Ranks.

The Infinite Powers of the ſecond Rank are $= \frac{1}{6} = \frac{1}{2 \times 3}$

Thoſe of the third Rank are $= \frac{1}{12} = \frac{1}{3 \times 4}$

Thoſe of the fourth Rank are $= \frac{1}{20} = \frac{1}{4 \times 5}$

Thoſe of the fifth Rank are $= \frac{1}{30} = \frac{1}{5 \times 6}$

&c. *in infinitum.*

viz. equal to the reſpective mean proportional Number between the Square Numbers reſpectively. *e. g.*

4,	6,	9
9,	12,	16
16,	20,	25.
25,	30,	36.

&c. *in infinitum.*

2. The Infinite Powers of the two firſt Ranks are $= \frac{2}{3}$

Thoſe of the three firſt are $= \frac{3}{4}$

Thoſe of the four firſt are $= \frac{4}{5}$

Thoſe of the five firſt are $= \frac{5}{6}$

&c. *in infinitum.*

3. All the Powers of all the Infinite Ranks, except the firſt, are $= \frac{1}{2}$

all, except the two firſt, are $= \frac{1}{3}$

all, except the three firſt, are $= \frac{1}{4}$

all, except the four firſt, are $= \frac{1}{5}$

&c. *in infinitum.*

The later *Corollaries* may all appear by ſimple Addition and Subduction; and ſo may many more.

XXVII. 1. That the Numeral Figures now in uſe, with the manner of Computation by them (and the names of Algoriſm, appropriated to that way of Computation) came to us from the *Arabs* (but ſomewhat altered, as to the ſhape of the Figures in ſucceeding Ages) and to them from the *Indians*, is generally agreed. But it is not ſo generally agreed, of what Antiquity the Uſe of them, in our *European* Parts, hath been.

The Antiquity of the Numeral Figures; by Dr. J. Wallis. N. 154. p. 399. Dec. An. 1683.

Jo. Gerard Voſſius (De Scientiis Mathematicis) thinks they came not in Uſe here, till about the Year of our Lord 1300; or at the fartheſt, later than the Year 1250. And *P. Mabillion (De Re Diplomatica)* tells us that he hath not found them any where uſed ſooner than the 14 Century. But I think their Uſe in theſe Parts was as old at leaſt as the times of *Hermannus Contractus,* who lived about the Year of our Lord 1050. (that is about the middle of the 11th Century,) If not ſo frequently in ordinary Affairs; yet at leaſt in Mathematical things, and eſpecially in Aſtronomical Tables.

But I do not remember, that I have any where ſeen any Monument of them more Antient than the Mantle-tree of the Parlour Chimney at the Dwelling-Houſe of Mr. *Will. Richards,* the Rector of *Helmdon* in *Northampton-ſhire.*

Fig. 55.

The ſides of the Chimney, by which the Mantle-tree is ſupported, are of Stone; but the Mantle-tree itſelf is of Wood. It is all over as black as Ink, having by Age and Smoak contracted that Colour. It may yet continue many hundreds of Years; for I did not diſcern in it, any thing either of Worm, or Rottenneſs, or any tendency to it. The length of it is 5 Foot 9 Inches: Its breadth or depth at the ends, (A B) is 11 $\frac{1}{2}$ Inches, but as the middle, as C D, ſomewhat leſs. It is alſo carved from end to end: and the lower part of it is abated, as in the mouldings of other Chimneys. On the Front of the upper part there is (beginning at the middle) on 3 Squares parted from each other by a deep Furrow of Channel, the Date (I ſuppoſe, when it was firſt made) deſcribed partly in Numeral Figures A° Do I M° 133; on a fourth a Flower, and on a 5th the two Letters W. R. with an Eſcutcheon, repreſenting (I ſuppoſe) the Name of him to whom it did then belong. Both the Letters and the Figures are of an Antique Form, agreeing well enough with that Age. They are not engraved or cut in, but prominent on their ſeveral Squares (by way of *bas-relief*) the Wood being abated round about them. The o over the A, is a round o, but that over the M is a ſquare ▫.

Hence it appears, that the Uſe of ſuch Figures here in *England*, even on ordinary Occaſions, is at leaſt as ancient as the Year 1133. And I judge it to have been yet ſomewhat ancienter, becauſe the ſhape of the Figures, though not come juſt to the ſhape which we now uſe, was even then conſiderably varied from the ſhape of the *Arabick* Figures; which argues they had then been for ſome time in uſe; ſuch change of ſhape in Figures and Letters coming on gradually with time.

It need not move any ſcruple at all, that part of the Numbers is expreſſed by the Numeral Letter M (or the word *Milleſimo*, of which M° is but a Contraction) while the reſt is expreſſed in Numeral Figures. For the like doth oft occur in old *Manuſcripts*; and ſometimes even at this Day. And it doth rather favour the ſimplicity of that Age, (not very nice in ſuch things, eſpecially amongſt Mechanicks) than any deſign of Impoſture.

By Mr. Tho. Luffkin. N. 255. p. 287. N. 266. p. 677. Aug. An. 1699.

2. Over-againſt the *Market-place* in *Colcheſter*, ſtands the Houſe of Mr. *Furley*, a Linnen Draper; ſome of the backermoſt part of which is an ancient *Roman* Building, but the Front is of leſſer ſtanding, and timbred. Upon the bottom Cell (which is almoſt in the form of a Triangular Priſm) of one of the Windows of the Front, between two carved Lions, ſtands an Eſcutcheon, containing only theſe Figures 1090. The Periphery of the Cyphers, and Nine, are rather Fracted than Flected, prominent, large, and very fair; but to make them the more perſpicuous, they are gilded by the Proprietor. The Window looks directly North, the Date being thereby preſerved from the ſcorching heat of the Sun; and by its Inclination (falling from the *Vertex* or *Perpendicular* by an Angle of about 60 Degrees) from Rain, Snow, &c. If it be objected, that the ſecond and fourth Figures, may repreſent that among the *Arabians*, which is with us as 5; I anſwer, that the o is not uſed with all the *Arabs* for 5, but with ſome for a Cypher, and ſo it was uſed by the *Moors* in *Spain*, who firſt brought theſe Figures into our Parts; nor is the Square ▫ an *Arabick* Letter, but an *Engliſh* Letter of that Age. And the form of theſe Figures ſoon degenerated from that of the *Arabs*, into ſuch as we now uſe, if not at the firſt reception from the *Arabs*, [or *Moors*] certainly long before 1595, as this *Conſtruction* would make it.

Fig. 56.

The Conſtruction of Logarithms; *by Mr.* Edm. Halley. N. 216. p. 58. Mar. An 1695.

XXVIII. The old Definition of *Logarithms*, that they are *Numerorum proportionalium æqui-differentes comites*, is too ſcanty to define them fully: For they may much more properly be ſaid to be *Numeri Rationum Exponentes*; wherein *Ratio* is conſider'd as a *Quantitas ſui generis*, beginning from the Ratio of equality, or 1 to 1 = 0; being Affirmative when the Ratio is increaſing, as of Unity to a greater Number, but Negative when decreaſing; and theſe Rationes we ſuppoſe to be meaſured by the Number of the Ratiunculæ contained in each. Now theſe Ratiunculæ are to be ſo underſtood as in a continued Scale of Proportionals, infinite in Number between the two Terms of the Ratio; which infinite Number of mean Proportionals is to that infinite Number of the like and equal Ratiunculæ between any other two Terms, as the Logarithm of the one Ratio is to the Logarithm of the other. Thus if there be ſuppoſed between 1 and 10 an infinite Scale of mean Proportionals, whoſe Number is 100000, &c. *in infinitum*; between 1 and 2 there ſhall be 30102, &c. of ſuch Proportionals, and between 1 and 3 there will be 47712, &c. of them, which Numbers therefore are the Logarithms of the Rationes, of 1 to 10, 1 to 2, and 1 to 3; and not ſo properly to be called the Logarithms of 10, 2, and 3.

This being laid down, it is obvious that if between Unity and any Number propoſed, there be taken any infinity of mean Proportionals, the infinitely little Augment or Decrement of the firſt of thoſe Means from Unity, will be a Ratiuncula, that is the Momentum or Fluxion of the Ratio of Unity to the ſaid Number: And ſeeing that in theſe Continual Proportionals all the Ratiunculæ are equal, their Sum, or the whole Ratio will be as the ſaid Momentum is directly; that is, the Logarithm of each Ratio will be as the Fluxion thereof. Wherefore if the Root of any infinite Power be extracted out of any Number, the Differentiola of the ſaid Root from Unity, ſhall be as the Logarithm of that Number. So that Logarithms thus produced, may be of as many forms as you pleaſe to aſſume infinite Indices of the Power whoſe Root you ſeek: as if the Index be ſuppoſed 100000, &c. infinitely; the Roots ſhall be the Logarithms invented by the Lord *Napier*; but if the ſaid Index were 2302585, &c. Mr. *Brigg*'s Logarithms would immediately be produced. And if you pleaſe to ſtop at any Number of Figures, and not to continue them on, it will ſuffice to aſſume an Index of a Figure or two more than your intended Logarithm is to have, as Mr. *Briggs* did, who to have his Logarithms true to 14 Places, by continual extraction of the Square Root, at laſt came to have the Root of the 140737488355328*th.* Power; but how operoſe that Extraction was, will be eaſily judged by whoſo ſhall undertake to examine his *Calculus*.

Now, tho' the Notion of an infinite Power may ſeem very ſtrange, and to thoſe that know the difficulty of the Extraction of the Roots of High Powers, perhaps impracticable; yet by the help of that admirable Invention of Mr. *Newton*, whereby he determines the Unciæ or Numbers prefixt to the Members compoſing Powers, (on which chiefly depends the *Doctrine of Series*) the Infinity of the Index contributes to render the Expreſſion much more eaſy: For if the Infinite

Power to be reſolved be put (after Mr. *Newton*'s Method) $\overline{p+pq}$

$$\overline{p+pq}|^{\frac{1}{m}}$$

$\overline{p+pq}|^{\frac{1}{m}}$, or $\overline{1+q}|^{\frac{1}{m}}$, inſtead of $1+\frac{1}{m}q+\frac{1-m}{2mm}qq+\frac{1-3m+2mm}{6m^3}q^3+\frac{1-6m+11mm-6m^3}{24m^4}q^4$, &c. (which is the Root when m is finite,) becomes $1+\frac{1}{m}q-\frac{1}{2m}qq+\frac{1}{3m}q^3+\frac{1}{4m}q^4+\frac{1}{5m}q^5$, &c. mm being *infinitè* infinite, and conſequently whatever is divided thereby vaniſhing. Hence it follows that $\frac{1}{m}$ multiplied into $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5$, &c. is the Augment of the firſt of our mean Proportionals between Unity and $1+q$, and is therefore the Logarithm of the Ratio of 1 to $1+q$; and whereas the Infinite Index m may be taken at pleaſure, the ſeveral Scales of Logarithms to ſuch Indices will be as $\frac{1}{m}$, or reciprocally as the Indices. And if the Index be taken 10000, &c. as in the caſe of *Napier*'s Logarithms, they will be ſimply $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5-\frac{1}{6}q^6$, &c.

Again, If the Logarithm of a decreaſing Ratio be ſought, the Infinite Root of $1-q$, or $\overline{1-q}|^{\frac{1}{m}}$ is $1-\frac{1}{m}q-\frac{1}{2m}q^2-\frac{1}{3m}q^3-\frac{1}{4m}q^4-\frac{1}{5m}q^5-\frac{1}{6m}q^6$, &c. whence the Decrement of the firſt of our Infinite Number of Proportionals will be $\frac{1}{m}$ into $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+\frac{1}{6}q^6$, &c. which therefore will be as the Logarithm of the Ratio of Unity to $1-q$. But if m be put 10000, &c. then the ſaid Logarithm will be $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+\frac{1}{6}q^6$, &c.

Hence the terms of any Ratio being a and b, q becomes $\frac{b-a}{a}$, or the difference divided by the leſſer term, when 'tis an Increaſing Ratio, or $\frac{b-a}{b}$ when 'tis Decreaſing, or as b to a. Whence the Logarithm of the ſame Ratio may be doubly expreſt, for putting x for the difference of the terms a and b, it will be either

$$\frac{1}{m} \text{ into } \frac{x}{b} + \frac{x^2}{2bb} + \frac{x^3}{3b^3} + \frac{x^4}{4b^4} + \frac{x^5}{5b^5} + \frac{x^6}{6b^6} \text{ \&c. or}$$

$$\frac{1}{m} \text{ into } \frac{x}{a} - \frac{x^2}{2aa} + \frac{x^3}{3b^3} - \frac{x^4}{4a^4} + \frac{x^5}{5a^5} - \frac{x^6}{6a^6} \text{ \&c.}$$

But if the Ratio of a to b be ſuppoſed to be divided into two parts, *viz.* into the Ratio of a to the *Arithmetical Mean* between the Terms, and the Ratio of the ſaid *Arithmetical Mean* to the other Term b, then will the Sum of the Logarithms of thoſe two Rationes be the Logarithm of the Ratio of a to b; and ſubſtituting $\frac{1}{2}z$ inſtead of $\frac{1}{2}a + \frac{1}{2}b$ the ſaid *Arithmetical Mean*, the Logarithms of thoſe Rationes will be by the foregoing Rule

$$\frac{1}{m} \text{ into } \frac{x}{z} + \frac{xx}{2zz} + \frac{x^3}{3z^3} + \frac{x^4}{4z^4} + \frac{x^5}{5z^5} + \frac{x^6}{6z^6} \text{ \&c. and}$$

$$\frac{1}{m} \text{ into } \frac{x}{z} - \frac{xx}{2zz} + \frac{x^3}{3z^3} - \frac{x^4}{4z^4} + \frac{x^5}{5z^5} - \frac{x^6}{6z^6}, \text{ \&c.}$$

the Sum whereof $\frac{1}{m}$ into $\frac{2x}{z} * + \frac{2x^3}{3z^3} * + \frac{2x^5}{5z^5} * + \frac{2x^7}{7z^7}$ &c. will be the Logarithm of the Ratio of a to b, whoſe difference is x, and ſum z. And this Series converges twice as ſwift as the former, and therefore is more proper for the Practice of making Logarithms: which it performs with that expedition, that where x the difference is but the hundredth part of the Sum, the firſt Step $\frac{2x}{z}$ ſuffices to ſeven Places of the Logarithm, and the ſecond Step to twelve; but if *Briggs*'s firſt Twenty *Chiliads* of Logarithms be ſuppoſed made, as he very carefully computed them, to fourteen Places, the firſt Step alone is capable to give the Logarithm of any intermediate Number, true to all the Places of thoſe Tables.

After the ſame manner, may the Difference of the ſaid two Logarithms be very aptly applied to find the Logarithms of Prime Numbers, having the Logarithms of the two next Numbers above and below them: For the difference of the Ratio of a to $\frac{1}{2}z$, and of $\frac{1}{2}z$ to b, is the Ratio of ab to $\frac{1}{4}zz$, and the half of that Ratio is that of $\sqrt{ab}$ to $\frac{1}{2}z$, or of the Geometrical Mean to the Arithmetical. And conſequently the Logarithm thereof will be the half difference of the Logarithms of thoſe Rationes, *viz.*

$$\frac{1}{m} \text{ into } \frac{xx}{2zz} + \frac{x^4}{4z^4} + \frac{x^6}{6z^6} + \frac{x^8}{8z^8}, \text{ \&c.}$$

Which is a *Theorem* of good Diſpatch to find the Logarithm of $\frac{1}{2}z$. But the ſame is yet much more advantageouſly performed by a Rule derived from the foregoing, and beyond which in my Opinion nothing better can be hoped. For the Ratio of ab to $\frac{1}{4}zz$, or $\frac{1}{4}aa + \frac{1}{2}ab + \frac{1}{4}bb$, has the difference of its Terms, $\frac{1}{4}aa - \frac{1}{2}ab + \frac{1}{4}bb$, or the Square of $\frac{1}{2}a - \frac{1}{2}b = \frac{1}{4}xx$, which in the preſent caſe of finding the Logarithms of Prime Numbers is always Unity, and calling the Sum of the Terms $\frac{1}{4}zz + ab = yy$, the Logarithm of the Ratio of $\sqrt{ab}$ to $\frac{1}{2}a + \frac{1}{2}b$, or $\frac{1}{2}z$ will be found

$$\frac{1}{m} \text{ into } \frac{1}{yy} + \frac{1}{3y^{6}} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} + \frac{1}{9y^{18}}, \text{ \&c.}$$

which converges very much faſter than any *Theorem* hitherto publiſhed for this purpoſe.

Here note that $\frac{1}{m}$ is all along applied to adapt theſe Rules to all ſorts of Logarithms. If *m* be 10000, &c. it may be neglected, and you will have *Napier*'s Logarithms, as was hinted before; but if you deſire *Brigg*'s Logarithms, which are now generally received, you muſt divide your Series by 2,302585092994045684017991454684364207610101488628772976033328, or multiply it by the reciprocal thereof, *viz.* 0,434294481903251827651128918916605082294397005803666566114454.

But to ſave ſo operoſe a Multiplication (which is more than all the reſt of the Work) it is expedient to divide this Multiplicator by the Powers of *z* or *y* continually, according to the Direction of the *Theorem*, eſpecially where *x* is ſmall and Integer, reſerving the proper Quotes to be added together, when you have produced your Logarithm to as many Figures as you deſire; of which Method I will give a Specimen, in the Logarithms of the firſt Prime Numbers under 20 to ſixty places, computed by Mr. *Abraham Sharp*, as they were communicated to me by our common Friend, Mr. *Euclid Speidal*.

Num.	Logarithms,
2.	0, 301029995663981195213738894724493026768189881462108541310427
3.	0, 477121254719662437295027903255115309200128864190695864829866
7.	0, 845098040014256830712216258592636193483572396323965406503835
11.	1, 041392685158225040750199971243024241706702190466453094596539
13.	1, 113943352306837769206541895026246254561189005053673288598083
17.	1, 230448921378273028540169894328337030007567378425046397380368
19.	1, 278753600952828961536333475756929317951129337394497598906819

The next Prime Number is 23, which I will take for an Example of the foregoing Doctrine; and by the firſt Rules, the Logarithm of the Ratio of 22 to 23 will be found to be either.

$$\frac{1}{22} - \frac{1}{968} + \frac{1}{31944} - \frac{1}{937024} + \frac{1}{25768160}, \text{ \&c. or}$$

$$\frac{1}{23} + \frac{1}{1058} + \frac{1}{36501} + \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c.}$$

As likewiſe that of the Ratio of 23 to 24 by a like Proceſs.

$$\frac{1}{23} - \frac{1}{1058} + \frac{1}{36501} - \frac{1}{1119364} + \frac{1}{32181715},\ \&c.\ \text{or}$$

$$\frac{1}{24} + \frac{1}{1152} + \frac{1}{41471} + \frac{1}{1327104} + \frac{1}{39813120},\ \&c.$$

And this is the Reſult of the Doctrine of *Mercator*, as improv'd by the Learned Dr. *Wallis*. But by the ſecond *Theorem*, *viz.* $\frac{2x}{z} + \frac{2x^3}{3z^3} + \frac{2x^5}{5z^5}$, &c. the ſame Logarithms are obtained by fewer ſteps. To wit,

$$\frac{2}{45} + \frac{2}{273375} + \frac{2}{922640625} + \frac{2}{2615686171875},\ \&c.\ \text{And}$$

$$\frac{2}{47} + \frac{2}{311469} + \frac{2}{1146725035} + \frac{2}{3546361843241},\ \&c.$$

which was Invented and Demonſtrated in the Hyperbolical Spaces Analogous to the Logarithms, by the Excellent Mr. *James Gregory*, in his *Exercitationes Geometricæ*, and ſince further proſecuted by the aforeſaid Mr. *Speidall*, in a late Treatiſe in *Engliſh* by him Publiſh'd on this Subject. But the Demonſtration, as I conceive, was never till now Perfected without the conſideration of the Hyperbola, which in a matter purely Arithmetical as this is, cannot ſo properly be applied. But what follows, I think, I may more juſtly claim as my own, *viz.* *That the Logarithm of the Ratio of the Geometrical Mean to the Arithmetical*, between 22 and 24, or of $\sqrt{528}$ to 23, will be found to be either,

$$\frac{1}{1058} + \frac{1}{1119364} + \frac{1}{888215334} + \frac{1}{626487882248},\ \&c.\ \text{or}$$

$$\frac{1}{1057} + \frac{1}{3542796579} + \frac{1}{6596765584852 85},\ \&c.$$

All theſe Series being to be Multiplied into 0,4342944819, &c. if you deſign to make the *Logarithm* of *Briggs*. But with great Advantage in reſpect of the Work, the ſaid 0,4342944819, &c. is divided by 1057, and the Quotient thereof again divided by three times the Square of 1057, and that Quotient again by $\frac{5}{3}$ of that Square, and that Quotient by $\frac{7}{5}$ thereof, and ſo forth, till you have as many Figures of your Logarithm as you deſire. As for Example, the Logarithm of the Geometrical Mean between 22 and 24 is found by the Logarithms of 2, 3, and 11, to be

1,3613169612669061294500917266980ſ
1057) 43429 &c. (----41087462810146814347315886368
3 *in* 1117249) 41087 &c. (-------- 12258521544181829460074
$\frac{5}{3}$ *in* 1117249) 12258 &c. (------------- 6583235184376175
$\frac{7}{5}$ *in* 1117249) 65832 &c. (---------------- 4208829765
$\frac{9}{7}$ *in* 1117249) 42088 &c. (---------------------- 2930

Summa 1,3617278360175928788677771122ſ117

Which is the Logarithm of 23 to Thirty two Places, and obtained by 5 Diviſions with very ſmall Diviſors, all which is much leſs Work than ſimply Multiplying the Series into the ſaid Multiplicator 0,43429, &c.

From the Logarithm given to find what Ratio it expreſſes, is *a Problem* that has not been ſo much conſidered as the former, but which is ſolved with the like Eaſe, and demonſtrated by a like Proceſs, from the ſame general *Theorem* of Mr. *Newton*. For as the Logarithm of the Ratio of 1 to $1+q$ was proved to be $\overline{1+q}|^{\frac{1}{m}} - 1$, and that of the Ratio of 1 to $1-q$ to be $1 - \overline{1-q}|^{\frac{1}{m}}$: ſo the Logarithm, which we will from henceforth call L, being given, $1+L$ will be equal to $\overline{1+q}|^{\frac{1}{m}}$ in the one Caſe; and $1-L$ will be equal to $\overline{1-q}|^{\frac{1}{m}}$ in the other: Conſequently $\overline{1+L}|^{m}$ will be equal to $1+q$, and $\overline{1-L}|^{m}$ to $1-q$; that is, according to Mr. *Newton*'s ſaid Rule, $1 + mL + \frac{1}{2}m^2L^2 + \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 + \frac{1}{120}m^5L^5$, &c. will be = to $1+q$, and $1 - mL + \frac{1}{2}m^2L^2 - \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 - \frac{1}{120}m^5L^5$, &c. will be equal to $1-q$, m being any infinite Index whatſoever; which is a full and general *Propoſition* from the Logarithm given to find the Number, be the Species of Logarithms what it will. But if *Napier*'s Logarithm be given, the Multiplication by m is ſaved, (which Multiplication is indeed no other than the reducing the other Species to his) and the Series will be more ſimple, *viz.* $1 + L + \frac{1}{2}L^2 + \frac{1}{6}L^3 + \frac{1}{24}L^4 + \frac{1}{120}L^5$, &c. or $1 - L + \frac{1}{2}L^2 - \frac{1}{6}L^3 + \frac{1}{24}L^4 - \frac{1}{120}L^5$, &c. This Series, eſpecially in great Numbers, Converges ſo ſlowly, that is were to be wiſhed it could be contracted.

If one Term of the Ratio, whereof L is the Logarithm, be given, the other Term will be had eaſily by the ſame Rule: For if L were *Napier*'s Logarithm.

rithm of the Ratio of a the leſſer, to b the greater Term, b would be the Product of a into $1 + L + \frac{1}{2}L^2 + \frac{1}{6}L^3$, &c. $= a + aL + \frac{1}{2}aL^2 + \frac{1}{6}aL^3$, &c. But if b were given, a would be $= b - bL + \frac{1}{2}bL^2 - \frac{1}{6}bL^3$, &c. Whence by the help of the *Chiliads*, the Number appertaining to any Logarithm will be exactly had to the utmoſt Extent of the Tables. If you ſeek the neareſt next Logarithm, whether greater or leſſer, and call its Number a if leſſer, or b if greater, then the given L, and the difference thereof from the ſaid neareſt Logarithm you call l; it will follow that the number anſwering to the Logarithm L will be either a into $1 + l + \frac{1}{2}l^2 + \frac{1}{6}l^3 + \frac{1}{24}l^4 + \frac{1}{120}l^5$, &c. or elſe b into $1 - l + \frac{1}{2}l^2 - \frac{1}{6}l^3 + \frac{1}{24}l^4 - \frac{1}{120}l^5$, &c. wherein as l is leſs the Series will converge the ſwifter. And if the firſt 20000 Logarithms be given to 14 places, there is rarely occaſion for the three firſt Steps of this Series to find the Number to as many places. But for *Vlacq*'s great *Canon* of 100000 Logarithms, which is made but to ten places, there is ſcarce ever need for more than the firſt ſtep $a + al$, or $a + mal$ in one Caſe, or elſe $b - bl$, or $b - mbl$ in the other, to have the Number true to as many Figures as thoſe Logarithms conſiſt of.

There is another Series which is not indeed ſo ſimple and uniform, yet the firſt ſtep thereof is moſt commodious for Practice, and exact enough for Tables not exceeding 14 Places. It is thus: $a + \frac{al}{1 - \frac{1}{2}l}$, or $b - \frac{bl}{1 + \frac{1}{2}l}$ will be the Number anſwering to the Logarithm given, differing from the Truth but by one half of the third ſtep from the former Series. But that which renders it yet more eligible is, that with equal facility it ſerves for *Briggs*'s or any other ſort of Logarithms, with the only variation of Writing $\frac{1}{m}$ inſtead of 1, that is

$$a + \frac{al}{\frac{1}{m} - \frac{1}{2}l}\text{, and } b - \frac{bl}{\frac{1}{m} + \frac{1}{2}l}\text{, or } \frac{\frac{1}{m}a + \frac{1}{2}la}{\frac{1}{m} - \frac{1}{2}l} \text{ and } \frac{\frac{1}{m}b - \frac{1}{2}lb}{\frac{1}{m} + \frac{1}{2}l}\text{,}$$

which are eaſily reſolved into Analogies, *viz.*

$$\left.\begin{array}{l}\text{As } 43429 \text{ \&c. } - \frac{1}{2}l : \text{ to } 43429 + \frac{1}{2}l :: \text{ ſo is } a : \\ \text{Or As } 43429 \text{ \&c. } + \frac{1}{2}l : \text{ to } 43429 - \frac{1}{2}l :: \text{ ſo is } b : \end{array}\right\} \text{to the Number ſought.}$$

If more Steps of this Series be deſired, it will be found as follows,

$a + \frac{al}{1 - \frac{1}{2}l} - \frac{\frac{1}{13}al^3}{1 - l} + \frac{\frac{1}{17}al^5}{1 - 2l}$, *&c.* as may eaſily be demonſtrated by working out the Diviſions in each Step, and collecting the Quotes, whoſe Sum will be found to agree with our former Series, which is no other than an eaſy *Corollary* to Mr. *Newton*'s general *Theorem* for forming *Roots* and *Powers.*

XXIX. *Papers of leſs General Uſe Omitted.*

Tangents to Curves. N. 81. p. 4010. Mar. An. 1672.

1. A Breviat of Dr. *Wallis*'s two Methods of drawing Tangents; Extracted by him from his *Con. Sect.* and other Parts of his *Mathematical Works.*

Rectification of Curves. N. 98. p. 6146, 6149. Nov. An. 1673.

2. M. *Huygens* in his *Hor. Oſcill.* having given M. *Huraet* the Honour of Inventing a Curve equal to a Straight Line in the Year 1659; Dr. *Wallis* here aſſerts this Invention to Mr. *William Neile* (Son of Sir *Paul Neile*), who diſcovered and demonſtrated the Equality of a *Paraboloid* to a Straight Line two Years before. The ſame was ſoon after otherwiſe Demonſtrated by my Lord *Brounker*, and Sir *Christopher Wren*, in *June* and *July* 1657; and the Demonſtrations inſerted by Dr. *Wallis*, in his Tract *de Cycloide* 1659, with a fair Relation of that whole Matter. Beſides, Sir *Chriſtopher Wren* found a Straight Line equal to that of a *Cycloide* in the Year 1658: Yet he freely confeſſes Mr. *Neile*'s Invention of a Curve capable of *Rectification* the Year before.

Ib. p. 6150.

Transformation of Curves. N. 214. p. 233. Nov. An. 1694.

3. The Abbot *Galloys* having, in the Year 1693, aſſerted that Mr. *James Gregory* and Dr. *Barrow* ſtole their General Propoſitions concerning the *Transformation of Curves* from Mr. *Robervall*; Dr. *David Gregory* here fully refutes that Aſſertion. For Mr. *Gregory* Publiſh'd his Book at *Padua* 1668, and Dr. *Barrow* his *Lectiones Geometricæ* 1674, which Mr. *Robervall* doubtleſs had a Sight of before he dy'd (which was not till *October* 1675), yet he never complain'd of any ſuch Injury done him.

Cycloidal Spaces perfectly Quadrable. N. 217. p. 111. Oct. An. 1695.

4. Beſides that Segment of the *Semicycloidal Figure*, firſt obſerved by Sir *Chriſtopher Wren*, and after him by Mr. *Huygens*, and a Trilinear Part of it, which are capable of being Geometrically Squared, Dr. *Wallis* here produces from his Tracts *de Cycloide*, and *de Motu*, ſome other Portions thereof equally capable of Quadrature.

The Cycloid conſider'd long ago. N. 229. p. 561. June An. 1697.

5. Dr. *Wallis* finds among the Mathematical Works of *Bovillus*, Publiſh'd at ſeveral times between the Years 1501 and 1510, that the Curve (which is now call'd the *Cycloid*, was then conſider'd. But he alſo finds that *Bovillus* was not the firſt who conſider'd it: For Cardinal *Cuſanus*, as appears by an Ancient Manuſcript of his Works (tranſcrib'd by *J. Scoblant* in the Year 1451) has conſider'd it ſome time before. The Figure indeed (thro' the Unskilfulneſs of the Tranſcriber) both in the MS. and the *Baſil* Edition, A. 1565, is very ill drawn; but being Corrected according to the true Meaning of that *Cardinal*'s own Words, it evidently repreſents the Modern *Cycloid.* From hence 'tis manifeſt, that this Curve was not firſt taken into Conſideration

either by *Merſennus* or *Galilæo*, but ſome Ages before, tho' never well underſtood till this preſent Age.

6. Some Papers ſent by Mr. *Jo. Collins* to Dr. *Wallis*, giving his Thoughts about ſome Defects in Algebra, which he did not live to finiſh. *Defects in Algebra.* N. 159. *p.* 575. May An. 1684.

XXX. *Accounts of Books with Editions, Emendations, &c. Omitted.*

1. EUclidis *Elementa Geometrica, novo ordine ac Methodo demonſtrata.* Lond. 1666. N. 15. *p.* 261.

2. Archimedis *Opera*; Apollonii Perg: *Conic. Libri* 4; Theodoſii *Spharica*; *Methodo nova illuſtrata, & ſuccincte demonſtrata: ab* Iſa. Barrow, *R. S. S.* Lond. 1675. *in* 4*to*. N. 114. *p.* 314.

3. Ἀρχιμήδȣς τȣ̃ Συρακȣσίȣ Ψαμμίτης, ϗ Κύκλȣ Μέτρησις: Εὐτοκίȣ Ἀσκαλωνίτȣ εἰς αὐτὴν ὑπόμνημα, &c. *Cum Verſione & Notis* Jo. Wallis, S. S. Th. D. *Oxon.* 1676. N. 123. *p.* 567.

4. *Theon Smyrnæus*, publiſhed at *Paris* by *Iſmael Bulialdus* in Greek and Latin. N. 80. *p.* 3095.

5. Diophanti Alexandrini *Arithmeticorum Libri 6, & de Numeris Multangulis Liber unus; cum Commentariis* C. G. Bacheti, *& Obſervationibus*, D. P. de Fermat, *Senatoris* Tholoſani: *cui acceſſit Doctrina Analytica Inventum Novum.* Toloſæ 1670. in *Folio.* N. 72. *p.* 2185.

6. The Works of Monſieur *de Fermat.* N. 1. *p.* 15.

7. Franciſci du Laurens *Specimina Mathematica, duobus Libris comprehenſa. Horum prior Syntheticus agit de Genuinis Matheſeos Principiis in genere; in ſpecie autem de veris Geometriæ Elementis hucuſque nondum traditis. Poſterior Analyticus de Methodo Compoſitionis atque Reſolutionis fuſe diſſerit, & multa nova complectitur, quæ ſubtiliſſimam Analyſeos Artem mirum in modum promovent.* This Book is here Cenſured, ſome Miſtakes in it Corrected, and the Cenſure vindicated, by Dr. *Wallis.* N. 30. *p.* 580. N. 34. *p.* 654. N. 38. *p.* 744. N. 39. *p.* 775. N. 41. *p.* 825.

8. R. P. Andreæ Taquet, *è S. J. Opera Mathematica.* Antwerp. 1669. in *Folio.* N. 43. *p.* 869.

9. A Mathematical Compendium, Collected out of the Notes and Papers of Sir *Jonas Moore*, by *Nicholas Stevenſon. Lond.* 1674. in 12*mo*. N. 104. *p.* 83.

10. R. P. Claudii Franc. Milliet de Chales, è S. J. *Curſus ſeu Mundus Mathematicus, univerſam Matheſin tribus Tomis complectens.* Lugd. 1674. in *Folio.* N. 110. *p.* 229.

11. The Mathematical Works of Dr. *Jo. Wallis, Savilian Profeſſor of Geometry* in the Univerſity of *Oxford*, & F. R. S. in three Volumes in *Folio. Oxon.* N. 216. *p.* 73. N. 254. *p.* 259.

12. An Introduction to Algebra, Tranſlated out of *High Dutch* into *Engliſh* by *Tho. Branker*, M. A. much altered and augmented by Dr. *J. Pell.* Alſo a Table of ſuch odd Numbers, as are leſs than one Hundred Thouſand, ſhowing thoſe that are *Incompoſit*, and reſolving the reſt into their *Factors*, or *Coefficients. Lond.* in 4*to*. N. 35. *p.* 688.

13. *Labyrinthus Algebræ. Auth.* Joh. Jac. Ferguſon. 1667. in 4*to*. N. 49. *p.* 996.

N. 90. *p.* 5152. 14. The Elements of that Mathematical Science call'd *Algebra*; by *Jo. Kerſey.*
N. 95. *p.* 6073. Lond. 1673. in *Folio.*
N. 108. *p.* 192.
N. 143. *p.* 21. 1. A Treatiſe of *Algebra*, both Hiſtorical and Practical; by *Jo. Wallis.*
N. 17 . *p.* 1095. D. D. In the 109th Chapter there are ſome Numbers miſtaken, which are
N. 233. *p.* 730. here rectify'd by the Authot.

N. 14. *p.* 253. 16. *De Principiis & Ratiocinatione Geometrarum*; *contra Faſtum Profeſſorum*
N. 16. *p.* 239. *Geometriæ. Authore* Tho. Hobbes. This Book is here Animadverted on, and
Anſwered; by Dr. *Wallis.*

N. 48. *p.* 971. 17. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi,*
confutata. Auth. Jo. Wallis. S. T. D. Oxon. 1669. *in Quarto.*

N. 55. *p.* 1121. 18. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi,*
(ſecundo edita) denuo refutata Auth. Jo. Wallis. S. T. D. Oxon. 1669.

N. 72. *p.* 2185. 19. *Roſetum Geometricum, cum Cenſura brevi* Doctrinæ Walliſianæ *de Motu.*
Auth. Tho. Hobbes *Malmesburienſi.* Lond. 1671. in *Quarto.* This Book is
N. 73. *p.* 2202. here anſwered by Dr. *Wallis.*

N. 75. *p.* 2241. 20. Four Papers of Mr. *Hobbs*'s, Publiſhed in the Months of *Auguſt* and
September 1671. which are here anſwered by -----

N. 86. *p.* 5047. 21. *Lux Mathematica, Colliſionibus* Johannis Walliſii S. T. D. *&* Thomæ
Hobbeſii Malmsburienſis, *excuſſa, multis & fulgentiſſimis aucta radiis. Auth.*
R. R. *Adjuncta Cenſura Doctrinæ* Walliſianæ *de Libra, una cum* Roſeto Hobbeſii.
N. 87. *p.* 5067. Lond. 1672, *in Quarto.* This Book is here anſwered by Dr. *Wallis.*

N. 97. *p.* 6131. 22. *Principia & Problemata aliquot Geometrica, ante deſperata, nunc breviter*
explicata & demonſtrata. Auth. T. H. Malmsburienſi. Lond. 1673. in
Quarto.

N. 185. *p.* 245. 23. *Le Grand & Fameux Probleme de la Quadrature du Cercle reſolu Geometriquement par le Cercle & la Ligne droit, par* M. Mallement de Meſſange.
à Paris. 1686. *in Twelves.* This Book is here Refuted by M. D. *Cluverius.*
R. S. S.

N. 32. *p.* 625. 24. *Nouveaux Elemens de Geometrie:* Or a Mathematical Treatiſe, entituled
New Elements of Geometry. Paris. 1667. *in Quarto.*

N. 79. *p.* 3064 25. *Elemens de Geometrie; par le* P. Ignace Gaſton Pardies, *de la Comp. de J.*
à Paris 1671. *in Twelves.*

N. 33. *p.* 640. 26. 1. *Vera Circuli & Hyperbolæ Quadratura, in propria ſua Proportionis*
Specie inventa & Demonſtrata, a Jac. Gregorio Scoto. Patavii. *in Quarto.* This
Ib. p. 641. Subject is here further conſidered, and the Area of an Hyperbole explain'd;
by Mr. *J. Collins.*

N. 37. *p.* 732. 2. *M. Huygens* having Publiſh'd Animadverſions upon this Book, in the
Journal de Scavans 1668. Mr. *Gregory* here anſwers them. To this M. *Huygens*
N. 44. *p.* 882. Reply'd in a following Journal of that Year; and Mr. *Gregory*, further to elucidate the Controverſy, here returns a ſecond Anſwer.

N. 216. *p.* 65. 3. In the 48th Page of this Book, Mr. *Halley* has Diſcovered and Corrected a ſmall Miſtake in the Logarithm of 10.

N. 35. *p.* 685. 27. *Geometriæ pars Univerſalis, Quantitatum Curvarum Tranſmutationi & Menſura inſerviens. Auth.* Jac. Gregorio *Scoto.* Patavii 1668. *in Quarto.*

N. 37. *p.* 378. 28. *De Infinitis Spiralibus inverſis, Infinitiſque Hyperbolis, aliiſque Geometricis.*
Auth. F. Stephano de Angelis *Veneto.* Patavii. in *Quarto.*

29. *Michaeliis*

29. *Michaelis Angeli Ricci Exercitatio Geometrica.* Romæ. *in 4to.* Reprinted at *London*, and annext to *Mercator's Logarithmotechnia.* N. 37. p. 738.

30. Renati Franc. Sluſii *Meſolabum. Cui acceſſit pars altera de Analyſi, & Miſcellanea.* Leodii Eburonum 1668. *in 4to.* N. 45. p. 903.

31. *Elementa Geometriæ Planæ. Authore* Ægidio Franciſco de Gottignies *Bruxellenſi. S. J.* Romæ. 1669. *in 12mo.* N. 67. p. 2054.

32. *Synopſis Geometrica; cum tribus Opuſculis, de Linea Sinuum & Cycloide; de Maximis & Minimis, Centuria; & Synopſis Geometriæ Planæ. Auth.* Honor. Fabry. *S. J.* Lugduni Galliarum 1669. *in 12mo.* N. 67. p. 2055.

33. *Lectiones* 13. *Geometricæ; in quibus (præſertim) Generalia Linearum Curvarum Symptomata declarantur, ab* Iſaaco Barrow. Lond. 1669. *in 4to.* To theſe Lectures the Author here adds ſeveral *Corollaries* and *Theorems.* N. 75. p. 2260.

34. Eraſmi Bartholini *Selecta Geometrica.* Hauniæ. 1674. *in 4to.* N. 105. p. 137.

35. *Elemens des Mathematiques, ou Principes Generaux de toutes les Sciences qui ont les Grandeurs pour Object. Par* J. P. à Paris 1675. *in 4to.* N. 126. p. 638.

36. *Nouvelle Methode en Geometrie pour les Sections des Superficies Coniques & Cylindriques; qui ont pour Baſe des Circles, ou des Paraboles, des Ellipſes, & des Hyperboliques; par* Ph. de la Hire. à Paris 1673. *in 4to.* N. 129. p. 745.

37. *De Cycloide & Sectionibus Conicis.* Ph. de la Hire. *Ibid.* p. 746.

38. The Geometrical Key, or Conſtruction of all Equations Linear, Quadratick, Cubick, and Biquadratick, by a Circle and one only Parabola; by Mr. *Tho. Baker.* N. 157. p. 549.

39. *Exercitatio Geometrica de Dimenſione Figurarum. Auth.* Davide Gregorio. Edinb. 1684. *in 4to.* N. 163. p. 730.

40. *Methodus Figurarum Lineis Rectis & Curvis comprehenſarum Quadraturas determinandi, Auth.* J. Craige. Lond. 1685. *in 4to.* N. 183. p. 185. To this Tract the Author here makes an Addition; *Ibid.* p. 186. and takes notice of ſome Remarks made on it in the *Act. Lipſ.* by M. *Leibnitz*, and M. *J. Bernoulli.* N. 235. p. 786.

41. *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis & Locis Geometricis. Auth.* J. Craig. Lond. 1693. *in 4to.* N. 209. p. 113.

42. *Tractatus de Principiis Calculi Exponentialis. Auth.* D. Bernoullio; wherein a Miſtake is here Diſcovered and Corrected, by Mr. *Craig.* N. 245. p. 374.

43. *Analyſis Geometrica, ſive nova & vera Methodus Reſolvendi, tam Problemata Geometrica, quam Arithmeticas Quæſtiones. Pars prima, de Planis. Auth.* D. Antonio Hugone de Omerique *Sanlucarenſe.* N. 257. p. 351.

44. Stereometrical Propoſitions, variouſly applicable, but particularly intended for *Gauging*; by *Rob. Anderſon. Lond.* 1668. *in 8vo.* N. 39. p. 785.

45. *Gauging* promoted; being an Appendix to Stereometrical Propoſitions; by *Rob. Anderſon. Lond.* 1669. *in 8vo.* N. 47. p. 960.

46. *Gauging* Epitomized; by *Mich. Dary. Lond.* 1699. upon one Folio Page. N. 52. p. 1054.

47. *Tabula Numerorum Quadratorum decies Millium, una cum ipſorum Lateribus ab Unitate incipientibus, & Ordine Naturali uſque ad* 10000 *progredientibus.* Lond. 1672. N. 82. p. 4050.

48. The Deſcription and Uſe of two Arithmetick Inſtruments, &c. by Sir *Sam. Moreland. Lond.* 1673. N. 95. p. 6048.

N 139. p. 980. 49. Johannis Walliſii S. T. D. *Exercitationes tres.* 1. *De Cometarum Diſtantiis inveſtigandis.* 2. *De Rationum & Fractionum Reductione.* 3. *De Periodo*
N. 38. p. 753. *Juliano.* Lond. 1678.

Ibid. p. 756. 50. *Logarithmotechnia* Nicolai Mercatoris. Lond. 1668. *in Quarto.* This
Author's Method of ſquaring the Hyperbola, and of finding the Sum of the
Ibid. p. 759. Logarithms is here improved by Dr. *Wallis*; and further Explicated by the
Author himſelf.

CHAP. II.

Trigonometry, Surveying.

A Chronographical Problem propoſed by Mr. Rich. Townley, *ſolved by Mr.* John Collins. N. 69. p. 2093. Mar. An. 1674.

I. Prob. THE *Diſtances of three Objects in the ſame Plain being given, as A, B, C; The Angles made at a fourth Place in the ſame Plain as at S, are obſerved: the Diſtances from the Place of Obſervation to the reſpective Objects, are required.*

The *Problem* hath ſix *Caſes*.

Fig. 57. CASE I. If the Station be taken without the Triangle made by the Objects, but in one of the Sides thereof produced, as at S: find the Angle ACB; then in the Triangle ACS, all the Angles and the Side AC are known, whence either or both the Diſtances SA, or SC, may be found.

Fig. 58. *Caſe* 2. If the Station be in one of the Sides of the Triangle, as at S: then having the three Sides, AC, CB, BA, given, find the Angle CAB; then again in the Triangle SAB all the Angles, and the Side AB, are known, whence may be found either AS, or SB, Geometrically, if you make the Angle CAD equal to the obſerved Angle CSB, and draw BS parallel to DA, you determine the Point of Station S.

Fig. 59. *Caſe* 3. If the three Objects lie in a right Line as ACD (ſuppoſe it done), and that a Circle paſſeth through the Station S, and the two Exteriour Objects A, B: then is the Angle ABD, equal to the Obſerved Angle ASC (by 21.3.*E.*) as Inſiſting on the ſame Arch AD: And the Angle BAD in like manner equal to the obſerved Angle CSB: By this means the point D is determined. Joyn DC, and produce the ſame, then a Circle paſſing through the Points A, B, D, interſects DC, produced at S, the place of Station.

Calculation] In the Triangle ABD, all the Angles and the Side AB, are known, whence may be found the Side AD.

Then in the Triangle CAD, the two Sides CA, and AD, are known, and their contained Angle CAD is known; whence may be found the Angles CDA, and ACD, the Complement whereof to a Semicircle is the Angle SCA: in which Triangle the Angles are now all known, and the Side AC: whence may be found either of the Diſtances SC, or SA.

Fig. 60. *Caſe* 4. If the Station be without the Triangle made by the Objects, the Sum of the Angles obſerved is leſs than four Right Angles. The *Conſtruction* is the ſame as in the laſt *Caſe*, and the *Calculation* likewiſe; ſaving that you muſt make one Operation more, having the three Sides A C, C B, B A, thereby find the Angle C A B, which add to the Angle E A D, then you have the two Sides, *viz.* A C, being one of the Diſtances, and A D, (found as in the former *Caſe*) with their contained Angle C A D given, to find the Angles C D A, and A C D, the Complement whereof to a Semicircle is the Angle S C A: Now in the Triangle S C A, the Angle at C being found, and at S obſerved, and given by Suppoſition, the other at A is likewiſe known, as being the Complement of the two former to a Semicircle, and the Side A C given; hence the Diſtances C S, or A S, may be found.

Fig. 61. *Caſe* 5. If the Place of Station be at ſome Point within the Plain of the Triangle, made by the three Objects, the *Conſtruction* and *Calculation* is the ſame as in the laſt, ſaving only that inſtead of the obſerved Angle A S C, the Angle A B D is equal to the Complement thereof to a Semicircle, to wit, it is equal to the Angle A S D; both of them inſiſting on the ſame Arch A D: And in like manner the Angle B A D is equal to the Angle D S B, which is the Complement of the obſerved C S B; and in this *Caſe* the Sum of the three Angles obſerved, is equal to four Right Angles.

In theſe three latter *Caſes* no uſe is made of the Angle obſerved between the two Objects, as A and B, that are made the Baſe-line of the *Conſtruction*; yet the ſame is of ready uſe for finding the third Diſtance or laſt Side ſought; as *Fig.* 60. in the Triangle S A B, there is given the Diſtance A B, its oppoſite Angle equal to the Sum of the two obſerved Angles, and the Angle S A B attained, as in fourth *Caſe:* Hence the third Side, or laſt Diſtance S B, may be found.

And here it may be noted, that the three Angles C A S, A S B, S B C, are together equal to the Angle A C B; for the two Angles C S B and C B S are equal to E C B, as being the Complement of S C B to two Right Angles; and the like in the Triangle on the other Side. *Ergo*, &c.

Fig. 62. *Caſe* 6. If the three Objects be A, B, C, and the Station at S, as before, it may happen, according to the former *Conſtructions*, that the Points C and D may fall cloſe together, and ſo a right Line joyning them, ſhall be produced with Uncertainty; in ſuch *Caſe* the Circle may be conceived to paſs through the Place of Station at S, and any two of the Objects, as through B and C, wherein making the Angle D B C equal to the obſerved Angle A S C, and B C D equal to the Complement to 180 deg. of both the obſerved Angles in D S B; thereby the Point D is determined, through which, and the Points C, B, the Circle is to be deſcribed, and joyning D A, (produced when need requireth) where it interſects the Circle, as at S, is the Place of Station ſought.

This *Problem* may be of good uſe for the due Situation of Sands and Rocks, that are within ſight of three Places upon Land, whoſe Diſtances are well known; or for *Chorographical Uſes*, &c. eſpecially now there is a Method of obſerving Angles nicely accurate by the Aid of a Teleſcope.

Three Chorographic Problems solved by a Member of the Philoſophical Society *at* Oxford. N. 177. *p.* 1231. Dec. An. 1685.

II. The three following *Problems* may occur at Sea, in finding the Diſtance and Poſition of Rocks, Sands, *&c.* from the Shore; or in Surveying the Sea-Coaſt; when only two Objects, whoſe Diſtance from each other is known, can be ſeen at one Station: But eſpecially they may be uſeful to one, that would make a Map of a Country by a Series of Triangles derived from one or more meaſured Baſes; which is the moſt exact Way of finding the Bearing and Diſtance of Places from each other, and thence their true Longitude and Latitude; and may conſequently occur to one that would in that Matter meaſure a Degree on the Earth.

Fig. 63.

Prob. 1.] *There are two Objects B and C, whoſe Diſtance B C is known, and there are two Stations at A and E, where the Objects B, C, being viſible, and the Stations one from another, the Angles B A C, B A E, A E B, A E C, are known by Obſervation,* (which may be made with an ordinary *Surveying Semicircle*, or *Croſs-Staff*; or if the Objects beyond the View of the naked Eye, with a *Teleſcopic Quadrant*) *to find the Diſtances or Lines A B, A C, A E, E C.*

Fig. 64.

Conſtruction.] In each of the Triangles, B A E, C A E, two Angles at A, E, being known, the third is alſo known; then take any Line $\alpha\epsilon$ at pleaſure, on which conſtitute the Triangles, $\beta\alpha\epsilon$, $\alpha\epsilon\gamma$, reſpectively equiangular to the Triangles B A E, A E C, joyn $\beta\gamma$. Then upon B C conſtitute the Triangles B C A, B C E, equiangular to the correſpondent Triangles $\beta\gamma\alpha$, $\beta\gamma\epsilon$, joyn A E, and the thing is manifeſtly done.

The Calculation.] Aſſuming $\alpha\epsilon$, of any Number of Parts, in the Triangles $\alpha\beta\epsilon$, $\alpha\gamma\epsilon$, the Angles being given, the Sides $\alpha\beta$, $\alpha\gamma$, $\epsilon\beta$, $\epsilon\gamma$, may be found by Trigonometry: Then in the Triangle $\beta\alpha\gamma$, having the Angle $\beta\alpha\gamma$, and the Legs $\alpha\beta$, $\alpha\gamma$, we may find $\beta\gamma$. Then $\beta\gamma$: BC : : $\beta\alpha$: BA : : $\beta\epsilon$: BE : : $\gamma\alpha$: CA : : $\gamma\epsilon$: CE.

Fig. 65.

Prob. 2.] *Three Objects B, C, D, are given, or (which is the ſame) the Sides and conſequently Angles, of the Triangle B C D, are given; alſo there are two Points or Stations A, E, ſuch that at A may be ſeen the three Points B, C, E, but not D, and at the Station E, may be ſeen A, C, D, but not B; that is the Angles B A C, B A E, A E C, A E D, (and conſequently E A C, A E C) are known by Obſervation: To find the Lines A B, A C, A E, E C, E D.*

Fig. 66.

Conſtruction.] Take any Line $\alpha\epsilon$ at pleaſure, and at its Extremities make the Angles $\epsilon\alpha\gamma$, $\epsilon\alpha\beta$, $\alpha\epsilon\gamma$, $\alpha\epsilon\delta$, equal to the correſpondent obſerved Angles E A C, E A B, A E C, A E D. Produce $\beta\alpha$, $\delta\epsilon$, till they meet in φ, joyn $\varphi\gamma$; then upon C B deſcribe (according to 33. 3. E) a Segment of a Circle, that may contain an Angle $= \gamma\varphi\beta$; and upon C D deſcribe a Segment of a Circle capable of an Angle $= \gamma\varphi\delta$: Suppoſe F the common Section of theſe 2 Circles; joyn F B, F C, F D; then from the point C, draw for the Lines C A, C E, ſo that the Angle F C A may be $= \varphi\gamma\alpha$, and F C E $= \varphi\gamma\epsilon$; ſo A, E, the common Sections of C A, C E, with F B, F D, will be the Points required, from whence the reſt is eaſily deduced.

Calculation.] Aſſuming α ε of any Number, in the Triangles α γ, α ε, φ ε, all the Angles being given, with the Side α ε aſſum'd the Sides α γ, ε γ, α φ, ε φ, will be known; then in the Triangle γ α φ, the Angle γ α φ, with the Legs α γ, α φ, being known, the Angles α φ γ, α γ φ, with the Side φ γ, will be known: Then as for the reſt of the Work, the Triangle B C D having all its Sides and Angles known, and the Angles B F C, B F D, being equal to the found β φ γ, β φ δ; how to find F B, F C, F D, by *Calculation* (and alſo *Protraction*) has been already ſhewn above by Mr. *Collins*, as to all its *Caſes*. Sect. 1.

But it muſt here be noted, that if the Sum of the obſerved Angles B A E, A E D, is 180 deg. then A B, and E D, cannot meet, becauſe they are parallel, and conſequently the given Solution cannot take place; for which Reaſon I here ſubjoyn another.

Another Solution.] Upon B C deſcribe a Segment B A C, of a Circle, ſo that the Angle of the Segment may be equal to the obſerved Angle β α γ, (which is ſhewn 33. 3. *E.*) and upon C D deſcribe a Segment C E D, of a Circle, capable of an Angle equal to the obſerved C E D; from C draw the Diameters of theſe Circles C G, C H; then upon C G deſcribe a Segment of a Circle G F C, capable of an Angle equal to the obſerved Angle A B C; likewiſe upon C H, deſcribe a Circle's Segment C F H, capable of an Angle equal to the obſerved Angle C A E: ſuppoſe F the common Section of the two laſt Circles H F C, G F C; joyn F H, cutting the Circle H E C in E; joyn alſo F G, cutting the Circle G A C in A: I ſay, that A, E, are the Points required. Fig. 67.

Demonſtration.] For the Angle B A C is = β α γ, by *Conſtruction* of the Segment; alſo the Angles C E H, C A G, are right, becauſe each exiſts in a Semicircle: Therefore a Circle being deſcribed upon C F, as a Diameter, will paſs thro' E, A, therefore the Angle C A E = C F E = C F H (by *Conſtruction*) to the obſerved Angle γ α ε. In like manner the Angle C E A = C F A = C F G = obſerv'd Angle γ ε α.

If the Stations A, E, fall in a right Line with the Point C; the Lines G A, H E, being parallel, cannot meet; but in this *Caſe* the Problem is indeterminate, and capable of infinite Solutions. For, as before, upon C G, deſcribe a Segment of a Circle capable of the obſerved Angle γ ε α, and upon C H, deſcribe a Segment capable of the obſerved Angle γ α ε: then through C draw a Line any way, cutting the Circles in A, E, theſe Points will anſwer the Queſtion.

Problem 3.] *Four Points, B, C, D, F, or the four Sides of a Quadrilateral, with the Angles comprehended, are given; alſo there are two Stations A and E, ſuch, that at A, only B, C, E, are viſible, and at E, only A, D, F; that is, the Angles B A C, B A E, A E D, D E F, are given, to find the Places of the two Points A, E; and conſequently the Lengths of the Lines A B, A C, A E, E D, E F.* Fig. 68.

Conſtruction.] Upon B C (by 33. 3. *E.*) deſcribe a Segment of a Circle, that may contain an Angle equal to the obſerved Angle B A C, then from C draw the Chord C M, or a Line cutting the Circle in M, ſo that the Angle B C M, may be equal to the Supplement of the obſerved Angle B A E, *i. e.* its Reſidue to 180 deg. In like manner on D F deſcribe a Segment of a Circle, capable of an Angle equal to the obſerved D E F, and from D draw the Chord D N, ſo that the Angle F D N may be equal to the Supplement of the obſerved Angle A E F; joyn M N, cutting the two Circles in A, E: I ſay, A, E, are the two Points required.

Demonſtration.] Joyn A B, A C, E D, E F, then is the Angle M A B = B C M, (by 21. 3. E.) = Supplement of the obſerved Angle B A E, by *Conſtruction*; therefore the conſtructed Angle B A E, is equal to that which was obſerv'd. Alſo the Angle B A C, of the Segment, is, by Conſtruction of the Segment, equal to the obſerved Angle B A C. In like manner the conſtructed Angles A E F, and D E F, are equal to the Correſpondent obſerved Angles A E F, D E F; therefore A, E, are the Points required.

Calculation.] In the Triangle B C M, the Angle B C M, (= Supplement of B A E) and Angle B M C, (= B A C) are given, with the Side B C; thence M C may be found; in like manner D N, in the Triangle D N F, may be found. But the Angle M C D (= B C D − B C M) is known, with its Legs M C, C D, therefore its Baſe M D, and Angle M D C, may be known. Therefore the Angle M D N (= C D F − C D M − E D N) is known, with its Legs M D, D N; thence M N, with the Angles D M N, D N M, will be known. Then the Angle C M A (= D M C + D M N) is known, with the Angle M A C (= M A B + B A C) and M C, before found; therefore M A, and A C, will be known. In like manner in the Triangle E D N, the Angles E, N, with the Side D N, being known, the Sides E N, E D, will be known; therefore A E (= M N − M A − E N) is known. Alſo in the Triangle A B C, the Angle A, with its Sides B C, C A, being known, the Side A B will be known, with the Angle B C A; ſo in the Triangle E F D, the Angle E, with the Sides E D, D F, being known, E F will be found, with the Angle E D F. Laſtly, in the Triangle A C D, the Angle A C D (= B C D − B C A) with its Legs A C, C D, being known, the Side A D will be known; and in like manner E C, in the Triangle E D C.

Note, That in this *Problem*, as alſo in the firſt and ſecond, if the two Stations fall in a right Line with either of the given Objects, the *Locus* of A or E being a Circle, the particular Point of A or E cannot be determined from the Things given.

As to the other *Caſes* of this third *Problem*, wherein A and E may ſhift Places, *i. e.* only D, F, E, may be viſible at A, and only A, B, C, at E; or wherein B, D, E, may be viſible at A, and only C, F, A, at E; or wherein A may be on one ſide of the Quadrilateral, and E on the other; or one of the Stations within the Quadrilateral, and the other without it; I preſume that the *Surveyor* will eaſily direct himſelf, by what has been already ſaid.

The *Solution* of this third *Problem* is general, and ſerves alſo for both the precedent. For ſuppoſe C, D, the ſame Point in the laſt Figure, and it gives the *Solution* of the ſecond *Problem*; but if B, C, be ſuppoſed the ſame Points with D, F, by proceeding as in the laſt, you may directly ſolve the firſt *Problem*.

An Error of Common Surveyors, in comparing Surveys taken at long Intervals of Time with the magnetick Needle, demonſtrated: by Mr. William Molyneux. N. 230. p. 625. July, An. 1697.

III. The Variation of the magnetick Needle is ſo commonly known, that I need not inſiſt much on the Explication thereof; 'tis certain that the true ſolar Meridian, and the Meridian ſhewn by a Needle, agree but in very few Places of the World; and this too, but for a little Time (if a Moment) together; the Difference between the true Meridian and magnetick Meridian perpetually varying and changing in all Places, and at all Times; ſometimes to the Eaſtward, and ſometimes to the Weſtward.

On which Account 'tis impoſſible to compare two *Surveys* of the ſame Place, taken at diſtant Times, by magnetick Inſtruments (ſuch as the Circumferentor, by which the *Down Survey*, or Sir *William Petty*'s *Survey* of *Ireland* was taken) without due Allowance be made for this Variation. To which purpoſe, we ought to know the Difference between the magnetick Meridian and true Meridian, at that Time of the *Down Survey*, and the ſaid Difference at the Time when we make a *New Survey* to compare with the *Down Survey*.

But here I would not be underſtood, as if I propoſed hereby to ſhew, that a Map of the ſame Place, taken by magnetick Inſtruments at never ſo diſtant Times, ſhould not at one Time give the ſame Figure and Contents as at another Time. This certainly it will do moſt exactly, the Variation of the Needle having nothing to do either in the Shape or Contents of the *Survey*. All that is affected thereby, is the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cauſe a conſiderable Error (unleſs due Allowance be made for it) is what I ſhall prove moſt fully.

In order to which, let us ſuppoſe that about the Year 1657 (at which Time the *Down Survey* was taken) the magnetick Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to ſay, that there was no Variation. And indeed by Experiment it was at that Time found, as I am well aſſured, that at *Dublin* it was hardly half a Degree.

Let us ſuppoſe, that in the Year 1695 the Variation was 7 Degrees from the North to the Weſtward: That it was really ſo, I believe I am pretty well aſſured, from an Experiment made by *my ſelf* with all Diligence. But this is not material; let us now only ſuppoſe it.

Fig. 69.

Let A, B, repreſent the Survey of two Town Lands, one in the Poſſeſſion of A, and the other in Poſſeſſion of B, taken by the *Down Survey*, *Anno* 1657, when there was no Variation.

Let the Line N S, running thro' the Point P, be the true Meridian, and conſequently the magnetick Meridian alſo at that Time, becauſe of the ſuppoſed no Variation; and let this Line N S, be alſo the Boundary between the two Town Lands A, and B.

In the Year 1695, when the Variation is 7 Degrees from the North to the Weſtward, B having a Map of the *Down Survey*, and being ſuſpicious that his Neighbour A had encroached on him by a Ditch P Q, employs a Surveyor to enquire into the Matter: The Surveyor finds by his Map, that the Boundary between B and his Neighbour A, run from the Point P, through a Meadow, directly according to the magnetick Meridian S P N; but obſerving the Ditch P Q caſt up much to the Eaſtward of the preſent magnetick Meridian, he concludes that A has encroached upon B, and that the Ditch ought to have been caſt up along the Line P *q*, the Angle Q P *q* being an Angle of 7 deg. that is, the preſent Variation of the Needle, and the Line P *q*, the preſent magnetick Meridian; for which Variation not making any Allowance, he poſitively determines that B has all the Land in the Triangle Q P *q*, more than he ought to have; and that his Ditch ought to run along the Line P *q*.

'Tis true indeed, if the *Surveyor* go the whole Surround of the Land A, and B, he will find their Figure and Contents exactly agreeable to the Map here expreſſed. But then the Bearings of the Lines are all 7 deg. different from the Bearings in the Map, and they will run in and out upon the adjacent neighbouring Lands, and cauſe endleſs Differences between their Poſſeſſors; as is manifeſt from the Figure: Wherein the prick'd Lines repreſent the Diſagreement in the Bearings of the Lines, protracted from the Point P; and we ſee A encroaching upon his Neighbours on the Weſtward, as he encroaches upon B, and B's Eaſtward Neighbours encroaching on him, and ſo forward and clear round. Whereas by a due Allowance for the Variation of the Needle, all this Confuſion and Diſagreement is avoided, and every thing hits right.

Thus, for Inſtance, in the Caſe before us, knowing that the magnetick Variation has cauſed the preſent magnetick Meridian to fall in the Line *n q* P *s*, 7 deg. from the North to the Weſtward; to reduce this to the magnetick Meridian at the Time of the *Down Survey*, I muſt make the Meridian of my Map to fall 7 deg. to the Eaſtward of my magnetick Meridian; as we ſee the Line P Q, falls 7 deg. to the Eaſtward of the Line P *q*.

What is here ſaid on Suppoſition that the Magnet had no Variation at the Time of the firſt Survey taken, and that it had 7 deg. Variation Weſtward at the Time of the ſecond Survey, may eaſily be accommodated to the Suppoſal of any other Variations at the firſt and ſecond Surveys, *Mutatis Mutandis*, for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle Q P *q*, by which we reduce them to each other. The beſt Way therefore to make Maps invariable, conſtant and everlaſting, were for the Surveyors, who uſe magnetick Inſtruments, to make always Allowance for the magnetick Variation, and to protract and lay down Plats by the true Meridian.

Perhaps it may be objected, That Surveys may be taken without magnetick Inſtruments, and that therefore this Error ariſing from the magnetick Variation, and Change of the Bearing of Lines, may be avoided. To which I anſwer, firſt, That granting a Survey may be taken without magnetick Inſtruments, this is nothing againſt what we have laid down, relating to Surveys that are taken with magnetick Inſtruments, as the *Down Survey* actually was, and moſt *Surveys* at preſent actually are taken therewith. Secondly, Tho' a Survey may be taken truly without magnetick Inſtruments, ſo as to ſhew the exact Angles and Lines of the Plat, and conſequently the true Contents, yet this will not give the true Bearings of the Lines, or ſhew my Poſition in relation to my Neighbours, or other Parts of the Country. This muſt be ſupplied by the Magnet, or ſomething equivalent thereto, as finding a true meridian Line on your Land by celeſtial Obſervation. And I doubt not but the antient *Ægyptians*, before the Diſcovery of the Magnet, were forced to ſome ſuch Expedient in their Surveys, and Applotments of Lands, between Neighbour and Neighbour, after the *Inundations* of the *Nile*; which, we are told, gave the firſt Original to *Geometry* and *Surveying*: Abſolute Neceſſity and Uſe having introduced theſe, as Delight and Diverſion introduced *Aſtronomy* amongſt the *Chaldeans*.

And this brings me to another Objection, which may be made againſt the Inſtance before laid down: It may be ſaid, That certainly the Surveyor which B employed was very ignorant, who would chuſe to judge of the Line P Q rather by its Bearing, than by determining the Point Q by meaſuring from H and G. To this I anſwer, What if both the Points H and G were vaniſhed ſince the *Down Survey* was taken? What if the whole Face of the Country were changed, ſave only the Point P, and the Line P Q? How ſhall the Surveyor then judge of the Line P Q, but by its Bearing? That this is no extravagant Suppoſition, we have an Example in *Ægypt* abovementioned, where the *Nile* lays all flat before it, and ſo uniformly covers all with Mud, that there is no Diſtinction. In ſuch a Caſe your Bearing muſt certainly help you out, there is no other Way.

But I anſwer, ſecondly, To ſay that the Surveyor might have determined the Point Q by Admeaſurement from G and H, or any other adjoyning noted Points, as from F, K, I, *&c.* 'Tis very true; but then 'tis againſt our *Suppoſition*. I am upon ſhewing an Error that ariſes from judging of the Line P Q by *magnetick Bearing*; and to tell me that this might be avoided by another Way, is to ſay nothing. I my ſelf ſhew how it may be avoided, by allowing for the Variation; but ſtill it is an Error till it be avoided.

But, thirdly, if B's Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points G, F, H, K, *&c.* or any other Points in the Plat; but inſtead thereof, will fall on the Points *g*, *h*, *f*, *k*.

From what has been laid down, we may ſee the abſolute Neceſſity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which, great Diſputes may ariſe between neighbouring Proprietors of Lands: And it were to be wiſhed, that our honourable and learned Judges would take this Matter into their Conſideration, whenever any Buſineſs of this Kind comes before them.

IV. I have invented a *Level* with a Tube, with Glaſſes and a Thread, hanging between four Points, with a Weight in a Box ſo contrived, that as ſoon as the Inſtrument is ſet down, you have your *Point* of Horizon with a great deal of Exactneſs. I am making another, which playeth on one ſteel Point, ſtanding on a Diamond.

A new Level; *by Mr.* Butterfield. N. 141. *p.* 1026. Sept. An. 1678.

V. *An Account of a Book omitted*; *viz.* The Art of Levelling; by *M. Mariotte*.

N. 74. *p.* 2217.

CHAP. III.

OPTICKS.

A new Theory about Light and Colours; by Mr. Iſaac Newton. N. 80. p. 3075. Feb. An. 1672.

I. IN the Year 1666. (at which time I applied my ſelf to the Grinding of Optick Glaſſes of other Figures than Spherical) I procured me a Triangular Glaſs-Priſm, to try therewith the celebrated Phænomena of Colours. And in order thereto, having darkned my Chamber, and made a ſmall Hole in my Window-ſhuts, to let in a convenient Quantity of the Sun's Light, I placed my Priſm at its Entrance, that it might be thereby refracted to the oppoſite Wall. It was at firſt a very pleaſing Divertiſement, to view the vivid and intenſe Colours produced thereby; but after a while applying my ſelf to conſider them more circumſpectly, I became ſurpriſed to ſee them in an oblong Form; which, according to the receiv'd Laws of Refractions, I expected ſhould have been circular. They were terminated at the Sides with ſtreight Lines, but at the Ends, the Decay of Light was ſo gradual, that it was difficult to determine juſtly what was their Figure, yet they ſeemed Semicircular.

Comparing the Length of this colour'd *Spectrum* with its Breadth, I found it about five times greater; a Diſproportion ſo extravagant, that it excited me to a more than ordinary Curioſity of examining from whence it might proceed. I could ſcarce think, that the various Thickneſs of the Glaſs, or the Termination with Shadow or Darkneſs, could have any Influence on Light to produce ſuch an Effect; yet I thought it not amiſs, firſt to examine thoſe Circumſtances, and ſo try'd what would happen by tranſmitting Light through Parts of the Glaſs of divers Thickneſſes, or through Holes in the Window of divers Bigneſſes, or by ſetting the Priſm without, ſo that the Light might paſs throught it, and be refracted, before it was terminated by the Hole: But I found none of thoſe Circumſtances material. The Faſhion of the Colours was in all theſe Caſes the ſame.

Then I ſuſpected, whether by any Unevenneſs in the Glaſs, or other contingent Irregularity, theſe Colours might be thus dilated. And to try this, I took another Priſm like the former, and ſo placed it, that the Light paſſing through them both, might be refracted contrary ways, and ſo by the latter returned into that Courſe from which the former had diverted it: For by this Means I thought the regular Effects of the firſt Priſm would be deſtroy'd by the ſecond Priſm, but the irregular Ones more augmented, by the Multiplicity of Refractions. The Event was, that the Light, which by the firſt Priſm was diffuſed into an oblong Form, was by the ſecond reduced into an orbicular One, with as much Regularity as when it did not at all paſs through them.

N. 83. p. 4061. That this Experiment may be better apprehended; let E G, deſign the Window; F, the Hole in it, thro' which the Light arrives at the Priſms; A B C, the firſt Priſm, which refracts the Light towards P T, painting there the Colour in an Oblong, *Fig.* 70.

and $\alpha\beta\gamma$, the ſecond Priſm, which refracts back again the Rays to Q, where the long Image P T is contracted into a Round one. I ſuppoſe the Plane $\alpha\gamma$ parallel to B C, and $\beta\gamma$ to A C, that the Rays may be equally refracted contrary ways in both Priſms. The Priſms alſo muſt be placed very near to one another; for if their Diſtance be ſo great, the Colours begin to appear in the Light, before its Incidence on the ſecond Priſm, thoſe Colours will not be deſtroyed by the contrary Refractions of that Priſm. And if a *Lens* be placed in the Hole F, or immediately after the Priſms, ſo that its *Focus* be at the Image Q, or P T, the Perimeter of the Image Q, and the ſtraight Sides of the Image P T, will become much better defined than otherwiſe. So that, whatever was the Cauſe of that Length, 'twas not any Contingent Irregularity.

N. 80. p. 3076.
Feb. An. 1672.

I then proceeded to examine more critically, what might be effected by the Difference of the Incidence of Rays coming from divers Parts of the Sun; and to that end, meaſured the ſeveral Lines and Angles belonging to the Image. Its Diſtance from the Hole or Priſm was 22 Foot; its utmoſt Length 13 ¼ Inches; its Breadth 2 ⅝; the Diameter of the Hole ¼ of an Inch; the Angle which the Rays, tending towards the middle of the Image, made with thoſe Lines, in which they would have proceeded without Refraction, was 44 deg. 56 min. And the Vertical Angle of the Priſm, 63 deg. 12 min. Alſo the Refractions on both ſides the Priſm, that is of the Incident and Emergent Rays, were, as near as I could make them, equal, and conſequently about 54 deg. 4 min. And the Rays fell perpendicularly upon the Wall. Now ſubducting the Diameter of the Hole from the Length and Breadth of the Image, there remains 13 Inches in the Length, and 2 ⅜ the Breadth, comprehended by thoſe Rays, which paſſed through the Center of the ſaid Hole, and conſequently the Angle of the Hole, which that Breadth ſubtended, was about 31 min. anſwerable to the Sun's Diameter; but the Angle which its Length ſubtended, was more than 5 ſuch Diameters, namely 2 deg. 49 min.

Having made theſe Obſervations, I firſt computed from them the refractive Power of that Glaſs, and found it meaſured by the Ratio of the Sines, 20 to 31; and then by that Ratio I computed the Refractions of two Rays flowing from oppoſite Parts of the Sun's *Diſcus*, ſo as to differ 31 min. in their Obliquity of Incidence, and found that the emergent Rays ſhould have comprehended an Angle of about 31 min. as they did before they were incident.

But becauſe this Computation was founded on the Hypotheſis of the Proportionality of the Sines of Incidence and Refraction, which though by my own Experience I could not imagine to be ſo erroneous, as to make that Angle but 31 min. which in reality was 2 deg. 49 min. yet my Curioſity cauſed me again to take my Priſm. And having placed it at my Window, as before, I obſerved, that by turning it a little about its Axis to and fro, ſo as to vary its Obliquity to the Light, more than an Angle of 4 or 5 degrees, the Colours were not thereby ſenſibly tranſlated from their Place on the Wall, and conſequently by that Variation of Incidence, the Quantity of Refraction was not ſenſibly varied. By this Experiment therefore, as well as by the former

Computation, it was evident, that the Difference of the Incidence of Rays, flowing from divers Parts of the Sun, could not make them after Decuſſation diverge at a ſenſibly greater Angle, than that at which they before converged; which being, at moſt, but about 31 or 32 min. there ſtill remained ſome other Cauſe to be found out, from whence it could be 2 deg. 49 min.

Then I began to ſuſpect, whether the Rays, after their Trajection through the Priſm, did not move in curve Lines, and according to their more or leſs Curvity tend to divers Parts of the Wall. And it increaſed my Suſpicion, when I remember'd that I had often ſeen a Tennis-Ball ſtruck with an oblique Racket, deſcribe ſuch a curve Line. For, a circular as well as a progreſſive Motion being communicated to it by that Stroke, its Parts on that Side, where the Motions conſpire, muſt preſs and beat the contiguous Air more violently than on the other, and there excite a Reluctancy and Re-action of the Air proportionably greater. And for the ſame Reaſon, if the Rays of Light ſhould poſſibly be globular Bodies, and by their oblique Paſſage out of one Medium into another, acquire a circulating Motion, they ought to feel the greater Reſiſtance from the ambient Æther, on that Side, where the Motions conſpire, and thence be continually bowed to the other. But notwithſtanding this plauſible Ground of Suſpicion, when I came to examine it, I could obſerve no ſuch Curvity in them. And beſides (which was enough for my purpoſe) I obſerved, that the Difference betwixt the Length of the Image, and the Diameter of the Hole, through which the Light was tranſmitted, was proportionable to their Diſtance.

The gradual Removal of theſe Suſpicions at length led me to the *Experimentum Crucis*, which was this; I took two Boards, and placed one of them cloſe *behind the Priſm at the Window*, ſo that the Light might paſs through a ſmall Hole, made in it for the purpoſe, and fall on the other Board, which I placed at about 12 Feet diſtance, having firſt made a ſmall Hole in it alſo, for ſome of that incident Light to paſs through. Then I placed another Priſm behind this ſecond Board, ſo that the Light trajected through both the Boards, might paſs through that alſo, and be again refracted before it arrived at the Wall. This done, I took the firſt Priſm in my Hand, and turned it to and fro ſlowly about its Axis, ſo much as to make the ſeveral Parts of the Image, caſt on the ſecond Board, ſucceſſively paſs through the Hole in it, that I might obſerve to what Places on the Wall the ſecond Priſm would refract them. And I ſaw by the Variation of thoſe Places, that the Light, tending to that End of the Image, towards which the Refraction of the firſt Priſm was made, did in the ſecond Priſm ſuffer a Refraction conſiderably greater than the Light tending to the other End. And ſo the true Cauſe of the Length of that Image was detected to be no other, than that *Light* is not ſimilar or homogenial, but conſiſts of *difform Rays, ſome of which are more refrangible than others*; ſo that without any Difference in their Incidence on the ſame Medium, ſome ſhall be more *refracted* than others; and therefore that, according to their *particular Degrees of Refrangibility*, they were tranſmitted through the Priſm to divers Parts of the oppoſite Wall.

Ibid. p. 3081.

I ſhall now proceed to acquaint you with another more notable *Difformity* in its Rays, wherein the Origin of Colours is unfolded: Concerning which I ſhall lay down the Doctrine firſt; and then, for its Examination, give you an Inſtance or two of the Experiments, as a Specimen of the reſt.

The Doctrine you will find comprehended and illuſtrated in the following Propoſitions.

1. As the Rays of Light differ in Degrees of Refrangibility, ſo they alſo differ in their Diſpoſition to exhibit this or that particular Colour. Colours are not Qualifications of Light, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed) but original and connate Properties, which in divers Rays are divers. Some Rays are diſpoſed to exhibit a Red Colour, and no other; ſome a Yellow, and no other; ſome a Green, and no other; and ſo of the reſt. Nor are there only Rays proper and particular to the more eminent Colours, but even to all their intermediate Gradations.

2. To the ſame Degree of Refrangibility ever belongs the ſame Colour, and to the ſame Colour ever belongs the ſame Degree of Refrangibility. The leaſt refrangible Rays are all diſpoſed to exhibit a Red Colour; and contrarily, thoſe Rays which are diſpoſed to exhibit a Red Colour, are all the leaſt refrangible: So the moſt refrangible Rays are all diſpoſed to exhibit a deep Violet Colour; and contrarily, thoſe which are apt to exhibit ſuch a Violet Colour, are all the moſt refrangible: And ſo to all the intermediate Colours in a continued Series belong intermediate Degrees of Refrangibility. And this Analogy betwixt Colours and Refrangibility is very preciſe and ſtrict; the Rays always either exactly agreeing in both, or proportionally diſagreeing in both.

3. The Species of Colour, and Degree of Refrangibility proper to any particular Sort of Rays, is not mutable by Refraction, nor by Reflection from natural Bodies, nor by any other Cauſe that I could yet obſerve. When any one Sort of Rays hath been well parted from thoſe of other Kinds, it hath afterwards obſtinately retained its Colour, notwithſtanding my utmoſt Endeavours to change it. I have refracted it with Priſms, and reflected it with Bodies, which in Day-light were of other Colours; I have intercepted it with the coloured Film of Air, interceeding two compreſſed Plates of Glaſs, tranſmitted it through coloured Mediums, and through Mediums irradiated with other Sorts of Rays, and diverſly terminated it, and yet could never produce any new Colour out of it. It would by contracting or dilating become more brisk, or faint, and by the Loſs of many Rays, in ſome Caſes very obſcure and dark; but I could never ſee it changed in Specie.

4. Yet ſeeming Tranſmutations of Colours may be made, where there is any Mixture of divers Sorts of Rays: For in ſuch Mixtures, the component Colours appear not; but, by their mutual allaying each other, conſtitute a midling Colour. And therefore, if by Refraction, or any other of the aforeſaid Cauſes, the difform Rays, latent in ſuch a Mixture, be ſeparated, there ſhall emerge Colours different from the Colour of the Compoſition. Which Colours are not new generated, but only made apparent by being parted; for

if they be again intirely mix'd and blended together, they will again compoſe that Colour, which they did before Separation. And for the ſame Reaſon, Tranſmutations made by the convening of divers Colours are not real; for when the difform Rays are again ſevered, they will exhibit the very ſame Colours which they did before they enter'd the Compoſition; as you ſee Blue and Yellow Powders, when finely mix'd, appear to the naked Eye, Green, and yet the Colours of the component Corpuſcles are not thereby really tranſmuted, but only blended. For when viewed with a good Microſcope, they ſtill appear Blue and Yellow interſperſedly.

5. There are therefore two ſorts of Colours. The one Original and Simple; the other compounded of theſe. The original or primary Colours are, Red, Yellow, Green, Blue, and a Violet-Purple, together with Orange, Indico, and an indefinite Variety of intermediate Gradations.

6. The ſame Colours in Specie with theſe primary Ones, may be alſo produced by Compoſition. For a Mixture of Yellow and Blue makes Green; of Red and Yellow makes Orange; of Orange and Yellowiſh Green makes Yellow. And in general, if any two Colours be mix'd, which in the Series of thoſe generated by the Priſm are not too far diſtant one from another, they by their mutual Alloy compound that Colour, which in the ſaid Series appeareth in the Midway between them. But thoſe which are ſituated at too great a Diſtance, do not ſo. Orange and Indico produce not the intermediate Green, nor Scarlet and Green the intermediate Yellow.

7. But the moſt ſurprizing and wonderful Compoſition was that of Whiteneſs. There is no one ſort of Rays which alone can exhibit this. 'Tis ever compounded; and to its Compoſition, are requiſite all the aforeſaid primary Colours, mix'd in a due Proportion. I have often with Admiration beheld, that all the Colours of the Priſm being made to converge, and thereby to be again mix'd, as they were in the Light before it was incident upon the Priſm, reproduced Light, intirely and perfectly White, and not at all ſenſibly differing from a direct Light of the Sun, unleſs when the Glaſſes, I uſed, were not ſufficiently clear; for then they would a little incline it to their Colour.

8. Hence therefore it comes to paſs, that Whiteneſs is the uſual Colour of Light; for Light is a confuſed Aggregate of Rays indued with all ſorts of Colours, as they were promiſcuouſly darted from the various Parts of luminous Bodies. And of ſuch a confuſed Aggregate, as I ſaid, is generated Whiteneſs, if there be a due Proportion of the Ingredients; but if any one predominate, the Light muſt incline to that Colour; as it happens in the blue Flame of Brimſtone; the yellow Flame of a Candle; and the various Colours of the Fixed Stars.

9. Theſe things conſidered, the manner how Colours are produced by the Priſm is evident. For, of the Rays, conſtituting the incident Light, ſince thoſe which differ in Colour proportionally differ in Refrangibility, they by their unequal Refractions muſt be ſevered and diſperſed into an oblong Form in an orderly Succeſſion, from the leaſt refracted Scarlet, to the moſt refracted Violet. And for the ſame Reaſon it is, that Objects when looked upon thro' a Priſm, appear coloured. For the difform Rays, by their unequal

Refractions, are made to diverge towards ſeveral Parts of the *Retina*, and there expreſs the Images of things coloured, as in the former Caſe they did the Sun's Image upon a Wall. And by this Inequality of Refractions, they become not only coloured, but alſo very confuſed and indiſtinct.

10. Why the Colours of the Rainbow appear in falling Drops of Rain, is alſo from hence evident. For thoſe Drops which refract the Rays, diſpoſed to appear Purple, in greateſt Quantity to the Spectator's Eye, refract the Rays of other ſorts ſo much leſs, as to make them paſs beſide it; and ſuch are the Drops on the Inſide of the primary Bow, and on the Outſide of the ſecondary or exterior One. So thoſe Drops, which refract in greateſt Plenty the Rays, apt to appear Red, toward the Spectator's Eye, refract thoſe of other ſorts ſo much more, as to make them paſs beſide it; and ſuch are the Drops on the exterior Part of the primary, and interiour Part of the ſecondary Bow.

11. The odd Phænomena of an Infuſion of *Lignum Nephriticum*, Leaf-Gold, Fragments of coloured Glaſs, and ſome other tranſparently coloured Bodies, appearing in one Poſition of one Colour, and of another in another, are on theſe Grounds no longer Riddles. For thoſe are Subſtances apt to reflect one ſort of Light, and tranſmit another; as may be ſeen in a dark Room, by illuminating them with ſimilar or uncompounded Light. For then they appear of that Colour only, with which they are illuminated; but yet in one Poſition more vivid and luminous than in another, accordingly as they are diſpoſed more or leſs to reflect or tranſmit the incident Colour.

12. From hence alſo is manifeſt the Reaſon of an unexpected Experiment, which Mr. *Hook*, ſomewhere in his *Micrography*, relates to have made with two Wedge-like tranſparent Veſſels, filled the one with a Red, the other with a Blue Liquor: namely, that though they were ſeverally tranſparent enough, yet both together became opake; for if one tranſmitted only Red, and the other only Blue, no Rays could paſs through both.

13. I might add more Inſtances of this Nature, but I ſhall conclude with this general One. That the Colours of all natural Bodies have no other Origin than is, that they are variouſly qualified, to reflect one ſort of Light in greater plenty than another. And this I have experimented in a dark Room, by illuminating thoſe Bodies with uncompounded Light of divers Colours. For by that means any Body may be made to appear of any Colour. They have there no appropriate Colour, but ever appear of the Colour of the Light caſt upon them, but yet with this Difference, that they are moſt briſk and vivid in the Light of their own Day-light Colour. Minium appeareth there of any Colour indifferently, with which it is illuſtrated, but yet moſt luminous in Red; and ſo Biſe appeareth indifferently of any Colour, with which its illuſtrated, but yet moſt luminous in Blue: and therefore Minium reflecteth Rays of any Colour, but moſt copiouſly thoſe endowed with Red, and conſequently when illuſtrated with Day-light; that is, with all ſorts of Rays promiſcuouſly blended, thoſe qualified with Red ſhall abound moſt in the reflected Light, and by their Prevalence cauſe it to appear of that Colour. And for the ſame Reaſon Biſe, reflecting Blue moſt copiouſly, ſhall appear Blue by the Exceſs of thoſe Rays in its reflected Light; and the like of other Bodies. And that this is the intire

tire and adequate Cauſe of their Colours, is manifeſt, becauſe they have no Power to change or alter the Colours of any ſort of Rays incident apart, but put on all Colours indifferently, with which they are enlightned.

Theſe things being ſo, it can be no longer diſputed, whether there be Colours in the Dark, or whether they be the Qualities of the Objects we ſee, no nor perhaps, whether Light be a Body. For, ſince Colours are the Qualities of Light, having its Rays for their intire and immediate Subject, how can we think thoſe Rays Qualities alſo, unleſs one Quality may be the Subject of, and ſuſtain another; which, in effect, is to call it Subſtance. We ſhould not know Bodies for Subſtances, were it not for their ſenſible Qualities; and the Principal of thoſe being now found due to ſomething elſe, we have as good Reaſon to believe that to be a Subſtance alſo.

Beſides, Who ever thought any Quality to be a heterogeneous Aggregate, ſuch as Light is diſcovered to be? But to determine more abſolutely, what Light is, after what Manner refracted, and by what Modes or Actions it produceth in our Minds the Phantaſms of Colours, is not ſo eaſy: And I ſhall not mingle Conjectures with Certainties.

Reviewing what I have written, I ſee the Diſcourſe itſelf will lead to divers Experiments ſufficient for its Examination: And therefore I ſhall not trouble you further than to deſcribe one of thoſe which I have already inſinuated.

In a darkned Room make a Hole in the Shut of a Window, whoſe Diameter may conveniently be about a third Part of an Inch, to admit a convenient Quantity of the Sun's Light: And there place a clear and colourleſs Priſm, to refract the entring Light towards the further part of the Room; which, as I ſaid, will thereby be diffuſed into an oblong coloured Image. Then place a Lens of about 3 Foot Radius (ſuppoſe a broad Object-Glaſs of a three Foot Teleſcope) at the Diſtance of about 4 or 5 Foot from thence, through which all thoſe Colours may at once be tranſmitted, and made by its Refraction to convene at a further Diſtance of about 10 or 12 Feet. If at that Diſtance you intercept this Light with a Sheet of white Paper, you will ſee the Colours converted into Whiteneſs again by being mingled. But it is requiſite that the Priſm and Lens be placed ſteddy, and that the Paper, on which the Colours are caſt, be moved to and fro; for by ſuch Motion you will not only find at what Diſtance the Whiteneſs is moſt perfect, but alſo ſee how the Colours gradually convene and vaniſh into Whiteneſs; and afterwards, having croſſed one another in that Place where they compound Whiteneſs, are again diſſipated and ſevered, and in an inverted Order retain the ſame Colours which they had before they entred the Compoſition. You may alſo ſee, that if any of the Colours at the Lens be intercepted, the Whiteneſs will be changed into the other Colours. And therefore, that the Compoſition of Whiteneſs be perfect, Care muſt be taken that none of the Colours fall beſides the Lens. *Fig. 71.* Thus, in the Deſign of this Experiment, A B C expreſſeth the Priſm ſet endwiſe to ſight, cloſe by the Hole F, of the Window E G. Its vertical Angle A C B may conveniently be about 60 deg. M N deſigneth the Lens. Its Breadth 2½, or 3 Inches. S F, one of the ſtraight Lines, in which *difform Rays* may be conceived to flow ſucceſſively from the Sun. F P, and F R, two of thoſe Rays unequally refracted, which the Lens makes to converge towards Q, and after Decuſſation to diverge again. And H I, the Paper, at divers Diſtances, on which the Colours are projected; which in Q conſtitute Whiteneſs, but are Red and Yellow in R, *r*, and *ρ*, and Blue and Purple in P, *p*, and *π*.

If you proceed further to try the Impoſſibility of changing any uncompounded Colour (which I have aſſerted in the third and thirteenth Propoſitions) 'tis requiſite that the Room be made very dark, leſt any ſcattering Light, mixing with the Colour, diſturb and allay it, and render it compound, contrary to the Deſign of the Experiment. 'Tis alſo requiſite, that there be a perfecter Separation of the Colours, than, after the Manner above deſcribed, can be made by the Refraction of one ſingle Priſm; and how to make ſuch further Separations, will ſcarce be difficult to them that conſider the diſcovered Laws of Refractions. But if Tryal ſhall be made with Colours not throughly ſeparated, there muſt be allowed Changes proportionable to the Mixture. Thus if compound Yellow Light fall upon the Blue Biſe, the Biſe will not appear perfectly Yellow, but rather Green; becauſe there are in the Yellow Mixture many Rays endued with Green, and Green being leſs remote from the uſual Blue Colour of Biſe than Yellow, is the more copiouſly reflected by it.

In like manner, if any one of the priſmatick Colours, ſuppoſe Red, be intercepted, on Deſign to try the aſſerted Impoſſibility of reproducing that Colour out of the others which are pretermitted, 'tis neceſſary, either that the Colours be very well parted before the Red be intercepted, or that together with the Red, the neighbouring Colours, into which any Red is ſecretly diſperſed, (that is, the Yellow, and perhaps Green too) be intercepted; or elſe, that Allowance be made for the emerging of ſo much Red out of the Yellow-Green, as may poſſibly have been diffuſed, and ſcatteringly blended in thoſe Colours. And if theſe Things be obſerved, the new Production of Red, or any intercepted Colour, will be found impoſſible.

II. 1. To contract the Beams of the Sun without the Hole of the Window, and to place the Priſm between the Focus of the Lens and the Hole.

Some Experiments propoſed in relation to this Theory.
N. 83. *p.* 4059.
May, An. 1672.

2. To cover over both Ends of the Priſm with Paper at ſeveral Diſtances from the Middle; or with moveable Rings, to ſee how that will vary or divide the Length of the Figure.

3. To move the Priſm ſo, as the End may turn about, the Middle being ſteddy.

4. To move the Priſm by ſhoving it, till firſt the one Side, then the Middle, then the other Side paſs over the Hole, obſerving the ſame Paralleliſm.

2. I ſuppoſe the Deſign of the *Propoſer* of theſe Experiments is, to have their Events expreſſed, with ſuch Obſervations as may occur concerning them. Touching the *firſt*, I have obſerved, that the ſolar Image falling on a Paper placed at the Focus of the Lens, was by the interpoſed Priſm drawn out in Length proportional to the Priſm's Refraction or Diſtance from that Focus. And the chief Obſervable here, which I remember, was, that the ſtreight Edges of the oblong Image were diſtincter than they would have been without the Lens.

Obſervations on this Propoſal by Mr. Newton.
N. 83. *p.* 4060.
May, An. 1672.

Conſidering that the Rays coming from the Planet *Venus*, are much leſs inclined one to another, than thoſe which come from the oppoſite Parts of thn Sun's Disk; I once tried an Experiment or two with her Light. And to make it ſufficiently ſtrong, I found it neceſſary to collect it firſt by a broad Lens, and then interpoſing a Priſm between the Lens and its Focus, at ſuch Diſtance that all the Light might paſs through the Priſm, I found the Focus, which before appeared like a lucid Point, to be drawn out into a long ſplendid Line by the Priſm's Refraction.

Concerning the *ſecond* Experiment, I have occaſionally obſerved, that by covering both Ends of the Priſm with Paper at ſeveral Diſtances from the Middle, the Breadth of the ſolar Image will be increaſed or diminiſhed as much, as is the Aperture of the Priſm, without any Variation of the Length: Or, if the Aperture be augmented on all Sides, the Image on all Sides will be ſo much and no more augmented.

Of the *third* Experiment I have occaſion to ſpeak in my Anſwer to another Perſon; where you will find the Effects of two Priſms, in all croſs Poſitions of one to another, deſcribed, But if one Priſm alone be turned about, the coloured Image will only be tranſlated from Place to Place, deſcribing a Circle, or ſome other conick Section on the Wall, on which it is projected, without ſuffering any Alteration in its Shape, unleſs ſuch as may ariſe from the Obliquity of the Wall, or caſual Change of the Priſm's Obliquity to the Sun's Rays.

The Effect of the *fourth* Experiment I have already inſinuated, telling you that Light paſſing through Parts of the Priſm of divers Thickneſſes, did ſtill exhibit the ſame Phænomena.

The Genuine Method of examining this Theory; by Mr. Newton. N. 85. *p.* 5024. July, An. 1672.

III. I cannot think it effectual for determining Truth, to examine the ſeveral Ways by which Phænomena may be explained, unleſs where there can be a perfect Enumeration of all thoſe Ways. You know, the proper Method for enquiring after the Properties of Things, is to deduce them from Experiments. And I told you, that the *Theory* which I propounded, was evinced to me, not by inferring *'tis thus, becauſe 'tis not otherwiſe*; that is, not by deducing it only from a Confutation of contrary Suppoſitions, but by deriving it from Experiments concluding poſitively and directly. The Way therefore to examine it is, by conſidering, Whether the Experiments which I propound do prove thoſe Parts of the *Theory* to which they are applied, or by proſecuting other Experiments which the *Theory* may ſuggeſt for its Examination? And this I would have done in a due Method; the Laws of Refraction being thoroughly enquired into and determined, before the Nature of Colours be taken into Conſideration. It may not be amiſs to proceed according to the Series of theſe Queries; which I could wiſh were determined by the Event of proper Experiments, declared by thoſe that may have the Curioſity to examine them.

1. Whether *Rays*, that are alike incident on the ſame Medium, have *unequal Refractions*? And how great are the *Inequalities* of their Refractions at any Incidence?

2. What is *the Law* according to which each Ray is *more or leſs Refracted*; whether it be that the ſame Ray is ever Refracted according to the *ſame Ratio* of the Sines of Incidence and Refraction; and divers Rays, according to divers Ratio's; or that the Refraction of each Ray is greater or leſs without any certain Rule? That is, whether each Ray have a certain degree of Refrangibility, according to which its Refraction is performed; or is Refracted without that Regularity?

3. Whether *Rays*, which are endued with particular degrees of *Refrangibility*, when they are by any means ſeparated, have particular *Colours conſtantly belonging to them*; *viz.* the leaſt Refrangible, Scarlet; the moſt Refrangible, deep Violet; the middle, Sea-Green; and others, other Colours? And on the contrary?

4. Whether *the Colour* of any ſort of *Rays apart* may be changed by *Refraction?*

5. Whether *Colours* by coaleſcing do really *Change* one another to produce a *New Colour*, or produce it by *Mixing* only?

6. Whether *a due Mixture of Rays*, indued with all variety of Colours, produces *Light* perfectly *like that of the Sun*, and which hath all the ſame properties, and exhibits the ſame Phænomena?

7. Whether the *Competent Colours* of each Mixture be really *Changed*; or be only *Separated*, when from that Mixture various Colours are produced again by Refraction?

8. Whether there be any *other Colours* produced by *Refraction* than ſuch, as ought to reſult from the *Colours* belonging to the *diverſly Refrangible Rays*, by their being ſeparated or mixed by *that Refraction?*

To determine by Experiments theſe and ſuch like Queries, which involve the propounded Theory, ſeems the moſt proper and direct way to a Concluſion. And therefore I could wiſh all Objections were ſuſpended, taken from Hypotheſes or any other Heads than theſe two; Of ſhewing the inſufficiency of Experiments to determine theſe Queries, or prove any other parts of my Theory, by aſſigning the flaws and defects in my Concluſions drawn from them; or of producing other Experiments which directly contradict me, if any ſuch may ſeem to occur. For if the Experiments, which I urge, be Defective, it cannot be difficult to ſhew the Defects; but if Valid, then by proving the Theory, they muſt render all Objections Invalid.

Animadverſions upon this Theory, *by* R. P. Ign. Gaſton Pardies. N. 84. *p.* 4087. June. An. 1672.

IV. 1. Iſtæc tam extraordinaria Hypotheſis, quæ Dioptricæ fundamenta evertit, praxeſque hactenus inſtitutas inutiles reddit, tota nititur illo Experimento Priſmatis Cryſtallini, ubi Radii per foramen feneſtræ intra obſcurum Cubiculum ingreſſi, ac deinde in Parietem impacti, aut in charta recepti, non in rotundum conformati, ut *Cl. Newtono*, ad Regulas Refractionum receptas, attendenti, expectandum videbatur, ſed in Oblongam figuram extenſi apparuerunt: Unde concluſit, Oblongam ejuſmodi figuram ex eo eſſe, quod nonnulli Radij minus, nonnulli magis Refringerentur.

Sed mihi quidem videtur juxta communes & receptas Dioptricæ Leges figuram illam, non Rotundam, ſed Oblongam eſſe oportere. Cum enim Radij

ex oppoſitis diſci Solaris partibus procedentes, variam habeant in ipſo tranſitu Priſmatis Inclinationem, varie quoque Refringi debent; ut cum unorum Inclinatio 30 ſaltem minutis major ſit Inclinatione aliorum, major quoque evadat illorum Refractio.

Igitur Radii oppoſiti, ex altera ſuperficie Priſmatis Emergentes magis Divergunt & Divaricantur, quam ſi nullatenus, aut ſaltem æqualiter, omnes Infracti proceſſiſſent. Refractio autem iſta Radiorum fit ſolummodo verſus eas partes quæ fingi poſſunt in planis ad Axem Priſmatis rectis; nulla autem Refractionis inæqualitas contingit verſus eas partes, quæ intelliguntur in planis Axi parallelis; ut facile demonſtrari poteſt: Superficies enim duæ Priſmatis cenſeri poſſunt inter ſe Parallelæ, ratione habita ad Inclinationem Axis, cum ſingulæ ipſi Axi Parallelæ ſint. Refractio autem per duas Parallelas planas ſuperficies nulla computatur, quia quantum à prima ſuperficie Radius in unam partem torquetur, tantum ab altera in oppoſitam partem detorquetur. Igitur cum Radii Solares è foramine per Priſma tranſmiſſi ad latera quidem non frangantur, procedunt ulterius, perinde ac ſi nulla Priſmatis ſuperficies obſtitiſſet, (habita, inquam, ratione ſolum ad lateralem illam Divaricationem;) at vero cum ijdem Radii ad ſuperiores ſeu inferiores partes, alij quidem magis, alij vero minus, utpote inæqualiter Inclinati, Infringantur; neceſſe eſt eos magis inter ſe Divaricari, adeoque & in Longiorem Figuram extendi.

Quin ſi Calculus rite obeatur; ut Radii laterales inventi ſunt à *Cl. Newtono* in ea latitudine quæ ſubtendit Arcum 31. min. qui Arcus reſpondet Diametro Solis; ita nullus dubito, quin illa inventa quoque Altitudo Imaginis quæ 2 gr. & 49 min. ſubtendit, ſit illa ipſa quæ eidem Diametro Solis poſt inæquales Refractiones in illo ipſo caſu reſpondeat.

Fig. 72. Et revera, poſito Priſmate ABC, cujus Angulus A ſit 60 gr. Radio DE, qui faciat cum perpendiculari EH, Angulum 30 grad. invenio illum, dum Emergit per FG, facere cum perpendiculari FI Angulum 76 gr. 22 min. At vero poſito alio Radio *d*E, qui cum perpendiculari EH, faciat Angulum 29 gr. 30 min. invenio illum, dum Emergit per *fg*, facere cum perpendiculari *fi*, Angulum 78 gr. 45 min. Unde iſti duo Radii DE, *d*E, qui procedere ſupponuntur ex oppoſitis partibus diſci Solaris, faciuntque inter ſe Angulum 30 min. ijdem dum Emergunt per Lineas F*g*, *fg*, ita Divergunt ut conſtituant Angulum inter ſe 2 gr. 23 min. Quod ſi duo alii Radii aſſumerentur magis accedentes ad perpendicularem EH, (*v. g.* qui cum eadem perpendiculari facerent, unus quidem Angulum, 29 gr. 30 min. alter vero, 29 gr. 0 min.) tunc ijdem Radij emergentes magis adhuc Divergerent, conſtituerentque Angulum majorem etiam aliquando plus quam trium Graduum. Et præterea augetur ulterius iſta intercapedo Refractorum Radiorum ex eo, quod duo Radii DE, *d*E, concurrentes in E, illico incipiunt Divaricari, atque impingunt in duo puncta disjuncta alterius ſuperficiei, nempe in F & in *f*. Quapropter non ſufficit ab obeundum rite Calculum, & Longitudine Imaginis impactæ in chartam ſubtrahere magnitudinem Foraminis Feneſtræ; quandoquidem etiam poſito Foramine Indiviſibili E, adhuc fieret aliud veluti Foramen Latum in alia ſuperficie, nempe F*f*.

Quod etiam vocat *Experimentum Crucis*, mihi quidem videtur quadrare cum vulgaribus & receptis Refractionum Regulis. Nam, ut modo oſtendi, Radii Solares qui accedentes & Convergentes faciunt Angulum 30 min. Egredientes deinde etiam poſt Indiviſibile Foramen, Divergunt in Angulum duorum & trium Graduum. Quapropter non mirum, ſi iſti Radii, ſigillatim impingentes in alterum Priſma, perexiguo item apertum Foramine, inæqualiter Infringantur, cum ſit inæqualis illorum Inclinatio. Neque refert, quod iſti Radii attollantur aut deprimantur per converſionem primi Priſmatis, manente immoto ſecundo Priſmate, (quod tamen in omni caſu fieri non poteſt) vel quod manente primo Immobili, ſecundum moveatur, ut ſucceſſive Radios Coloratos totius Imaginis excipiat, & per proprium Foramen tranſmittat; utrolibet enim modo neceſſe eſt Radios illos extremos, hoc eſt, Rubrum & Violaceum, incidere in ſecundum Priſma ſub inæquali Angulo, adeoque eorundum Refractionem eſſe inæqualem, ut Violaceorum ſit major.

Cum igitur manifeſta cauſa appareat Oblongæ ejuſmodi Figuræ Radiorum, cauſaque illa ex ipſa natura Refractionis oriatur; non videtur neceſſe recurrere ad aliam Hypotheſin, aut admittere diverſam illam Radiorum Frangibilitatem.

Quod deinde excogitavit de *Coloribus*, illud quidem egregie conſequitur ex præcedente Hypotheſi; veruntamen nonnullas & ipſum patitur difficultates. Nam quod ait, nullum Colorem, ſed potius Candorem apparere, ubi omnes omnium Colorum Radii promiſcue confunduntur, id vero non videtur conforme omnibus Phænomenis. Certe quæ Variationes cernuntur in permiſtione diverſorum corporum, diverſis Coloribus imbutorum, eædem omnino obſervantur in permiſtione diverſorum Radiorum diverſis item Coloribus imbutorum: Atque optime ipſe advertit, quod quemadmodum ex Flavo & Cæruleo corpore exſurgit Viridis Color; ita ex Flavo & Cæruleo Radio Viridis item Color efficitur. Quare ſi omnes omnium Colorum Radii ſimul confunderentur, neceſſe eſſet in iſta Hypotheſi, ut ille Color appareret, qui revera apparet in permiſtione omnium Pigmentorum. Atqui ſi iſta, hoc eſt, Rubrum ſimul & Flavum una cum Cæruleo & Purpureo aliiſque omnibus, ſi quæ ſint, conterantur & confundantur, non jam Candidus, ſed Obſcurus & Satur Color exſurget. Ergo ſimilis Color appareret in Lumine Ordinario, quod conſtaret ex aggregatione omnium Colorum.

Præterea nihil primo aſpectu magis Ingenioſum magiſque aptum videtur, quam quod ait Circa Experimentum Acutiſſimi *Hookii*, quo duo diverſi Liquores, quorum alter Rubeus, alter Cæruleus, uterque ſigillatim Pellucidus, ſimul permixti, Opaci evadunt. Id autem ait Clariſſimus *Newtonus* ex eo oriri, quod unus Liquor ſolos Rubeos natus ſit tranſmittere, alter vero ſolos Flavos; unde permiſti nullos tranſmittent. Hoc, inquam, videtur ſtatim valde oppoſitum; nihilominus tamen ex eo conficeretur, quod ſimilis opacitas fieret in permiſtione quorumcunque Liquorum qui eſſent diverſi Coloris; quod tamen verum non eſt.

2. Refractiones à diverſa parte Priſmatis quantum poteſt Inæquales ſtatuit, R. P. *Pardies*, cum tamen ego tum in Experimentis, tum in Calculo de Experimentis iſtis inito, Æquales adhibuerim. Si autem A B C, Priſmatis Se-

Anſwered by Mr. Newton, N. p. 84. 4091. June. An. 1672.

Fig. 73. ctio ad Axem ejus Perpendicularis, F L, & K G, Radii duo in x (medio Foraminis) decuſſantes, & in Priſma illud Incidentes ad G, & L; ſintque eorum Refracti G H, & L m, ac denuo H I, & $m\,n$. Et cum Refractiones ad Latus A C, æquales eſſe Refractionibus ad Latus B C, quam proxime ſuppoſuerim; ſi A C, & B C, ſtatuantur æqualia, ſimilis erit Radiorum G H, & L m, ad A B, baſin Priſmatis Inclinatio; adeoque Ang. C L m = Ang. CHG, & Ang. C m L = Ang. C G H. Quare etiam Refractiones in G, & m, æquales erunt, ut & in L, & H; atque adeo Ang. K G A = Ang. $n\,m$ B, & Ang. F L A = Ang. B H I; & proinde Refractorum H I, & $m\,n$, eadem erit ad invicem inclinatio ac eſt Incidentium Radiorum F L, & K G. Sit ergo Angulus F x K, 30 min. æqualis nempe Solari Diametro, & erit Angulus, quem H I, & $m\,n$, comprehendunt, etiam 30 min. ſi modo Radii F L, & K G, æqualiter Refrangibiles ſtatuantur. At mihi Experienti prodiit Angulus ille circiter 2. gr. 49 min. quem Radius H I, extremum Violaceum Colorem, & $m\,n$, Cæruleum exhibens, conſtituere; ac proinde Radios illos diverſimode Refrangibiles eſſe, ſive Refractiones ſecundum Diſparem ſinuum Incidentiæ & Refractionis Rationem peragi neceſſario concedendum eſt.

Addit præterea R. P. quod non ſufficit ad obeundum rite Calculum, ex Longitudine Imaginis impacta in Chartam ſubtrahere magnitudinem Foraminis Feneſtræ; quandoquidem etiam poſito Foramine Indiviſibili, adhuc fieret aliud veluti Foramen Latum in poſteriori ſuperficie Priſmatis. Mihi tamen videtur, his non obſtantibus, quod Refractiones Radiorum, in anteriori æque ac poſteriori ſuperficie Priſmatis decuſſantium, ex adhibitis Principiis poſſint rite computari. Sed ſi res ſecus eſſet latitudo hiatus in poſteriori ſuperficie, quod ad inſtar Foraminis eſt, haud efficeret errorem duorum minutorum ſecundorum; & in rebus practicis non operæ pretium duco ad minutias iſtas attendere.

Illi inſuper *Experimento*, quod *Crucis* vocaveram, nihil adverſatur R. P. dum contendit, inæquales Radiorum diverſis Coloribus imbutorum, Refractiones ex Inæqualibus Incidentiis effectas fuiſſe. Nam Radiis pro duo admodum parva, ab invicem diſtantia, & immota Foramina, tranſeuntibus, Incidentia illæ, prout ergo Experimentum inſtitui, omnino Æquales erant, & tamen Refractiones liquido Inæquales. Sin ille de Experimentis noſtris dubitet, oro, ut Radiorum diverſis Coloribus præditorum Refractiones ex Incidentiis paribus menſuret, & ſentiet inæquales eſſe. Si modus ille, quem ego ad hoc negotium adhibui minus placeat (quo tamen nullus poteſt eſſe Luculentior, (facile eſt alios excogitare; ſicut & alios ipſe haud paucos cum fructu expertus ſum.

Contra *Theoriam* de Coloribus objicitur, quod Pulveres diverſorum Colorum permiſti, non Candidum ſed Subobſcurum & Fuſcum Colorem exhibent. Mihi vero Albus, Niger & omnes intermedii Fuſci, qui ab Albo & Nigro permiſtis componi poſſunt, non ſpecie Coloris ſed Quantitate Lucis tantum differre videntur. Et cum in miſtione Pigmentorum, ſingula corpuſcula non niſi proprium Colorem Reflectant, adeoque maxima pars Lucis incidentis ſupprimatur & retineatur; Lux Reflexa Subobſcura evadet, & quaſi cum tenebris permiſta, adeo ut non intenſum Alborem, ſed qualem Nigredinis permiſtio conficit, hoc eſt Fuſcum, exhibere debeat.

Objicitur deinde, quod à Liquoribus quibuſcunque diverſi Coloris in eodem vaſe commiſtis, æque ac in diverſis vaſis contentis, opacitas oriri debet; quod tamen, ait, verum non eſſe. Sed non video conſequentiam. Nam plurimi Liquores agunt in ſe invicem & novam ſibi mutuo partium contexturam ſecreto inducunt; unde Opaci, Diaphani, vel variis Coloribus, ex Coloribus permiſtorum nullo modo oriundis, præditi evadere poſſunt. Et hac de cauſa Experimenta hujuſmodi minus apta ſemper exiſtimavi, à quibus concluſiones deduci poſſint. Subnoto tamen, quod ad hoc Experimentum requiruntur Liquores ſaturis & intenſis Coloribus præditi, qui per paucos niſi proprii Coloris Radios tranſmittant; quales raro occurrunt, ut videbitur illuminando Liquores cum diverſis Coloribus Priſmatis in Obſcurato Cubiculo. Nam pauci reperientur, qui in propriis Coloribus ſatis Diaphani appareant, inque alienis Opaci. Convenit præterea, ut adhibiti Colores ſint inter ſe oppoſiti, quales exiſtimo fore Rubrum & Cæruleum, vel Flavum & Violaceum, vel etiam Viridem & Purpureum illum qui Coccineo affinis eſt. Et ex hujuſmodi Liquoribus nonnulli (quorum partes tingentes non congrediuntur) fortaſſe permiſti evadunt Opaciores. Sed de eventu nihil ſum ſollicitus, tum quod luculentius eſt Experimentum in Liquoribus ſeorſim exiſtentibus, tum quod Experimentum illud (ſicut & Iridis, Tincturæ Nephriticæ, & aliorum Corporum naturalium Phænomena) non ad Probandam ſed ad illuſtrandam tantum Doctrinam propoſui.

Quod R. P. *Theoriam* noſtram *Hypotheſin* vocat, amice habeo, ſiquidem ipſi nondum conſtet. Sed alio tamen conſilio propoſueram, & nihil aliud continere videtur quam proprietates quaſdam Lucis, quas jam inventas probare haud difficile exiſtimo, & quas ſi non veras eſſe cognoveram, pro futuli & inani ſpeculatione mallem repudiare, quam pro mea *Hypotheſi* agnoſcere.

Some further Objections by R. P. Pardies. N. 85. *p.* 5012. July. 1 An. 1672.

3. In ea *Hypotheſi*, quam fuſe explicat noſter *Grimaldus*, in qua ſupponitur Lumen eſſe Subſtantia quædam rapidiſſime mota, poſſet fieri aliqua diffuſio Luminis poſt tranſitum Foraminis, & decuſſationem Radiorum; Item in ea *Hypotheſi*, qua Lumen ponitur progredi per certas quaſdam Materiæ Subtilis Undulationes, ut explicat Subtiliſſimus *Hookius*, poſſunt explicari Colores per certam quandam *diffuſionem atque Expanſionem Undulationum, quæ fiat ad latera Radiorum ultra Foramen, ipſo Contagio ipſaque materiæ continuatione*. Certe ego talem adhibeo *Hypotheſin* in Diſſertatione de *Motu Undulationis*, quæ eſt *Sexta Pars meorum Mechanicorum*; ut ponam, Colores iſtos apparentes fieri ex ſola illa Communicatione Motionis, quæ ab Undulationibus directe procedentibus ad latera effundatur: Ut ſi Radii intrantes per Foramen *a*, progrediantur verſus *b*, Undulationes quidem directe terminari deberent (habendo rationem ad motum Rectum & Naturalem) ad lineam rectam *a b*; nihilominus tamen, propter continuitatem materiæ, ſit aliqua communicatio commotionis verſus Latera *e c*, ubi tremula quædam & criſpans ſucceſſio excitatur: Atque ſi in illa laterali criſpatione conſiſtere Colores ſupponatur, exiſtimo omnia Phænomena Colorum explicari poſſe, ut fuſius in ea, quam dixi, diſſertatione expono. Quibus item poſitis apparet etiam, cur ultra quam ferat Radiorum ipſorum Divaricatio, expandi Colorum Latitudinem neceſſe ſit. *Fig.* 74.

Circa *Experimentum Crucis*, nequaquam dubito, quo minus in ſuo Experimento talem ſitum adhibuerit, in quo æqualis Inclinatio fuerit Radiorum incidentium, quandoquidem id ita à ſe præſtitum expreſſe affirmat. Verum id non ego poteram conjicere ex iis quæ ſuperius Legeram; ubi ponuntur duo exigua & maxime diſtantia Foramina, & unum Priſma prope primum Foramen quod eſt in Feneſtra; per quod Priſma Radii Colorati erumpentes, Incidunt in alterum diſtans Foramen: Addebatur autem, quod ad hoc, ut omnes illi Radii ſucceſſive Inciderent in Secundum illud Foramen, convertebatur Primum Priſma ſupra Axem: Atqui hoc modo neceſſe eſt mutari Inclinationem Radiorum, qui Incidunt in ſecundum Foramen: Atque indicavi ego, quod perinde ſeſe res haberet, ſive manente Primo Priſmate Immobili, Secundum Foramen attolleretur aut deprimeretur, ut poſſit ſucceſſive Radios omnes depictæ Imaginis Solaris excipere; ſive manente iſto Secundo Foramine Immobili, Primum Priſma converteretur, ut ita eadem Imago ſitum mutaret, atque in Foramen impingere Secundum omnes ſucceſſive partes poſſet. Sed alias ſine dubio adhibuit cautiones Solertiſſimus *Newtonus*.

Quæ circa Colores objeceram, optime ſoluta exiſtimo. Quod autem *Theoriam* iſtam, appellarim *Hypotheſin*, id certe ego nullo adhibito conſilio feci; atque nomen uſurpavi quod primum occurrit: quapropter velim ut ne per contemptum adhibitam vocem ejuſmodi exiſtimet.

Anſwered by Mr. Newton. *Ibid.* p. 5014. July. An. 1672.

4. Ait R. P. quod abſq; varia Diverſorum Radiorum Refrangibilitate, poſſibile ſit explicare Longitudinem Colorum; puta ex Hypotheſi *P. Grimaldi*, per diffuſionem Luminis, quod ſupponitur eſſe ſubſtantia quædam rapidiſſime mota, vel ex Hypotheſi *Hookii* noſtri, per diffuſionem vel expanſionem Undulationum, quas ſtatuit in æthere à Lucidis corporibus excitatus quaquaverſum propagari. Addo quod ex Hypotheſi *Carteſiana* poteſt etiam effingi conſimilis diffuſio conatus vel preſſionis Globulorum, perinde ut in explicatione Caudæ Cometæ ſupponitur. Et eadem Diffuſio vel Expanſio juxta aliam quamvis Hypotheſin, in qua Lumen ſtatuitur eſſe Vis, Actio, Qualitas, vel Subſtantia quælibet à Luminoſis corporibus undique emiſſa, effingi poteſt.

Ut his reſpondeam, animadvertendum eſt, quod Doctrina illa, quam de Refractione & Coloribus explicui, in quibuſdam Lucis proprietatibus ſolummodo conſtitit, neglectis Hypotheſibus per quas proprietates illæ explicari debent. Quamobrem ab Hypotheſium contemplatione, tanquam improprio Argumentandi loco, hic abſtinendum eſſe cenſui, & Vim Objectionis abſtrahendam, ut pleniorem & magis generalem reſponſionem accipiat.

Itaque per Lumen intelligo quodlibet Ens vel Entis poteſtatem (ſive ſit Subſtantia, ſive quævis ejus Vis, Actio, vel Qualitas) quod à corpore lucido recta pergens aptum ſit ad excitandam Viſionem; & per Radios Luminis intelligo minimas vel quaſlibet indefinite parvas ejus partes quæ ab invicem non dependent, quales ſunt illi omnes Radii, quos Lucentia corpora vel ſimul vel ſucceſſive ſecundum Rectas Lineas emittunt. Nam illæ tum collaterales tum ſucceſſivæ partes Luminis ſunt independentes; ſiquidem unæ abſque aliis intercipi poſſint, & in quaſlibet plagas ſeorſim Reflecti vel Refringi. Et hoc præcognito, Objectionis Vis omnis in eo ſita erit; quod Colores per aliquam Luminis ultra Foramen diffuſionem, quæ non oritur ab inæquali diverſorum Radiorum (ſeu Luminis independentium partium Refrangibilitate, in Longum diduci poſſint.

Quod autem non aliunde Oblongentur ſuperius monſtravi: & ut omnia ſumme confirmarem, adjeci *Experimentum* illud quod jam nomine *Crucis* paſſim inſignitur: de cujus conditionibus cum R. P. dubitaverit, placuit jam deſignare Schemate. Sit B C Anterior Tabula, cui Priſma A, immediate præfigitur, ſitque D E Altera Tabula, quaſi 12 pedibus abinde diſtans, cui ſuffigitur alterum Priſma F. Tabulæ autem ad x & y, ita perforentur, ut aliquantulum Lucis ab anteriori Priſmate Refractæ trajici poſſit per utrumque Foramen ad Secundum Priſma, inque eo denuo Refringi. Jam Priſma anterius circa Axem reciproco motu convertatur, & Colores in Tabulam Poſteriorem D E procidentes, per vices attollentur ac deprimentur, eoque pacto alius atque alius Color ſucceſſive pro arbitrio trajici poteſt per Foramen ejus y, ad Poſterius Priſma, dum cæteri Colores in Tabulam impingunt: Et videbis, Radios diverſis Coloribus præditos diverſam pati Refractionem in illo poſteriori Priſmate, ex eo quod ad diverſa loca parietis vel cujuſvis obſtaculi G H, pedibus aliquot ulterius remoti, allabentur; puta Violacei Radii ad H, Rubri ad G, & Intermedii ad loca intermedia: & tamen propter determinatam Poſitionem foraminum neceſſe eſt ut Similis ſit Incidentia Radiorum cujuſque Coloris per utrumque trajecti. Atque ita ex menſura conſtat Radios, diverſis Coloribus affectos, habere diverſas Leges Refractionum. Fig. 75.

Sed ſuſpicor unde adductus ſit R. P. in dubitationem; nempe videtur collocaſſe Primum Priſma A, poſt Tabulam B C, atque ita convertendo circa Axem, veriſimile eſt Inclinationem Radiorum qui interjacent Foramina propter Intermediam Refractionem fuiſſe mutatam. At, ex deſcriptione prius expoſita, debuit Tabula illa collocari poſt Priſma, ut Radii inter Foramina in directum jacerent, quemadmodum, ex verbis, *I took two Boards and placed one of them cloſe behind the Priſm at the Window*, conſtare poteſt. Et uſus Experimenti idem innuit.

Ex abundanti placet obſervare, quod in hoc Experimento Colorata Lux ob Refractionem Secundi Priſmatis longe minus diffunditur ac divaricat, quam cum Alba exiſtit, adeo ut Imago ad G, vel H, ſit pene Circularis; præſertim ſi Priſmata ſtatuantur parallela, & in contrario ſitu Angulorum, prout in Schemate deſignantur. Quinetiam, ſi præterea diameter foraminis y, adæquet Latitudinem Colorum, nulla erit ejuſdem Coloratæ lucis in Longum diffuſio; ſed Imago, quæ à quopiam Colore ad G, vel H, effingitur, (poſitis Circularibus foraminibus, & Refractione poſterioris Priſmatis non majori quam prioris, Radiiſque ad obſtaculum quam proxime perpendicularibus) erit plane Circularis. Id quod arguit diffuſionem, de qua ſupra egimus, non ex contagione vel continuitate materiæ undulantis aut celerrime motæ vel ſimilibus cauſis ortam eſſe, ſed ex certa Refractionum cujuſque Generis Radiorum Lege. Cur autem Imago illa in uno caſu ſit Circularis, & in aliis nonnihil Oblongata, & quomodo diffuſio lucis in Longitudinem in quolibet caſu pro arbitrio minui poſſit, à Geometris determinandum, & cum experientia conferendum, relinquo.

Poſtquam proprietates Lucis his & ſimilibus Experimentis ſatis exploratæ fuerint, ſpectando Radios tanquam ejus ſive collaterales ſive ſucceſſivas partes, de quibus experti ſumus per independentiam quod ſint ab invicem diſtinctæ;

Hypotheſes exinde dijudicandæ ſunt, & quæ non poſſunt conciliari rejiciendæ. Sed leviſſimi negotij eſt accommodare Hypotheſes ad hanc Doctrinam. Nam ſi quis Hypotheſin Carteſianam defendere velit, dicendum eſt, Globulos eſſe inequales; vel preſſiones Globulorum eſſe alias aliis fortiores, & inde diverſimode Refrangibiles, & aptas ad excitandam ſenſationem diverſorum Colorum. Et ſic juxta Hypotheſin Cl. *Hookii* dicendum eſt, Undulationes Ætheris eſſe alias majores ſive craſſiores aliis. Atque ita in cæteris. Hæc enim videtur eſſe ſumme neceſſaria Lex & Conditio Hypotheſium, in quibus naturalia corpora ponuntur conſtare ex quamplurimis corpuſculis acervatim contextis, ut à diverſis Lucentium corpuſculis, vel ejuſdem corpuſculi diverſis partibus (prout Motu, Figura, Mole, aut aliis Qualitatibus differunt) inæquales preſſiones, motiones aut mota corpuſcula per Æthera quaquaverſum trajiciantur, ex quibus confuſe miſtis, Lux conſtitui ſupponetur. Et nihil durius eſſe poteſt in iſtis Hypotheſibus quam contraria ſuppoſitio.

Ex Apertura ſive Dilatatione Lucis in poſteriori facie Priſmatis, quam R. P. dixit eſſe veluti Foramen, ſufficit, quod error non emerget ſenſibilis, ſi modo aliquis emergeret. Quod ſi Calculus juxta obſervationes præciſe ineatur, error erit nullus. Nam diametro foraminis à Longitudine Imaginis ſubducta, reſtabit Longitudo quam Imago haberet, ſi modo foramen ante Priſma eſſet indiviſibile, idque non obſtante præfata Lucis dilatatione in poſteriori facie Priſmatis; ut facile oſtenditur. Deinde ex data illa Longitudine Imaginis, ac diſtantia à foramine indiviſibili, ut & poſitione & forma Priſmatis, & ad id inclinatione Incidentium Radiorum, ac angulo, quem Refracti Radii ad medium imaginis tendentes, cum à centro Solis incidentibus conſtituunt, cætera omnia determinantur. Et quæ determinant Refractiones & Poſitiones Radiorum, ſufficiunt ad Calculum iſtarum Refractionum rite ineundum. Sed res non tanti eſſe videtur ut moram inferat.

Quod R. P. Doctrinam noſtram Hypotheſin vocaverit, non aliunde factum eſſe credo, quam quod vocabulum uſurpavit quod primum occurrit; ſiquidem mos obtinuit ut quicquid exponitur in Philoſophia dicatur Hypotheſis. Et ego ſane non alio Conſilio vocabulum iſtud reprehendi, quam ut ne invaleſceret appellatio quæ recte Philoſophantibus præjudicio eſſe poſſet.

To the Satisfaction of P. Ig. Gaſt. Pardies. N. 85. *p.* 5018. July. An. 1672.

5. Omnino mihi ſatisfecit noviſſima reſponſio, à D. *Newtono* ad meas Inſtantias data. Noviſſimus Scrupulus, qui mihi hærebat circa *Experimentum Crucis*, penitus fuit exemptus. Atque nunc plane ex Figura ipſius intelligo, quod non intellexeram ante. Experimentum peractum cum fuerit iſto modo, nil habeo quod in eo deſiderem amplius.

Some Conſiderations upon this Theory; by . . . Anſwer'd by Mr. Newton. N. 88. *p.* 5086. Nov. An. 1672.

V. The Conſideration on my *Theories* conſiſt in aſcribing an Hypotheſis to me, which is not mine; in Aſſerting an Hypotheſis, which, as to the principal Parts, is not againſt me; in Granting the greateſt part of my Diſcourſe, if explicated by that Hypotheſis; and in Denying ſome things, the truth of which would have appear'd by an Experimental Examination.

Of theſe particulars I ſhall diſcourſe in order. And *firſt* of the Hypotheſis, which is aſcribed to me in theſe Words: *But grant this firſt Suppoſition, that Light is a Body, and that as many Colours or Degrees as there may be, ſo many Bodies*

there may be; all which compounded together would make White, &c. This, it ſeems, is taken for my Hypotheſis. 'Tis true, that from my *Theory* I argue the Corporiety of Light; but I do it without any abſolute Poſitiveneſs, as the Word *perhaps* intimates; and make it at moſt but a very plauſible Conſequence of the Doctrine, and not a fundamental Suppoſition, nor ſo much as any part of it; which was wholly comprehended in the precedent Propoſitions. And I ſomewhat wonder, how the Objector could imagine, that, when I had aſſerted the *Theory* with the greateſt Rigour, I ſhould be ſo forgetful as afterwards to aſſert the fundamental Suppoſition itſelf with no more than a *perhaps*. Had I intended any ſuch Hypotheſis, I ſhould ſomewhere have explained it: But I knew, that the Properties, which I declared of Light, were in ſome meaſure capable of being explicated, not only by that, but by many other mechanical Hypotheſes; and therefore I choſe to decline them all, and to ſpeak of Light in general Terms, conſidering it abſtractly, as ſomething or other propagated every way in ſtreight Lines from luminous Bodies, without determining what that thing is; whether a confuſed Mixture of difform Powers, or Beings whatſoever. And for the ſame Reaſon I choſe to ſpeak of Colours according to the Information of our Senſes, as if they were Qualities of Light without us. Whereas by that Hypotheſis, I muſt have conſider'd them rather as Modes of Senſation, excited in the Mind by various Motions, Figures, or Sizes of the Corpuſcles of Light, making various mechanical Impreſſions on the Organ of Senſe; as I expreſs'd it in that Place, where I ſpake of the Corporiety of Light.

But ſuppoſing I had propounded that Hypotheſis, I underſtand not why the Objector ſhould ſo much endeavour to oppoſe it: For certainly it has a much greater Affinity with his own Hypotheſis, than he ſeems to be aware of; the Vibrations of the Æther being as uſeful and neceſſary in this, as in his; for aſſuming the Rays of Light to be ſmall Bodies, emitted every way from ſhining Subſtances, thoſe, when they impinge on any refracting or reflecting Superficies, muſt as neceſſarily excite Vibrations in the Æther, as Stones do in Water when thrown into it. And ſuppoſing theſe Vibrations to be of ſeveral Depths or Thickneſſes, accordingly as they are excited by the ſaid corpuſcular Rays of various Sizes and Velocities; of what Uſe they will be for explicating the Manner of Reflection or Refraction, the Production of Heat by the Sun-Beams, the Emiſſion of Light from Burning, Putrifying, or other Subſtances, whoſe Parts are vehemently agitated, the Phænomena of thin tranſparent Plates and Bubbles, and of all natural Bodies, the Manner of Viſion, and the Difference of Colours, as alſo their Harmony and Diſcord; I ſhall leave to their Conſideration, who may think it worth their Endeavour, to apply this Hypotheſis to the Solution of Phænomena.

In the *ſecond* Place, I told you, That the Objector's Hypotheſis, as to the fundamental Part of it, is not againſt me. That fundamental Suppoſition is; *That the Parts of Bodies, when briskly agitated, do excite Vibrations in the Æther, which are propagated every way from thoſe Bodies in ſtreight Lines, and cauſe a Senſation of Light by beating and daſhing againſt the Bottom of the Eye,* *ſomething after the manner, that Vibrations in the Air cauſe a Senſation of Sound by beating againſt the Organs of Hearing.* Now, the moſt free and natural Application of this Hypotheſis, to the Solution of Phænomena, I take to be this: That the agitated Parts of Bodies, according to their ſeveral Sizes, Figures and Motions, do excite Vibrations in the Æther of various Depths, or Bigneſſes, which being promiſcuouſly propagated through that Medium to our Eyes, effect in us a Senſation of Light of a white Colour; but if by any means thoſe of unequal Bigneſſes be ſeparated from one another, the largeſt beget a Senſation of a red Colour; the leaſt or ſhorteſt, of a deep Violet; and the intermediate ones, of intermediate Colours; much after the manner that Bodies, according to their ſeveral Sizes, Shapes and Motions, excite Vibrations in the Air of various Bigneſſes, which, according to thoſe Bigneſſes, make ſeveral Tones in Sound: That the largeſt Vibrations are beſt able to overcome the Reſiſtance of a refracting Superficies, and ſo break through it with leaſt Refraction; whence the Vibrations of ſeveral Bigneſſes, that is, the Rays of ſeveral Colours, which are blended together in Light, muſt be parted from one another by Refraction, and ſo cauſe the Phænomena of Priſms, and other refracting Subſtances: And that it depends upon the Thickneſs of a thin tranſparent Plate or Bubble, whether a Vibration ſhall be reflected at its further Superficies, or tranſmitted; ſo that, according to the Number of Vibrations interceding the two Superficies, they may be reflected or tranſmitted for many ſucceſſive Thickneſſes. And ſince the Vibrations which make Blue and Violet, are ſuppoſed ſhorter than thoſe which make Red and Yellow, they muſt be reflected at a leſs Thickneſs of the Plate: Which is ſufficient to explicate all the ordinary Phænomena of thoſe Plates or Bubbles, and alſo of all natural Bodies, whoſe Parts are like ſo many Fragments of ſuch Plates.

Theſe ſeem to be the moſt plain, genuine, and neceſſary Conditions of this Hypotheſis. And they agree ſo juſtly with my Theory, that if the Animadverſor think fit to apply them, he need not, on that Account, apprehend a Divorce from it. But yet how he will defend it from other Difficulties, I know not; for to me the fundamental Suppoſition itſelf ſeems impoſſible, namely, that the Waves or Vibrations of any Fluid, can, like the Rays of Light, be propagated in ſtreight Lines, without a continual and very extravagant ſpreading and bending every way into the quieſcent Medium, where they are terminated by it. I miſtake, if there be not both Experiment and Demonſtration to the contrary. And as to the other two or three Hypotheſes, which he mentions, I had rather believe them ſubject to the like Difficulties, than ſuſpect the Animadverſor ſhould ſelect the worſt for his own.

What I have ſaid of this, may be eaſily applied to all other mechanical Hypotheſes, in which Light is ſuppoſed to be cauſed by any Preſſion or Motion whatſoever, excited in the Æther by the agitated Parts of luminous Bodies; for it ſeems impoſſible that any of thoſe Motions or Preſſions can be propagated in ſtreight Lines, without the like ſpreading every way into the ſhadowed Medium on which they border. But yet, if any Man can think it poſſible, he muſt at leaſt allow, that thoſe Motions, or Endeavours to Motion, cauſed in the Æther by the ſeveral Parts of any lucid Body, that differ

fer in Size, Figure, and Agitation, muſt neceſſarily be unequal: Which is enough to denominate Light an Aggregrate of *Difform Rays*, according to any of thoſe Hypotheſes. And if thoſe original Inequalities may ſuffice to difference the Rays in Colour and Refrangibility, I ſee no reaſon, why they, that adhere to any of thoſe Hypotheſes, ſhould ſeek for other Cauſes of theſe Effects, unleſs (to uſe the Objector's Argument) *they will multiply Entities without Neceſſity*.

The *third* Thing to be conſider'd is, the Condition of the Animadverſor's Conceſſions, which is, That I would explicate my Theories by his Hypotheſis: And if I could comply with him in that Point, there would be little or no Difference between us: For he grants, That, without any reſpect to a different Incidence of Rays, there are different Refractions; but he would have it explicated, not by the different Refrangibility of ſeveral Rays, but by the ſplitting and rarefying Æthereal Pulſes. He grants my third, fourth, and ſixth Propoſitions; the Senſe of which is, That uncompounded Colours are unchangeable, and that compounded Ones are changeable only by reſolving them into the Colours, of which they are compounded; and that all the Changes, which can be wrought in Colours, are effected only by variouſly mixing or parting them: But he grants them on Condition, that I will explicate Colours by the two Sides of a ſplit Pulſe, and ſo make but two Species of them, accounting all other Colours in the World to be but various Degrees and Dilutings of thoſe two. And he further grants, that Whiteneſs is produced by the Convention of all Colours; but then I muſt allow it to be not only by Mixture of thoſe Colours, but by a farther Uniting of the Parts of the Ray ſuppoſed to be formerly ſplit.

If I would proceed to examine theſe his Explications, I think it would be no difficult Matter to ſhew, that they are not only inſufficient, but in ſome reſpects to me (at leaſt) unintelligible: For though it be eaſy to conceive, how Motion may be dilated, and ſpread, or how parallel Motions may become diverging; yet I underſtood not, by what Artifice any linear Motion can by a refracting Superficies be infinitely dilated and rarefied, ſo as to become ſuperficial: Or, if that be ſuppoſed, yet I underſtand as little, why it ſhould be ſplit at ſo ſmall an Angle only, and not rather ſpread and diſperſed through the whole Angle of Refraction. And further, though I can eaſily imagine, how unlike Motions may croſs one another; yet I cannot well conceive, how they ſhould coaleſce into one uniform Motion, and then part again, and recover the former Unlikeneſs; notwithſtanding that I conjecture the Ways, by which the Animadverſor may endeavour to explain it. So that the direct, uniform, and undiſturb'd Pulſes ſhould be ſplit and diſturbed by Refraction; and yet the oblique and diſturbed Pulſes perſiſt without ſplitting, or further Diſturbance, by following Refractions, is (to me) as unintelligible, and there is as great a Difficulty in the Number of Colours; as you will ſee hereafter.

But whatever be the Advantages or Diſadvantages of this Hypotheſis, I hope I may be excuſed from taking it up, ſince I do not think it needful to explicate my Doctrine by any Hypotheſis at all: For if Light be conſider'd abſtractedly without reſpect to any Hypotheſis, I can as eaſily conceive, that the ſeveral Parts of a ſhining Body may emit Rays of different Colours and other Qualities, of all which Light is conſtituted, as that the ſeveral Parts of a falſe or uneven String, or of unevenly agitated Water in a Brook or Cataract, or the ſeveral Pipes of an Organ inſpir'd all at once, or all the Variety of ſounding Bodies in the World together, ſhould produce Sounds of ſeveral Tones, and propagate them thro' the Air confuſedly intermix'd. And if there were any natural Bodies that could reflect Sounds of one Tone, and ſtifle or tranſmit thoſe of another; then, as the Echo of a confuſed Aggregate of all Tones would be that particular Tone, which the echoing Body is diſpoſed to reflect; ſo, ſince (even by the Animadverſor's Conceſſions) there are Bodies apt to reflect Rays of one Colour, and ſtifle or tranſmit thoſe of another; I can as eaſily conceive, that thoſe Bodies, when illuminated by a Mixture of all Colours, muſt appear of that Colour only which they reflect.

But when the Objector would inſinuate a Difficulty in theſe Things, by alluding to Sounds in the String of a muſical Inſtrument before Percuſſion, or in the Air of an Organ-Bellows before its Arrival at the Pipes; I muſt confeſs, I underſtand it as little, as if one had ſpoken of Light in a Piece of Wood before it be ſet on Fire, or in the Oyl of a Lamp before it aſcend up the Match to feed the Flame.

You ſee therefore how much it is beſides the Buſineſs in Hand to diſpute about Hypotheſes. For which Reaſon I ſhall now, in the laſt Place, proceed to abſtract the Difficulties in the Animadverſor's Diſcourſe, and without having regard to any Hypotheſis, conſider them in general Terms. And they may be reduced to theſe three Queries:

1. *Whether the unequal Refractions, made without reſpect to any Inequality of Incidence, be cauſed by the different Refrangibility of ſeveral Rays; or by the ſplitting, breaking, or diſſipating the ſame Ray into diverging Parts?*
2. *Whether there be more than two Sorts of Colours?*
3. *Whether Whiteneſs be a Mixture of all Colours?*

The *firſt* of theſe *Queries* you may find determined above by an Experiment: The Deſign of which was to ſhew, that the Length of the colour'd Image proceeded not from any Unevenneſs in the Glaſs, or any other contingent Irregularity in the Refractions. Amongſt other Irregularities, I know not, what is more obvious to ſuſpect, than a fortuitous Dilating and Spreading of Light after ſome ſuch manner, as *Des Cartes* has deſcribed in his *Æthereal Refractions* for explicating the Tail of a Comet; or as the Animadverſor now ſuppoſes to be effected by the ſplitting and rarefying of his æthereal Pulſes. And to prevent the Suſpicion of any ſuch Irregularities, I told you, that I refracted the Light contrary ways, with two Priſms ſucceſſively, to deſtroy thereby the regular Effects of the firſt Priſm by the ſecond, and to diſcover the irregular Effects by augmenting them with iterated Refractions. Now, amongſt other Irregularities, if the firſt Priſm had ſpread and diſſipated every Ray into an indefinite Number of diverging Parts, the ſecond ſhould in like manner have ſpread and diſſipated every one of thoſe Parts into a further indefinite Number, whereby the Image would have been ſtill more dilated, contrary to the Event. And this ought to have happen'd, becauſe thoſe linear diverging Parts depend not on one another for the manner of their Refraction, but are every one of them as truly and compleatly Rays, as the whole was before its Incidence; as may appear by intercepting them ſeverally.

The Reaſonableneſs of this Proceeding, will, perhaps, better appear by acquainting you with this further Circumſtance. I ſometimes placed the ſecond Priſm in a Poſition tranſverſe to the firſt, on Deſign to try if it would make the long Image become four-ſquare, by Refractions croſſing thoſe that had drawn the round Image into a long one. For if, amongſt other Irregularities, the Refraction of the firſt Priſm, did, by ſplitting, dilate a linear Ray into a ſuperficial; the croſs Refractions of that ſecond Priſm ought, by further ſplitting, to dilate and draw that ſuperficial Ray into a pyramidial Solid. But, upon Tryal, I found it otherwiſe; the Image being as regularly oblong as before, and inclined to both the Priſms at an Angle of 45 degrees.

I tried alſo all other Poſitions of the ſecond Priſm, by turning the Ends about its middle Part; and in no caſe could obſerve any ſuch Irregularity. The Image was ever alike inclined to both Priſms, its Breadth anſwering to the Sun's Diameter, and its Length being greater or leſs, according as the Refractions more or leſs agreed or contradicted one another.

And by theſe Obſervations, ſince the Breadth of the Image was not augmented by the croſs Refraction of the ſecond Priſm, that Refraction muſt have been performed without any ſplitting or dilating of the Ray; and therefore at leaſt the Light incident on that Priſm muſt be granted an Aggregate of Rays *Unequally Refrangible* in my Senſe. And ſince the Image was equally inclined to both Priſms, and conſequently the Refractions alike in both, it argues, that they were performed according to ſome conſtant Law, without any Irregularity.

To determine the *ſecond Query*, The Animadverſor refers to an Experiment made with two Wedge-like Boxes, recited in the *Micrography* of the Ingenious Mr. *Hook*, *Obſerv.* 10. *Pag.* 73. The Deſign of which was to produce all Colours out of a Mixture of two. But there is, I conceive, a double Defect in this Inſtance: For it appears not, that by this Experiment all Colours can be produced out of two; and if they could, yet the Inference would not follow.

That all Colours cannot by that Experiment be produced out of two, will appear by conſidering, that the Tincture of Aloes, which afforded one of thoſe Colours, was not all over of one uniform Colour, but appear'd Yellow near the Edge of the Box, and Red at other Places where it was thicker; affording all Variety of Colours, from a pale Yellow to a deep Red or Scarlet, according to the various Thickneſs of the Liquor. And ſo the Solution of Copper, which afforded the other Colour, was of various Blues and Indicoes. So that inſtead of two Colours, here is a great Variety made uſe of for the Production of all others. Thus, for Inſtance, to produce all Sorts of Greens, the ſeveral Degrees of Yellow and pale Blue muſt be mix'd; but to compound Purples, the Scarlet and deep Blue are to be the Ingredients.

Now if the Animadverſor contend, that all the Reds and Yellows of the one Liquor, or Blues and Indicoes of the other, are only various Degrees and Dilutings of the ſame Colour, and not divers Colours, that is a begging of the Queſtion: And I ſhould as ſoon grant, that the two Thirds or Sixths in Muſick are but ſeveral Degrees of the ſame Sound, and not divers Sounds. Certainly it is much better to believe our Senſes, informing us, that Red and Yellow are divers Colours; and to make it a philoſophical Query, Why the ſame Liquor doth, according to its various Thickneſs, appear of thoſe divers Colours, than to ſuppoſe them to be the ſame Colour, becauſe exhibited by the ſame Liquors? For if that were a ſufficient Reaſon, then Blue and Yellow muſt alſo be the ſame Colour, ſince they are both exhibited by the ſame Tincture of Nephritick Wood. But that they are divers Colours, you will more fully underſtand by the Reaſon, which in my Judgment is this: The Tincture of Aloes is qualified to tranſmit moſt eaſily the Rays indued with Red, moſt difficultly the Rays indued with Violet, and with intermediate Degrees of Facility, the Rays indued with intermediate Colours. So that where the Liquor is very thin, it may ſuffice, to intercept moſt of the Violet, and yet tranſmit moſt of the other Colours; all which together muſt compound a middle Colour, that is, a faint Yellow. And where it is ſo much thicker, as alſo to intercept moſt of the Blue and Green, the remaining Green, Yellow and Red, muſt compound an Orange. And where the Thickneſs is ſo great, that ſcarce any Rays can paſs through it beſides thoſe indued with Red, it muſt appear of that Colour, and that ſo much the deeper, and obſcurer, by how much the Liquor is thicker. And the ſame may be underſtood of the various Degrees of Blue, exhibited by the Solution of Copper, by reaſon of its Diſpoſition to intercept Red moſt eaſily, and tranſmit a deep Blue or Indico Colour moſt freely.

But ſuppoſing that all Colours might, according to this Experiment, be produced out of two by Mixture; yet it follows not, that thoſe two are the only original Colours; and that for a double Reaſon: Firſt, Becauſe thoſe two are not themſelves original Colours, but compounded of others; there being no Liquor, nor any other Body in Nature, whoſe Colour in Day-light is wholly uncompounded. And then, becauſe though thoſe two were original, and all others might be compounded of them, yet it follows not that they cannot be otherwiſe produced. For I ſaid, that they had a double Origin, the ſame Colours to Senſe being in ſome Caſes compounded, and in others uncompounded; and ſufficiently declared in my *third* and *fourth Propoſitions*, and in the *Concluſion*, by what Properties the one might be known and diſtinguiſh'd from the other. But becauſe I ſuſpect, by ſome Circumſtances, that the Diſtinction might not be rightly apprehended, I ſhall once more declare it, and further explain it by Examples.

That Colour is primary or original, which cannot by any Art be changed, and whoſe Rays are alike refrangible: And that compounded, which is changeable into other Colours, and whoſe Rays are not alike refrangible. For Inſtance, To know whether the Colour of any green Object be compounded or not, view it through a Priſm, and if it appear confuſed, and the Edges tinged with Blue, Yellow, or any Variety of other Colours, then is that Green compounded of ſuch Colours as at its Edges emerge out of it: But if it appear diſtinct, and well defined, and entirely Green to the very Edges, without any other Colours emerging, it is of an original and uncompounded Green. In like manner, if a refracted Beam of Light being caſt on a white Wall exhibit a green Colour, to know whether that be compounded, refract the Beam with an interpoſed Priſm, and if you find any Difformity in the Refractions, and the Green be transformed into Blue, Yellow, or any Variety of other Colours, you may conclude that it was compounded of thoſe that emerge: But if the Refractions be uniform, and the Green perſiſt without any Change of Colour, then is it original and uncompounded. And the Reaſon why I call it ſo is, becauſe a Green indued with ſuch Properties cannot be produced by any mixing of other Colours.

Now if two green Objects may to the naked Eye appear of the ſame Colour, and yet one of them through a Priſm ſeem confuſed and variegated with other Colours at the Edges, and the other diſtinct and entirely green; or if there may be two Beams of Light, which falling on a white Wall, do to the naked Eye exhibit the ſame green Colour, and yet one of them, when tranſmitted through a Priſm, be uniformly and regularly refracted, and retain its Colour unchanged, and the other be irregularly refracted, and made to divaricate into a Multitude of other Colours: I ſuppoſe theſe two Greens will in both Caſes be granted of a different Origin and Conſtitution. And if by mixing Colours, a Green cannot be compounded with the Properties of the unchangeable Green, I think I may call that an uncompounded Colour, eſpecially ſince its Rays are alike refrangible, and uniform in all Reſpects.

The ſame Rule is to be obſerved in examining, whether Red, Orange, Yellow, Blue, or any other Colour, be compounded or not. And, by the way, ſince all white Objects through the Priſm appear confuſed and terminated with Colours, Whiteneſs muſt, according to this Diſtinction, be ever compounded; and that the moſt of all Colours, becauſe it is the moſt confuſed and changed by Refractions.

There now remains the *third* Query to be conſider'd, which is, Whether Whiteneſs can be an uniform Colour, or a diſſimilar Mixture of all Colours? The Experiment which I brought to decide it, the Animadverſor thinks may be otherwiſe explained, and ſo concludes nothing. But he might eaſily have ſatisfied himſelf by trying what would be the Reſult of a Mixture of all Colours. And that very Experiment might have ſatisfied him, if he had pleaſed to examine it by the various Circumſtances. One Circumſtance I there declared, of which I ſee no Notice taken; and it is, That if any Colour at the Lens be intercepted, the Whiteneſs will be changed into other Colours: If all the Colours but Red be intercepted, that Red alone in the Concourſe or croſſing of the Rays, will not conſtitute Whiteneſs, but continues as much Red as before; and ſo of the other Colours. So that the Buſineſs is not only to ſhew how Rays, which before the Concourſe exhibit Colours, do in the Concourſe exhibit White; but to ſhew, how, in the ſame Place, where the ſeveral Sorts of Rays apart exhibit ſeveral Colours, a Confuſion of all together make White. For Inſtance, If Red alone be firſt tranſmitted to the Paper at the Place of Concourſe, and then the other Colours be let fall on that Red, the Queſtion will be, Whether they convert it into White by mixing with it only, as Blue falling on a Yellow Light is ſuppoſed to compound Green? Or whether there be ſome further Change wrought in the Colours by their mutual Acting on one another, until, like contrary *peripatetick Qualities*, they become aſſimilated? And he that ſhall explicate this laſt Caſe mechanically, muſt conquer a double Impoſſibility. He muſt firſt ſhew, that many unlike Motions in a Fluid can by claſhing ſo act on one another, and change each other, as to become one uniform Motion; and then, that an uniform Motion can of itſelf, without any new unequal Impreſſions, depart into a great Variety of Motions regularly unequal. And after this he muſt further tell me, why all Objects appear not of the ſame Colour; that is, why their Colours in the Air, where the Rays that convey them every way are confuſedly mixt, do not aſſimilate one another, and become uniform before they arrive at the Spectator's Eye?

But if there be yet any doubting, 'tis better to put the Event on further Circumſtances of the Experiment, than to acquieſce in the Poſſibility of any hypothetical Explications. As for Inſtance, by trying what will be the Apparition of theſe Colours in a very quick Conſecution of one another. And this may be eaſily performed by the rapid Gyration of a Wheel with many Spokes or Coggs in its Perimeter, whoſe Interſtices and Thickneſſes may be equal, and of ſuch a Largeneſs, that, if the Wheel be interpoſed between the Priſm and the white Concourſe of the Colours, one half of the Colours may be intercepted by a Spoke or Cogg, and the other half paſs through an Interſtice. The Wheel being in this Poſture, you may firſt turn it ſlowly about, to ſee all the Colours fall ſucceſſively on the ſame Place of the Paper, held at their aforeſaid Concourſe; and if you then accelerate its Gyration, until the Conſecution of thoſe Colours be ſo quick, that you cannot diſtinguiſh them ſeverally, the reſulting Colour will be a Whiteneſs perfectly like that, which an unrefracted Beam of Light exhibits, when in like manner ſucceſſively interrupted by the Spokes or Coggs of that circulating Wheel. And that this Whiteneſs is produced by a ſucceſſive Intermixture of the Colours, without their being aſſimilated, or reduced to any Uniformity, is certainly beyond all doubt, unleſs Things that exiſt not at the ſame time may notwithſtanding act on one another.

There are yet other Circumſtances, by which the Truth might have been decided; as by viewing the white Concourſe of the Colours through another Priſm placed cloſe to the Eye, by whoſe Refraction that Whiteneſs may appear again transformed into Colours: And then, to examine their Origin, if an Aſſiſtant intercept any of the Colours at the Lens before their Arrival at the Whiteneſs, the ſame Colours will vaniſh from amongſt thoſe, into which that Whiteneſs is converted by the ſecond Priſm. Now, if the Rays which diſappear be the ſame with thoſe that are intercepted, then it muſt be acknowledged, that the ſecond Priſm makes no new Colours in any Rays, which were not in them before their Concourſe at the Paper. Which is a plain Indication, that the Rays of ſeveral Colours remain diſtinct from one another in the Whiteneſs, and that from their previous Diſpoſitions are derived the Colours, of the ſecond Priſm. And, by the way, what is ſaid of their Colours may be applied to their Refrangibility.

The aforeſaid Wheel may alſo here be made uſe of; and, if its Gyration be neither too quick nor too ſlow, the Succeſſion of the Colours may be diſcern'd thro' the Priſm, whilſt to the naked Eye of a By-ſtander they exhibit Whiteneſs.

There is ſomething ſtill remaining to be ſaid of this Experiment. But this, I conceive, is enough to enforce it, and ſo to decide the Controverſy. However I ſhall now proceed to ſhew ſome other Ways of producing Whiteneſs by Mixtures, ſince I perſuade my ſelf, that this Aſſertion above the reſt appears paradoxical, and is with moſt Difficulty admitted. And becauſe the Animadverſor deſires an Inſtance of it in Bodies of divers Colours, I ſhall begin with that. But in order thereto it muſt be conſider'd, that ſuch colour'd Bodies reflect but ſome part of the Light incident on them; as is evident by the 13*th Propoſition:* And therefore the Light reflected from an Aggregate of them will be much weaken'd by the Loſs of many Rays. Whence a perfect and intenſe Whiteneſs is not to be expected, but rather a Colour between thoſe of Light and Shadow, or ſuch a Grey or dirty Colour as may be made by mixing White and Black together.

And that ſuch a Colour will reſult, may be collected from the Colour of Duſt found in every Corner of an Houſe, which hath been obſerv'd to conſiſt of many colour'd Particles. There may alſo be produced the like dirty Colour, by mixing ſeveral Painters Colours together. And the ſame may be effected by painting a Top (ſuch as Boys play with) of divers Colours. For when it is made to circulate by whipping it, it will appear of ſuch a dirty Colour.

Now the compounding of theſe Colours is proper to my purpoſe, becauſe they differ not from Whiteneſs in the Species of Colour, but only in a Degree of Luminouſneſs: Which (did not the Animadverſor concede it) I might thus evince. A Beam of the Sun's Light being tranſmitted into a darkned Room, if you illuminate a Sheet of white Paper by that Light, reflected from a Body of any Colour, the Paper will always appear of the Colour of that Body, by whoſe reflected Light it is illuminated. If it be a Red Body, the Paper will be Red; if a Green Body, it will be Green; and ſo of the other Colours. The Reaſon is, that the Fibres or Threads, of which the Paper conſiſts, are all tranſparent and ſpecular; and ſuch Subſtances are known to reflect Colours without changing them. To know, therefore, to what Species of Colour a Grey belongs, place any Grey Body (ſuppoſe a Mixture of Painters Colours) in the ſaid Light, and the Paper being illuminated by its Reflection, ſhall appear White. And the ſame thing will happen, if it be illuminated by Reflection from a Black Subſtance.

Theſe therefore are all of one Species; but yet they ſeem diſtinguiſh'd not only by Degrees of Luminouſneſs, but alſo by ſome other Inequalities, whereby they become more harſh or pleaſant. And the Diſtinction ſeems to be, that Greys, and perhaps Blacks, are made by an uneven Defect of Light, conſiſting as it were of many little Veins or Streams, which differ either in Luminouſneſs, or in the unequal Diſtribution of diverſly colour'd Rays; ſuch as ought to be cauſed by Reflection from a Mixture of White and Black, or of diverſly coloured Corpuſcles. But when ſuch imperfectly mix'd Light is by a ſecond Reflection from the Paper more evenly and uniformly blended, it becomes more pleaſant, and exhibits a faint or ſhadowed Whiteneſs. And that ſuch little Irregularities as theſe may cauſe theſe Differences, is not improbable, if we conſider, how much Variety may be cauſed in Sounds of the ſame Tone, by irregular and uneven Jarrings. And beſides, thoſe Differences are ſo little, that I have ſometimes doubted, whether they be any at all, when I have conſidered, that a Black and White Body being placed together, the one in a ſtrong Light, and the other in a very faint Light, ſo proportioned that they might appear equally luminous, it has been difficult to diſtinguiſh them, when viewed at Diſtance, unleſs when the Black ſeemed more Bluiſh, and the White Body in a Light ſtill fainter, hath, in Compariſon of the Black Body, itſelf appeared Black.

This leads me to another Way of compounding Whiteneſs; which is, that, if four or five Bodies of the more eminent Colours, or a Paper painted all over in ſeveral Parts of it with thoſe ſeveral Colours in a due Proportion, be placed in the ſaid Beam of Light, the Light reflected from thoſe Colours to another White Paper, held at a convenient Diſtance, ſhall make that Paper appear White. If it be held too near the Colours, its Parts will ſeem of thoſe Colours that are neareſt them; but by removing it further, that all its Parts may be equally illuminated by all the Colours, they will be more and more diluted, until they become perfectly White. And you may further obſerve, that if any of the Colours be intercepted, the Paper will no longer appear White, but of the other Colours which are not intercepted. Now that this Whiteneſs is a Mixture of the ſeverally coloured Rays, falling confuſedly on the Paper, I ſee no reaſon to doubt of; becauſe, if the Light became uniform and ſimilar before it fell confuſedly on the Paper, it muſt much more be uniform, when at greater Diſtance it falls on the Spectator's Eye; and ſo the Rays, which come from ſeveral Colours, would in no Qualities differ from one another, but all of them exhibit the ſame Colour to the Spectator, contrary to what he ſees.

Not much unlike this Inſtance it is, that if a poliſh'd Piece of Metal be ſo placed that the Colours appear in it as in a Looking-Glaſs, and then the Metal be made rough, that by a confuſed Reflection thoſe apparent Colours may be blended together, they ſhall diſappear, and by their Mixture cauſe the Metal to look White.

But further to enforce this Experiment: If inſtead of the Paper, any White Froth conſiſting of ſmall Bubbles, be illuminated by Reflexion from the aforeſaid Colours, it ſhall to the naked Eye ſeem White, and yet through a good Microſcope the ſeveral Colours will appear diſtinct on the Bubbles, as if ſeen by Reflexion from ſo many ſpherical Surfaces. With my naked Eye, being very near, I have alſo diſcerned the ſeveral Colours on each Bubble; and yet at a greater Diſtance, where I could not diſtinguiſh them apart, the Froth hath appeared entirely White. And at the ſame Diſtance, when I look'd intently, I have ſeen the Colours diſtinctly on each Bubble; and yet by ſtraining my Eyes as if I would look at ſomething afar off beyond them, thereby to render the Viſion confuſed, the Froth has appeared without any other Colour than Whiteneſs. And what is here ſaid of Froths, may eaſily be underſtood of the Paper or Metal, in the foregoing Experiments. For their Parts are ſpecular Bodies, like theſe Bubbles; and perhaps with an excellent Microſcope, the Colours may alſo be ſeen intermixedly reflected from them.

In proportioning the ſeveral colour'd Bodies to produce theſe Effects, there may be ſome Niceneſs; and it will be more convenient, to make uſe of the Colours of the Priſm, caſt on a Wall, by whoſe Reflection the Paper, Metal, Froth, and other white Subſtances may be illuminated. And I uſually made my Trials this Way, becauſe I could better exclude any ſcattering Light from mixing with the Colours to dilate them.

To this Way of compounding Whiteneſs, may be referred that other, by mixing Light after it hath been trajected through tranſparently colour'd Subſtances. For Inſtance, if no Light be admitted into a Room, but only through colour'd Glaſs, whoſe ſeveral Parts are of ſeveral Colours in a pretty equal Proportion; all white Things in the Room ſhall appear white, if they be not held too near the Glaſs: And yet this Light, with which they are illuminated, cannot poſſibly be uniform; becauſe, if the Rays, which at their Entrance are of divers Colours, do in their Progreſs through the Room ſuffer any Alteration to be reduced to an Uniformity, the Glaſs would not in the remoteſt Parts of the Room appear of the very ſame Colour, which it doth when the Spectator's Eye is very near it: Nor would the Rays, when tranſmitted into another dark Room through a little Hole in an oppoſite Door or Partition-Wall, project on a Paper the Species or Repreſentation of the Glaſs in its proper Colours.

And, by the by, this ſeems a very fit and cogent Inſtance of ſome other Parts of my *Theory*, and particularly of the 13th *Propoſition*. For in this Room all natural Bodies whatever appear in their proper Colours. And all the Phænomena of Colours in Nature, made either by Refraction or without it, are here the ſame as in the open Air. Now the Light in this Room being ſuch a diſſimilar Mixture, as I have deſcribed in my *Theory*, the Cauſes of all theſe Phænomena muſt be the ſame that I have here aſſign'd. And I ſee no reaſon to ſuſpect, that the ſame Phænomena ſhould have other Cauſes in the open Air.

The Succeſs of this Experiment may be eaſily conjectured by the Appearances of Things in a Church or Chapel, whoſe Windows are of colour'd Glaſs; or in the open Air, when it is illuſtrated with Clouds of various Colours.

There are yet other Ways by which I have produced Whiteneſs; as by caſting ſeveral Colours from two or more Priſms upon the ſame Place; by refracting a Beam of Light with two or three Priſms ſucceſſively, to make the diverging Colours converge again; by reflecting one Colour to another, and by looking through a Priſm on an Object of many Colours; and (which is equivalent to the above-mention'd Way of mixing Colours by concave Wedges fill'd with colour'd Liquors) I have obſerv'd the Shadows of a painted Glaſs-Window to become White, where thoſe of many Colours have at a great Diſtance interfered. But yet for further Satisfaction, the Animadverſor may try, if he pleaſe, the Effects of four or five of ſuch Wedges fill'd with Liquors of as many ſeveral Colours.

Beſides all theſe, the Colours of Water-bubbles, and other thin pellucid Subſtances, afford ſeveral Inſtances of Whiteneſs produc'd by their Mixture; with one of which I ſhall conclude this Particular. Let ſome Water, in which a convenient Quantity of Soap or Waſhball is diſſolv'd, be agitated into Froth, and after that Froth has ſtood a while without further Agitation, 'till you ſee the Bubbles, of which it conſiſts, begin to break, there will appear a great Variety of Colours all over the Top of every Bubble, if you view them near at hand; but if you view them at ſo great a Diſtance that you cannot diſtinguiſh the Colours one from another, the Froth will appear perfectly White.

Thus much concerning the Deſign and Subſtance of the Animadverſor's Conſiderations. There are yet ſome Particulars to be taken notice of, before I conclude; as the Denial of the *Experimentum Crucis*. On this I choſe to lay the whole Streſs of my Diſcourſe; which therefore was the principal Thing to have been objected againſt. But I cannot be convinced of its Inſufficiency by a bare Denial, without aſſigning a Reaſon for it. I am apt to believe, it has been miſunderſtood; for otherwiſe it would have prevented the Diſcourſes about rarefying and ſplitting of Rays; becauſe the Deſign of it is to ſhew, that Rays of divers Colours conſider'd apart, do at equal Incidences ſuffer unequal Refractions, without being ſplit, rarefy'd, or any ways dilated.

Some Conſiderations upon this Doctrine of Colours; from Paris by . . . . N. 96. *p.* 6086. July, An. 1673.

VI. 1. Methinks that the moſt important Objection which is made againſt Mr. *Newton* by way of Query, is that, Whether there be more than two ſorts of Colours? For my part, I believe, that an Hypotheſis, that ſhould explain Mechanically, and by the Nature of Motion, the Colours Yellow, Green and Blue, would be ſufficient for all the reſt, in regard that thoſe others, being only more deeply charged, (as appears by the Priſms of Mr. *Hook*) do produce the dark or deep Red and Blue; and that of theſe four all the other Colours may be compounded. Neither do I ſee why Mr. *Newton* doth not content himſelf with the two Colours, Yellow and Blue; for it will be much more eaſy to find any Hypotheſis by Motion, that may explicate theſe two Differences, than for ſo many Diverſities as there are of other Colours. And 'till he hath found this Hypotheſis, he hath not taught us, what it is wherein conſiſts the Nature and Difference of Colours, but only this Accident (which certainly is very conſiderable) of their *different Refrangibility*.

As for the Compoſition of White made by all the Colours together, it may poſſibly be, that Yellow and Blue might alſo be ſufficient for that: Which is worth while to try; and it may be done by the Experiment which Mr. *Newton* propoſeth, by receiving againſt a Wall of a darkned Room the Colours of the Priſm, and to caſt their reflected Light upon white Paper. Here you muſt hinder the Colours of the Extremities, *viz.* the Red and Purple, from ſtriking againſt the Wall, and leave only the intermediate Colours, Yellow, Green and Blue, to ſee whether the Light of theſe alone would not make the

Paper appear White, as well as when they all gave Light. I even doubt, whether the lighteſt Place of the Yellow Colour may not all alone produce that Effect, and I mean to try it at the firſt Conveniency; for this Thought never came into my Mind but juſt now. Mean time you may ſee, that if theſe Experiments do ſucceed, it can no more be ſaid, that all the Colours are neceſſary to compound White, and that 'tis very probable, that all the reſt are nothing but Degrees of Yellow and Blue, more or leſs charged.

Anſwer'd, by
Mr. Newton:
N. 97. *p.* 6108.
Aug. An. 1673.

2. It ſeems to me, that *N.* takes an improper Way of examining the Nature of Colours, whilſt he proceeds upon compounding thoſe that are already compounded. Perhaps he would ſooner ſatisfy himſelf by reſolving Light into Colours, as far as may be done by Art, and then by examining the Properties of thoſe Colours apart, and afterwards by trying the Effects of re-conjoining two or more, or all of thoſe; and laſtly, by ſeparating them again, to examine what Changes that Re-conjunction had wrought in them. I have formerly ſhew'd, That all Colours cannot practically be deriv'd out of the Yellow and Blue, and conſequently that thoſe Hypotheſes are groundleſs which imply they may. If you aſk what Colours cannot be derived out of Yellow and Blue? I anſwer, None of thoſe which I defined to be Original; and if he can ſhew by Experiment how they may, I will acknowledge my ſelf in an Error. Nor is it eaſier to frame an Hypotheſis by aſſuming only two original Colours, rather than an indefinite Variety; unleſs it be eaſier to ſuppoſe that there are but two Figures, Sizes, and Degrees of Velocity or Force of the Æthereal Corpuſcles or Pulſes, rather than an indefinite Variety; which certainly would be a harſh Suppoſition. No Man wonders at the indefinite Variety of Waves of the Sea, or of Sands on the Shore; but, were they all but two Sizes, it would be a very puzzling Phænomenon. And I ſhould think it as unaccountable, if the ſeveral Parts or Corpuſcles, of which a ſhining Body conſiſts, which muſt be ſuppoſed of various Figures, Sizes and Motions, ſhould impreſs but two Sorts of Motion on the adjacent Æthereal Medium, or any other way beget but two Sorts of Rays. But to examine how Colours may be explain'd hypothetically, is beſides my purpoſe. I never intended to ſhew wherein conſiſts the Nature and Difference of Colours, but only to ſhew, that *de facto* they are original and immutable Qualities of the Rays which exhibit them; and to leave it to others to explicate by mechanical Hypotheſes, the Nature and Difference of theſe Qualities; which I take to be no difficult Matter. But I would not be underſtood, as if their Difference conſiſted in the *different Refrangibility* of thoſe Rays; for that *different Refrangibility* conduces to their Production no otherwiſe, than by ſeparating the Rays whoſe Qualities they are. Whence it is, That the ſame Rays exhibit the ſame Colours when ſeparated by any other Means; as by their *different Reflexibility*, a Quality not yet diſcourſed of.

In the next Particular, where *N.* would ſhew, that it is not neceſſary to mix all Colours for the Production of White; the Mixture of Yellow, Green and Blue, without Red and Violet, which he propounds for that End, will not produce White, but Green; and the brighteſt Part of the Yellow will afford no other Colour but Yellow, if the Experiment be made in a Room well darkned, as it ought; becauſe the colour'd Light is much weakned by the Reflection, and ſo apt to be diluted by the mixing of any other ſcattering Light. But yet there is an Experiment or two formerly mention'd, by which I have produced White out of two Colours alone, and that variouſly; as out of Orange and a full Blue, and out of Red and pale Blue, and out of Yellow and Violet, as alſo out of other Pairs of intermediate Colours. The moſt convenient Experiment for performing this, was that of caſting the Colours of one Priſm upon thoſe of another, after a due Manner. But what *N.* can deduce from hence, I ſee not; for the two Colours were compounded of all others, and ſo the reſulting White (to ſpeak properly) was compounded of them all, and only decompounded of thoſe two. For Inſtance, the Orange was compounded of Red, Orange, Yellow, and ſome Green; and the Blue, of Violet, full Blue, light Blue, and ſome Green, with all their intermediate Degrees; and conſequently the Orange and Blue together made an Aggregate of all Colours to conſtitute the White. Thus if one mix Red, Orange, and Yellow Powders to make an Orange; and Green, Blue, and Violet Colours to make a Blue; and laſtly, the two Mixtures to make a Grey; that Grey, though decompounded of no more than two Mixtures, is yet compounded of all the ſix Powders, as truly as if the Powders had been all mix'd at once.

This is ſo plain, that I conceive there can be no further Scruple; eſpecially to them who know how to examine, whether a Colour be ſimple or compounded, and of what Colours it is compounded; which having explain'd in another Place, I need not now repeat. If therefore *N.* would conclude any thing, he muſt ſhew how White may be produced out of two uncompounded Colours: which when he hath done, I will further tell him, why he can conclude nothing from that. But I believe there cannot be found an Experiment of that Kind; becauſe, as I remember, I once try'd, by gradual Succeſſion, the Mixture of all Pairs of uncompounded Colours; and though ſome of them were paler, and nearer to White than others, yet none could be truly call'd White. But it being ſome Years ſince this Tryal was made, I remember not well the Circumſtances, and therefore recommend it to others to be try'd again.

A Reply, by Monſieur N. N. 97. *p.* 6112.

3. Seeing that Mr. *Newton* maintains his Opinion with ſo much Concern, I liſt not to diſpute. But what means it, I pray, that he ſaith, *Though I ſhould ſhew him, that the White could be produced only of two uncompounded Colours, yet I could conclude nothing from that?* And yet he hath affirm'd, that to compoſe the White, all primitive Colours are neceſſary.

Anſwered, by Mr. Newton. N. 96. *p.* 6087. July, An. 1673.

4. In my ſaying, that when *Monſieur N.* hath ſhewn *how White may be produced out of two uncompounded Colours, I will tell him, why he can conclude nothing from that*; my Meaning was, That ſuch a White (were there any ſuch) would have different Properties from the White which I had reſpect to, when I deſcribed my *Theory*, that is, from the White of the Sun's immediate Light, of the ordinary Objects of our Senſes, and of all white Phænomena that have hitherto fallen under my Obſervation. And thoſe different Properties would evince it to be of a different Conſtitution: Inſomuch, that ſuch a Production

of White would be ſo far from contradicting, that it would rather illuſtrate and confirm my Theory; becauſe by the Difference of that from other Whites it would appear, that other Whites are not compounded of only two Colours like that. And therefore if *Monſieur N.* would prove any thing, it is requiſite that he do not only produce out of two primitive Colours a White, which to the naked Eye ſhall appear like other Whites, but alſo ſhall agree with them in all other Properties.

But to let you underſtand wherein ſuch a White would differ from other Whites, and why from thence it would follow that other Whites are otherwiſe compounded, I ſhall lay down this Poſition.

That a compounded Colour can be reſolved into no more ſimple Colours than thoſe of which it is compounded.

This ſeems to be ſelf-evident, and I have alſo tried it ſeveral Ways, and particularly by this which follows: Let α repreſent an oblong Piece of White Paper about [illegible] or ¼ of an Inch broad, and illuminated in a dark Room, with a Mixture of two Colours caſt upon it from two Priſms, ſuppoſe a deep Blue and Scarlet, which muſt ſeverally be as uncompounded as they can conveniently be made; then, at a convenient Diſtance, ſuppoſe of ſix or eight Yards, view it through a clear Triangular Glaſs or Cryſtal Priſm held parallel to the Paper, and you ſhall ſee the two Colours parted from one another in the Faſhion of two Images of the Paper, as they are repreſented at β and γ, where ſuppoſe β the Scarlet, and γ the Blue, without Green, or any other Colour between them. *Fig.* 76.

Now from the aforeſaid *Poſition* I deduce theſe two *Concluſions*; 1. That if there were found out a Way to compound White of two ſimple Colours only, that the White would be again reſolvable into no more than two. 2. That if other Whites, as that of the Sun's Light, *&c.* be reſolvable into more than two ſimple Colours (as I find by Experiment that they are) then they muſt be compounded of more than two.

To make this plainer, ſuppoſe that A repreſents a white Body, illuminated by a direct Beam of the Sun tranſmitted through a ſmall Hole into a dark Room, and α ſuch another Body, illuminated by a Mixture of two ſimple Colours; which, if poſſible, may make it alſo appear of a white Colour exactly like A: Then, at a convenient Diſtance, view theſe two Whites through a Priſm, and A will be changed into a Series of all Colours, Red, Yellow, Green, Blue, Purple, with their intermediate Degrees ſucceeding in Order from B to C. But α, according to the aforeſaid Experiment, will only yield thoſe two Colours of which it was compounded, and thoſe not conterminate like the Colours at B C, but ſeparate from one another, as at β and γ, by means of the different Refrangibility of the Rays to which they belong. And thus by comparing theſe two Whites, they would appear to be of a different Conſtitution, and A to conſiſt of more Colours than α. So that what *Monſieur N.* contends for, would rather advance my Theory by the Acceſs of a new Kind of White, than conclude againſt it. But I ſee no Hopes of compounding ſuch a White.

As for *Monſieur N.* his Expreſſion, *That I maintain my Doctrine with ſome Concern*, I confeſs it was a little ungrateful to me, to meet with Objections which had been anſwer'd before, without having the leaſt Reaſon given me why thoſe Anſwers were inſufficient. Thoſe Anſwers were to ſhew, that there are other ſimple Colours beſides Blue and Yellow; I inſtanced in a ſimple or homogeneal Green, ſuch as cannot be made by mixing Blue and Yellow, or any other Colours. And I alſo ſhew'd why, ſuppoſing that all Colours might be produced out of two, yet it would not follow that thoſe two are the only original Colours. The Reaſons I deſire you would compare with what hath now been ſaid of White. And ſo the Neceſſity of all Colours to produce White, might have appear'd by that Experiment, where I ſay, That if any Colour at the Lens be intercepted, the Whiteneſs (which is compounded of them all) will be changed into (the Reſult of) the other Colours.

Vid. Suppl. p. 149. & ſeq.

However, ſince there ſeems to have happen'd ſome Miſunderſtanding between us, I ſhall endeavour to explain my ſelf a little further in theſe Things, according to the following Method.

Definitions.] 1. I call that Light homogeneal, ſimilar or uniform, whoſe Rays are equally refrangible.

2. And that heterogeneal, whoſe Rays are unequally refrangible.

Note. There are but three Affections of Light, in which I have obſerved its Rays to differ, *viz. Refrangibility*, *Reflexibility*, and *Colour*; and thoſe Rays which agree in Refrangibility agree alſo in the other two, and therefore may well be defined homogeneal, eſpecially ſince Men uſually call thoſe Things homogeneal, which are ſo in all Qualities that come under their Knowledge, tho' in other Qualities, that their Knowledge extends not to, there may poſſibly be ſome Heterogeneity.

3. Thoſe Colours I call ſimple, or homogeneal, which are exhibited by homogeneal Light.

4. And thoſe compound or heterogeneal, which are exhibited by heterogeneal Light.

5. Different Colours, I call not only the more eminent Species, Red, Yellow, Green, Blue, Purple, but all other the minuteſt Gradations; much after the ſame manner that not only the more eminent Degrees in Muſick, but all the leaſt Gradations are eſteemed different Sounds.

Propoſitions.] 1. The Sun's Light conſiſts of Rays differing by indefinite Degrees of Refrangibility.

2. Rays which differ in Refrangibility when parted from one another, do proportionally differ in the Colours which they exhibit. Theſe two *Propoſitions* are Matter of Fact.

3. There are as many ſimple or homogeneal Colours, as Degrees of Refrangibility: For to every Degree of Refrangibility belongs a different Colour, by *Prop.* 2. and that Colour is ſimple, by *Def.* 1, and 3.

4. Whiteneſs, in all Reſpects like that of the Sun's immediate Light, and of all the uſual Objects of our Senſes, cannot be compounded of two Simple Colours alone: For ſuch a Compoſition muſt be made by Rays that have only two Degrees of Refrangibility, by *Def.* 1, and 3; and therefore it cannot be like that of the Sun's Light, by *Prop.* 1; nor, for the ſame Reaſon, like that of ordinary white Objects.

5. Whiteneſs, in all Reſpects like that of the Sun's immediate Light, cannot be compounded of Simple Colours, without an indefinite Variety of them: For to ſuch a Compoſition there are requiſite Rays endued with all the indefinite Degrees of Refrangibility, by *Prop.* 1. And thoſe infer as many Simple Colours, by *Def.* 1, and 3. and *Prop.* 2, and 3.

To make theſe a little plainer, I have added alſo the *Propoſitions* that follow.

6. The Rays of Light do not act on one another in paſſing through the ſame *Medium.* This appears by ſeveral former Paſſages, and is capable of further Proof.

7. The Rays of Light ſuffer not any Change of their Qualities from Refraction.

8. Nor afterwards from the adjacent quiet Medium. Theſe two Propoſitions are manifeſt *de facto* in homogeneal Light, whoſe Colour and Refrangibility is not at all changeable, either by Refraction; or by the Contermination of a quiet Medium. And as for heterogeneal Light, it is but an Aggregate of ſeveral Sorts of homogeneal Light; no one Sort of which ſuffers any more Alteration than if it were alone, becauſe the Rays act not on one another, by *Prop.* 6. and therefore the Aggregate can ſuffer none. Theſe two Propoſitions alſo might be further proved apart by Experiments, too long to be here deſcribed.

9. There can no homogeneal Colours be educed out of Light by Refraction, which were not commixt in it before; becauſe by *Prop.* 7 and 8, Refraction changeth not the Qualities of the Rays, but only ſeparates thoſe which have diverſe Qualities, by means of the different Refrangibility.

10. The Sun's Light is an Aggregate of an indefinite Variety of homogeneal Colours; by *Prop.* 1, 3, and 9. And hence it is, that I call homogeneal Colours alſo primitive or original.

Animadverſions on this Theory of Light and Colours; by Mr. Fr. Linus. N. 110. *p.* 217. Jan. An. 1675.

VII. 1. I doubt not of what Mr. *Newton* affirms; and have my ſelf ſometimes in like Circumſtances obſerved the like Difference between the Length and Breadth of the coloured *Spectrum*; but never found it ſo when the Sky was clear and free from Clouds, near the Sun: But then only appeared this Difference of Length and Breadth, when the Sun either ſhined thro' a white Cloud, or enlightned ſome ſuch Clouds near unto it. And then indeed it was no marvel, the ſaid *Spectrum* ſhould be longer than broad; ſince the Cloud or Clouds ſo enlightned, were in order to thoſe Colours like to a great Sun, making a far greater Angle of Interſection in the Hole, than the true Rays of the Sun do make; and therefore are able to enlighten the whole Length of the Priſm, and not only ſome ſmall part thereof as we ſee enlightned by the true Sun-Beams coming thro' the ſame little Hole. And this we behold alſo in the true Sun-Beams, when they enlighten the whole Priſm; for altho' in a clear Heaven, the Rays of the Sun paſſing thro' the ſaid Hole, never make a Spectrum longer than broad, becauſe they then can occupy but a ſmall Part of the Priſm; yet if the Hole be ſo much bigger as to enlighten the whole Priſm, you ſhall preſently ſee the Length of the Spectrum much exceed its Breadth; which Exceſs will be always ſo much the greater, as the Length of the Priſm exceeds its Breadth. From whence I conclude, That the Spectrum, this *Learned Author* ſaw, much longer than broad, was not affected by the true Sun-Beams, but by Rays proceeding from ſome bright Cloud, as is ſaid; and by Conſequence, that the *Theory of Light* grounded upon that *Experiment* cannot ſubſiſt.

What I have here ſaid, needs no other Confirmation than meer Experience, which any one may quickly try: neither have I only tried the ſame upon this Occaſion, but near 30 Years ago ſhew'd the ſame, together with divers other Experiments of Light, to that worthy Promoter of Experimental Philoſophy, Sir *Kenelm Digby*, who coming into theſe Parts to take the *Spaw*-Waters, reſorted oftentimes to my darkned Chamber *, to ſee the various Phænomena of Light, made by divers Refractions and Reflections, and took Notes upon them; which Induſtry if they alſo had us'd, who endeavour to explicate the aforeſaid Difference between the Length and Breadth of this colour'd *Spectrum*, by the received Laws of Refraction, would never have taken ſo impoſſible a Task in Hand.

* *At* Leige.

Anſwered by .... N. 110. *p.* 219. Jan. An. 1675.

2. Theſe Animadverſions ſeem to need no other Anſwer but this, that you would be pleaſed to conſider the Scheme in Mr. *Newton*'s ſecond Anſwer to *P. Pardies*, and reſt aſſured, that the Experiment, as 'tis repreſented, was tried in clear Days, and the Priſm placed cloſe to the Hole in the Window, ſo that the Light had no room to diverge, and the coloured Image made not parallel (as in that Conjecture) but tranſverſe to the Axis of the Priſm.

A Reply by Mr. Fr. Linus. N. 121. *p.* 499. Jan. An. 1676.

3. *If theſe Aſſertions be admitted*, they do indeed directly cut off what I ſaid of Mr. *Newton*'s being deceived by a *bright Cloud*. But if we compare them with Mr. *Newton*'s firſt Relation of the Experiment, it will evidently appear, they cannot be admitted, as being directly contrary to what is there delivered. For there he tells us, *The Ends of the coloured Image, he ſaw on the oppoſite Wall, near five times as long as broad, ſeemed to be ſemicircular*. Now theſe *ſemicircular Ends* are never ſeen in a clear Day, as Experience ſhews. From whence follows againſt the *firſt* Aſſertion, that the Experiment was not made in a clear Day. Neither are thoſe ſemicircular Ends ever ſeen when the Priſm is placed cloſe to the Hole; which contradicts the *ſecond* Aſſertion. Neither are they ever ſeen when the Image is tranſverſe to the Length or Axis of the Priſm, which directly oppoſes the *third* Aſſertion. But if in any of theſe three Caſes, the Image be made ſo much longer than broad (as eaſily it may, by turning the Priſm a little about its Axis, near *five times* as long as broad) then the one End thereof will run out into a ſharp Cone or Pyramis like the Flame of a Candle, and the other into a Cone ſomewhat more blunt; both which are far from ſeeming ſemicircular: Whereas, if the Image be made not in a clear Day but with a bright Cloud, and the Priſm not

placed cloſe to the Hole but in a competent Diſtance from the ſame, then theſe ſemicircular Ends always appear, with the Sides thereof, ſtreight Lines, juſt as Mr. *Newton* deſcribes them. Neither is the Length of the Image tranſverſe, but parallel to the Length of the Priſm. Out of all which evidently follows, that the Experiment was not made in a clear Day; nor with the Priſm cloſe to the Hole; nor yet with the Image tranſverſe, but by a bright Cloud and a parallel Image (as I conjectured); and I hope you will alſo now ſay, I had good Reaſon ſo to conjecture, ſince it ſo well agrees with the Relation. And Experience will alſo ſhew you, if you pleaſe to make Tryal, as it was made in a dark Chamber, and obſerve the Difference between ſuch an Image made by a bright Cloud, and another made by the immediate Rays of the Sun: For the former you ſhall always find parallel, with the Ends ſemicircular; but the latter you ſhall find tranſverſe, with the Ends pyramidical, as aforeſaid, whenſoever it appears ſo much longer than broad.

More might be ſaid out of the ſame Relation, to ſhew that the Image was not tranſverſe. For if it had been tranſverſe, Mr. *Newton*, ſo well skill'd in *Opticks*, could not have been ſurpriſed (as he ſays he was) to ſee the Length thereof ſo much to exceed the Breadth; it being a Thing ſo obvious and eaſy to be explicated by the ordinary Rules of Refraction. That other Place alſo (where he ſays the *Incident* Refractions were made in the Experiment *equal to the Emergent*) proves again that the ſaid oblong Image was not tranſverſe, but parallel. For it is impoſſible the tranſverſe Image ſhould be ſo much longer than broad, unleſs thoſe two Refractions be made very unequal, as both the Computation according to the common Rules of Refraction and Experience teſtify.

Anſwered by Mr. Newton.
N. 121. *p.* 501.
N. 123. *p.* 556.
Mar. An. 1676.

4. What it is that impoſes upon Mr. *Line* I cannot imagine; but I ſuſpect he has not try'd the Experiment ſince he acquainted himſelf with my Theory, but depends upon his old Notions, taken up before he had any Hint given to obſerve the Figure of the coloured Image. I ſhall deſire him therefore, before he returns any Anſwer, to try it once more for his Satisfaction, and according to this Manner.

Let him take any Priſm, and hold it ſo that its Axis may be perpendicular to the Sun's Rays, and in this Poſture let it be placed as cloſe as may be to the Hole through which the Sun ſhines into a dark Room, which Hole may be about the Bigneſs of a Pea. Then let him turn the Priſm ſlowly about its Axis, and he ſhall ſee the Colours move upon the oppoſite Wall, firſt towards that Place to which the Sun's direct Line would paſs, if the Priſm were taken away, and then back again. When they are in the Middle of theſe two contrary Motions, that is, when they are neareſt that Place to which the Sun's direct Ray tends, there let him ſtop; for then are the Rays equally refracted on both Sides the Priſm. In this Poſture of the Priſm let him obſerve the Figure of the Colours, and he ſhall find it not round, as he contends, but oblong, and ſo much the more oblong as the Angle of the Priſm, comprehended by the refracting Plains, is bigger, and the Wall, on which the Colours are caſt, more diſtant from the Priſm; the Colours Red, Yellow, Green, Blue, Purple, ſucceeding in Order, not from one Side of the Figure to the other, as in Mr. *Line*'s Conjecture, but from one End to the other;

other; and the Length of the Figure being not parallel but tranſverſe to the Axis of the Priſm. After this manner I uſed to try the Experiment; and it will not ſucceed well if the Day be not clear, and the Priſm placed cloſe to the Hole, or ſo near at leaſt, that all the Sun's Light that comes from the Hole may paſs through the Priſm alſo, ſo as to appear in a round Form, if intercepted by a Paper immediately after it has paſſed the Priſm.

When Mr. *Line* has try'd this, I could wiſh he would proceed a little further, to try that which I call'd the *Experimentum Crucis.* For when he has try'd them (which by his denying them, I know he has not done yet as they ſhould be try'd) I preſume he will reſt ſatisfied. It may be try'd (tho' not ſo perfectly) even without darkning a Room, or the Expence of any more Time than half a Quarter of an Hour.

The Experiment of Mr. Line *affirmed by Mr.* Gaſcoigne, N. 121. *p.* 503. *marg.* Jan. An. 1676.

5. Mr. *Linus* (now deceaſed) *try'd the Experiment* again and again, and called divers on purpoſe to ſee it, nor ever made Difficulty to ſhew it to any one, who either by Chance came into his Chamber as he was doing it, or ſhewed the leaſt Deſire to ſee the ſame; ſo that for Point of Experience, Mr. *Newton* cannot be more confident on his Side, than we are here on the other; who are fully perſuaded, that, unleſs the Diverſity of placing the Priſm, or the Bigneſs of the Hole, or ſome other ſuch Circumſtance, be the Cauſe of the Difference betwixt them, Mr. *Newton*'s Experiment will hardly ſtand.

Anſwered by Mr. Newton, *Ibid.* N. 123. *p.* 556. Mar. An. 1676.

6. By Mr. *Gaſcoigne*'s Letter one might ſuſpect, that Mr. *Linus* try'd the Experiment ſome other Way than I did; and therefore I ſhall expect, till his Friends have try'd it according to my late Directions. In which Tryal it may poſſibly be a further Guidance to them, to acquaint them that the Priſm caſts from it ſeveral Images. One is, that oblong One of Colours which I mean; and this is made by two Refractions only. Another there is made by two Refractions and an intervening Reflection; and this is round and colourleſs, if the Angles of the Priſm be exactly equal; but if the Angles at the reflecting Baſe be not equal, it will be colour'd, and that ſo much the more, by how much unequaller the Angles are, but yet much more unround, unleſs the Angles be very unequal. A third Image there is, made by one ſingle Reflection, and this is always round and colourleſs. The only Danger is, in miſtaking the *Second* for the *Firſt*. But they are diſtinguiſhable not only by the Length and lively Colours of the *Firſt*, but by its different Motion too: For, whilſt the Priſm is turned continually the ſame Way about its Axis, the *Second* and *Third* move ſwiftly, and go always on the ſame Way, till they diſappear; but the *Firſt* moves ſlow, and grows continually ſlower, till it be Stationary, and then turns back again, and goes back faſter and faſter, till it vaniſh in the Place where it began to appear.

If, without darkning their Room, they hold the Priſm at their Window in the Sun's open Light, in ſuch a Poſture that its Axis be perpendicular to the Sun-Beams, and then turn it about its Axis, they cannot miſs of ſeeing the firſt Image; which having found, they may double up a Paper once or twice, and make a round Hole in the middle of it, about $\frac{1}{3}$ or $\frac{3}{4}$ of an Inch broad, and hold the Paper immediately before the Priſm, that the Sun may ſhine on the Priſm through that Hole; and the Priſm being ſtayed, and held ſteady in that Poſture which makes the Image ſtationary; if the Image then fall directly on an oppoſite Wall, or on a Sheet of Paper placed at the Wall, ſuppoſe 15 or 20 Foot from the Priſm, or further off; they will ſee the Image in ſuch an oblong Figure as I have deſcribed, with the Red at one End, the Violet at the other, and the bluiſh Green in the Middle: And if they obſcure their Room as much as they can, by drawing Curtains, or otherwiſe, it will make the Colours the more conſpicuous.

This Direction I have ſet down, that no Body, into whoſe Hands a Priſm ſhall happen, may find Difficulty and Trouble in trying it. But when Mr. *Linus*'s Friends have try'd it thus, they may proceed to repeat it in a dark Room, with a leſs Hole made in their Window-ſhut. And then I ſhall deſire that they will ſend a full and clear Deſcription how they try'd it. I ſhould be glad too, if they will favour me with a Deſcription of the Experiment as it hath been hitherto try'd by Mr. *Linus*, that I may have an Opportunity to conſider what there is in that which makes againſt me.

7. Mr. *Gaſcoigne* wanting Convenience to make the Experiment, according to the freſh Directions from Mr. *Newton*, requeſted me to ſupply this Want.

Exceptions by Mr. Lucas. N. 128. *p.* 692. Aug. An. 1676.

The vertical Angle of my Priſm was 60 Degrees; the Diſtance of the Wall, whereon the coloured *Spectrum* appeared, from the Window, about 18 Foot; the Diameter of the Hole in the Window-ſhuts about $\frac{1}{4}$ Inch, which, upon Occaſions, I contracted to half the ſaid Diameter, but ſtill with equal Succeſs as to the main of the Experiment. The Refractions on both Sides the Priſm were, as near as I could make them, equal, and conſequently about 48 Deg. 40 Min. the refractive Power of Glaſs being computed according to the Ratio of the Sines 2 to 3. The Diſtance of the Priſm from the Hole in the Shuts was about 2 Inches; the Room darkned to that Degree as to equal the darkeſt Night, while the Hole in the Shuts was covered.

Now as to the Iſſue of my Tryals; I conſtantly found the Length of the coloured Image (tranſverſe to the Axis of the Priſm) conſiderably greater than its Breadth, as often as the Experiment was made on a clear Day; but if a bright Cloud were near the Sun, I found it ſometimes exactly as Mr. *Line* wrote to you, namely, Broader than Long, eſpecially while the Priſm was placed at a great Diſtance from the Hole: Which Experiment will not, I conceive, be queſtion'd by Mr. *Newton*, it being ſo agreeable to the received Laws of Refractions. And indeed the Obſervations of theſe two Learned Perſons, as to this Particular, are eaſily reconcileable to each other, and both to Truth; Mr. *Newton* contending only for the Length of the Image (tranſverſe to the Axis of the Priſm) in a very clear Day; whereas Mr. *Line* only maintained the Exceſs of Breadth, parallel to the ſame Axis, while the Sun is in a bright Cloud. Tho' as to what is further deliver'd by Mr. *Newton*, and oppoſed by Mr. *Line*, namely, that the Length of the coloured Image was *five times* the Diameter of its Breadth; I never yet have found the Exceſs above thrice the Diameter, or at moſt $3\frac{1}{2}$, while the Refractions on both Sides the Priſm were equal. So much as to the Matter of Fact.

Now as to Mr. *Newton*'s *Theory of Light* and *Colours*, I confeſs his neat Sett of very Ingenious and Natural Inferences, was to me, upon the firſt

Peruſal, a ſtrong Conjecture in Favour of his new Doctrine; I having formerly obſerved the like Chain of Inferences upon Search into natural Truths. But ſince ſeveral Experiments of Refractions remain ſtill untouched by him, I conceived a further Search into them would be very proper, in Order to a further Diſcovery of the Truth of his Aſſertion. For accordingly as they are found, either agreeing with, or diſagreeing from his new Theory, they muſt needs much ſtrengthen, or wholly overthrow the ſame. The Experiments I pitched upon for this purpoſe are as follow:

1. Having frequently obſerved, that the Form of Objects viewed in the Microſcope (or rather of the Microſcope itſelf) conſiſts almoſt in an indiviſible Point, I concluded, two very ſmall Pieces of Silk, the one Scarlet, the other Violet Colour, placed near together, ſhould, according to Mr. *Newton*'s Theory, appear in the Microſcope in a very different Degree of Clarity, in regard their unequal Refrangibility muſt cauſe the Scarlet Rays, or Species, to over-reach the Retina, while placed in the due Focus of the Violet ones, and conſequently muſt occaſion a ſenſible Confuſion in the Viſion of the former, one and the ſame Point of the Scarlet Object affecting ſeveral Nerves in the Retina. Yet upon frequent Tryals I have not been able to perceive any Inequality in this Point.

2. The Second Experiment I made in Water. I took a Braſs Ruler, and faſtning thereunto ſeveral Pieces of Silk, Red, Yellow, Green, Blue, and Violet, I placed it at the Bottom of a Square Veſſel of Water: Then I retired from the Veſſel ſo far as not to be able to ſee the aforeſaid Ruler and coloured Silks, otherwiſe than by the Help of the refracted Ray. Now, did Mr. *Newton*'s Doctrine hold, I conceived I ſhould not ſee all the mentioned Colours in a ſtreight Line with the Ruler, in regard the unequal Refrangibility of different Rays muſt needs diſplace ſome more than others. Yet in Effect, upon many Tryals, I conſtantly found them in as ſtreight a Line, as the bare Ruler had appeared in.

3. To advance this Experiment, I adjoined a ſecond Refraction to the former of Water, by placing my Priſm ſo as to receive perpendicularly the refracted Species of the Silk and Ruler; whereby only the emergent Species ſuffered a ſecond Refraction. But ſtill with equal Succeſs, as to their appearing in a ſtreight Line to the Eye placed behind the Priſm.

4. To theſe two Refractions I further added a third, by receiving the coloured Species obliquely upon the Priſm; whereby both incident and emergent Species ſuffered their reſpective Refractions. But ſtill with the ſame Succeſs as formerly, as to the ſtreight Line they appeared in.

For further Aſſurance in this Experiment, leſt Prepoſſeſſion, occaſion'd from previous Knowledge of the Silk's Situation in a ſtreight Line, might poſſibly prejudice the Judgment of the Eye (as ſometimes I have obſerv'd to happen to the Judgment the Eye paſſeth upon the Diſtance of Objects) I call'd into the Room ſome unconcern'd Perſons, wholly ignorant of what the Experiment aim'd at; and demanding whether they ſaw not the coloured Silks and Ruler in a crooked Line, they anſwer'd in the Negative.

5. The next Experiment I made in uncompounded Colours (as Mr. *Newton* terms them, *Prop.* 5. & 13.) as follows. Having caſt two coloured Images upon the Wall, ſo as the Scarlet Colour of the one did fall in a ſtreight Line (parallel to the Horizon) with the Violet of the other: I then looked upon both through another Priſm, and found them ſtill appear in a ſtreight Line parallel to the Horizon, as they had formerly done to the naked Eye. Now according to Mr. *Newton*'s Aſſertion of different Refrangibility in different Rays, I conceive the Violet Rays ſhould ſuffer a greater Refraction in the Priſm at the Eye, than the Scarlet ones; and conſequently both Colours ſhould not appear in a ſtreight Line parallel to the Horizon.

6. Another Experiment I made, in order to ſome further Diſcovery of that ſurprizing Phænomenon of the coloured Image, which occaſioned Mr. *Newton*'s ingenious Theory of Light and Colours, as alſo of his excellent Invention of the reflecting Teleſcope and Microſcope. Having then ſometimes ſuſpected, that not only the direct Sun-Beams, but alſo other extraneous Light, might poſſibly influence the coloured *Spectrum*, I hoped to diſcover the Truth of this Suſpicion by means of the Sun-Spots, made to appear in the coloured Image, by placing a Teleſcope behind the Priſm. But my Endeavours proving ineffectual herein, by reaſon of ſome intervening Difficulties, I thought at length of a more feaſible Method, in order to the deſigned Diſcovery; as in the following Experiment.

I faſtned a very white Paper-Circle (about an Inch in Diameter) upon my Window-ſhuts; and beholding it thro' my Priſm, I found a coloured Image painted thereby upon my Retina, anſwerable in almoſt all reſpects to the former of the Sun-Beams upon the Wall, eſpecially when the Paper-Circle was indifferently well illuminated. This Image indeed appeared contrary to the former, as to the Situation of Colours, that is, the Scarlet appearing above, the Violet below, tho' but faint. But this I was not ſurprized at, having obſerved upon diſſecting the Eye, that Objects are painted on the Retina after a contrary Poſture to what they appear to ſight. Having thus rendered the coloured Image much more tractable than formerly it was, I conceived good Hopes of ſome further Diſcovery in the Point mentioned. *Fig.* 77.

In purſuance then of my former Suſpicion, having fixed my Priſm in a ſteady Poſture, I cauſed the Paper C to be applied cloſe up to the Paper-Circle *a b d*; whereupon the former Violet *d*, and the Scarlet Colour of C, vaniſhed into Whiteneſs. Next I removed the mentioned Circle from the Shuts, and placed it in the open Window, ſupported only by the Edge *d*: whereupon, to my Aſtoniſhment, all the former Colours exchanged Poſtures in the Retina, the Scarlet now appearing below, the Violet above, the intermediate Colours ſcarce diſcernable. And here, on the by, 'tis very remarkable, that during this Obſervation I clearly perceived both Blue and Scarlet Light to be tranſparent, I being able to diſcern ſeveral Objects thro' both, namely Steeples oppoſite to my Window. Whence it follows, that theſe Colours do in great Part ariſe from the neighbouring Light. Laſtly, I placed the Paper-Circle anew, ſo as the one half *b* was faſtned to the Shuts, the other Semicircle *a* being expoſed to the open Air. Whereupon the Semicircle *a* became bordered with Violet above, Scarlet below; but the other Semicircle *b*, quite contrary. Hence I make the following Inferences.

Firſt, That not only the Light reflected from the Paper-Circle, but alſo from the ambient Air, hath great Influence upon the coloured Image, eſpecially as to the Violet and Scarlet Colours. Whence, perchance, it will not hereafter ſeem ſtrange, that the coloured Spectrum on the Wall is ſo long, but only that the Breadth is not greater. Secondly, Were there a more luminous Body behind the Sun, we ſhould in all likelihood have the Colours of the Spectrum in a contrary Situation to what they appear in at preſent; Whence (Thirdly) it ſeems to follow, that the preſent Situation and Order of Colours ariſeth not from any intrinſical Property of *Refrangibility*, (as maintained by Mr. *Newton*) but from contingent and extrinſical Circumſtances of neighbouring Objects: For accordingly as the Body behind the Paper-Circle was more or leſs illuminated than the Circle itſelf, all the ſeveral Colours changed their Situation.

8. The next Experiment was made in Order to Mr. *Newton*'s Doctrine of primary Colours, as *Prop.* 5. Having covered the Hole in the Window-ſhuts with a thin Slice of Ivory, the tranſmitted Light appeared Yellow; but upon adding three, four, or more Slices, it became Red. Whence it ſeems to follow, that Yellowneſs of Light is not a primary Colour, but a Compound of Red, *&c.*

9. The laſt Experiment was made in reference to Mr. *Newton*'s 12th *Prop.* where from his own Principles he renders a very plauſible Reaſon of a ſurprizing Phænomenon, related by Mr. *Hooke*; namely, of two Liquors, the one Blue, and the other Red, both ſeverally tranſparent; yet both, if placed together, became opake. The Reaſon whereof, ſaith Mr. *Newton*, is, becauſe if one Liquor tranſmitted only Red, the other only Blue, no Rays could paſs thro' both.

In reference then to this Point, I filled two ſmall Glaſſes with flat poliſhed Bottoms, the one with *Aqua fortis* deeply died Blue, the other with Oyl of Turpentine died Red, both to that Degree as to repreſent all Objects thro' them reſpectively Blue or Red: Then placing the one upon the other, I was able to diſcern ſeveral Bodies thro' both. Whereas, according to Mr. *Newton*'s Theory, no Object ſhould appear thro' both Liquors; becauſe if one tranſmit only Red, the other only Blue, no Rays can paſs thro' both.

P. S. *Juſt upon the Cloſe of the adjoyned Letter, I received from Mr.* Gaſcoigne *yours of* May *the 4th; wherein you are pleaſed to favour us with an exact Account of the famous Experiment of the coloured Spectrum, lately exhibited before the* Royal Society. *I was much rejoyced to ſee the Tryals of that Illuſtrious Company agree ſo exactly with ours here, tho' in ſomewhat ours diſagree from Mr.* Newton.

Anſwered by Mr. Newton. *Ibid. p.* 693.

8. The Things oppoſed by Mr. *Line* being upon Tryals found true and granted me, I begin with the new Queſtion about the Proportion of the Length of the Image to its Breadth: And it is no wonder, that Mr. *Lucas* found the

Image ſhorter than I did, ſeeing he tryed the Experiment with a leſs Angle.

The Angle indeed which I uſed was but about 63 degrees, 12 min. and his is ſet to 60 degrees: the difference of which from mine, being but 3 deg. 12 min. is too little to reconcile us, but yet it will bring us conſiderably nearer together. And if this Angle was not exactly meaſured, but the round number of 60 degrees ſet down by gueſs, or by a leſs accurate Meaſure (as I ſuſpect by the conjectural Meaſure of the Refraction of his Priſm, by the Ratio of the Sines to 2 to 3, ſet down at the ſame time, inſtead of an Experimental one,) then might it be two or three degrees leſs than 60 deg. if not ſtill leſs: And all this, if it ſhould be ſo, would take away the greateſt part of the difference between us.

But however it be, I am well aſſured my own Obſervation was exact enough. For I have repeated it divers times ſince the receipt of Mr. *Lucas*'s Letter, and that without any conſiderable difference of my Obſervations, either from one another, or from what I wrote before. And that it might appear experimentally, how the increaſe of the Angle increaſes the length of the Image, and alſo that no body, who has a Mind to try the Experiment exactly, might be troubled to procure a Priſm which has an Angle juſt of the Bigneſs aſſigned by me; I tryed the Experiment with divers Angles, and have ſet down my Tryals in the following Table; where the firſt Column expreſſes the Angles of two Priſms which I uſed, which were meaſured as exactly as I could by applying them to the Angle of a Sector; and the ſecond Column expreſſes in Inches the length of the Image made by each of thoſe Angles; its Breadth being two Inches, its Diſtance from the Priſm 18 Feet and 4 Inches, and the Breadth of the Hole in the Window-ſhut ¼ of an Inch.

	Angles	*Lengths*		*Angles*	*Lengths*
The firſt Priſm	56°. 10′	7¼	The ſecond Priſm	54°. 0′	7⅛
	60 . 24	9½		62 . 12	10⅛
	63 . 26	10⅓		63 . 48	10¼

You may perceive, that the Length of the Images, in reſpect of the Angles that made them, are ſomething greater in the ſecond Priſm than in the firſt; but that was becauſe the Glaſs, of which the ſecond Priſm was made, had the gretter Refractive Power.

The Days in which I made theſe Tryals were pretty clear, but not ſo clear as I deſired; and therefore afterwards meeting with a Day as clear as I deſired, I repeated the Experiment with the ſecond Priſm, and found the Lengths of the Image made by its ſeveral Angles, to be about ¼ of an Inch greater than before, the Meaſures being thoſe ſet down in the Table.

	Angles	*Lengths*
The ſecond Priſm	54°. 0′	7⅓
	62 . 12	10½
	63 . 48	11

The reaſon of this difference, I apprehend, was, that in the cleareſt Days the Light of the White Skies, which dilutes and renders inviſible the fainteſt Colours at the Ends of the Image, is a little diminiſhed in a clear Day, and ſo gives leave to the Colours to appear to a great Length; the Sun's Light at the ſame time becoming Brisker, and ſo ſtrengthning the Colours, and making the faint ones at the two ends more conſpicuous. For I have obſerved, that in Days ſomething Cloudy, whilſt the Priſm has ſtood unmoved at the Window, the Image would grow a little longer or a little ſhorter, accordingly as the Sun was more or leſs obſcur'd by thin Clouds which paſſed over it; the Image being ſhorteſt when the Cloud was brighteſt, and the Sun's Light fainteſt. Whence 'is eaſie to apprehend, that if the Light of the Clouds could be quite taken away, ſo that the Sun might appear ſurrounded with Darkneſs, or if the Sun's Light were much ſtronger than it is, the Colours would ſtill appear to a greater Length.

In all theſe Obſervations the Breadth of the Image was juſt two Inches. But obſerving that the Sides of the two Priſms I uſed were not exactly plain, but a little Convex, (the Convexity being about ſo much as that of a double Convex-Glaſs of a ſixteen or eighteen Foot Teleſcope) I took a third Priſm, whoſe Sides were as much Concave as thoſe of the other were Convex; and this made the breadth of the Image to be two Inches and a third part of an Inch, the Angles of this Priſm, and the Lengths of the Image made by each of thoſe Angles, being thoſe expreſt in this Table.

Angles	*Lengths*
58°.	$8\frac{1}{2}$
$59\frac{1}{2}$	9
$62\frac{1}{2}$	$10\frac{1}{3}$

In this caſe you ſee, the Concave figure of the ſides of the Priſm by making the Rays Diverge a little, cauſes the Breadth of the Image to be greater in proportion to its Length than it would be otherwiſe. And this I thought fit to give you notice of, that Mr. *Lucas* may examine, whether his Priſm have not this fault. If a Priſm may be had with ſides exactly plain, it may do well to try the Experiment with that; but its better, if the ſides be about ſo much Convex as thoſe of mine are, becauſe the Image will thereby become much better defined. For this Convexity of the ſides does the ſame effect, as if you ſhould uſe a Priſm with ſides exactly plain, and between it and the Hole in the Window-ſhut, place an Object-Glaſs of an 18 Foot Teleſcope, to make the round Image of the Sun appear diſtinctly defin'd on the Wall when the Priſm is taken away, and conſequently the long Image made by the Priſm to be much more diſtinctly defined (eſpecially at its ſtreight ſides) than it would be otherwiſe.

One thing more I ſhall add: That the utmoſt length of the Image, from the fainteſt Red at one End, to the fainteſt Blue at the other, muſt be meaſured. For in my firſt Letter about Colours, where I ſet down the Length to be five times the Breadth, I called that Length the utmoſt Length of the Image; and I meaſured the utmoſt Length, becauſe I account all that Length to be cauſed by the immediate Light of the Sun, ſeeing the Colours (as I noted above) become viſible to the greateſt Length in the cleareſt Days, that is, when the Light of the Sun tranſcends moſt the Light of the Clouds. Sometimes there will happen to ſhoot out from both Ends of the Image a glaring Light a good way beyond theſe Colours, but this is not to be regarded, as not appertaining to the Image. If the Meaſures be taken right, the whole Length will exceed the Length of the ſtreight Sides by about the Breadth of the Image.

By theſe things ſet down thus circumſtantially, I preſume Mr. *Lucas* will be enabled to accord his Tryals of the Experiment with mine; ſo nearly at leaſt that there ſhall not remain any very conſiderable difference between us. For if ſome little difference ſhould ſtill remain, that need not trouble us any further, ſeeing there may be many various Circumſtances which may conduce to it; ſuch as are not only the different Figures of Priſms, but alſo the different Refractive Power of Glaſſes, the different Diameters of the Sun at divers Times of the Year, and the little Errors that may happen in Meaſuring Lines and Angles, or in placing the Priſm at the Window; though for my part, I took Care to do theſe things as exactly as I could. However Mr. *Lucas* may make ſure to find the Image as long or longer than I have ſet down, if he take a Priſm whoſe Sides are not hollow ground, but plain, or (which is better) a very little Convex, and whoſe Refracting Angle is as much greater than that I uſed, as that he hath hitherto tryed it with is leſs; that is, whoſe Angle is about 66 or 67 Degrees, or (if he will) a little greater.

Concerning Mr. *Lucas*'s other Experiments, I am much obliged to him that he would take theſe things ſo far into Conſideration, and be at ſo much Pains for Examining them; and I thank him ſo much the more, becauſe he is the firſt that hath ſent me an Experimental Examination of them. But yet it will conduce to his more ſpeedy and full Satisfaction, if he a little change the Method which he has propounded, and inſtead of a Multitude of Things try only the *Experimentum Crucis*. For it is not Number of Experiments but Weight to be regarded; and where one will do, what need many?

The main thing he goes about to examine is, the *Different Refrangibility of Light*; and this I demonſtrated by the *Experimentum Crucis*. Now if this Demonſtration be good, there needs no further Examination of the thing; if not good, the Fault of it is to be ſhewn: For the only way to examine a Demonſtrated Propoſition is, to examine the Demonſtration. Let that Experiment therefore be examined in the firſt place, and that which it proves be acknowledged, and then if Mr. *Lucas* wants my Aſſiſtance, to unfold the difficulties which he fanſies to be in the Experiments he has propounded, he ſhall freely have it. At preſent I ſhall ſay nothing in Anſwer to his Experimental Diſcourſe, but this in general, that it has proceeded partly from ſome miſunderſtanding of what he writes againſt, and partly for want of due caution in trying Experiments; and that amongſt his Experiments there is one, which when duly tryed, is, next to the *Experimentum Crucis*, the moſt conſpicuous Experiment, I know, for proving the different Refrangibility of Light, which he brings it to prove againſt.

By the Poſt-ſcript of Mr. *Lucas*'s Letter, one not acquainted with what has paſſed, might think, that he quotes the Obſervation of the *Royal Society* againſt me; whereas the Relation of their Obſervation, which you ſent to *Liege*, contained nothing at all about the juſt Proportion of the Length of the Image to its Breadth according to the Angle of the Priſm, nor any thing more (ſo far as I can perceive by your laſt) than what was pertinent to the things then in diſpute, *viz.* that they found them ſucceed as I had affirmed. And therefore, ſince Mr. *Lucas* has found the ſame Succeſs, I ſuppoſe, that when he expreſſed, That *he much rejoyced to ſee the Tryals of the* R. Society *agree ſo exactly with his*, he meant only ſo far as he agreed with mine.

P. S. *I had like to have forgotten to adviſe, that the* Experimentum Crucis, *and ſuch others as ſhall be made for knowing the Nature of Colours, be made with Priſms that Refract ſo much, as to make the Length of the Images five times its Breadth, and rather more than leſs; for otherwiſe Experiments will not ſucceed ſo plainly with others as they have done with me.*

An Optical Experiment; by Mr. Steph. Gray. N. 221. *p.* 286. June. An. 1696.

VIII. I took a ſtiff piece of brown Paper, and pricking a ſmall Hole therein, I held it at a little diſtance from me: then applying a Needle to my Eye, I was ſurprized to ſee the Point of it inverted. The nearer the Needle was to the Hole, it was ſo much the more magnify'd, but leſs diſtinct; and if it were ſo held, as that its Image was near to the Edge of the Hole, its Point ſeemed crooked. So that, it ſeems theſe ſmall Holes, or ſomewhat in them, perform the Effects of a Concave Speculum, and ſo I take leave to call them *Aerial Speculums.*

A Problem of Alhazen, *ſolved by Mr.* Chr. Hugens, N. 97. *p*, 6119. Oct. An. 1673. *Fig.* 78.

IX. 1. Mitto tibi hac occaſione Conſtructionem Problematis *Alhazeni* nuper à me inventam, & à Collegis meis felicem ſatis judicatam. Problema eſt;

Dato Speculo Cavo aut Convexo, itemque Oculo & Puncto Rei viſæ, invenire Punctum Reflexionis.

Eſto Speculum ex Sphæra quæ Centrum habeat A Punctum, Oculus vero ſit in B, & Punctum Viſibile in C, Planumque ductum per A, B, C, faciat in Sphæra Circulum D *d*, in quo invenienda ſint Reflexionis Puncta. Per tria Puncta A, B, C, deſcribatur Circuli Circumferentia, cujus ſit Centrum Z; occurrat autem ei producta A E, Perpend. B C, in R, & ſit duabus R A, O A, tertia Proportionalis N A, eritque N M, Parallela B C, altera Aſymptoton. Rurſus ſint Proportionales E A, $\frac{1}{2}$ A O, A I, & Summa I Y æquali I N, ducatur Y M Parallela A Z; eaque erit altera Aſymptotos. Denique ſumptis I X, I S, quæ ſingulæ poſſint dimidium Quadratum A O, una cum Quadrato A I; erunt Puncta *x* & *s* in Hyperbola, aut Sectionibus oppoſitis D *d*, ad inventas Aſymptotos deſcribendis, quarum Interſectiones cum Circumferentia D O, oſtendent Puncta Reflexionis quæſita. Conſtructio hæc, in omni Caſu, quo Problema ſolidum eſt, locum habet, præterquam in uno, ubi non Hyperbola

ſed Parabola deſcribenda eſt; cum nimirum Circumferentia per Puncta A, B, C, deſcripta, tangit Rectam A E.

By M. Sluſius, *ibid.*

2. Cum Nobiliſſimi *Hugenii* Conſtructionem ad Calculos revocarem, eandem omnino mecum Analyſin ſecutum eſſe deprehendi; ſed cum ex illa duæ naſcantur Effectiones, utraque per Hyperbolam circa Aſymptotos; Ille unam, Ego alteram, uti faciliorem, ſelegeram. Evidens eſt autem, nihil aliud quæri hoc Problemate (ſi illud ad Terminos mere Geometricos revocemus) niſi in dato Circulo, cujus Centrum A, Radius A P) Punctum aliquod ut P, a quo ductis ad Puncta data E, B, inæqualiter à Centro A diſtantia, Rectis P E, P B, Recta A P, producta biſecet Angulum E P B. Quod quidem varios Caſus recipit. Vel enim Normalis ex A in Rectum E B, nimirum A O, cadit inter E & B, vel ultra B. Si ultra, vel Rectangulum E O B æquale eſt Quadrato A O, vel majus vel minus. De caſu æqualitatis videbimus infra; nunc vero tres alios Caſus eadem fere Conſtructione complectemur. Per tria puncta A, E, B, tranſeat Circulus, ad cujus Circumferentiam producatur A O in D. Ac ſi quidem punctum O cadat inter E & B, Recta A O verſus O producenda erit; ſin autem ultra B, ſitque Rectangulum E O B majus Quadrato A O, producenda erit verſus A; at ſi Rectangulum quadrato minus fuerit, Circulus in ipſo puncto D, rectam A O ſecabit. Tum ducta A X, parallela E B, ſecante Circulum datum in N, fiat ut Rectangulum D A O ad quadratum A N, ita $\frac{1}{2}$ A X ad A H, quæ ſumenda erit verſus X, ſi O cadat inter E & B, aut Rectangulum E O B minus ſit quadrato O A; at ex parte contraria, ſi ſit majus. Ponatur nunc O Q æqualis A H (in directum E B primo & ſecundo caſu, tertio vero verſus E:) tum fiant proportionales X A, N A, H K, ſumenda omni caſu verſus X: ſectaque A O in V, ut ſit eadem Ratio K A ad A V, quæ A D ad A X; jungatur K V, ac producatur donec occurrat recta E M Parallelæ O A, indefinite productæ, in puncto L; erunt omni caſu K L & Q L Aſymptoti Hyperbolæ, quæ per punctum O deſcripta, propoſito ſatisfaciet: hoc tantum diſcrimine, quod primo & ſecundo caſu Hyperbola per O, Problema ſolvet in ſpeculo Convexo, Sectio vero ei oppoſita in Concavo; at tertio caſu contra, Hyperbola per O ſerviet Concavo, ejus oppoſita Convexo. Atque id quidem, cum punctum V cadit inter A & O; nam ſi ultra O caderet, unica Hyperbola inter eaſdem Q L, K L, deſcripta, tam ſpeculo Convexo quam Concavo ſatisfaceret. Cæterum ſi V caderet in ipſum punctum O; Problema tunc Planum eſſet, & ipſæ Rectæ L Q, L K, illud abſolverent. Unde patet Problematis hujus dari Caſus infinitos, qui per Locum Planum ſolvi poſſunt: quo magis venia digni videntur ii, qui illud per eundem Locum univerſe ſolvi poſſe cenſuerunt, quod ipſis aliquoties Calculus feliciter cecidiſſet. Nulla enim dari poteſt trium punctorum A, E, B, poſito, (de caſu æqualitatis Rectanguli E O B, & Quadrati O A, mox videbimus,) quæ non admittat Circulum aliquem ex Centro A deſcribendum, ad cujus Circumferentiam Problema per Locum Planum ſolvi queat. Hujus autem Circuli Radius, ſi tanta eſt, ita invenietur: in primo & ſecundo caſu ſuperioris Conſtructionis fiat ut Quadratum A X una cum duplo rectangulo O A D, ad duplum Quadratum A D; ita Quadratum A O, ad Quadratum A N, erit A N Radius quæſitus. At in tertio caſu, faciendum eſt, ut Quadratum A X minus

Fig. 79, 80, &c.

Duplo Rectangulo O A D, ad Duplum Quadratum A D; ita Quadratum A O, ad Quadratum A N.

Conſtruendus nunc ſupereſt alius caſus, æqualitatis nempe Rectanguli E O B, & Quadrati A O, ſive in quo Circulus, per puncta A, B, E, deſcriptus, tangit Rectam A O. Recte autem monuit Clariſſimus *Hugenius* hoc caſu deſcribendam eſſe Parabolam; quod tamen non ita intelligendum eſt quaſi per Hyperbolam ſolvi non poſſit, cum & Hyperbolam & Ellipſin, imo Infinitas (ſi quis Methodo noſtra uti velit) admittat; ſed quod Parabolam quoque recipiat, quem alii caſus reſpuunt. Eadem ratione temperandum eſt quod ait; Conſtructionem ſuam omni caſu quo Problema ſolidum eſt; locum habere; intelligit enim, levi mutatione ſemper inveniri Hyperbolam quæ propoſito ſerviat: quod caſus à nobis ſuperius Conſtructos cum ejus Conſtructione comparanti planum fiet. Ut autem ad caſum æqualitatis redeam, & nequid temere aſſeruiſſe videar, Ecce tibi non unam, ſed duas Parabolas, ac præterea Hyperbolas Oppo-
Fig. 82. ſitas quæ Propoſitum abſolvunt. Sint, ut prius, puncta data E, B, Circulus ex Centro A, ac alius per tria puncta A, E, B, cujus Tangens ſit A O, Centrum D. Ducta Diametro N A D X, fiant tres Proportionales X A, N A, Z A, cujus dimidium ſit A L. Fiant iterum tres Proportionales 2 O A, N A, I A, cujus dimidium ſit K A, & perficiatur Rectangulum L A O V; productaque L V in S, donec V S ſit tertia Proportionalis ipſarum A I, O V; Axe S L, Latere Recto A I, Vertice S, deſcribatur Parabola; hæc enim Circulum ſecabit in punctis P, P, quæſitis. Tantundem faciet alia, ſic perfecto Rectangulo D A K C, & producta K C in T, ita ut C T ſit tertia Proportionalis ipſarum A Z, D C; deſcribatur circa Axem T K, Vertice T, Latere Recto, Z A: occurret enim Circulo in iiſdem punctis P, P. Facilior adhuc eſt Conſtructio
Fig 83. per Sectiones Oppoſitas; factis enim, ut prius, tribus Proportionalibus X A, N A, Z A, demittatur Z I Normalis, tertia Proportionalis Duplæ A O, & A N. Erit itaque Z I major Z A, cum Dupla A O minor ſit X A: tum in puncto I, inclinetur utrinque Angulo Semirecto ad lineam I Z, rectæ I Q, I M, & ab utraque parte indefinite producantur; demum circa illas tanquam Aſymptotos deſcribatur per A Hyperbola, & alia ipſi Oppoſita; hæc enim ſatisfaciet Problemati in Speculo Convexo, illa in Concavo. Cum vero, ut oſtendimus, Z I ſemper major ſit Recta Z A, Recta I M nunquam tranſibit per A. Non dabitur itaque Caſus, quo ex hac Conſtructione, velut in præcedentibus, Problema per ipſas Aſymptotos ſolvi poſſit: & tamen hoc quoque aliquando Locum Planum admittit; cum ſcilicet accidit, ut Recta X O ducta ad Centrum D, Tangat Circulum N P P; ipſum enim punctum Contactus quæſtionem ſolvit. Et hæc quidem de Problemate, quod hactenus multorum ingenia exercuit, & cujus ſolutionem ante aliquot Annos abſolvi.

* Otherwiſe, by M. Sluſius. *ibid.* p. 6123. Accipe quæ circa *Alhazeni* Problema, curis ſecundis meditatus ſum.

Fig. 84. Datus ſit Circulus, cujus Centrum A; puncta data ſunt D & *d*. Supponatur factum quod quæritur; ſitque Radius Incidens D E, Reflexus E *d*; & ex Puncto Reflexionis E cadat in junctam D A, Normalis E I, & in eandem, ex *d*, Normalis *d* N, occurrantque eidem Tangens E C & Radius *d* E, productis in B. Sit nunc $DA = z$. $AI = a$. $NA = n$. $EI = e$. $dN = b$. $BA = y$. $AE = q$. $CA = x$. Igitur, cum Anguli, D E C, C E B, ſint æquales, &

Angulus C E A, Rectus, ex Hypotheſi erunt tres, DA, CA, BA, Harmonice Proportionales (hoc enim facile oſtenditur). Erit itaque ut D A, ad B A, ita D C, ad C B; ſive in terminis Analyticis, $z : y :: z - x : x - y$; & $2zy - xy = zx$, ſive $\frac{2zy}{z + y} = x$. Cum autem Rectangulum C A I, ſive xa, ſit æquale Quadrato A E, ſive qq, erit $x = \frac{qq}{a}$, & per conſequens $\frac{2zy}{z + y} = \frac{qq}{a}$, ſive $\frac{zqq}{2za - qq} = y$. Porro, eſt ut dN, ad E I: ita N B, ad I B; ſive $b : e :: y - n : y - a$. Itaque $ye - ne = by - ba$; & $y = \frac{ba - ne}{b - e}$. Igitur $\frac{zqq}{2za - qq} = \frac{ba - ne}{b - e}$ ſive $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$. Quæ æquatio eſt ad Hyperbolam circa Aſymptotos, cujus Conſtructio cum Circulo dato, Problemati ſatisfacit. Cum vero, ob Circulum, ſit $qq = aa + ee$, ſi loco $2bzaa$, ponatur ejus Valor $2bzqq - 2bzee$, habebitur alia pariter ad Hyperbolam circa Aſymptotos $bzqq - 2bzee - 2znae - qqba + qqne = -zqqe$. Et hac Methodo, atque illa, quam in Libello noſtro de *Analyſi* expoſuimus, prodibunt infinitæ Æquationes ad Hyperbolas & Ellipſes, quæ cum Circulo dato Problema abſolvent; niſi quod Effectiones plerumque intricatiores evadunt quam ut operæ pretium ſit illas aggredi: conſtrui tamen poterunt eo modo, quo ibi uſi ſumus in Ellipſi.

Retulimus, ut vides, Calculi noſtri Summam ad lineam D A, ſed ſatis animadvertis, non majori difficultate referri potuiſſe ad d A (quæ pariter data eſt) ductis *ſcil.* lineis, quas in Schemate punctis adumbravimus. Verum novo Calculi labore non eſt opus. Si enim Rectæ dA, ejuſque partibus, eoſdem ac prius terminos Analyticos adhibeas, *h. e.* ſi ipſam d A facias æqualem z. D$n = b$. nA $= n$. AI $= a$. iE $= e$. &c. prodibit eadem Æquatio quæ prius; & infinitas alias Hyperbolas & Ellipſes obtinebis, quæ cum Circulo dato Problemati ſatisfacient. Φορτικὸς eſſem ſi ſingulos Caſus proſequi vellem, cum illorum Æquationes ſola Signorum + & − variatione diſcernantur. Unum tamen excipio, nimirum cum Angulus d A D eſt Rectus; ejus enim æquatio habetur, ex punctis à priori æquatione partibus, in quibus n (quæ in nihilum abit) invenitur: nempe hæc, $2zbaa - qqba = bzqq - zqqe$, vel (pro $2zbaa$, poſito ejus valore) $zbqq - qqba = 2zbee - zqqe$.

Sed animadvertendum eſt, quod licet referendo Analyſin ad Rectam D A, ſtatim ſeſe offerant in æquatione duæ Hyperbolæ; & aliæ totidem à prioribus diverſæ, cum refertur ad Rectam dA; eaſdem tamen omnino Parabolas haberi, ad utramvis Rectarum dA, vel D A, referatur Analyſis: cujus rei ratio levi conſideratione tibi occurret.

Patere nunc, *V. Cl.* ut ſuperiorem Analyſin omnibus, quæ circa Speculorum Sphæricorum Reflectionem proponi ſolent, Problematibus applicem, novo facto Schemate. Sit igitur, ut prius, Circulus, cujus Centrum A, Punctum D datum, & ab eo Radius Incidens D E, cujus Reflexus ſit E Q. Juncta D A, ducatur ad illam Tangens E C, & Normalis E I; & producatur ad eandem, Recta Q E B; Denominentur partes ut prius. D A $= z$. C A $= x$. A E $= q$. B A $= y$. A I $= a$. I E $= e$. Igitur, propter tres D A, C A, B A, Harmonice Proportionales, & tres C A, A E, AI, Geometrice, ſemper habebitur æquatio $y = \frac{zqq}{2za - qq}$, in quodcunque Circuli punctum cadat Radius D E. Itaque ſi quæratur punctum E, in quod ſi Radius D E Incidat, Reflectatur παραλλήλως Diametro L A V normali ad D A; Reflexus Q E, productus tranſibit per I, ut patet; & I ac B coincident. Igitur $a = y = \frac{zqq}{2za - qq}$; ſive, $aa - \frac{1}{2}\frac{qqa}{z} = \frac{1}{2}qq$, & Problema per Plana ſolvetur.

Fig. 85.

Si quæratur punctum, à quo Radius Reflectatur parallelus alteri cuilibet lineæ, ut A K (ductæ ex Centro A;) ducatur ad illam, ex puncto I, Tangens K L $= d$. Evidens eſt, Triangula A K L, E I B, fore ſimilia, cùm omnia Latera unius parallela ſint Lateribus alterius, &c. Itaque A L, ad L K; ut E I, ad I B, ſive $q : d :: e : a - y$; & $\frac{qa - de}{q} = y = \frac{zqq}{2za - qq}$; & $zq^3 = 2qzaa - 2zdae - q^3a + qqde$; ſive, pro aa poſito $qq - ee$, $zq^3 = 2zq^3 - 2zqee - 2zdae - q^3a + qqde$. Utraque autem æquatio eſt ad Hyperbolicam circa Aſymptotos, quæ cum Circulo dato Problema abſolvit.

Proponatur nunc efficere, ut Radius Reflexus tranſeat per datum punctum N, (ut in Problemate *Alhazeni*) vel ut productus verſus punctum Reflexionis E, occurrat dato puncto N. Ex N cadat in A L Normalis N O $= n$, ſitque A O $= b$. Patet eſſe, ut A O, ad differentiam ipſarum O N, A B; ita E I, ad I B, $b . e . b : n - y :: e : a - y$; vel $b : y - n :: e : y - a$. Igitur $\frac{ba - ne}{b - e} = y = \frac{zqq}{2za - qq}$. Unde $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$; nim. illa ipſa æquatio Problematis *Alhazeniani* quam ſupra innuimus: Vel, ſecundo caſu, $\frac{ba + ne}{b + e} = y = \frac{zqq}{zza - qq}$, ſive $2zbaa + 2znae - qqba - qqne = zbqq + zqqe$.

Atque hæc ſunt Problemata, quæ circa punctum Reflexionis proponi ſolent, in quibus tamen Finitam puncti D dati diſtantiam ſuppoſuimus. Sed facilior erit Analyſis, ſi ſupponamus Infinitam. Secta enim C A bifariam in G, conſtat ex proprietate trium, D A, C A, B A, Harmonice Proportionalium, tres D G, C G, B G, fore Geometrice Proportionales, ſuppoſita quacunque punćti D diſtantia. Itaque, ſi ſupponatur Infinita, B G abibit in nihilum, & punćtum B cum punćto G coincidet. Igitur A B erit perpetuo æqualis B C; erit itaque C A $= 2y$, & Rećtangulum C A I, æquale Quadrato A E, dabit, in terminis Analyticis, $2ay = qq$, ſive $y = \frac{qq}{2a}$: Cumque diſtantia punćti D ſupponatur infinita, erit E D Parallela A C. Itaque, ſi quæratur Radius Reflexus parallelus A L, quoniam eo caſu a & y coincidunt, erit $a = y = \frac{qq}{2a}$, ſive $aa = \frac{1}{2}qq$: Si quæratur ut Parallelus ſit A K, erit rurſus $q : d :: e : a - y$; & $\frac{qa - de}{q} = y = \frac{qq}{2a}$, ſive $2qaa - 2dae = q^3$. Si petatur ut tranſeat per N, erit, ut ſupra, $\frac{ba + ne}{b \pm e} = y = \frac{qq}{2a}$, & $2baa \pm 2nae = bqq \pm qqe$: Quæ æquationes ſunt quoque ad Hyperbolas circa Aſymptotos, niſi N punćtum eſſe ſupponatur in A L; nam cum tunc n abeat in nihilum, ſublatis ab æquatione partibus, in quibus n continetur, reſiduæ dant æquationem ad Parabolam, ut ſupra quoque monuimus.

Non exſpećtas, *V. Cl.* ut cum Specula Concava hactenus in Exemplum adduxerim, nunc agam de Convexis. Scis enim, eandem eſſe prorſus Analyſin, & Æquationes ſola Signorum + & − variatione diſtingui. Scis Parabolam vel Ellipſin quæ uni ſatisfacit, ſatisfacere alteri; & ſi Hyperbola in Convexo Problema abſolvat, ejus Oppoſitam paria facere in Concavo. His itaque omiſſis, addo tantum, eadem Analyſi haberi in Speculis Concavis Focos & Spatia, quæ Radii occupant in Axe, data qualibet Punćti Lucentis diſtantia: Sed mira facilitate, cum Radii ſupponuntur paralleli; quod tamen nonnullo circuitu à quibuſdam demonſtrari vidi. *Fig.* 86. Nam in Speculo Concavo E E, cujus Centrum A, ſi Radius extremus Reflećti intelligatur ad Axem A R in B, dućta Tangente E C, erit C B = B A. Biſecetur Semi-axis A R in Q; erit itaque Q Focus. Et Q B Spatium quæſitum. Eſt autem Q B dimidia C R (ob æquales A Q, Q R, A B, B C,) *h.e.* dimidia exceſſus ſecantis Arcus E R ſupra Sinum totum. Igitur ſi Arcus E R ſit (*e.g.*) grad. 9, erit A C, 101246, & B Q $\frac{623}{100000}$ ipſius A R.

4. Compendium, quod eodem tempore inveni circa primam Conſtrućtionem, ab initio tibi communicatam, tale eſt: Dućta linea A T, parallela C B, eaque biſecta in V, punćtum hoc eſt illud, per quod tranſire debet una Hyperbolarum Oppoſitarum, quarum Aſymptoti inventæ fuerunt Y M, M N.

Otherwiſe, by M Hugens. N. 98. *p.* 6143. *Fig.* 87.

Sed en Tibi bonam illam Conſtrućtionem, quæ in omnibus Caſibus obtinet. *Fig.* 88. Sit Circulus datus E D, cujus Centrum eſt A; Punćta data, B & C.

Dućtis Lineis A B, A C, fiant Proportionales B A (Radius Circuli) & F A: Eodem modo C A, (Radius Circuli) & G A. Tum jungatur F G, eaque biſecetur in H; & per hoc punćtum ducantur Lineæ L H K, M H N, ſe invicem interſecantes ad Angulos Rectos, quarumque LHK ſit Parallela ei quæ biſecat Angulum BAC. Hæ ſunt duæ Aſymptoti Hyperbolarum deſcribendarum per puncta F & G, & quarum una tranſibit etiam per Centrum A, quarum Interſectiones cum Circuli Peripheria notabunt puncta Reflexionis quæſita.

Futrher conſidered, by Mr. Sluſius. *ib. p.* 6141.

5. Videt hic Nob. *Hugenius* qua ratione ad omnes Caſus extendi poſſet Hyperbola Æqualium Laterum, quam in caſu Anguli Recti ſeſe ſtatim offerre præcedentibus meis inſinuaveram. Poſſet quoque ex infinitis Ellipſibus, quæ adhiberi poſſunt, una ſeligi non difficilis Conſtructionis: ſed piget tamdiu in eodem Problemate hærere. Supereſt tamen aliquid, quod contemplationem habet non injucundam; *nim.* cum Sectiones, quæ cum Circulo dato ad Problematis ſolutionem adhibentur, illum in quatuor punctis ſecent, quorum duo tantum Reflexioni ſerviunt, quæri poſſet, quodnam Problema ſolvent duo reliqua; & quanam verborum forma concipienda ſit Propoſitio, ut quatuor illos Caſus complectatur. Deinde annon etiam ijdem quatuor Caſus occurrant cum Puncta data æqualiter diſtant à Centro?

Again. ib.

Clar. *Hugenius* non alia utitur Analyſi quam mea, quæ Parabolam uno tantum caſu admittit. Quod ut evidentius tibi conſtet, Æquationem quam Conſtruxit hic adſcribam. Repete memoria, ſi placet, quæ ſecundis curis ad te ſcripſi, & invenies, me duas æquationes, Problemati per Hyperbolam circa Aſymptos ſolvendo idoneas, aſſignaſſe has nimirum;

$$2zbaa - 2znae - qqba + qqne = bzqq - zqqe,$$

Et $$bzqq - 2znae - qqba + qqne = 2zbee - zqqe;$$

ac ſubjeciſſe, levi mutatione, (ſubſtituendo, ex gr. pro qq, ejus Valorem $aa + ee$) invenire poſſe infinitas Hyperbolas & Ellipſes, quæ cum Circulo dato Problema ſolverent. Nunc in priori ex his æquationibus pro $bzqq$ ponatur ejus Valor, fiet

$$zbaa - 2znae - qqba + qqne = bzee - zqqe;$$

Sive $$aa - \frac{qqa}{z} = ee - \frac{qqe}{b} + \frac{2nae}{b} - \frac{qqne}{zb}.$$

Atque hæc eſt Æquatio, quam magno ingenii acumine, ac pari facilitate, conſtruxit Vir Doctiſſimus.

Again. ib. *Fig.* 89.

Incidi nuper in ſequentem Conſtructionem, qua breviorem cum dari poſſe vix credam, committere nolui, quin eam judicio ac cenſuræ tuæ ſubmitterem. Sint igitur Puncta data EB, Circulus cujus Centrum A; junctis EA, BA, Secantibus Circulum in F & C; fiant tres Proportionales EA, FA, VA, & tres iterum BA, CA, XA: Tum juncta VX, ac producta utcunque, (Vertite X, Latere Tranſverſo VX, ac Recto ipſi æquali) deſcribatur Hyperbola XP, cujus applicatæ ad Diametrum VXG, Parallelæ ſint Rectæ AB: Illa enim ſatisfacit propoſito in caſu Speculi Convexi, ut ejus Oppoſita in caſu Concavi. Si Aſymptotos deſideres, facile reperiri poſſunt, producta VX, donec cum EB, pariter producta, concurrat in L; deinde biſecta VX in I, ac ſumpta LD aquali LI; juncta enim DI erit Aſymptoton una, in quam alia normaliter incidit ad punctum I.

Sed fortaſſe ingratum tibi non erit intelligere, quà via ad hanc Conſtructionem pervenerim. Scias itaque, me ex priori mea Analyſi deduxiſſe hoc modo. Datis iiſdem quæ prius, cadat in E B, Normalis A O, ſitque punctum quæſitum P, ex quo in A O cadat Normalis P R. Si A O ſit b; E O, z; O B, d; A P, q; P R, e; A R, a; facile colligitur hæc Æquatio, $\frac{2zdae + 2bbae - 2bqqe}{zb - bd} + ee = aa - \frac{qqa}{b}$, quæ mutari poteſt in has, $\frac{zdae + bbae - bqqe}{zb - bd} = aa - \frac{1}{2}qq - \frac{\frac{1}{2}qqa}{b}$; Et $\frac{zdae + bbae - bqqe}{zb - bd} + ee = \frac{1}{2}qq - \frac{\frac{1}{2}qqa}{b}$. Hujus ultimæ Conſtructionem olim ad te miſi; alterius vero, Cl. *Hugenius*. Primam autem, licet ſe ſtatim in conſpectum dediſſet; ferme neglexeram, quod difficilioris Conſtructionis eſſe præſumerem. Sed me vano timore deluſum agnovi, cum in hanc, quam ad te mitto, Conſtructionem deſinere nuper ſum expertus. Sit enim, brevioris Calculi cauſa, $z - d = k$, $zd + bb = bm$; fiet $ee \frac{-2qqe + 2mae}{k} = aa \frac{-qqa}{b}$. Et additis utrinque $\frac{q^4 + mmaa - 2qqma}{kk}$, erit $ee + \frac{mae - 2qqe}{k} + \frac{q^4 + mmaa - 2qqma}{kk}$, (hoc eſt quadratum ex $e - \frac{qq + ma}{k}$,) æquale $aa - \frac{qqa}{b} + \frac{q^4 + mmaa - 2qqma}{kk}$. Fiet igitur, ἀναλογισμὸς $kk : kk + mm :: aa - \frac{kkqqa}{bkk + bmm} - \frac{2qqma + q^4}{kk + mm}$; & quadratum $e - \frac{qq + ma}{k}$: qui ad æquationem faciliorem reduci poteſt, ſi, poſito $kk + mm = pp$, fiat $\frac{ky}{p} = a$; fit enim tandem, quadratum ex $e \frac{-qq}{k} + \frac{my}{p} = yy \frac{-qqky}{bp} - \frac{2qqmy}{kp} + \frac{q^4}{kk}$; quam Æquationem ſuperiori Conſtructioni reſpondere animadvertes, ſi Calculos applicueris; ac ſimul obſervabis, ad quamcunque linearum E A, A B, B E, referatur Analyſeos ſumma, eaſdem ſemper habere poſſe Sectiones, quamvis longiori circuitu & æquationibus valde diverſis.

Fig. 9.

Ex hac Conſtructione, κατ᾽ ἀναλογίαν deducere licet alterius Problematis effectionem, cum ſcil. quæritur Punctum, a quo Radius Reflexus Parallelus ſit cuilibet lineæ datæ; ut, ſi dato Puncto Luminoſo B, Circulo ex Centro A, quæreretur Radius Reflexus parallelus rectæ A E. Fig. 91. Idem enim eſt, ac ſi in alio Problemate, diſtantia punctorum A & E ſupponeretur infinita; quo caſu Tertia Proportionalis ipſarum E A, F A, abiret in nihilum, & puncta A & V. Coinciderent: Itaque V X eſſet æqualis A X, & A E parallela P E. Applica igitur ſuperiorem Conſtructionem & Problema abſolves. Deſcripta ſcil. (Vertice X, Latere Tranſverſo V X, vel A X, & Recto ipſi æquali,) Hyperbola X P, cujus Applicatæ ad Diametrum A X, parallelæ ſint rectæ A E.

By M. Hugens. Ibid. p. 614.

6. Verum eſt, quin imo mirandum, Conſtructionem quam antehac ad te miſi inveniri quoque per Calculum quem Dn. *Sluſius* de ea inſtituit poſt mutationem qq in $aa + ee$; at hoc videtur fieri caſu, nec ibi apparet Conſtructionis ſimplicitatis niſi poſtquam eam peragere ſategimus.

Problema Alhazeni.] *Dato Circulo, cujus Centrum A, Radius AD, & punctis duobus B, C; invenire punctum H in Circumferentia Circuli dati, unde ductæ HB, HC, faciant ad Circumferentiam Angulos æquales.* Fig. 92.

Ponatur Inventum, ductaque A M recta, quæ biſariam ſecet Angulum B A C, ducatur ei Perpendicularis H F, itemque B M, C L. Jungatur porro A H, cui Perpendicularis ſit H E, Rectiſque B H, H C, occurrat A M in punctis K, G.

Sit jam $AM = a$
$MB = b$
$AL = c$
$LC = n$
Radius $AD = d$
$AF = x$
$FH = y$

Quia ergo æquales Anguli K H E & C H Z, ſive E H G; eſtque E H A Angulus Rectus, erit ut K E, ad E G; ita K A, ad A G. Quia vero B M, ad M D; ut H F, ad F K, erit,

ut B M + H F ad H E, ita M F ad F K,

i. e. $b + y : y :: a - x : \frac{ay - xy}{b + y}$. add. $FA = x$ fit $KA = \frac{ay + bx}{b + y}$.

Rurſus, quia C L ad L G, ut H F ad F G, erit permutando & dividendo C L − H F ad H F, ut L F ad F G, $n - y : y :: c - x : \frac{cy - xy}{n - y}$, qua ablata ab $AF = x$, fit $GA = \frac{nx - cy}{n - y}$. Eſt autem $EA = \frac{dd}{x}$, quia Proportionales F A, A H, A E: Ergo E A − G A, hoc eſt, E G, $= \frac{dd}{x} - \frac{nx + cy}{n - y}$. Et K A − E A, hoc eſt, $KE = \frac{ay + bx}{b + y} - \frac{dd}{x}$.

Sed diximus, quod KE ad EG, ut KA ad AG; i. e. $\frac{ay+bx}{b+y} - \frac{dd}{x} : \frac{dd}{x} - \frac{nx+cy}{n-y} :: \frac{ay+bx}{b+y} : \frac{nx-cy}{n-y}$.

Unde invenitur $2anxxy + 2bnx^3 - ddbnx - ddnxy = naddy + nbddx$
$- 2acxyy - 2bcxxy + ddbcy + ddcyy - addyy - bddxy$.

Et quia $n = \frac{bc}{a}$, fit $\frac{2bbc}{a}x^3 - \frac{bbddcx}{a} - \frac{2bbcyyx}{a}$, quia $xx = dd - yy$. Eſt autem $\frac{2bbc}{a}x^3 = \frac{2bbcddx}{a} - \frac{2bbcyyx}{a}$, quia $xx = dd - yy$. Ergo $\frac{-2bbcxyy}{a} - \frac{ddbcyx}{a} - 2acxyy + ddcyy = - adddyy - bddxy$.

Et diviſis omnibus per y & ductis in a

$$-2bbcxy - ddbcx - 2aacxy + ddcay = -aaddy - bddax$$
$$abddx - cbddx + acddy + aaddy = 2aacxy + 2bbcxy$$
$$\frac{abddx - cbddx + acddy + aaddy}{2aac + 2bbc} = xy,$$ quæ æquatio eſt ad Hyperbolam.

Vel quia $bc = na$, $\frac{abdd - anddx + acddy + aaddy}{2aac + 2bbc} = xy$.

Sit $\frac{add}{aa+bb} = p$; Ergo $\frac{pbx - pnx + pcy + pay}{2c} = xy$.

Unde porro non difficulter invenitur ſequens Conſtructio: Jungantur BA, AC, & applicato ſeorſim ad utramque Quadrato Radii AD, fiant inde AP, AQ; & juncto PQ, dividatur ipſa bifariam in R, & per punctum R ducantur RD, RN, ſeſe ad Rectos Angulos ſecantes, quorumque RD, ſit parallela AD, quæ dividet bifariam Angulum BAC. Erunt jam RD, RN Aſymptoti Oppoſitarum Hyperbolarum, quarum altera per Centrum A tranſire debet, quæque ſecabunt Circumferentiam in punctis H quæſitis. Tranſibunt autem Hyperbolæ per Puncta P, G. Fig. 93.

Ratio Conſtructionis apparet, ductis P γ, & Q ζ, perpendicularibus in AM. Fit enim A $\gamma = \frac{add}{aa+bb}$ ſive p; & A $\zeta = \frac{ap}{c}$. Item P $\gamma = \frac{pn}{c}$, & Q $\zeta = \frac{pb}{c}$. Quare AO $= \frac{pc+pa}{2c}$, & OR $= \frac{pb-pn}{2c}$. Unde Cætera facilia.

By Mr. Sluſius ib. p. 6145.

Fig. 94.

7. Mirari deſine, Vir Clariſſime, eandem in *Alhazeniano Problemate* Conſtructionem ex diverſis Æquationibus deduci, quandoquidem illæ omnes, quibus hactenus uſi ſumus in una eademque generali Analyſi contineantur. Quod ut oſtendam, datus ſit Circulus cujus Centrum A, puncta H & I; ſitque Punctum quæſitum K, ad quod ex punctis I & H ducantur Rectæ HK, IK, & Tangens KD. Tum ex A ducatur quælibet AG, occurrens HK in E, IK in B, Tangenti KD in D (iis nim. productis, quas produci eſt opus). His poſitis evidens eſt, ob Angulos EKD, DKB, æquales, & Angulum AKD Rectum, tres AE, BE, DE fore ſemper Harmonice proportionales. Itaque ductis ad AE Normalibus KC, IF, HG, ac denominatis partibus,

$AK = q$
$AC = a$
$CK = e$
$HG = b$
$AG = d$
$FA = z$
$FI = n$

habebitur, methodo, quam in ſecunda hujus Problematis Analyſi olim adhibui, hæc generalis Æquatio,

$$ndaa - bzaa - nqqa + bqqa = ndee - zbee + 2bnae + 2zdae - dqqe - zqqe.$$

Finge nunc, AG eſſe perpendicularem ad HI, nihil varietatis erit in æquatione; niſi quod AF & AG, hoc eſt, d & z, erunt æquales. Poſito itaque d pro z, fiet

$$ndaa - bdaa - nqqa + bqqa = ndee - dbee + 2bnae + 2ddae - 2dqqe.$$

Sive applicatis omnibus ad $nd - db$,

$$aa \frac{-qqa}{d} = ee \frac{+2bnae + 2ddae - 2dqqe}{nd - bd};$$

Eadem nempe, quam ex prima mea Analyſi, licet alia via, deduxeram, & quam nuper, modo facili conſtructam, ad te miſi.

Pone deinde, AG coincidere cum AH; abibit igitur HG, ſive b, in nihilum. Expunctis itaque ab æquatione partibus, in quibus b reperitur, remanebit, $ndaa - nqqa = ndee + 2zdae - dqqe - qqze$. Hanc autem, ſi meminiſti, curis ſecundis inveni, & aliam huic ſimilem, in Caſu pro Recta AG tranſire intelligitur per I.

Supponamus demum, Rectam AG, ſecare biſariam Angulum HAI; erit ob ſimilitudinem Triangulorum HAG, IAF, ut HG ad GA, ita IF ad FA, ſive ut b ad d, ita n ad z, & $nd = bz$. Ablatis igitur æqualibus, fit $bqqa - nqqa = 2bnae + 2zdae - dqqe - qqze$: Illa ipſa, quam ut ex literis tuis nuper intellexi, Cl. *Hugenius* conſtruxit.

Intelligatur tandem eadem Recta HG, ſecare biſariam Rectam HI; erunt igitur æquales HG, IG, hoc eſt, $b = n$; fietque, ablatis æqualibus, $bdaa - bzaa = bdee - bzee + 2bbae + 2zdae - dqqe - qqze$; quam licet non admodum difficilem, nemo noſtrum hactenus conſtruxit. Hæ autem, ut & ipſa Generalis Æquatio, in duas alias dividi poſſunt, poſito, ut noſti, pro aa vel ee, ejus valore, $qq - ee$, vel $qq - aa$.

Vides igitur, quicquid hactenus præſtitum eſt, in eandem Analyſin reſolvi; quæ & infinitas alias Conſtructiones per Circulum datum & Hyperbolam complectatur. Sed eas inveſtigare non eſt tanti, cum in hoc Problemate, ut olim fortaſſis inopia, ſic nunc copia laboremus. Addam tantum Conſtructionem per Parabolam, idque via duplici; quæ licet aliis per Hyperbolam operoſior videatur, Lineæ tamen ſimplicitate, qua Parabola inter reliquas Sectiones commendatur, operam compenſat.

Fig. 95.

Iiſdem igitur Datis, Jungatur A I, & producatur in S, donec A S fiat æqualis A H, junctaque H S, & biſecta I S in M, ducatur per M Recta R M Q normalis ad H S, in quam cadat ex A Normalis A Q, & cui Parallelus ducatur Radius A C. Tum factis tribus Proportionalibus I A, A C, A E, fiat ut S A ad A E, ita M Q ad A D, & R S ad A P (in recta A Q verſus Q;) & in eadem ab alia parte ſumatur D O æqualis D C. Demum biſecta P D in X, inclinetur per X, Angulo Semi-recto ad A X. Recta V X L, occurrens Normali in D erectæ in puncto V, & in quam ex O cadat Normalis O B. Aio, ſi fiat ut V X ad X B, ita X B ad B L, punctum L eſſe Verticem, L V, Axem, X V Latus Rectum Parabolæ, quæ Problemati ſatisfacit omni caſu; Secans nimirum Circulum datum in punctis K, quorum ſupremum & ultimum ad *Problema Alhazenianum* pertinent, reliqua ad aliud.

Datur, ut ſupra indicavi, alia quoque Parabola, quæ cum hac paria facit, & cujus deſcriptio ex hac adeo facile deducitur, ut nova non ſit opus. Sumatur enim A δ, in Directum D A, & ipſi æqualis, & in Directum O A, ipſi quoque æqualis A ω. Tum biſecta P δ in ξ, ducatur per ξ Recta ν ξ β, Normalis ad X B, concurrens cum δ ν, Normali ad O A, in ν, & in quam cadat Normalis ω ν; ac fiat ut ν ξ ad ξ β, ita hæc ad β λ: Erit λ Vertex, λ ξ Axis, ν ξ Latus Rectum Parabolæ, quæ in iiſdem cum priore punctis Circulum datum ſecabit.

To find the Principal Focus of Optick Glaſſes univerſally; by Mr. Edm. Halley. N. 205. p. 960.

Fig. 96.

X. Let B E β be a double Convex Lens, C the Center of the Segment E B, and K the Center of the Segment E β, B β the Thickneſs of the Lens, D a Point in the Axis of the Lens; and it is required to find the Point F, at which the Beams proceeding from the point D, are collected therein, the Ratio of Refraction being as *m* to *n*. Let the diſtance of the Object D B = D A = *d*, (the point A being ſuppoſed the ſame with B, but taken at a diſtance therefrom, to prevent the Coincidence of ſo many Lines) the Radius of the Segment towards the Object C B, or C A = *r*, and the Radius of the Segment from the Object K β, or K α = ρ, and let B β, the thickneſs of the Lens, be = *t*, and then let the Sine of the Angle of Incidence D A G, be to the Sine of the Refracted Angle H A G, or C A φ, as *m* to *n*; and in very ſmall Angles the Angles themſelves will be in the ſame Proportion; whence it will follow, that, As *d* to *r*, ſo the Angle at C to the Angle at D, and $d + r$ will be as the Angle of Incidence G A D; and again, as *m* to *n*, ſo $d + r$ to $\frac{dn + rn}{m}$, which will be as the Angle G A H = C A φ. This being taken from A C D, which is as *d*, will leave $\frac{m - nd - nr}{m}$ Analogous to the Angle A φ D, and the Sides being in this Caſe proportional to the Angles they ſubtend, it

will follow, that as the Angle A φ D, is to the Angle A D φ, ſo is the Side A D, or B D, to A φ, or B φ: That is, B φ will be $= \frac{mdr}{m - nd - nr}$, which ſhews in what Point the Beams proceeding from D would be collected by means of the firſt Refraction; but if nr cannot be ſubſtracted from $m - nd$, it follows, that the Beams after Refraction do ſtill paſs on Diverging, and the Point φ is on the ſame ſide of the Lens beyond D. But if nr be equal to $m - nd$, then they proceed Parallel to the Axis, and the Point φ is infinitely diſtant.

The Point φ being found as before, and B φ − B β being given, which we will call δ, it follows by a Proceſs like the former, that β F, or the Focal Diſtance ſought, is equal to $\frac{\delta \rho n}{m - n\delta + m\rho} = f$. And in the room of δ ſubſtituting B φ − B β $= \frac{mdr}{m - nd - nr} - t$, putting p for $\frac{n}{m - n}$, after due Reduction this Equation will ariſe, $\frac{mpdr\rho - nd\rho t + npr\,t}{mdr + md\rho - mpr\rho - m - ndt + nrt} = f$.

Which *Theorem*, however it may ſeem Operoſe, is not ſo, conſidering the great number of Data that enter the Queſtion, and that one half of the Terms ariſe from our taking in the thickneſs of the Lens, which in moſt Caſes can produce no great Effect; however, it was neceſſary to conſider it, to make our Rule perfect. If therefore the Lens conſiſt of Glaſs, whoſe Refraction is as 3 to 2, 'twill be $\frac{6dr\rho - 2d\rho t + 4r\rho t}{3dr + 3d\rho - 6r\rho - dt + 2rt} = f$. If of Water, whoſe Refraction is as 4 to 3, the *Theorem* will ſtand thus: $\frac{12dr\rho - 3d\rho t + 9r\rho t}{4dr + 4d\rho - 12r\rho - dt + 3rt} = f$. If it could be made of Diamant, whoſe Refraction is as 5 to 2, it would be

$$\frac{\frac{10}{3}dr\rho - 2d\rho t + \frac{4}{3}r\rho t}{5dr + 5d\rho - \frac{10}{3}r\rho - 3dt + 2rt} = f.$$

And this is the Univerſal Rule for the Foci of Double Convex Glaſſes expoſed to Diverging Rays. But if the thickneſs of the Lens be rejected as not ſenſible, the Rule will be much ſhorter, *viz.* $\frac{pdr\rho}{dr + d\rho - pr\rho} = f$; or in Glaſs, $\frac{2dr\rho}{dr + d\rho - 2r\rho} = f$. All the Terms wherein t is found being omitted, as equal to nothing. In this Caſe, if d be ſo ſmall, as that $2r\rho$ exceed $dr + d\rho$, then will it be $-f$, or the Focus will be Negative,

gative, which ſhew that the Beams after both Refractions ſtill proceed diverging.

To bring this to the other Caſes, as of Converging Beams, or of Concave Glaſſes, the Rule is ever compoſed of the ſame Terms, only changing the Signs of + and − ; for the diſtance of the Point of Concourſe of Converging Beams, from the Point B, or the firſt Surface of the Lens, I call a Negative Radius, or $-r$ if it be the firſt Surface, and $-\rho$ if it be the ſecond Surface. Let then Converging Beams fall on a double Convex of Glaſs, and the *Theorem* will ſtand thus $\frac{-2dr\rho}{-dr-d\rho-2r\rho}=+f$, which ſhews, that in this Caſe the Focus is always Affirmative.

If the Lens were a *Meniſcus* of Glaſs, expoſed to diverging Beams, the Rule is, $\frac{-2dr\rho}{-dr+d\rho+2r\rho}=f$. Which is Affirmative, when $2r\rho$ is leſs than $dr-d\rho$, otherwiſe Negative: But in the Caſe of Converging Beams falling on the ſame *Meniſcus*, 'twill be $\frac{+2dr\rho}{+dr-d\rho+2r\rho}=f$. And it will be $+f$ whilſt $d\rho-dr$ is leſs than $2r\rho$, but if it be greater than $2r\rho$, it will always be found Negative or $-f$. If the Lens be double Concave, the Focus of Converging Beams is Negative, where it was Affirmative in the Caſe of Diverging Beams on a double Convex, *viz.* $\frac{-2dr\rho}{+dr+dr-2r\rho}=f$. which is Affirmative only when $2r\rho$ exceeds $dr+d\rho$: But Diverging Beams paſſing a double Concave, have always a Negative Focus, *viz.* $\frac{2dr\rho}{+dr+d\rho+2r\rho}=-f$.

The *Theorems* for Converging Beams are principally of uſe to determine the Focus reſulting from any ſort of Lens placed in a Teleſcope, between the Focus of the Object-Glaſs and the Glaſs itſelf, the diſtance between the ſaid Focus of the Object-Glaſs and the interpoſed Lens being made $=-d$.

In caſe the Beams are Parallel, as coming from an infinite Diſtance, (which is ſuppoſed in the caſe of Teleſcopes) then will d be ſuppoſed infinite, and in the Theorem $\frac{pdr\rho}{dr+d\rho-pr\rho}$ the Term $pr\rho$ vaniſhes, as being finite, which is no part of the other infinite Terms, and dividing the Remainder by the infinite part d, the *Theorem* will ſtand thus $\frac{p\rho r}{r+\rho}=f$, or in Glaſs $\frac{2r\rho}{r+\rho}=f$.

In caſe the Lens were Plano-Convex expoſed to diverging Beams, inſtead of $\frac{p\,d\,r\,\rho}{d\,r + d\,\rho - p\,r\,\rho}$, r being infinite, it will be $\frac{p\,d\,\rho}{d - p\,\rho} = f$, or $\frac{2\,d\,r}{d - 2\,\rho} = f$, if the Lens be Glaſs

If the Lens be Double-Convex, and r be equal to ρ, as being formed of Segments of equal Spheres, then will $\frac{p\,d\,\rho\,r}{d\,r + d\,\rho - p\,r\,\rho}$ be reduced to $\frac{p\,d\,r}{2\,d - p\,r} = f$; and in caſe d be infinite, then it it will be yet farther contracted to $\frac{1}{2}\,p\,r$, and p being $= \frac{n}{m - n}$, the Focal Diſtance in Glaſs will be $= r$, in Water $1\frac{1}{2}\,r$, but in Diameter $\frac{1}{3}\,r$.

This is not only uſeful to diſcover the Focus from the other propoſed Data, but from the Focus given, we may thereby determine the Diſtance of the Object, or from the Focus and Diſtance given we may find of what Sphere it is requiſite to take another Segment, to make any given Segment of another Sphere caſt the Beams from the Diſtance d to the Focus f. As likewiſe from the Lens, Focus, and Diſtance given, to find the Ratio of Refraction, or of m to n, requiſite to anſwer thoſe Data. All which, it is obvious, are fully determined from the Equation we have hitherto uſed, *viz.* $p\,d\,\rho\,r = d\,r\,f + d\,\rho\,f - + p\,r\,\rho\,f$. For to find d, the *Theorem* is, $\frac{p\,r\,\rho\,f}{r\,f + \rho\,f - p\,\rho\,r} = d$, the Diſtance of the Object; for ρ, the Rule is $\frac{d\,r\,f}{p\,d\,r + d\,f + p\,r\,f} = \rho$; but for p, it will be $\frac{d\,r\,f + d\,\rho\,f}{d\,\rho\,r + f\,\rho\,r} = p$; which latter determines the Ratio of Refraction, m being to n as $1 + p$ to p.

I ſhall not expatiate on theſe Particulars, but leave them for the Exerciſe of thoſe that are deſirous to be informed in Optical Matters, which I am bold to ſay are comprehended in theſe three Rules, as fully as the moſt Inquiſitive can deſire them, and in all poſſible Caſes, Regard being had to the Signs + and −, as in the former Caſes of finding the Focus I ſhall only ſhew two conſiderable Uſes of them; the one to find the Diſtance whereat an Object being placed ſhall by a given Lens be repreſented in a Species as large as the Object itſelf, which may be of ſingular Uſe in drawing Faces, and other things in their true Magnitude, by tranſmitting the Species by a Glaſs into a dark Room, which will not only give the true Figure and Shades, but even the Colours themſelves, almoſt as Vivid as the Life. In this Caſe d is equal to f and ſubſtituting d for f in the Equation, we ſhall have $p\,d\,r\,\rho = d\,d\,r + d\,d\,\rho - d\,p\,\rho\,r$, and dividing all by d, $p\,r\,\rho = d\,r + d\,\rho - p\,r\,\rho$, that is,

$2\,p\,r\,\rho$

$\frac{2p r \varrho}{r+\varrho} = d$; but if the two Convexities be of the ſame Sphere, ſo as $r = \varrho$, then will the Diſtance be $= p\,r$, that is, if the Lens be Glaſs $= 2\,r$, ſo that if an Object be placed at the Diameter of the Sphere diſtant, in this Caſe the Focus will be as far within as the Object is without, and the Species repreſented thereby will be as big as the Life; but if it were a Plano-Convex, the ſame diſtance will be $= 2\,p\,r$, or in Glaſs to four times the Radius of the Convexity.

A ſecond Uſe is to find what Convexity or Concavity is required to make a vaſtly diſtant Object be repreſented at a given Focus, after the one Surface of the Lens is formed; which is but a *Corollary* to our *Theorem* for finding ϱ, having p, d, r, and f, given; for d being Infinite, the Rule becomes $\frac{r f}{p r - f} = \varrho$, that is in Glaſs $\frac{r f}{2 r - f} = \varrho$, whence, if f be greater than $2\,r$, ϱ becomes Negative, and $\frac{r f}{f - 2 r}$ is the Radius for the Concave ſought.

But to return to their firſt *Theorem*, which accounting for the Thickneſs of the Lens, we will here again reſume, *viz.*

$$\frac{m p d r \varrho - n d \varrho t - n p r \varrho t}{m d r + m d \varrho - m p \varrho r - m - n d t - n r t} = f.$$

And let it be required to find the Focus where a whole Sphere will collect the Beams proceeding from an Object at the diſtance d. Here t is equal to $2\,r$, and $r = \varrho$; and after due Reduction, the *Theorem* will ſtand thus, $\frac{m p d r - 2 n d r + 2 n p r r}{2 n d + 2 n r - m p r} = f$: But if d be Infinite, it is contracted to $\frac{m p r}{2 n} - r = \frac{2 n - m}{2 m - 2 n} r = f$; wherefore a Sphere of Glaſs collects the Sun's Beams at half the Semidiameter of the Sphere without it, and a Sphere of Water at a whole Semidiameter. But if the Ratio of Refraction m to n be as 2 to 1, the Focus falls on the oppoſite Surface of the Sphere, but if it be of greater Inequality, it falls within.

Another Example ſhall be when a Hemiſphere is expoſed to Parallel Rays, that is, d and ϱ being Infinite, and $t = r$, and after due Reduction, the *Theorem* reſults $\frac{n n}{m m - m n} r = f$; that is, in Glaſs it is at $\frac{4}{3}\,r$, in Water at $\frac{9}{4}\,r$, but if the Hemiſphere were Diamant, it would collect the Beams at $\frac{4}{15}$ of the Radius beyond the Center.

Laſtly, As to the Effect of turning the two ſides of a Lens towards an Object; it is Evident, that if the thickneſs of the Lens be very ſmall, ſo as that you neglect it, or account $t = 0$, then in all Caſes the Focus of the ſame Lens, to whatſoever Beams, will be the ſame, without any difference upon the turning the Lens: But if you are ſo curious to conſider the thickneſs, (which is ſeldom worth accounting for) in the caſe of Parallel Rays falling on a Plano-Convex of Glaſs, if the Plain-ſide be towards the Object, t does occaſion no difference, but the Focal diſtance $f = 2r$. But when the Convex ſide is towards the Object, it is contracted to $2r - \frac{2}{3}t$, ſo that the Focus is nearer by $\frac{2}{3}t$. If the Lens be Double Convex, the difference is leſs; if a Meniſcus, grearer. If the Convexity on both ſides be equal, the Focal Length is about $\frac{1}{6}t$ ſhorter than when $t = 0$. In a Meniſcus, the Concave ſide towards the Object encreaſes the Focal Length, but the Convex towards the Object diminiſhes it. A general Rule for the difference ariſing on turning the Lens, where the Focus is Affirmative, is this $\frac{2at - 2\rho t}{3r + 3 \quad - t}$, for double Convexes of different Spheres. But for *Meuiſci* the ſame difference becomes $\frac{2rt + 2\rho t}{3r - 3\rho + t}$; of which I need give no other Demonſtration, but that by a due Reduction it will ſo follow from what is premiſed, as will the *Theorems* for all ſorts of Problems relating to the Foci of Optick Glaſſes.

The Generation of an Hyperbolical Cylindroide, by Sir Chriſtopher Wren. N. 48. *p.* 961. Jun. An. 1669.

Fig. 97.

XI. 1. Sint Hyperbolæ Oppoſitæ D B, E C, quarum Axis Tranſverſus eſt B C, Centrum A, & una ex Aſymptotis G P; item per Centrum ſit O M ducta & Angulos Rectos ipſi B C. Quare ſi circumducantur Hyperbolæ circa Axin O M, manifeſtum eſt, ex ea Revolutione generari Corpus Cylindroides Hyperbolicum, cujus baſes, ſectioneſque Baſi parallelæ ſunt Circuli. Dico inſuper, ſi idem Corpus ſecetur per Aſymptoton G P, erit ſectio Parallelogrammum.

Secetur per Axin Tranſverſum ſectione Circulari B N C; item per O & M in Circulos æquales & æqualiter à Centro diſtantes; item per Axin in figuram Genetricem cujus ſemiſſis eſt B D E C, in cujus Plano erit Aſymptotos G P, per quam ad Rectos Angulos planum B D E ſecetur in Plano F H P; jungantur denique H O.

Quoniam Triangulum O G H eſt Rectangulum, Ergo quadratum O H, ſive O D, minus quadraro O G, eſt æquale quadrato G H: & quoniam D O parallela eſt ipſi B A, Aſymptoton ſecat in G, erit, (ex proprietatibus Hyperbolæ, quæ in Conicis demonſtrantur) quadratum O G una cum quadrato B A, æquale quadrato O D, *h. e.* Quadratum O D minus quadrato O G, æquale quadrato A B, ſive quadrato A N. Ergo quadratum G H, æquale eſt quadrato A N. Quare G H & A N æquantur, & ſunt ad Angulos Rectos ipſi G A; idemque demonſtratur de omnibus aliis ſectionibus Baſi parallelis. Quare Cylindroides Hyperbolica rite ſecatur per Aſymptoton in Parallelogrammum. *q. e. d.*

Carol.

Corol.] Hinc patet, in ſuperficie Cylindroidis, quamvis è duplici flexura conſtet, rectas nihilominus innumeras duci poſſe: Patet etiam, aliam eſſe hujus Corporis Generationem, nimir. ex revolutione Parallelogrammi circa Axin manente Angulo ad Axin æquali G A O, vel denique manente Linea Generatrice H R immobili, & maſſam Volubilem formante aut ſecante.

Et ſi acies Dolabri acutiſſima & rectiſſima ita diſponatur ad Axin, ſicut ſe habeat Linea Generatrix, rotante interim Mamphure, manifeſtum eſt, Torno tam accuratas poſſe elaborari Hyperbolas quam Circulos, cum nihil aliud requiratur ad formandam Cylindroidem quam ad Cylindrum, niſi quod in Cylindris acies Dolabri eſt Axi Parallela, hic vero inclinata.

Itaque notandum eſt, pro Inclinatione Anguli G A O, variari ſpeciem Hyperbolæ; adeoque facilius accommodatur ad datam Hyperbolam quam ut demonſtratione opus habeat: At ſi manente Angulo Generatrix magis ad Centrum accedat, exſurgit inde minor Hyperbola, ſed priori prorſus ſimilis.

The Application thereof to the Grinding of Hyperbolical Optick Glaſſes, by Sir Chriſt. Wren. N. 53. *p.* 1059. Nov. An. 1669. *Fig.* 98.

2. Sint tria Corpora terendo idonea, P, Q, R; quorum P & Q ſint æqualia & Columnari forma, R vero Corpus Lenti-forme. P rotetur circa Axin A B; Q, circa C D; & R, circa E G. Sint autem A B, & C D, in diverſis Planis, ita tamen ut E G producta ſit ad Rectos Angulos utrique A B & C D: accedant denique ad ſe invicem Corpora, prout opus fuerit, ſervata tamen eadem Inclinatione & ſitu Axium.

Dico ex Revolutione & mutua attritione Corporum prius poſitorum exſurgere nova Corpora Geometrica, quorum P & Q erunt Cylindroidea Hyperbolica æqualia, R vero Conoides Hyperbolicum, ſpecie & magnitudine datum.

Demonſtrationem in promptu habemus, nec non modulum ipſius Machinæ; terendis Lentibus Hyperbolicis deſtinatæ; quam operoſa Pictura & prolixa Explicatione deſcribere, mihi & Artifici magis fuerit moleſtum, quam Dædalo cuivis ſagaci ſimilem adinvenire. Poſtquam enim expoſita jam ſunt Principia Geometrica, facile erit conjicere, quale ſit Inſtrumentum; nempe, tres ſunt Tabulæ Oblongæ, Planæ, Validæ, Labiles, & ſibi invicem Impoſitæ: Infima & Media ſuſtinent inæqualia Capitula (ſive Anſas Mamphur ſuſtinentes) alternatim poſita; id poſtulat utriuſque Mamphuris Obliquitas & quaſi decuſſatio: Summæ Tabulæ æqualia ſunt Capitula in longum Tabulæ diſpoſita; & perforato citimo Capitulo Mamphur tranſmittitur. Omitto Rotas, Rotulas, Lora, Pondera, Cochleas, & reliqua ad motum expeditum & Machinæ Firmitudinem neceſſaria. P pertinet ad infimam Tabulam; Q ad mediam; R ad ſummam. R Lens eſt vitrea; Q Modulus Lentem terens; P Formula Modulum corrigens; quæ dum motu obliquo, & diverſo a motu tam Lentis quam Moduli, fertur, delet continuo & deterit, quicquid Vitii imprimitur in Modulum ex Lentis & Materiæ attritione.

Quare, cum adeo ſimplex & ſpontanea ſit iſta Hyperbolici Conoidis genitura, ex ſolis nempe Motibus Circularibus; cumque Motus ſit duplex & varius, credibile eſt, Lentes Hyperbolicas ex hiſce Principiis vel nullis fore explicandas.

Why four Convex Glaſſes in a Teleſcope, ſhew Objects Erect; by Mr. William Molineux. N. 183. p. 169. July An. 1686.

XII. This Phænomenon appears very eaſily explicable, from the Conſideration of placing Glaſſes in a Tube, which is thus; After the Object-Glaſs, the firſt Eye-glaſs is placed ſo much diſtant (towards the Eye) from the Focus of the Object-Glaſs, as is the Focus of the Eye-Glaſs; then the middle Eye-

Glaſs is placed ſo much diſtant from the Focus of the firſt Eye-Glaſs, as is the Focus of the middle Eye-Glaſs; Laſtly, the neareſt Eye-Glaſs is placed ſo much diſtant from the Focus of the middle Eye-Glaſs, as is the Focus of this neareſt Eye-Glaſs; and the Eye looking through them all is placed in the Focus of this neareſt Eye-Glaſs.

I ſay therefore, 1. That one ſingle Convex-Glaſs, cannot properly be ſaid by itſelf to ſhew Objects Erect or Reverſe, but in reſpect of the placing of the Eye that looks through it. For if the Eye, that looks through a ſingle Convex-Glaſs, be placed nigher thereto than the Glaſs's Focus, the Objects are Erect; if the Eye be placed juſt in the Focus, the Objects are neither Erect nor Reverſed, but all in Confuſion between both; and if the Eye be placed further from the Glaſs than the Focus, the Objects are Reverſed. I mean here diſtant Objects, the Rays flowing from any Point whereof may be counted to come Parallel towards the Object-Glaſs.

2. The Object-Glaſs of a Teleſcope reverſes the Object, both to the Eye-Glaſs and the Eye that looks through it: For the Eye-Glaſs is placed farther from the Object-Glaſs than is the Focus of the Object-Glaſs. But the Eye-Glaſs does nothing towards the Rectification, or Reverſion; the Eye being placed juſt in its Focus.

3. If the ſecond Eye-Glaſs (the firſt being that next the Object-Glaſs) be placed, as it ought, in a Teleſcope, place the Eye nearer in this middle Eye-Glaſs than its Focus, and it ſees the Object inverted and confuſed: Place the Eye in the Focus, and it ſees the Objects all in Confuſion, neither Erect nor Reverſed; for here again there is a diſtinct Repreſentation of the Objects to be received on a piece of Paper, as in the Focus of the Object-Glaſs, and the Eye being placed at any time at this place (which is uſually call'd the diſtinct Baſe) ſees all in Confuſion: But then let the Eye be placed farther from this middle Glaſs than its Focus, and it perceives the Objects Erect and Confuſed.

Laſtly, The third, or immediate Eye-Glaſs, does nothing towards the Erecting or Reverſing the Species, which it receives Erect from the middle Eye-Glaſs; no more than in a Teleſcope of two Convex-Glaſſes, the Eye-Glaſs does to the Species it receives from the Object-Glaſs, as we have ſhewn before. The Reaſon that this laſt or immediate Eye-Glaſs has nothing to do in the Erecting or Reverſing the Species is the ſame, as in the Teleſcope of two Convex Glaſſes, *viz.* The Eye placed in its Focus, and therefore ſees the Species as 'tis repreſented in the diſtinct Baſe; that is, the Species is inverted in the diſtinct Baſe of the Object-Claſs, and therefore a ſingle Convex Eye-Glaſs brings it to the Eye Inverted; but in the diſtinct Baſe of the middle or ſecond Eye-Glaſs the Species is Erect, and therefore the third or immediate Eye-Glaſs brings it to the Eye Erect.

Wherefore we are to conſider the Teleſcope conſiſting of an Object-Glaſs and three Eye-Glaſſes, as two Teleſcopes, each conſiſting of two Convex Glaſſes. The firſt conſiſts of the Object-Glaſs, and firſt Eye-Glaſs, and this inverts the Species; that is, the Species is Inverted in the diſtinct Baſe of the Object-Glaſs, and ſo brought into the Eye. The ſecond Teleſcope conſiſts of two immediate Eye-Glaſſes, and this Erects what the former Inverted; that is, the Species in the diſtinct Baſe of the middle Eye-Glaſs, is

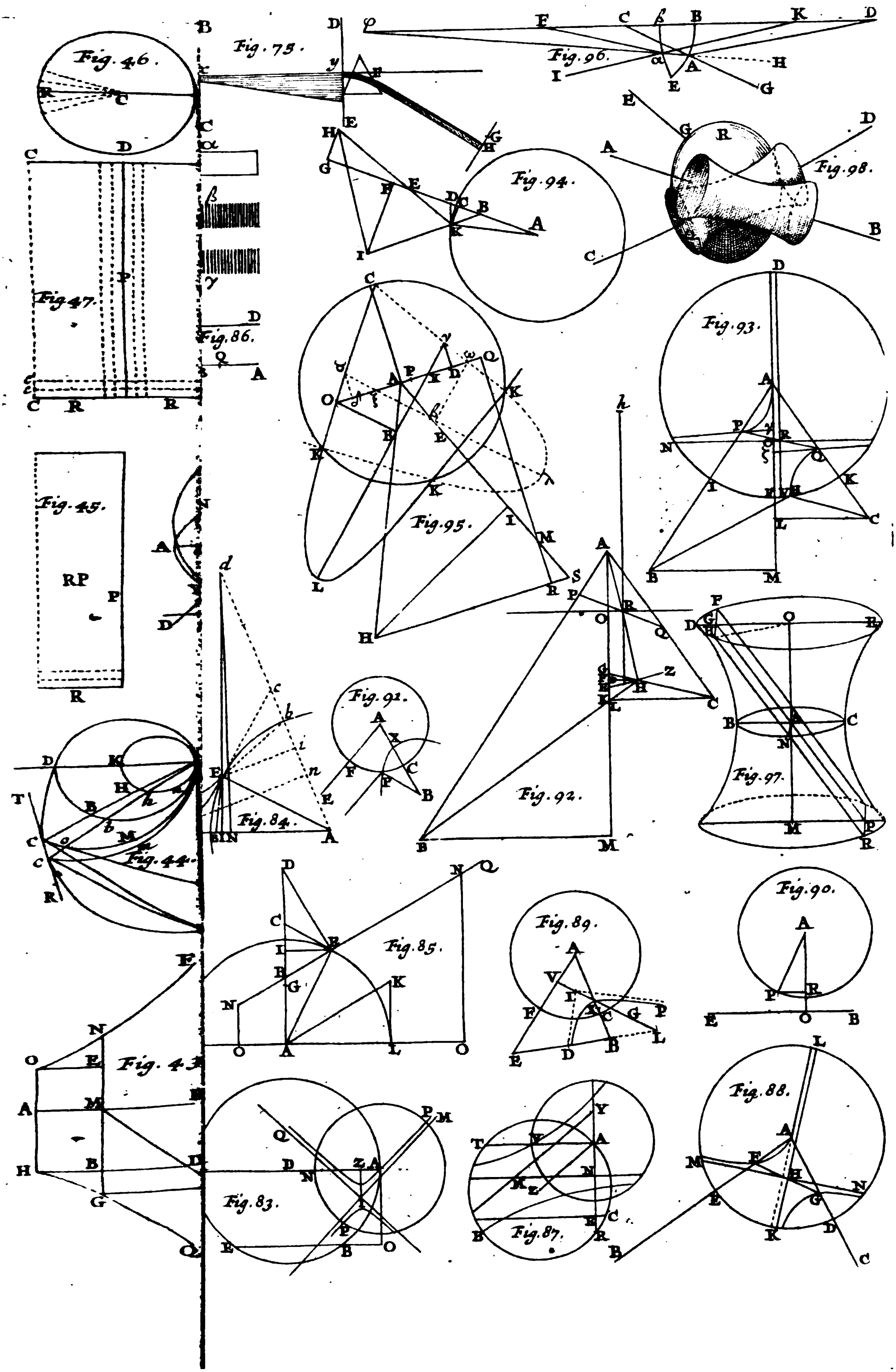


Fig. 46.
R
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C
D
B
C
Fig. 47.
C
D
P
c
e
C
R
R
Fig. 45.
RP
P
R
N
A
Y
D
Fig. 44.
D
K
H
h
B
b
M
T
C
c
o
R
Fig. 43.
F
N
O
E
A
M
H
B
D
G
Q
Fig. 75.
x
y
D
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E
G
H
α
β
γ
Fig. 86.
D
Q
A
Fig. 84.
d
c
b
i
n
E
B
I
N
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Fig. 96.
φ
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K
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Fig. 94.
H
G
F
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C
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A
K
I
C
Fig. 98.
E
G
R
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A
B
C
Fig. 95.
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Q
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Fig. 93.
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Fig. 92.
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Fig. 91.
A
X
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Fig. 97.
F
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Fig. 85.
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O
Fig. 89.
A
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P
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D
B
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Fig. 90.
A
P
R
E
O
B
Fig. 83.
Q
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M
D
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Z
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Fig. 87.
Y
T
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C
B
R
Fig. 88.
L
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B
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Erect, and is ſo brought into the Eye by the Eye-Glaſs; the Eye-Glaſſes themſelves in neither Caſe having any thing to do with the Erecting or Inverting, but merely in repreſenting the ſame Poſture the Species immediately before them. So that one Convex-Glaſs, as poſited in a Teleſcope, Inverts; the ſecond (that is the firſt Eye-Glaſs) does nothing towards the Erecting or Reverſing, but repreſents the Image as it is in the diſtinct Baſe of the Object-Glaſs before it, that is Inverted; the third Glaſs Erects, or rather Reſtores what was before Inverted; the fourth repreſents the Image as it receives it from the diſtinct Baſe of the third, that is, Erect.

The Apertures of Teleſcopes. By M. Auzout. N. 4. p. 55. June. An. 1665.

XIII. 1. Mr. *Auzout* has found that the Apertures, which Optick-Glaſſes can bear with Diſtinctneſs, are in about a ſubduplicate Proportion to their Lengths: And acccordingly he hath made the following Table.

Lengths of Glaſſes.		*Apertures for*						*Lengths of Glaſſes.*	*Apertures for*					
		Excellent		*Good.*		*Ordinary.*			*Excellent.*		*Good.*		*Ordinary.*	
Feet	*In.*	*In.*	*Lin.*	*In.*	*Lin.*	*In.*	*Lin.*	*Feet. In.*	*In.*	*Lin.*	*In.*	*Lin.*	*In.*	*Lin.*
	4		4		4		3	25	3	4	2	10	2	4
	6		5		5		4	30	3	8	3	2	2	7
	9		7		6		5	35	4	0	3	4	2	10
1	0		8		7		6	40	4	3	3	7	3	1
1	6		9		8		7	45	4	6	3	10	3	2
2	0		11		10		8	50	4	9	4	0	3	4
2	6	1	0		11		9	55	5	0	4	3	3	6
3	0	1	1	1	0		10	60	5	2	4	6	3	8
3	6	1	2	1	1		11	65	5	4	4	8	3	20
4	0	1	4	1	2	1	0	70	5	7	4	10	4	0
4	6	1	5	1	3	1	1	75	5	9	5	0	4	2
5	0	1	6	1	4	1	1	80	5	11	5	2	4	5
6		1	7	1	5	1	2	90	6	4	5	6	4	7
7		1	9	1	6	1	3	100	6	8	5	9	4	10
8		1	10	1	8	1	4	120	7	5	6	5	5	3
9		1	11	1	9	1	5	150	8	0	7	0	5	11
10		2	1	1	10	1	6	200	9	6	8	0	6	9
12		2	4	2	0	1	8	250	10	6	9	2	7	8
14		2	6	2	2	1	9	300	11	6	10	0	8	5
16		2	8	2	4	1	11	350	12	6	10	9	9	0
18		2	10	2	6	2	1	400	13	4	11	5	9	8
20		3	0	2	7	2	2							

Conſider'd by Dr. Hook. *ib. p.* 67.

This Theory of Apertures ſeems to me not very clear. For the ſame Glaſs will endure greater or leſſer Apertures, according to the leſſer or greater Light of the Object: If it be for looking on the Sun and *Venus*, or for ſeeing the Diameters of the fixed Stars, then ſmaller Apertures do better; if for the Moon in the Day-light, or on *Saturn*, or *Jupiter*, or *Mars*, then the largeſt. Thus I have often made uſe of a 12 Foot Glaſs to look on *Saturn* with an Aperture of almoſt 3 Inches, and with a ſingle Eye-Glaſs of 2 inches double Convex; but, when with the ſame Glaſs I looked on the Sun or *Venus*, I uſed both a ſmaller Aperture, and ſhallower Charge.

To meaſure Diſtances at one Station; by M. Auzout. N. 7. p. 125. Dec. An. 1665.

XIV. I have found long ſince a Way to meaſure, with a great Teleſcope, the Diſtance of Objects upon the Earth from one Station. The Practice indeed does not altogether anſwer the Theory, becauſe that the length of the Teleſcopes admits of ſome Latitude; yet one comes near enough, and perhaps as juſt as by moſt of the ways ordinarily uſed with Inſtruments. That which I am propoſing, I doubt not but Mr. *Hook* will ſoon underſtand, and ſee the Determination of all Caſes poſſible. I ſhall only ſay, That if we look upon the ſole Theory, we may make uſe of an ordinary Teleſcope, whereof the Eye-Glaſs is to be Convex: For by putting the Glaſſes at a little greater Diſtance, than they are, proportionably to the Diſtance for which it is to ſerve, and by adding to it a new Eye-Glaſs, the Object will be ſeen diſtinct, though Obſcure; and if the Eye-Glaſs be Conxex, the Object will appear Erect. They may be done two manner of ways; either by leaving the Teleſcope in its ordinary Situation, the Object-Glaſs before the Eye-Glaſs; or by inverting it, and putting this before that. But if any will make uſe of two Object-Glaſſes, whereof the Focus's are known, the Diſtances of them will be known. If it be ſuppoſed, that the Focus of the firſt be B, and that of the ſecond C, and the Diſtance given, $B + 2\,D$, and that $D - C$ be equal to F; for this Diſtance will be equal to $B + C + F - r\, F^{2}\, C^{2}$. And if you have the Focus of the firſt Object-Glaſs, equal to B, the Diſtance where you will put the ſecond Glaſs equal to $B + C + D$, the Focus of the ſecond Glaſs will be found equal to $\frac{C\,D}{C + D}$. And if you will that the Object ſhall be magnified as much with theſe two Glaſſes, as it would be with a ſingle one, whereof the Focus ſhould be of the diſtance given, having the Focus of the Object-Glaſs given equal to B, and the diſtance given to $B + D$; the diſtance between the firſt and ſecond Glaſs will be equal to $\frac{2\,B^{2} + 2\,B\,D}{2\,B + D}$, whence ſubducting B (the Focus of the Object given) there remains $\frac{B\,D}{2\,B + D}$: and if this Snm be compoſed equal to C, we ſhall eaſily know by the precedent Rule, the Focus of the ſecond Glaſs.

XV. Prepare two Glaſſes, the one exactly flat on both ſides, the other flat on the one ſide, and convex on the other, of what Sphere you pleaſe. Let the flat Glaſs be a little broader than the other. Then let there be made a Cell or Ring of Braſs, very exactly turned, into which theſe two Glaſſes may be ſo faſtned with Cement, that the plain Surfaces of them may lie exactly parallel, and that the Convex ſide of the Plano-convex Glaſs may lie inward; but ſo, as not to touch the Flat of the other Glaſs. Theſe being cemented into the Ring very cloſely about the Edges, by a ſmall Hole in the ſide of the Braſs Ring or Cell, fill the interpoſed Space between theſe two with Water, Oyl of Turpentine, Spirit of Wine, Saline Liquors, *&c.* then ſtop the Hole with a Skrew: And according to the differing Refraction of the interpoſed Liquors, ſo ſhall the Focus of this Compound Glaſs be longer or ſhorter.

To make a Plano-convex Glaſs of a ſmall Sphere, collect the Rays at a great Diſtance; by Dr Hook.
N. 4. *p.* 66.
N. 12. *p.* 202.
June, An. 1665.
May, An. 1666.

But this I would have only look'd upon as one Inſtance of many (for there may be others) of the Poſſibility of making a Glaſs, ground in a ſmaller Sphere, to conſtitute a Teleſcope of a much greater Length: Though (not to raiſe too great Expectation) I muſt add, That, of Spherical Object-Glaſſes, thoſe are the beſt which are made of the greateſt Sphere, and whoſe Subſtance hath the greateſt Refraction.

XVI. 1. *S. Campani* pretends to have found a Way to work great *Optick-Glaſſes* with a Turn-tool, without any Mould: And that he uſeth three Eye-Glaſſes for his great Teleſcopes, without finding any Rainbow Colours.

The Great Duke of *Tuſcany*, and Prince *Leopold* his Brother, upon Tryal made of the Glaſſes of *Campani* and *Divini*, have found that thoſe of *Campani* excel the other; and with them they have been eaſily able to diſtinguiſh People at 4 Leagues diſtance.

Teleſcopes and other Optick-Glaſſes; by Campani *and* Divini.
N. 1. *p.* 2.
Mar. An. 1665.
N. 8. *p.* 131.
Jan. An. 1666.

But *Euſtachio Divini* pretends, that in all the Tryals made with them, his great Glaſſes have performed better than thoſe of *Campani*; and that *Campani* was not willing to do what was neceſſary for well comparing one with the other, *viz.* to put equal Eye-Glaſſes in them, or to exchange the ſame Glaſſes.

N. 12. *p.* 209.

2. 'Tis now above ten Years ſince I invented a peculiar Way of Grinding Optick-Glaſſes, and reduced it alſo into Practice; by which 'tis eaſy, without any conſiderable Danger of failing, to make and poliſh Optick-Glaſſes of any *Conick Section*, and that (which is moſt notable) in any Diſh of any Section of a Sphere. I have already made ſeveral Glaſſes by it, which many Learned Men have ſeen and try'd.

*By M.*Hevelius.
N. 6. *p.* 93.
Nov. An. 1665.

M. *Huygens* alſo intends very ſhortly to try ſomething in that kind.

*By M.*Huygens.
Ibid.

3. M. *du Sons* doth at preſent employ himſelf in *London*, to bring Teleſcopes to Perfection, by grinding Glaſſes of a *Parabolical* Figure. I have ſeen two Eye-Glaſſes of that Shape, about one Inch and a half deep, and one Inch and a quarter broad, wrought by this *eminent Artiſt* with a rare Steel Inſtrument of his own Contrivance and Workmanſhip, and by himſelf alſo poliſh'd to Admiration. And certainly it will be wondred at by thoſe, who ſhall ſee theſe Glaſſes, how they could be truly wrought to ſuch a Figure, with

By M. du Sons.
Ibid. p. 99.
N. 7. *p.* 119.
Dec. An. 1665.

ſuch a Cavity; and yet more, when they ſhall hear the Author undertake to excavate other ſuch Eye-Glaſſes to above 2 Inches, and Object-Glaſſes of 5 Inches Diameter. He hath likewiſe already begun his Object-Glaſſes for the mentioned two ocular ones, of the ſame Figure of about 2 Inches Diameter, which are to be left all open, yet without cauſing any Colours.

By M. Burattini. N. 19. *p.* 348. N. 21. *p.* 374. 4. The Optick-Glaſſes of M. *Burattini* in *Poland*, are perfectly well wrought and poliſh'd. He hath ſent two to *Paris*, but they are only the one of 10, the other of 8 Foot. They bear a great Aperture in reſpect of their Length.

By Mr. Francis Smethwick. N. 33. *p.* 631. 5. Mr. *Fr. Smethwick* having found a Way of grinding Glaſſes *not Spherical*, produced before the Royal Society *(Feb.* 27. 166$\frac{7}{8}$.*)* certain *Specimina* of that Invention; which were, a Teleſcope, a Reading, and two Burning-Glaſſes.

The Teleſcope was about 4 Foot long, furniſhed with four Glaſſes, whereof the three ocular ones, Plano-convex, were of this newly invented *not Spherical* Figure, and the fourth a Spherical Object-Glaſs. This being compared with a common, yet very good Teleſcope, longer than it by about 4 Inches, and turn'd to ſeveral Objects, was found, by thoſe of the ſaid Society that look'd through them both, to exceed the other in Goodneſs, by taking in a greater Angle, and repreſenting the Objects more exactly in their reſpective Proportions, and enduring a greater Aperture free from Colours.

The Reading-Glaſs of the ſame Figure being compared with a common Spherical Glaſs did far excel it, by magnifying the Letters to which it was applied up to the very Edges, and by ſhewing them diſtinctly from one Brim through the Center to the other, which the Spherical Glaſs came far ſhort of.

Laſtly, The two Burning Concaves of this new-invented Figure, were the one of 6 Inches Diameter, its Focus 3 Inches diſtant from the Center thereof; the other of the ſame Diameter, but leſs Concave, and its Focus 10 Inches diſtant. Theſe, when approached to a large Candle lighted, did ſomewhat warm the Faces of thoſe that were 4 or 5 Foot diſtant at leaſt; and when held to the Fire, burned Gloves and Garments at the Diſtance of about 3 Foot from the Fire.

The Biſhop of *Salisbury*, Dr. *Seth Ward*, was by at another time, when the deeper of the two Concaves turned a Piece of Wood into Flame in the Space of 10 Sec. of Time, and the ſhallower in 5 Sec. at moſt, in the Seaſon of *Autumn*, about Nine of the Clock in the Morning, the Weather gloomy. The Inventor adds, That the deeper Concave, when held to a lucid Body, would caſt a Light ſtrong enough to read by at a conſiderable Diſtance; and that expoſing the ſame to a Northern Window, on which the Sun ſhined not at all, or very little, he had perceived that it would warm one's Hand ſenſibly, by collecting the warmed Air in the Day-time, which it would not do after Sun-ſet.

By an Artiſt at Paris. N. 40. *p.* 795. Oct. An. 1668. 6. We have an Artiſt at *Paris* that poliſhes Optick-Glaſſes on a Turn. I have ſeen a Glaſs of his Workmanſhip which is very good. He turns theſe Glaſſes as he does Wood, that is with the ſame Facility.

7. M. *Ba-*

By M. Borelli. N. 128. *p.* 691. Aug An. 1676.

7. M. *Borelli* hath found out a ſure and very eaſy Method to work all ſorts of great Glaſſes. He hath already made one of them very good of 200 Foot, wrought on both ſides on the ſame Rule. His Deſire of advancing Aſtronomical Diſcoveries hath induced him to make Preſents of them to ſeveral Perſons capable to make uſe of them. He hath intruſted the Secret to one of the *Royal Academy* of *Sciences.*

N. 140. *p.* 1005. July, An. 1678.

Campani and *Divini* have commonly ſold their Glaſſes at a Piſtole the Foot. Sometimes they have far exceeded that Price. One of *Divini*'s of 12 Foot was ſold for 400 Livres; and another of *Campani*'s, of 34 Foot, for 2000 Livres. Notwithſtanding which, *S. Borelli* is willing to part with the beſt of his own Glaſſes, of 50, 60, or 65 Foot, for 500 *(French)* Crowns; and the ſmall Glaſſes, from 6 to 12 Foot, at a *(French)* Crown a Foot; from 12 to 18, at half a Piſtole; and from 18 to 26, at a Piſtole.

Optick Lens of Rock Cryſtal; *by* Euſt. Divini. N. 20. *p.* 362. Dec. An. 1666.

8. Though it be commonly believed, that *Rock Cryſtal* is not fit for Optick-Glaſſes, becauſe there are many Veins in it; yet *Euſtachio Divini* made one of it, which he ſaid proved an excellent one, though full of Veins: But perhaps they were only ſuperficial Strictures and ſlight Scratches, not Veins.

Of Water; *by Mr.* Stephen Gray. N. 228. *p.* 539. May, An. 1697.

9. Drops of fair Water being let fall on a Piece of plain Glaſs, from themſelves into Plano-convexes, having a Convexity proportionable to the Heights from which they deſcend; from a greater Height a leſs, from a leſs a greater Degree of Convexity. I applied ſome of theſe as Reading-Glaſſes for ſingle Words of ſmall Letters, as on the Globes and Maps, and found no other Inconveniency, than that the Fluidity of the Water obliges one to keep the Glaſs Horizontal, which I after deviſed a Way to remedy. I took a ſufficient Quantity of Izing-Glaſs, and diſſolved it in Water over the Fire, and whilſt it was warm I dipt a Stick into the Solution, and let ſome Drops of it fall on the Glaſs as before; and in a quarter of an Hour they acquire a Conſiſtency, that permits them to be held in any Poſition, and though they are not altogether ſo tranſparent, yet this is little or no Impediment to their Uſe. The Drops of this Solution are more exactly defined than thoſe of common Water, having their Edges exactly circular, and one may make them of a much longer Focus than thoſe.

A thin flat Ring of Braſs, not exceeding 4 Tenths of an Inch Diameter in its interior Circle, being cemented to a plain Piece of Glaſs, and filled with Water, or the Solution now mentioned, then by preſſing the Finger into it, till what is ſuperfluous be taken off, there will be formed a Plano-concave, which may ſerve as an Eye-Glaſs to a Perſpective, or to any other optical Uſe Concave-Glaſſes are applicable.

I have tried what would be the Succeſs of combining Portions of Water by the help of Braſs Rings, and plain Pieces of Glaſs, to give them their true Figure and requiſite Apertures, and inſerted them at the Ends of Tubes of ſeveral Lengths; and find, that though theſe *Natural Lentes* may ſerve as Eye-Glaſſes, yet when uſed as Object ones, either to Teleſcopes, or double Microſcopes, the Effects will not compenſate the Trouble there is in uſing them.

The Advantages of Reflection to Optick Inſtruments; *by* Mr. Newton. N. 80. *p.* 3079. Feb. An. 1672.

XVII. 1. When I had found, that *Light conſiſts of Rays differently refrangible*, I left off my Glaſs-works; for I ſaw, that the Perfection of Teleſcopes was hitherto limited, not ſo much for want of Glaſſes truly figured according to the Preſcriptions of Optick Authors, (which all Men have hitherto imagined) as becauſe that *Light* itſelf is an heterogeneous Mixture of *differently Refrangible Rays*: So that were a Glaſs ſo exactly figured as to collect any one ſort of Rays into one Point, it could not collect thoſe alſo into the ſame Point, which having the ſame Incidence upon the ſame Medium, are apt to ſuffer a different Refraction. Nay, I wondred, that ſeeing the Difference of Refrangibility was ſo great as I found it, Teleſcopes ſhould arrive to that Perfection they are now at: For, meaſuring the Refractions in one of my Priſms, I found, that ſuppoſing the common Sine of Incidence upon one of its Plain was 44 Parts, the Sine of Refraction of the utmoſt Rays on the red End of the Colours, made out of the Glaſs into the Air, would be 68 Parts, and the Sine of Refraction of the utmoſt Rays on the other End 69 Parts; ſo that the Difference is about a 24th or 25th Part of the whole Refraction. And conſequently, the Object-Glaſs of any Teleſcope cannot collect all the Rays, which come from one Point of an Object, ſo as to make them convene at its Focus in leſs room than in a circular Space, whoſe Diameter is the 50th Part of the Diameter of its Aperture; which is an Irregularity, ſome hundreds of times greater than a circularly figured *Lens*, of ſo ſmall a Section as the Object-Glaſſes of long Teleſcopes are, would cauſe by the Unfitneſs of its Figure, were Light uniform.

This made me take Reflections into Conſideration; and finding them regular, ſo that the Angle of Reflection of all ſorts of Rays was equal to their Angle of Incidence, I underſtood that by their Mediation Optick Inſtruments might be brought to any Degree of Perfection imaginable, provided a reflecting Subſtance could be found, which would poliſh as finely as Glaſs, and reflect as much Light as Glaſs tranſmits, and the Art of communicating to it a parabolick Figure be alſo attained. But there ſeemed very great Difficulties, and I have almoſt thought them inſuperable, when I farther conſider'd that every Irregularity in a reflecting Superficies makes the Rays ſtray 5 or 6 times more out of their due Courſe, than the like Irregularities in a refracting one: So that a much greater Curioſity would be here requiſite, than in figuring Glaſſes for Refraction.

Amidſt theſe Thoughts I was forced from *Cambridge*, *Anno* 1666. by the intervening Plague, and it was more than two Years before I proceeded further. But then having thought on a tender Way of Poliſhing, proper for Metal, whereby, as I imagined, the Figure would be corrected to the laſt, I began to try what might be effected in this kind, and by degrees ſo far perfected an Inſtrument (in the eſſential Parts of it like that I ſent to *London*) by which I could diſcern *Jupiter*'s four Concomitants, and ſhew'd them divers times to two others of my Acquaintance. I could alſo diſcern the Moon-like *Phaſe* of *Venus*, but not very diſtinctly, nor without ſome Niceneſs in diſpoſing the Inſtrument.

From that time I was interrupted till this laſt *Autumn*, when I made another. And as that was ſenſibly better than the firſt, (eſpecially for Day-Objects) ſo I doubt not, but they will be ſtill brought to a much greater Perfection by their Endeavours, who, as you inform me, are taking care about it at *London*.

2. This new Inſtrument is compoſed of two Metalline Speculums, the one Concave (inſtead of an Object-Glaſs) the other Plain: And alſo of a ſmall Plano-convex Eye-Glaſs; as in the Figure, where A B is a Concave Speculum, of which the Radius or Semidiameter is $12\frac{2}{3}$ or 13 Inches.

A new Catadioptrical Teleſcope invented by Mr. Newton.
N. 81. *p.* 4004.
Mar. An. 1672.
Fig. 99.

C D, another Metalline Speculum, whoſe Surface is flat, and the Circumference oval.

G D, an Iron Wire, holding a Ring of Braſs, in which the Speculum C D is fixed.

F, a ſmall Eye-Glaſs, flat above, and convex below, of the 12th Part of an Inch Radius, if not leſs.

G G G, the fore Part of the Tube (which is open) faſtned to a Braſs Ring H I, to keep it immoveable.

P Q K L, the hinder Part of the Tube, faſtned to another Braſs Ring, P Q.

O, an Iron Hook faſtned to the Ring P Q, and furniſh'd with a Skrew N, thereby to advance or draw back the hinder Part of the Tube, and ſo by that means to put the Specula in their due Diſtance.

M Q G I, a crooked Iron ſuſtaining the Tube, and faſtned by the Nail R to the Ball and Socket S, whereby the Tube may be turned every Way.

The Center of the flat Speculum C D, muſt be placed in the ſame Point of the Tube's Axe, where falls the Perpendicular to this Axe, drawn to the ſame from the Center of the little Eye-Glaſs, which Point is here marked at T.

And to give the Reader ſome Satisfaction to underſtand, in what Degree it repreſents Things diſtinct, and free from Colours, and to know the Aperture by which it admits Light, he may compare the Diſtances of the Focus E from the Vertexes of the little Eye-Glaſs and the Concave Speculum; that is, E F, $\frac{1}{6}$ of an Inch, and E T V, $6\frac{1}{3}$ Inches, and the Ratio will be found as 1 to 38; whereby it appears, that the Objects will be magnified about 38 times; and be repreſented bigger by $2\frac{1}{2}$ times in Diameter, when ſeen thro' this, than thro' an ordinary Teleſcope of about 2 Foot long.

Thus far as to the Structure of this Teleſcope. Concerning the metalline Matter, fit for theſe Reflecting Speculums, the Inventor hath alſo conſider'd the ſame, and gives this Caution, That whilſt Men ſeek for a white, hard, and durable metalline Compoſition, they reſolve not upon ſuch an one as is full of ſmall Pores, only diſcoverable by a Microſcope: For tho' ſuch an one may, to Appearance, take a good Poliſh, yet the Edges of thoſe ſmall Pores will wear away faſter in the Poliſhing, than the other Parts of the Metal; and ſo, however the Metal ſeem polite, yet it ſhall not reflect with ſuch an accurate Regularity as it ought to do. Thus Tin-Glaſs mixt with ordinary Bell-Metal makes it more white, and apt to reflect a greater Quantity of

Light; but withal, its Fumes raiſed in the Fuſion, like ſo many aerial Bubbles, fill the Metal full of the microſcopical Pores. But white Arſenick both blanches the Metal, and leaves it ſolid, without any ſuch Pores, eſpecially if the Fuſion hath not been too violent. What the Stellate-Regulus of *Mars*, (which I have ſometimes uſed) or rather ſuch like Subſtance will do, deſerves particular Examination.

To this he adds this further Intimation, That Putty, or other ſuch like Powder, with which 'tis poliſh'd, by the ſharp Angles of its Particles, fretteth the Metal, if it be not very fine, and fills it full of ſuch ſmall Holes as he ſpeaketh of. Wherefore Care muſt be taken of that before Judgment be given, whether the Metal be throughout the Body of it porous or not.

But not having tried, as he ſaith, many Proportions of the Arſenick and Metal, he does not affirm which is abſolutely beſt, but thinks there may conveniently be uſed any Quantity of Arſenick equalling in Weight between a ſixth and an eighth Part of the Copper, a greater Proportion making the Metal brittle.

The Way which he uſed was this. He firſt melted the Copper alone, then put in the Arſenick, which being melted, he ſtirred them a little together, bewaring, in the mean time, not to draw in Breath near the pernicious Fumes. After this he put in Tin; and again, ſo ſoon as that was melted, (which was very ſuddenly) he ſtirred them well together, and immediately poured them off.

He ſaith, He knows not, whether by letting them ſtand longer on the Fire after the Tin was melted, a higher Degree of Fuſion would have made the Metal porous; but he thought that Way he proceeded to be the ſafeſt.

He adds, That in that Metal, which he ſent to *London*, there was no Arſenick, but a ſmall Proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much harm in making the Metal ſoft, and ſo leſs fit to be poliſh'd, as good in rendring it white and luminous.

At another time he mixed Arſenick one Ounce, Copper ſix Ounces, and Tin two Ounces; and this an Acquaintance of his hath, as he intimates, poliſh'd better than he did the other.

As to the Objection, That with this kind of Perſpectives, Objects are difficultly found; he anſwers, That that is the Inconvenience of all Tubes that magnify much; and that after a little Uſe, the Inconvenience will grow leſs, ſeeing that himſelf could readily enough find any Day Objects by knowing which Way they were poſited from other Objects that he accidentally ſaw in it. But in the Night to find Stars, he acknowledges it to be more troubleſome; which yet may, in his Opinion, be eaſily remedied by two Sights affixed to the Iron Rod, by which the Tube is ſuſtained, or by an ordinary Perſpective-Glaſs faſtned to the ſame Frame with the Tube, and directed towards the ſame Object, as *Des Cartes* in his *Dioptricks* hath deſcribed for remedying the ſame Inconvenience of his beſt Teleſcopes.

3. I

Approved by M. Hugens de Zulichem. *Ibid. p.* 4008.

3. I ſee by the Deſcription you have ſent me of Mr. *Newton*'s admirable Teleſcope, that he hath well conſidered the Advantage which a Concave Speculum hath above Convex-Glaſſes in collecting the parallel Rays; which certainly, according to the Calculation I have made thereof, is very great. Hence it is, that he can give a far greater Aperture to that Speculum, than to an Object-Glaſs of the ſame Diſtance of the Focus; and conſequently, that he can much more magnify Objects this Way, than by an ordinary Teleſcope. Beſides, by it he avoids an Inconvenience, which is inſeparable from Convex Object-Glaſſes, which is the Obliquity of both their Surfaces, which vitiateth the Refraction of the Rays that paſs towards the Sides of the Glaſs, and does more hurt than Men are aware of. Again, by the meer Reflection of the metalline Speculum there are not ſo many Rays loſt as in Glaſſes, which reflect a conſiderable Quantity by each of their Surfaces, and beſides intercept many of them by the Obſcurity of their Matter.

Mean time the main Buſineſs will be, to find a Matter for this Speculum that will bear ſo good and even a Poliſh as Glaſſes, and a Way of giving this Poliſh without vitiating the Spherical Figure. Hitherto I have found no Specula that had near ſo good a Poliſh as Glaſs: And if Mr. *Newton* hath not already found a Way to make it better than ordinarily, I apprehend his Teleſcope will not ſo well diſtinguiſh Objects as thoſe with Glaſſes. But 'tis worth while to ſearch for a Remedy to this Inconveniency, and I deſpair not of finding one. I believe that Mr. *Newton* hath not been without conſidering the Advantage, which a Parabolical Speculum would have above a Spherical one in this Conſtruction; but that he deſpairs, as well as I do, of working other Surfaces than Spherical ones with due Exactneſs; tho' elſe it be more eaſy to make a Parabolical, than Eliptical or Hyperbolical ones, by reaſon of a certain Propriety of the Parabolick Conoid, which is, that all the Sections parallel to the Axis make the ſame Parabola.

Ibid. p. 4009.

But though Mr. *Newton* (with M. *Hugens*) deſpairs of performing that work by Geometrical Rules, yet he doubts not but that the thing may in ſome meaſure be accompliſh'd by mechanical Devices.

A further Account of this Inſtrument; by Mr. Newton. *Ibid.*

4. In my laſt Letter I gave you occaſion to ſuſpect that the Inſtrument which I ſent you is in ſome reſpect or other indiſpoſed, or that the Metals are tarniſhed; and by yours I am fully confirm'd in that Opinion. For, whilſt I had it, it repreſented the Moon in ſome Parts of it as diſtinctly as other Teleſcopes uſually do which magnify as much as that. Yet I very well know, that that Inſtrument hath its Imperfections both in the Compoſition of the Metal, and in its being badly caſt, as you may perceive by a ſcabrous Place near the Middle of the Metal of it on the poliſh'd Side, and alſo in the Figure of that Metal near that ſcabrous Place: And in all thoſe Reſpects that Inſtrument is capable of further Improvement.

You ſeem to intimate, that the Proportion of 38 to 1, holds only for its magnifying Objects at ſmall Diſtances. But if for ſuch Diſtances, ſuppoſe 500 Feet, it magnify at that Rate, by the Rules of Opticks it muſt for the greater Diſtance imaginable magnify more than 37 ¾ to 1, which is ſo inconſiderable a Diminiſhing, that it may be even then as 38 to 1.

Here is made another Inſtrument like the former, which does very well. Yeſterday I compared it with a ſix Foot Teleſcope, and found it not only to magnify more, but alſo more diſtinctly: And to Day I found, that I could read in one of the Philoſophical Tranſactions, placed in the Sun's Light, at an 100 Foot Diſtance, and that at an 120 Foot Diſtance I could diſcern ſome of the Words. When I made this Tryal, its Aperture (defined next the Eye) was equivalent to more than an Inch and a third Part of the Object-Metal. This may be of ſome Uſe to thoſe that ſhall endeavour any thing in Reflections; for hereby they will in ſome meaſure be enabled to judge of the Goodneſs of their Inſtruments.

The Apertures and Charges of theſe Inſtruments, by Mr Newton. N. 82. *p.* 4032. April, An. 1672.

5. I know that the Aperture was 1 $\frac{1}{3}$ of an Inch, by trying that an Obſtacle of that Breadth was requiſite to intercept all the Light which came from one Point of the Object.

I ſhould tell you alſo, that the little plain Piece of Metal, next the Eye-Glaſs is not only truly figured: Whereby it happens, that Objects are not ſo diſtinct at the Middle as at the Edges. And I hope, that by correcting its Figure (in which I find more Difficulty than one would expect) they will appear all over diſtinct, and diſtincter in the Middle than at the Edges. And I doubt not but that the Performances will then be greater.

But yet I find, that there is more Light loſt by Reflection of the Metal which I have hitherto uſed, than by Tranſmiſſion through Glaſſes: For which Reaſon a ſhallower Charge would probably do better for obſcure Objects; ſuppoſe ſuch a one as would magnify 34 or 32 times. But for bright Objects at any Diſtance, it ſeems capable of magnifying 38 or 40 times, with ſufficient Diſtinctneſs. And for all Objects, the ſame Charge, I believe, may with Advantage be allowed, if the ſteely Matter, employ'd at *London*, be more ſtrongly reflective than this which I have uſed.

The Performances of one of theſe Inſtruments of any Length being known, it will appear by this following Table, what may be expected from thoſe of other Lengths by this Way, if Art can accompliſh what is promiſed by the Theory. In the firſt Column is expreſſed the Length of the Teleſcope in Feet; which doubled, gives the Semidiameter of the Sphere on which the Concave Metal is to be ground. In the ſecond Column are the Proportions of the Apertures for thoſe ſeveral Lengths. And in the third Column are the Proportions of the Charges, or Diameter of the Spheres, on which the Convex Superficies of the Eye-Glaſſes are to be ground.

Lengths.	*Apertures.*	*Charges.*	*Lengths.*	*Apertures.*	*Charges.*
$\frac{1}{2}$	100	100	8	800	200
1	168	119	10	946	211
2	283	141	12	1084	221
3	383	157	16	1345	238
4	476	168	20	1591	251
5	562	178	24	1824	263
6	645	186			

The Uſe of this Table will beſt appear by Example: Suppoſe therefore a half Foot Teleſcope may diſtinctly magnify 30 times with an Inch Aperture, and it being required to know, what ought to be the analogous Conſtitution and Performance of a four Foot Teleſcope: By the ſecond Column, as 100 to 476; ſo are the Apertures, as alſo the Number of Times which they magnify. And conſequently ſince the half Foot Tube hath an Inch Aperture and magnifieth 30 times, a four Foot Tube proportionally ſhould have 4 $\frac{74}{100}$ Inches Aperture, and magnify 143 times. And by the third Column, as 100 to 168; ſo have their Charges: And therefore if the Diameter of the Convexity of the Eye-Glaſs for a half Foot Teleſcope be $\frac{1}{5}$ of an Inch, that for a four Foot ſhould be $\frac{168}{500}$, that is about $\frac{1}{3}$ of an Inch; and ſo of other Lengths. But what the Event will really be, we muſt wait to ſee determined by Experience. Only this I thought fit to inſinuate, that they which intend to make Trial in other Lengths, may more readily know how to deſign their Inſtruments. Thus for a four Foot Tube, ſince the Aperture ſhould be 5 or 6 Inches, there will be required a Piece of Metal 7 or 8 Inches broad at leaſt, becauſe the Figure will ſcarcely be true to the Edges And the Thickneſs of the Metal muſt be proportional to the Breadth, leſt it bend in the Grinding. The Metals being poliſhed, there may be Trials made with ſeveral Eye-Glaſſes, to find what Charge may with beſt Advantage be made uſe of.

Some Objections of M. . . . . . *Anſwered, by Mr.* Newton. *ib. p.* 4034.

XVIII. 1. I doubt not but *M. A.* will allow the Advantage of Reflection in the Theory to be very great, when he ſhall have informed himſelf of the *Different Refrangibility* of the ſeveral Rays of Light. And for the practick Part, it is in ſome meaſure manifeſt by the Inſtruments already made, to what Degree of Vivacity and Brightneſs a metalline Subſtance may be poliſhed. Nor is it improbable but that there may be new Ways of Poliſhing found out for Metal, which will far excel thoſe that are yet in Uſe. And when a Metal is once well poliſhed, it will be a long while preſerved from tarniſhing, if Diligence be uſed to keep it dry and cloſe ſhut up from Air: For the principal Cauſe of Tarniſhing ſeems to be the condenſing of Moiſture on its poliſh'd Surface, which, by an acid Spirit, wherewith the Atmoſphere is impregnated, corrodes and ruſts it; or at leaſt at its exhaling leaves it cover'd over with a thin Skin, conſiſting partly of an earthly Sediment of that Moiſture, and partly of the Duſt, which, flying to and fro in the Air, had ſetled and adhered to it.

When there is not occaſion to make frequent Uſe of the Inſtrument, there may be other Ways to preſerve the Metal for a long time; as perhaps by immerging it in Spirit of Wine, or ſome other convenient Liquor. And if they chance to tarniſh, yet their Poliſh may be recover'd by rubbing them with a ſoft Piece of Leather, or other tender Subſtance, without the Aſſiſtance of any fretting Powders, unleſs they happen to be ruſty; for then they muſt be new poliſhed.

I am very ſenſible, that Metal reflects leſs Light than Glaſs tranſmits; and for that Inconvenience, I gave you a Remedy in my laſt Letter, by aſſigning a ſhallower Charge in Proportion to the Aperture, than is uſed in other Teleſcopes. But as I have found ſome metalline Subſtances to be more ſtrongly reflective, and to poliſh better, and be freer from tarniſhing than others; ſo I hope there may in time be found out ſome Subſtance much freer from theſe Inconveniencies than any yet known.

The Conſiderations of . . . . . Anſwer'd; by Mr. Newton. N. 88. *p.* 5084. Nov. An. 1672.

2. The *Conſiderer* is pleaſed to reprehend me for laying aſide the Thoughts of improving Opticks by Refractions. If he had obliged me by a private Letter on this Occaſion, I would have acquainted him with my Succeſs on the Trials I have made of that kind, which I ſhall now ſay have been leſs than I ſometimes expected, and perhaps than he at preſent hopes for. But ſince he is pleaſed to take it for granted, that I have let this Subject paſs without due Examination, I ſhall refer him to my former Letters, by which that Conjecture will appear to be ungrounded. For what I ſaid there, was in reſpect of Teleſcopes of the ordinary Conſtruction, ſignifying, that their Improvement is not to be expected from the well-figuring of Glaſſes, as Opticians have imagined; but I deſpaired not of their Improvement by other Conſtructions, which made me cautious to inſert nothing that might intimate the contrary. For although ſucceſſive Refractions that are all made the ſame Way, do neceſſarily more and more augment the Errors of the firſt Refraction; yet it ſeemed not impoſſible for contrary Refractions ſo to correct each others Inequalities, as to make their Difference regular; and if that could be conveniently effected, there would be no further Difficulty. Now to this end, I examined what may be done, not only by Glaſſes alone, but more eſpecially by a Complication of divers ſucceſſive Mediums, as by two or more Glaſſes or Cryſtals with Water or ſome other Fluid between them; all which together may perform the Office of one Glaſs, eſpecially of the Object-Glaſs, on whoſe Conſtruction the Perfection of the Inſtrument chiefly depends.

To the Aſſertion, That Rays are leſs true reflected to a Point by a Concave, than refracted by a Convex, I cannot aſſent; nor do I underſtand, That the Focus of the latter is a leſs Line than that of the former. The Truth of the contrary you will rather perceive by the following Table, computed for ſuch a Reflecting Concave, and the Refracting Convex, on Suppoſition that they have equal Apertures, and collect parallel Rays at an equal Diſtance from their Vertex; which Diſtance being divided into 15000 Parts, the Diameter of the Concave Sphere will be 60000 of thoſe Parts, and of the Convex 10000; ſuppoſing the Sines of Incidence and Refraction to be, in round Numbers, as 2 to 3. And this Table following ſhews, how much the exterior Rays, at ſeveral Apertures, fall ſhort of their principal Focus.

The

The Diameter of the Aperture.	*The Parts of the Axis intercepted betweeen the Vertex and the Rays.*		*The Errors by*	
	Reflected.	*Refracted.*	*Reflection.*	*Refraction.*
2000	$14991\frac{2}{3}$	14865	$8\frac{1}{3}$	135
4000	14966	14449	33	551
6000	14924	13699	76	1301
8000	14865	12475	135	2525
10000	14787	9472	213	5528

By this you may perceive, that the Errors of the Refracting Convex are ſo far from being leſs, that they are more than 16 times greater than the like Errors of the Reflecting Concave, eſpecially in great Apertures; and that without reſpect to the heterogeneous Conſtitution of Light. So that, however the contrary Suppoſition might make the Author of theſe Animadverſions reject Reflections as uſeleſs for the promoting of Opticks; yet I muſt for this, as well as other Conſiderations, prefer them in the Theory before Refractions.

Whether the *Parabola* be more difficult to deſcribe than the *Hyperbola*, or *Ellipſis*, may be a Quære; but I ſee no abſolute Neceſſity of endeavouring after any of their Deſcriptions. For if Metals can be ground truly Spherical, they will bear as great Apertures, as I believe Men will be well able to communicate an exact Poliſh to. And for Dioptrique Teleſcopes, I told you, that the Difficulty conſiſted not in the Figure of the Glaſs, but in the Deformity of Refractions; which if it did not, I could tell you a better and more eaſy Remedy than the Uſe of the *Conic Sections*.

3. We ſee that a Picture made by an Object-Glaſs of 12 Foot in a dark Room, is too diſtinct, and too well defined, to be produced by Rays, that ſhould ſtray the 50th Part of the Aperture.

Objections; by M. . . . . . .
N. 96. p. 6087:
July, An. 1673.

To take away this Difficulty, I muſt acquaint you, that though I put the greateſt Lateral Error of the Rays from one another to be about $\frac{1}{50}$ of the Glaſs's Diameter; yet their greater Error from the Points on which they ought to fall, will be but $\frac{1}{100}$ of that Diameter: And then, that the Rays, whoſe Error is ſo great, are but very few in Compariſon to thoſe, which are refracted more juſtly; for the Rays which fall upon the middle Parts of the Glaſs, are refracted with ſufficient Exactneſs, as alſo are thoſe that fall near the Perimeter, and have a mean Degree of Refrangibility; ſo that there remain only the Rays, which fall near the Perimeter, and are moſt or leaſt refrangible, to cauſe any ſenſible Confuſion in the Picture. And theſe are yet ſo much further weakned by the greater Space, through which they are ſcatter'd, that the Light which falls on the due Point, is infinitely more denſe than that which falls on any other Point round about it. And by this

Anſwer'd, by Mr. Newton.
N. 97. p. 6110.
Oct. An. 1673.

Exceſs of Denſity, the Light, which falls in or inviſibly near the juſt Point, may, I conceive, ſtrike the *Senſorium* ſo vigorouſly, that the Impreſs of the weak Light, which errs round about it, ſhall, in Compariſon, not be ſtrong enough to be animadverted, or to cauſe any more ſenſible Confuſion in the Picture than is found by Experience. But if this ſatisfy not, *N.* may try, if he pleaſe, how diſtinct the Picture will appear, when all the Lens is cover'd excepting a little Hole next its Edge on one ſide only: And if in this Caſe he pleaſe to meaſure the Breadth of the Colours thus made at the Edge of the Sun's Picture, he will perhaps find it to approach nearer to my Proportion than he expects.

A Reply by M. . . . . . *Ibid. p.* 6112.

4. I am ſatisfied with the Manner whereby Mr. *Newton* reconciles the Effect of Convex-Glaſſes with his Theory; but then he is alſo to acknowledge, that this Aberration of the Rays is not ſo diſadvantageous to Optick-Glaſſes, as he ſeems to have been willing to make us believe. His Invention is very good; but the Defect of the Metal ſeems to render it as impoſſible to execute, as the Difficulty of the Form obſtructs the Uſe of the *Hyperbole* of M. *Des Cartes.*

Anſwer'd, by Mr. Newton. N. 96. *p.* 6091. July, An, 1673.

If M. *N*. . . . . pleaſes to compare the Errors of a Glaſs and Speculum that collect Rays at equal Diſtances, he will find how much he is miſtaken; and that I have not been extravagant, as he imagines, in preferring Reflections. And as for what he ſays of the Difficulty of the *Praxis*, I know it is very difficult, and by thoſe Ways which he attempted it I believe it impracticable. But there is a Way inſinuated above by which it is not improbable, but that as much may be done in large Teleſcopes as I have thereby done in ſhort ones, but yet not without more than ordinary Diligence and Curioſity.

A Cata-Dioptrical Teleſcope; by M. Caſſegrain. N. 83. *p.* 4056. May, An. 1672. *Fig.* 100.

XIX. 1. M. *Caſſegrain* has communicated the Figure of a Teleſcope, almoſt like that of Mr. *Newton.*

A B C D is a ſtrong Tube, in the Bottom of which there is a great *Concave Speculum* C D, pierced in the middle E.

F is a *Convex Speculum*, ſo diſpoſed, as to its Convexity, that it reflects the Species, which it receives from the great Speculum, towards the Hole E, where is an Eye-Glaſs, which one looketh through.

The Advantage which I find in this Inſtrument above that of Mr. *Newton*, is, 1. That the Mouth or Aperture A B of the Tube may be of what Bigneſs you pleaſe; and conſequently you may have many more Rays upon the *Concave Speculum*, than upon that of which you have given us the Deſcription. 2. The Reflection of the Rays will be very natural, ſince it will be made upon the Axis itſelf, and therefore more vivid. 3. The Viſion of it will be ſo much the more pleaſing, in that you ſhall not be incommoded by the great Light, by reaſon of the Bottom C D, which hideth the whole Face. Beſides, you'll have leſs Difficulty in diſcovering the Objects, than in that of Mr. *Newton.*

Conſider'd by Mr. Newton. *Ibid. p.* 4057.

2. When I firſt applied my ſelf to try the Effects of Reflections, Mr. *Gregory's Optica Promota* (printed in the Year 1663.) being fallen into my Hands, where there is an Inſtrument (deſcribed *p.* 94.) like that of Mr. *Caſſegrain's*,

with a Hole in the midſt of the Object Metal, to tranſmit the Light to an Eye-Glaſs placed behind it: I had thence an Occaſion of conſidering that ſort of Conſtructions, and found theſe Diſadvantages in it; *viz.* 1. There will be more Light loſt in the Metal by Reflection from the little Convex Speculum, than from the oval Plane. For it is an obvious Obſervation, That Light is moſt copiouſly reflected from any Subſtance when incident moſt obliquely. 2. The Convex Speculum will not reflect the Rays ſo truly as the oval Plane, unleſs it be of an hyperbolick Figure; which is incomparably more difficult to form than a Plane; and if truly formed, yet would only reflect thoſe Rays truly which reſpect the Axis. 3. The Errors of the ſaid Convex will be too much augmented by the too great Diſtance, through which the Rays reflected from it muſt paſs, before their Arrival at the Eye-Glaſs. For which Reaſon, I find it convenient to make the Tube no wider than is neceſſary, that the Eye-Glaſs be placed as near to the oval Plane as is poſſible, without obſtructing any uſeful Light in its Paſſage to the Object-Metal. 4. The Errors of the Object-Metal will be more augmented by Reflection from the Convex than from the Plane, becauſe of the Inclination or Deflexion of the Convex on all ſides, from the Points on which every Ray ought to be incident. 5. For theſe Reaſons there is requiſite an extraordinary Exactneſs in the Figure of the little Convex, whereas I find by Experience that it is much more difficult to communicate an exact Figure to ſuch ſmall Pieces of Metal, than to thoſe that are greater. 6. Becauſe the Errors of the Perimeter of the Concave Object-Metal, cauſed by the Sphericalneſs of its Figure, are much augmented by the Convex, it will not with Diſtinctneſs bear ſo large an Aperture as in the other Conſtruction. 7. By reaſon that the little Convex conduces very much to the magnifying Virtue of the Inſtrument, which the oval Plane does not, it will magnify much more in Proportion to the Sphere, on which the great Concave is ground, than in the other Deſign; and ſo magnifying Objects much more than it ought to do in Proportion to its Aperture, it muſt repreſent them very obſcure and dark; and not only ſo, but alſo confuſed, by reaſon of its being over-charged. Nor is there any convenient Remedy for this. For if the little Convex be made of a larger Sphere, that will cauſe a greater Inconvenience, by intercepting too many of the beſt Rays; or if the Charge of the Eye-Glaſs be made ſo much ſhallower as is neceſſary, the Angle of Viſion will thereby become ſo little, that it will be very difficult and troubleſome to find an Object; and of that Object, when found, there will be but a very ſmall Part ſeen at once.

By this you may perceive, that the three Advantages, which Mr. *Caſſegrain* propounds to himſelf, are rather Diſadvantages. For according to his Deſign, the Aperture of the Inſtrument will be but ſmall, the Object dark and confuſed, and alſo difficult to be found. Nor do I ſee, why the Reflection is more upon the ſame Axis, and ſo more natural in one Caſe than in the other; ſince the Axis itſelf is reflected towards the Eye by the oval Plane, and the Eye may be defended from external Light, as well at the Side as at the Bottom of the Tube.

Mr. *Gregory* ſpeaking of theſe Inſtruments, in the aforeſaid Book, *p.* 95. ſaith; *De Mechanica horum Speculorum & Lentium, ab aliis fruſtra tentata, ego in Mechanicis minus verſatus nihil dico.* So that there have been Trials made of theſe Teleſcopes, but yet in vain. And I am informed, that about 7 or 8 Years ſince, Mr. *Gregory* himſelf, at *London* cauſed one of 6 Foot to be made by Mr. *Reive*, which I take to have been according to the aforeſaid Deſign deſcribed in his Book; but, though made by a skilful Artiſt, yet it was without Succeſs.

A Cata-Dioptrick Teleſcope; by S. Salvetti. N. 87. *p.* 5065. Sept. An. 1672.

XX. S. *Salvetti* hath made a Proſpective-Glaſs according to Mr. *Newton*'s new Invention. It was not above half a Foot long, it had the ſame Effect of one of two. He is now making another after the Conceit of Mr. *Caſſegrain*, though he agrees not with him in making Convex the little Speculum, which one looks into through the Eye-Glaſs, but believes the *French* Author only deviſed that to diſguiſe as much as was poſſible his pretended new Invention, which he endeavours to make anterior to Mr. *Newton*'s moſt noble one.

To make the Picture of any thing appear in a Light Room, by Dr. Hook. N. 38. *p.* 741. Aug An. 1688.

XXI. 1. Oppoſite to the Place or Wall, where the Apparition is to be, let a Hole be made of about a Foot in Diamerer, or bigger: If there be a high Window, that hath a Caſement in it, 'twill be ſo much the better. Without this Hole or Caſement open'd, at a convenient Diſtance (that it may not be perceiv'd by the Company in the Room) place the Picture or Object, which you will repreſent, inverted, and by means of Looking-Glaſſes placed behind, if the Picture be tranſparent, reflect the Rays of the Sun ſo, as that they may paſs through it towards the Place where it is to be repreſented; and to the End that no Rays may paſs beſides it, let the Picture be encompaſs'd on every ſide with a Board or Cloth. If the Object be a Statue, or ſome Living Creature, then it muſt be very much enlightned by caſting the Sun-Beams on it by Refraction, Reflection, or both. Between this Object and the Place where 'tis to be repreſented, there is to be placed a broad Convex-Glaſs, ground of ſuch a Convexity, as that it may repreſent the Object diſtinct on the ſaid Place; which any one, that hath an Inſight in the Opticks, may eaſily direct. The nearer it is placed to the Object, the more is the Object magnified on the Wall, and the further off the leſs; which Diverſity is effected by Glaſſes of ſeveral Spheres. If the Object cannot be inverted (as 'tis pretty difficult to do with Living Animals, Candles, *&c.*) then there muſt be two large Glaſſes of convenient Spheres, and they placed at their appropriated Diſtances (which are very eaſily found by Trials) ſo as to make the Repreſentations erect, as well as the Object.

Theſe Objects, Reflecting and Refracting Glaſſes, and the whole Apparatus; as alſo the Perſons employ'd to order, change, and make uſe of them, muſt be placed without the ſaid high Window or Hole, ſo that they may not be perceiv'd by the Spectators in the Room; and the whole Operation will be eaſily perform'd.

Whatſoever may be done by means of the Sun-Beams in the Day-time, the ſame may be done with much more eaſe in the Night, by the help of Torches, Lamps, or other bright Lights, placed about the Objects, according to the ſeveral ſorts of them.

2. There are every where made of theſe Lanthorns to repreſent and magnify Figures upon a Wall; but then 'tis only in the Dark; wherefore to give Variety of Colours, take Oyl of Spike, and therein mix the ſeveral Colours, wherewith you will have your Glaſs to be ſtained, paint them finely on, they dry preſently, and penetrate any Glaſs.

The Magick Lanthorn improv'd, by Sir Rob. Southwell.
N. 245. *p.* 364.
Oct. An. 1698.

XXII. Having found by many Trials, that ſome ſhort-ſighted Perſons could find little or no Relief by the Uſe of Concave-Glaſſes for ſeeing Objects at any Diſtance diſtinct, and that any one may be made ſhort-ſighted, and to be able to diſtinguiſh nothing but what is placed very near his Eye, but within certain Limits of Diſtance, by putting on and looking through a very deep Pair of Spectacles, ſuch as Ancient Men uſe: I concluded that what Glaſſes ſhould make this Man, whilſt looking thro' theſe Spectacles, to ſee Things at a greater Diſtance, would alſo help any other Perſon that ſhould be ſhort-ſighted by Nature. I then conſider'd, that by the help of a Convex-Glaſs, placed between the Object and the Eye, the Image of the Object may be made to appear at any Diſtance from the Eye; and conſequently all Objects may thereby be made to appear in any convenient Diſtance from the Eye: So that the ſhort-ſighted Eye ſhall contemplate the Picture of the Object, in the ſame manner as if the Object itſelf were in that Place. But then, becauſe the Pictures themſelves are ſo inverted, and therefore will be uncouth to one, not uſed to ſee them in that Poſture, I conſider'd of theſe Expedients to help that Defect alſo.

A Way to help Short-ſightedneſs; by Dr. Hook.
Ph. Coll.
N. 3. *p.* 59.
Dec. An. 1681.

Firſt, If it be only for Reading of a Book, or Writing, there needeth nothing but the Inverſion of the Book, and then holding the Convex at a due Diſtance, for the Picture of the Letters will appear erected in the due Place, for the Eye to ſee and diſtinguiſh them very plainly.

Secondly, For ſeeing to Write, I thought this would be the beſt Expedient, That the Perſon ſhort-ſighted ſhould firſt learn to Read with his naked Eye (both printed Letters and alſo written Hand) upſide downwards, which is quickly attined to by one that can do both the right Way.

Thirdly, For diſtinguiſhing Objects at a Diſtance, I can aſſert by my own Experience, that with a little Uſe of contemplating Objects inverted, one ſhall have as good an Idea, and as true a Knowledge of all manner of Objects, as if they were ſeen erected in their natural Poſture.

XXIII. 1. *Euſtachio Divini* hath made a Microſcope of a new Invention, wherein, inſtead of an Eye-Glaſs Convex on both ſides, there are two Plano-Convex Glaſſes, which are ſo placed, as to touch one another in the middle of their Convex Surface. It hath this Peculiar, that it ſhews the Objects flat and not crooked; and although it takes in much, yet nevertheleſs magnifieth extraordinarily.

Microſcopes, by S. Divini.
N. 42. *p.* 842.
Dec. An. 1668.

It is almoſt 16½ Inches high, and adjuſted at four different Lengths. In the firſt, which is the leaſt, it ſhews Lines 41 times bigger than they appear to the naked Eye; in the ſecond 90 times; in the third 111 times; and in the fourth 143 times: Whence one may eaſily calculate how much it augments Surfaces and Solidities.

By S. Piet. Salvetti. N. 87. *p.* 5065. Oct. An. 1672.

2. *S. Salvetti* lately ſhew'd one of his Microſcopes, made in Imitation of thoſe of *Divini* and *Campani*, to the Great Duke of *Tuſcany*, which was judged by all much better than any of the beſt his Highneſs hath. It was found, for Magnifying, Defining, and Clearneſs, to be very excellent.

By M. Leeuwenhoeck. N. 94. *p.* 6037. May, An. 1673.

3. M. *Leeuwenhoeck* hath lately contrived Microſcopes, excelling thoſe that have been hitherto made by *Euſtachio Divini*, and others.

By Mr. Butterfield. N. 141. *p.* 1026. Sept. An. 1678.

4. I have Microſcopes of the manner lately brought out of *Holland* by Mr. *Huygens*, of ſeveral Faſhions ready made. I have tried ſeveral Ways for the making of Glaſſes of the Bigneſs of a great Pin's Head and leſs; as in the Flame of a Tallow-Candle, and of one of Wax. But the beſt Way of all I have yet found, to make them clear and without Specks, is with the Flame of Spirit of Wine well rectified, and burned in a Lamp. Inſtead of Cotton I make uſe of very ſmall Silver-Wire, doubled up and down like a Skein of Thread; which being wet with the Spirit of Wine, and made to burn in the Lamp, giveth through the Veril of the Lamp, a very ardent Flame. Then take your beaten Glaſs, being firſt waſhed very clean, upon the Point of a Silver Needle filled very ſmall, and wet with Spittle. Hold it thus in the Flame till it be quite round, and no longer, for fear of burning it; and if the ſide of the Glaſs next the Needle be not melted, you may put it off, and take it up with the Needle on the round ſide, preſenting the rough ſide to the Flame, till it be every where round and ſmooth, then wipe and rub one or ſeveral of them together with ſoft Leather, which makes them much the better. Then put them between two Pieces of thin Braſs, the Apertures very round and without Bur, and that towards the Eye ſo big almoſt as the Diameter of the Glaſs, and ſo placed in a Frame with the Object conveniently for Obſervation.

By Mr. Stephen Gray. N. 221. *p.* 280. June, An. 1626.

5. I took a ſmall Particle of Glaſs, about the Bigneſs I deſigned my Globule, and laying it on the End of a Charcoal, I could, by the Help of a Blaſt-Pipe, with the Flame of a Candle, ſoon melt it into a Spherule; and by this means could make them indifferently clear, and the ſmalleſt very round, and I could make them much larger, than by the unaſſiſted Heat of the Candle; but theſe latter were attended with an Inconvenience, they were, on that ſide that reſted on the Coal, flatted, and received a rough Impreſſion from it. To remedy this Inconvenience, I was wont to grind them and poliſh them on a Braſs-Plane, and ſo reduce them to Hemiſpherules; but I found the clear ſmall Globules, not to mention that they magnify more, ſhew Objects more diſtinctly.

XXIV. 1. ABſ

A Water Microſcope, by Mr. Stephen Gray. N. 221. *p.* 281. N. 223. *p.* 353.

XXIV. 1. A B, I call the Frame of the Microſcope. It may be about $\frac{1}{18}$ of an Inch in Thickneſs. At A there is a ſmall Hole, near $\frac{1}{16}$ of an Inch Diameter, this ſerves for the Aperture of the Water, being in the Center of a larger Spherical Cavity, about $\frac{1}{8}$ of an Inch Diameter, and in Depth ſomewhat more than half the Thickneſs of the Braſs, Oppoſite to this, at the other ſide, there is another Concave, but half the Breadth of the former; which is ſo deep, as to reduce the Circumference of the ſmall Hole in the Center, to almoſt a ſharp Edge. In theſe Cavities the Water is to be placed, being taken upon a Pin, or large Needle, and convey'd into them till there be formed a double Convex Lens of Water; which, by the Concaves being of different Diameters, will be equivalent to a double Convex, of unequal Convexities. By this means, I find the Object is render'd more diſtinct than by a Planoconvex of Water, or by a double one, formed on the plain Surface of the Metal.

Fig. 101.

C D E, is the Supporter, whereon to place the Object; if it be Water, in the Hole G; if a ſolid Object, on the Point F. This is fix'd to the Frame of the Microſcope, by the Skrew E, where 'tis bent upwards, that its upper part may ſtand at a Diſtance from the Frame; 'tis moveable on the Skrew as a Center, to the end that either the Hole C, or the Point F, may be expoſed before the Microſcope; and that the Object may be brought to, and fix'd in its Focus. There is another Skrew, about half an Inch in Length, which goes through the round Plate into the Frame of the Microſcope A E, the Skrew and Plate taking hold of the Supporter about D, where there is a Slit ſomewhat larger than the Diameter of the Skrew, which is requiſite for the Admiſſion of the Hole C, or Point F, according to the Nature of the Object, into the Focus of the Glaſs; for by turning the Skrew G, the Supporter is carried to or from the ſame, which may be ſooner done, if whilſt one turn the Skrew with one Hand, the other hold the Microſcope by the end B, and one continue looking through the Water till the Object be ſeen moſt diſtinctly.

The Supporter muſt be made of a thin Piece of Braſs well hammer'd, that by its Spring it may the better follow the Motion of the Skrew, I choſe rather to fix the Supporter by the Skrew E, than by a Rivet; becauſe it may now, by help of a Knife, be unskrew'd, and by the other Skrew G, be brought cloſe to the Frame of the Microſcope without weakening its Spring, and ſo become more conveniently portable. If the Hole at G be fill'd with Water, but not ſo as to be Spherical; all Objects that will bear it, are ſeen therein more diſtinctly.

Another. Ibid. N. 221. *p.* 282.

2. Having obſerv'd ſome irregular Particles in Globules of Glaſs, and finding them diſtinct, but prodigiouſly magnified, when held cloſe to the Eye, I concluded that if I convey'd a ſmall Globule of Water to my Eye, and that there were any opacous or leſs tranſparent Particles than the Water therein, I might ſee them diſtinctly. I therefore took on a Pin a ſmall Portion of Water, which I knew to have in it ſome minute Animals, and laid it on the End of a ſmall Piece of Braſs Wire (there lay then by me) of about $\frac{1}{18}$ of

an Inch Diameter, till there was formed ſomewhat more than an Hemiſpherule of Water; then keeping the Wire erect, I applied it to my Eye, and ſtanding at a proper Diſtance from the Light, I ſaw them and ſome other irregular Particles, as I had predicted, but moſt enormouſly magnified; for whereas they are ſcarce diſcernable by my Glaſs-Microſcopes, or firſt Aqueous one, within the Globule they appeared not much different both in their Form, nor leſs in Magnitude than ordinary Peaſe. They cannot well be ſeen by Day-light, except the Room be darkned, after the manner of the famous Dioptrical Experiment, but moſt diſtinctly by Candle-light; they may be very well ſeen by the full Moon-light. If the Water be conveyed into the Hole B (which may be about $\frac{1}{10}$ of an Inch Diameter) till there remain near N. 223. p. 355. an Hemiſphere of Water on each ſide of the Hole, the Objects are ſeen more diſtinctly; and the Spherical Form of the Water is this way better ſecured, than on the Point of a Pin-Wire.

N. 221. p. 285. Fig. 102. The Reaſon of this Appearance may be thus explained. Let the Circle D B B D repreſent a Sphere of Water, A an Object placed in its Focus, ſending forth a Cone of Rays, two of which are A B, A B, which Opticians know coming into the Water at B and B, will be refracted from their direct Courſe, and become B D, B D; at D they will, at their paſſing into the Air, be again refracted into D E, D E, and ſo run parallel to one another, and to the Axis of the Sphere A F C G. Now 'tis a known and fundamental Principle in Opticks, that the Angle of Reflexion is equal to the Angle of Incidence; wherefore let the Rays B D, B D, be imagined to come from ſome Point of an Object placed within a Sphere of Water, by being reflected from the interior Surface of the Sphere at B B, C B D is the Angle of Reflexion, to which making C B F equal, F will be the place where an Object ſending forth a Cone of Rays, two of which are F B, F B, which are reflected into the Rays B D, B D, and then coming to the other ſide the Sphere at D and D, they are refracted into D E, D E, as before; and conſequently be as fit for diſtinct Viſion, whether the Object be placed in F within, or in A without the Sphere, if its interior Surface be conſidered as a Concave Reflecting Speculum.

Microſcopes improved; by Mr. Newton. N. 88. p. 5096. XXV. From the Diſtinction I have elſewhere given between *Compounded* and *Uncompounded Colours*, I take occaſion to communicate a Way for the Improvement of Microſcopes by Refraction; *viz.* By illuminating the Object in a darkned Room with Light of any convenient Colour not too much *Compounded*; for by that means the Microſcope will with Diſtinctneſs bear a deeper Charge and larger Aperture, eſpecially if its Conſtruction be ſuch as I may hereafter deſcribe; for the Advantage in ordinary Microſcopes will not be ſo ſenſible.

A Reflecting Microſcope, by Mr. Newton. N. 80. p. 3080. Fig. 103. XXVI. 1. I have ſometimes thought to make a Microſcope, which ſhould have, inſtead of an Object-Glaſs, a reflecting Piece of Metal. For theſe Inſtruments ſeem as capable of Improvement as Teleſcopes, and perhaps more; becauſe but one reflective Piece of Metal is requiſite in them, as you may perceive by the Diagram, where A B repreſenteth the Object Metal, C D the Eye-Glaſs, F their common Focus, and O the other Focus of the Metal, in which the Object is placed.

By Mr. Stephen Gray. N. 228. *p.* 541.

Fig. 104.

2. A repreſents a ſmall flat Ring of Braſs, whoſe interior Circle muſt not much exceed $\frac{4}{10}$ of an Inch Diameter, and about $\frac{1}{10}$ of an Inch thick: This we may call the Frame or Cell of the Glaſs; it muſt be prepared for Uſe after the following manner. Take a ſmall Globule of Quick-ſilver, and diſſolve it in a few Drops of *Aqua Fortis*, to which you may add 10 Parts of common Water; dip the End of a Stick in this Liquor, and rub the inward Circle of the Ring with it; ſo as it will have acquired a mercurial Tincture, and being wiped dry, be fit for Uſe. Then let it be laid on the Table, and pour a Drop of Quick-ſilver within it, which preſs gently with the Ball of the Finger, and it will adhere to the Ring; then cleanſe it with a Hare's Foot, and you will have a Convex Speculum. Take up the Ring and Speculum carrying it Horizontal, and lay it on the Brims of the hollow Cylinder B; ſo will the Mercury become a Concave Reflecting Speculum, which from the Smallneſs of the Sphere of which it ſeems to be a Section, may be uſed as a Microſcope. The Cylindrick Veſſel B has a Skrew-Hole at the Bottom, by which it is skrewed to the Top of the Pedeſtal C D; C E F G is the Supporter of the Object Plate, which, as you ſee, may be raiſed higher, or let lower, as there is Occaſion, by the Skrew on the Pedeſtal: The Object Plate muſt be of Glaſs cemented to the Ring G.

This Inſtrument, with a little Variation, may be made a Microſcope of Water, if, inſtead of the Ring G, there be only a ſmall Arm with a Hole in it to receive a Drop of Water, and the Cylindrick Veſſel B be either taken away, or skrewed on with its Bottom upwards, ſo as to make an Object Plate. This will be more convenient for viewing the Textures of opacous Objects, than that above deſcribed, which is more fit for fluid and tranſparent ones.

A Burning Concave at Lyons; *made by M.* de Vilette. N. 6. *p.* 95.

XXVII. 1. The Figure of it is round, being 30 Inches, and ſomewhat better in Diameter, On one ſide it hath a Frame of a Circle of Steel, to the end that it may keep its juſt Meaſure. 'Tis eaſy to remove it from Place to Place, tho' it be above an Hundred Weight, and 'tis eaſily put in all ſorts of Poſtures. The Burning Point is diſtant from the Center of the Glaſs about 3 Foot. The Focus is about half a *Louis d' Or* large. One may paſs one's Hand through it, if it be done nimbly; for if it ſtay there the time of a Second of a Minute, there is danger of receiving much hurt. Green Wood takes Fire in it in an inſtant, as do alſo many other Bodies.

	Seconds.
A ſmall Piece of Pot-Iron was melted, and ready to drop down in -	40
A Silver Piece of Fifteen Pence was pierced in - - - - - —	24
A Groſs Nail (called *le Clou de Paiſan*) was melted in - - - —	30
The End of a Sword-Blade of *Olinde* was burned in - - - —	43
A Braſs Counter was pierced in - - - - - - - - - -	06
A Piece of Red Copper was melted ready to drop down in —	42

	Seconds.
A Piece of a Chamber Quarry-Stone was vitrified, and put into a Glaſs-Drop, in	45
Steel, whereof Watchmakers make their Springs, was found melted, in	09
A Mineral-Stone, ſuch as is uſed in Harquebuſſes *à rouët*, was calcin'd and vitrified, in	01
A Piece of Mortar was vitrified, in	52

In ſhort, There is hardly any Body which is not deſtroy'd in this Fire. If one would melt by it any great Quantity of Metal, that would require much Time, the Action of Burning not being perform'd but within the Bigneſs of the Focus, ſo that ordinarily none but ſmall Pieces are expos'd to it. One M. *de Alibert* buys it, paying for it 1500 Livres.

Ibid. p. 97. You incline to believe, that the Glaſſes of *Maginus* and *Septalius* do approach to that of *Lyons*; but I can aſſure you they come very far ſhort of it. You may conſult *Maginus* his Book, where he deſcribes his; and there are ſome Perſons here who have ſeen one of his beſt, which had but about 20 Inches Diameter; ſo that this of *Lyons* muſt perform at leaſt twice as much. As to *Septalius*, we expect the Relation of it from Intelligent and Impartial Men. It cannot well be compared with that of *Lyons*, but in Bigneſs; and in this Caſe, if it have five Palms (as you ſay) that would be but 3½ Foot *French*, and ſo it were a Foot bigger, which would make it half as much greater in Surface: But as to the Effects, ſeeing it burns ſo far off, they cannot be very violent. And I have heard one ſay, that had ſeen it, that it did not ſet Wood on Fire but after the time of ſaying a *Miſerere*. You may judge of the Difference of the Effects, ſince that of *Lyons* gathers its Beams together within the Space of 7 or 8 Lines; and that of *Septalius* muſt ſcatter them in the Compaſs of 3 Inches.

N. 49. *p.* 986. It was diſpoſed of to the King of *Denmark*.

Another by the ſame. N. 49. *p.* 986. The ſame M. *de Vilette* of *Lyons* hath made another Burning Concave. It is of 34 Inches Diameter, and melts all ſorts of Metals, and Iron itſelf of the Thickneſs of a Silver Crown in leſs than a Minute of Time, and vitrifies Brick in the ſame time; and as for Wood, whether green or dry, it ſets it on Fire in a Moment. The King hath ſeen it, and the Performances of it, with great Satisfaction; and his Majeſty is likely to make it his, and then to beſtow it on his *Royal Academy* of Philoſophers, for making of farther Experiments with it.

By . . . . . *Ibid. p.* 987. This kind of Concaves burning the moſt forcibly of any Fire we know of, would be of great Uſe, if they could be ſo contrived as to have a Focus of any conſiderable Largeneſs, to take in a good Quantity of combuſtible Matter at once.

By S. Settalla. N. 40. *p.* 786. 3. *S. Settalla* at *Milan*, cauſeth to be made a Burning-Glaſs of 7 Foot in Diameter. He pretends to make it burn at the Diſtance of 50 Palms, which is about 33 Foot.

4. The outer Circle of the Concave Burning Speculum, which I lately caus'd to be made in *Luſace*, is near 3 *Leipſick* Ells in Diameter, exceeding that great one at *Paris* by $\frac{3}{8}$ of ſuch an Ell. It is made of a Copper Plate ſcarce twice ſo thick as the Back of an ordinary Knife, and may therefore be eaſily removed from Place to Place, and order'd for Uſe: And the Workmanſhip of it may, by the Contrivances I have invented, be eaſily, and in little time perform'd by one Man. The Poliſh thereof is very good, and repreſents by diſtinct Reflections all thoſe Appearances which ariſe from the Concave Figure thereof.

A Burning Concave in Germany; by . . . . N. 188. p. 52.

The Force of this Speculum is incredible. For, 1. A Piece of Wood put into the Focus (which is 2 Ells off) flames in a Moment, ſo as a freſh Wind can hardly put it out. 2. Water applied in an Earthen Veſſel preſently boils, ſo as to boil an Egg; and the Veſſel being held there ſome time, the Water evaporates all away. 3. A Piece of Tin or Lead 3 Inches thick, as ſoon as it is put into the Focus, melts away in Drops; and held there a little time is in a perfect Fluor, ſo as in two or three Minutes to be quite pierced through. 4. A Plate of Iron or Steel placed in the Focus immediately is ſeen to be red-hot on the Backſide, and ſoon after a Hole is burnt through: I have made three ſuch Holes in a Plate in ſix Minutes time. 5. Copper, Silver, and the like, applied to the Focus, melt; which I have tried with ſeveral ſorts of Coin; among the reſt, with a *Rix-Dollar*, and the ſame happen'd to it as to the aforeſaid Iron Plate in five or ſix Minutes. 6. Things not apt to melt, as Stones, Brick, and the like, ſoon become red-hot like Iron. 7. Slate at firſt is red-hot, but in a few Minutes turns into a fine ſort of black Glaſs; of which, if any part be taken in the Tongs and drawn out, it runs into Glaſs Threads. 8. Tiles, which had ſuffer'd the moſt intenſe Heat of Fire, in a little time melt down in a yellow Glaſs: As do, 9. Pot-Shreads, not only well burnt at firſt, but much us'd in the Fire, into a blackiſh yellow Glaſs. 10. Pumice-Stone, ſaid to be that of Burning Mountains, in this Solar Fire, melts into a white Tranſparent Glaſs. 11. A Piece of a very ſtrong Crucible put into the Focus, in eight Minutes, was melted into a Glaſs. 12. I have ſeen Bones turn'd into a kind of opake Glaſs, and a Clod of Earth into a yellow or greeniſh Glaſs.

Theſe Experiments were made in *Auguſt* and *September*, when the Sun has not the ſame Force as when he is about the Summer Solſtice. The Beams of the Full Moon, concentred by this Speculum, did not produce any Degree of Heat, tho' the Light was not a little encreas'd.

5. Some Years ago, Dr. *Hook* made a Propoſal to the *Royal Society* concerning the ſame thing. He conceives one may be made of many Foot Diameter, for a ſmall Price, being hammer'd out of a Copper Plate, and tinned over with a Mixture of Tin, Lead and Tin-Glaſs, which is found to bear a very good Poliſh. Such a Speculum might be of great Uſe in perfecting the Art of Paſtes, or fictitious Jewels, which require the moſt intenſe Degree of Heat, to bring them to an exact Mixture.

By Dr. Hook. *Ibid. p.* 354.

XXIII. A

Concave Specula nearly of a Parabolick *Figure; attempted by Mr.* Stephen Gray. N. 228. *p.* 542. N. 235. *p.* 787.

XXVIII. A Linnen-Cloth, firſt being wet in fair Water, and then laid on a Conclave Cylinder, as the Verge of a Sieve, Keeler, or the like, its central Parts will deſcend ſo as to form a very regular Concave Superficies. And a Thread, being firſt wet in common Water, and then ſuſpended with its two Ends, or any two Points nearer than their utmoſt Extent, ſo as it might touch the Center of the ſuſpended Cloth, and its two oppoſite Points on the Ring, was found to have the ſame Curvature. My Buſineſs was then to examine the Figure of the Thread thus ſuſpended, which I did in manner following. On the ſide of a Wall I deſcribed *Parabola's* of ſeveral Species, whoſe Axes were perpendicular, and Perimeter horizontal, to which the Line being applied, ſo as it might touch the Vertex, paſt very nearly through all the intermediate Points of the *Parabola*, much nearer than the Portion of a *Circle*, which paſt through the Extremity of the Perimeter, and Latus Rectum, would do.

From hence I conclude, That a ponderous and pliable Subſtance, being ſuſpended on a Ring or hollow Cylinder, ſo as that its central Parts may deſcend, will form itſelf into a Figure that is more commodious for Burning-Glaſſes than the Spherical, of which they are now made, being much nearer their moſt abſolute Figure, the *Parabola*.

Now if there may be a Way found to give to Cloth or Leather a metalline Surface, or a Varniſh that may bear a good Poliſh; or if this be found impracticable, perhaps Plates of Metal may be beat out ſo thin, as being ſuſpended on a large Ring, will by their own Gravity receive their true Figure, one may make Speculums of what Largeneſs he pleaſeth.

Upon this Conſideration, I deviſed the following Experiment. There was taken a ſufficient Quantity of Potter's Clay, of which there was formed a plain circular Plate, by help of an Iron Ring about 13 Inches Diameter. This was laid on a leſſer Ring, which was ſupported by four Feet, and it immediately became a very regular Concave on its upper, and Convex on its under Superficies; but notwithſtanding 'twas ſet to dry in the Shade, yet before it was dry enough, its central Parts extended ſo as to become almoſt plain, not without ſome Defects; if it had continued in its Regularity, I deſigned to have burned aud glazed it in a Potter's Furnace.

To make the Globe Looking-Glaſs; by Sir R. Southwell. N. 245. *p.* 363.

XXIX. Take Quick-ſilver, Marchaſite of Silver, each three Ounces; Tin and Lead, each half an Ounce; theſe two firſt throw on the Marchaſite, and laſt of all the Quick-ſilver; ſtir them well together; but they muſt be taken from the Fire, and be towards Cooling before the Quick-ſilver be added; let your Glaſs be well warmed, and pour in the Mixture, and roll it from ſide to ſide.

Note, This will do alſo when Cold, but 'tis beſt when the Glaſs is heated and very dry.

Note alſo, That if at the Glaſs-Houſe your Ball be of yellow Glaſs, then all will ſhine like Gold.

XXX. *Papers*

XXX. *Papers (of leſs General Uſe) omitted.*

1. DR. *Hook* having (in his *Micrographia*) deſcribed a new Engine for *Grinding Optick-Glaſſes* of very great Lengths, M. *Auzout* (in a ſmall *French Tract*) objects ſeveral Difficulties to this Engine itſelf: But however he thinks it impracticable to make any Glaſſes of above 300 or 400 Foot at moſt (and fears that neither Matter nor Art will go even ſo far) which will be very far from ſhowing us *Plants* or *Animals* in the *Moon*; and then propoſes Remedies to ſome of the Inconveniences of the Turn. To all this, Dr. *Hook* here replies; He anſwers the Objections, and rejects the propoſed Expedients. *Optick-Glaſſes, by a Turn-Lathe.* N. 2. *p.* 31. N. 4 *p.* 56. *Ibid. p.* 63.

2. *Carlo Ant. Mancini* having, in his *Occhiale all' Occhio*, deſcribed a particular Way for making Convex-Glaſſes upon a Plain, his Method is here tranſlated from the *Italian* into *Engliſh*. But 'tis added, That though the Contrivance be ingenious, yet it is conceiv'd by skilful Artiſts, that it will be very difficult to put it into Practice. *Upon a Plain.* N. 42. *p.* 838.

XXXI. *Accounts of Books omitted.*

1. P*Hyſico-Matheſis de Lumine, Coloribus, & Iride*, &c. *Auth.* Franc. Maria Grimaldo, *S. J.* Bononiæ 1665. *in* 4*to.* N. 79. *p.* 3068.

2. *Cogitationes Phyſico-Mechanica de Natura Viſionis. Auth.* Jo. Ott. *Schaphuſa Helvetio.* Heidelbergæ 1670. *in* 4*to.* N. 71. *p.* 2163.

3. *Synopſis Optica. Auth.* Honorato Fabri. *Soc. Jeſu.* Lugduni 1667. *in* 4*to.* N. 32. *p.* 626.

4. *L' Occhiale all' Occhio, overo Dioptrica Prattica del* Carlo Ant. Mancini, *in* Bologna 1660. *in* 4*to.* N. 42. *p.* 837.

4. *Lectiones* 18; *Cantabrigiæ in Scholis publicis habitæ, in quibus Opticorum Phænomenωn Genuinæ Rationes inveſtigantur, & exponuntur, ab* Iſaaco Barrow. Lond. 1669. *in* 4*to.* N. 75. *p.* 2258.

6. *La Dioptrique Occulaire, par le Pere* Cherubin *d'* Orleans, *Capucin.* A Paris, 1671. *in Folio.* N. 78. *p.* 3045.

7. Chriſtiani Hugenij *Aſtroſcopia Compendiaria, Tubi Optici molimine Liberata*: or, The Deſcription of an *Aërial Teleſcope.* Hague, 1684. *in* 4*to.* N. 161. *p.* 668.

8. A Treatiſe of Dioptricks. By *Will. Molineux*, Eſq; F. R. S. *in* 4*to.* N. 205. *p.* 967.

9. *Catoptricæ & Dioptricæ Elementa. Auctore* Davide Gregorio, *D. M.* Oxon, 1695. *in* 8*vo.* N. 219. *p.* 214.

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Chap. IV.

ASTRONOMY.

The Obſervatory of Tycho Brahe; *by* Mr Gourdon. N. 266. *p.* 692.

I. THE Iſland *Ween*, (vulgarly term'd the *Scarlet-Iſland*) famous for the Obſervations of *Tycho Brahe*, that renowned *Daniſh Aſtronomer*, (with all Submiſſion to better Judgments) was none of the fitteſt for Aſtronomical Obſervations of all ſorts, ſuch as the taking the exact Time of the true Riſing and Setting of Celeſtial Bodies, together with their reſpective Amplitudes; becauſe the Iſland lies low, and is Land-lock'd on all the Points of the Compaſs, ſave three. Beſides, the ſenſible Land-Horizon of the *Ween* is extreamly uneven and rugged, the North and Eaſtern Parts thereof being ſome riſing Hills in the Province of *Schonen*; and the Weſtern Part is moſtly overſpread with Trees on the Iſland *Zealand*; from the remoteſt of which Coaſts the *Ween* is not diſtant above three Leagues.

A new Aſtronomical Inſtrument; by M. Weighelius. N. 74 *p.* 2219.

II. M. *Weighelius* hath invented an Inſtrument, which he calls *Aſtrodicticum*, by the means whereof very many Perſons ſhall be able at one and the ſame time to behold one and the ſame Star. He hath alſo invented an exceeding great Globe of the World, capable of 10 Perſons to ſit in it all at once, and to behold the Motions of the celeſtial Bodies, *&c.*

A Celeſtial Globe; by M. Didier, P. Alleman. N. 136. *p.* 905.

III. The Bigneſs of this Globe is only of four Inches Diameter. The Body of the Globe of burniſh'd Steel, where all the Figures of the Conſtellations are deſigned in Silver-colour, but the Stars themſelves of all Magnitudes are put on in emboſs'd Gold.

This Globe moves from Eaſt to Weſt in 24 Hours; and you may there ſee the Sun exactly riſe and ſet as in the great World, together with the Moon, as alſo the Stars of the Conſtellations; likewiſe, how the Sun of this Globe comes to his Meridian, with an admirable Regularity, conform to the *Primum Mobile*. And you may alſo there perceive the mean Motions of the Sun and Moon from Weſt to Eaſt, and all the Lunations; and by the Diurnal Motion of the Moon, it ſhews the Flux and Reflux of the Sea.

The Meridian ſerveth for a Needle to ſhew the Hours, which are marked upon the Zodiack, which the Sun marcheth regularly, which hath two main Rays, one whereof goeth directly Northward, the other Southward. That of the North marks the Way or Degree, which the Sun maketh from Weſt to Eaſt upon the Signs of the Zodiack, and upon a Circle of Silver, where the 360 Degrees of the Circle are mark'd. The other Ray of the South, marks upon another Circle of Silver the Days of the Month, where the 365 Days are noted. The Circles of the Longitude of the Stars, which ſeparate the

Signs, and which come from the Poles of the Zodiack, are marked by Gold-Wires; as alſo the Equator, the Tropicks, and the Polar Circles.

There is but one great Spring, the *Pimum Mobile*, which puts all the reſt in Motion: It is wound up by the *Antartique* Pole, and you may wind it up to the Right or Left Hand, without wronging any contrary Motion: And by the *Arctique* Pole, you may advance and retard this Movement, if you ſhould find any Inequality, without altering at all the great Spring.

A Way to meaſure the Diameters of the Planets, and the Parallax of the Moon; by M. Auzout. N. 21. *p.* 373. Jan. An. 1666.

IV. I applied my ſelf the laſt Summer to the taking of the Diameters of the Sun, Moon, and the other Planets, by a Method which one M. *Picard* and my ſelf have, eſteemed by us the beſt of all thoſe that have been practis'd hitherto; ſince we can take the Diameters to Second Minutes, being able to divide one Foot into 24000 or 30000 Parts, ſcarce failing as much as in one only part, ſo as we can in a manner be aſſured, not to deceive our ſelves in 3 or 4 Seconds. I ſhall not now tell you my Obſervations; but I may very well aſſure you, that the Diameter of the Sun has not been much leſs in his *Apogee*, than 31 min. 37 or 40 ſec. and certainly not leſs than 31 min. 35 ſec. and that at preſent in his *Perigee* it paſſes not 32 min. 45 ſec. and may be leſs by a Second or two: That which is at the preſent troubleſome is, that the vertical Diameter, which is the moſt eaſy to take, is diminiſh'd, even at Noon, by 8 or 9 ſec. becauſe of the Refractions, which are much greater in Winter than Summer at the ſame Height; and that the horizontal Diameter is difficult, becauſe of the ſwift Motion of the Heavens.

As for the Moon, I never yet found her Diameter leſs than 29 min. 44 or 45 ſec. and I have not ſeen it paſs 33 min. or if it hath, it was only by a few Seconds. But I have not yet taken her in all the Kinds of Situations of the *Apogees* and *Perigees* which happen, with the Conjunctions and Quadratures. I do not mention all what can be deduced from thence; I ſhall only tell you, that I have found a Way to know the *Parallax* of the Moon, by the means of her Diameter: *viz.* If on a Day, when ſhe is to be in her *Apogee* or *Perigee*, and in the moſt boreal Signs, you take her Diameter towards the Horizon, and then towards the South, with her Altitudes above the Horizon. For if the Obſervation of the Diameters be exact, as in theſe Situations the Moon changes not conſiderably her Diſtance from the Earth in 6 or 7 Hours, the Difference of the Diameters will ſhew the Proportion there is of her Diſtance with the Semidiameter of the Earth. I do not enlarge, becauſe that as ſoon as one hath this Idea the reſt is eaſy. The ſame would yet be practiſed better in the Places where the Moon paſſes through the Zenith, than here; for the greater the Difference is of the Heights, the greater is that of the Diameters. I do not note (for it eaſily appears) that if one were under the ſame Meridian, or the ſame *Azimuth* in two very diſtant Places, and took at the ſame time the Diameter of the Moon, one would do the ſame thing; tho' this Method goes not to Preciſeneſs.

From what has been ſaid may be collected the Reaſon of the Obſervation, which M. *Hevelius* made in the laſt Eclipſe of the Sun (*July* 2. *St. N.* 1666.) touching the Encreaſe of the Moon's Diameter about the End. I am exceeding glad, that a Perſon, who probably knew not the Cauſe of it, has made the Experiment; but it is ſtrange, that until now no Aſtronomer has foreſeen that That ſhould happen, nor given any Precepts for the Change of the Moon's Diameter in the Eclipſes of the Sun, according to the Places where they ſhould happen, and according to the Hour and Height the Moon ſhould have: For what happen'd in that Eclipſe of Augmentation, would have fallen out contrarily, if it had been in the Evening; for the Moon, which in that Eclipſe that began in the Morning was higher about the End than at the Beginning, was nearer us, and conſequently was to appear bigger: But if the Eclipſe ſhould happen in the Evening, ſhe would be lower at the End, and therefore more diſtant from us, and conſequently appear leſſer. So alſo in two different Places, whereof one ſhould have the Eclipſe in the Morning, and the other at Noon, the Moon ſhould appear bigger to him that hath it at Noon: And ſhe muſt likewiſe appear to thoſe who ſhall have a leſſer Elevation of the Pole under the ſame Meridian, becauſe the Moon will be nearer them.

An Account of M. Gaſcoigne's *Micrometer; by Mr.* Richard Townley.
N. 25. *p.* 457.
May, An. 1667.

V. 1. I ſhould be look'd upon as a great Wronger of our Nation, ſhould I not let the World know, that I have, out of ſome ſcatter'd Papers and Letters that formerly came to my Hands of one Mr. *Gaſcoigne*'s, found out, that before our late *Civil Wars* he had not only deviſed an Inſtrument of as great a Power as M. *Auzout*'s, but had alſo for ſome Years made uſe of it; not only for taking the Diameters of the Planets, and Diſtances upon Land; but had farther endeavoured, out of its Preciſeneſs, to gather many Certainties in the Heavens; amongſt which I ſhall only mention one, *viz. The finding the Moon's Diſtance*, from two Obſervations of her Horizontal and Meridional Diameters; which I the rather mention, becauſe the *French Aſtronomer* eſteems himſelf the firſt that took any ſuch Notice, as thereby to ſettle the Moon's Parallax: For our Country-Man fully conſider'd it before, and imparted it to an Acquaintance of his, who thereupon propoſed to him the Difficulties that would ariſe in the Calculation; with Conſiderations upon the ſtrange Niceties, neceſſary to give him a Certainty of what he deſired. The very Inſtrument he firſt made I have now by me, and two others more perfected by him; which doubtleſs he would have infinitely mended, had he not been ſlain unfortunately in His late Majeſty's Service. He had a *Treatiſe of Opticks* ready for the Preſs; but though I have uſed my utmoſt Endeavour to retrieve it, yet I have in that Point been totally unſucceſsful: But ſome looſe Papers and Letters I have, particularly about this Inſtrument for taking of Angles, which was far from perfect. Nevertheleſs, I find it ſo much to exceed all others, that I have uſed my Endeavours to make it exact, and eaſily tractable; which above a Year ſince I effected to my own Deſire, by the Help of an ingenious and exact Watchmaker: Since which time, I have not altogether neglected it, but employ'd it particularly in taking the *Diſtances* (as Occaſion ſerved) of the *Circum-jovialiſts*, towards a perfect ſettling their Motion. I ſhall only ſay of it, That it is ſmall, not exceeding in Weight, nor much in Bigneſs, an ordinary Pocket-Watch, exactly marking above 40000 Diviſions in a Foot, by the help of two Indexes; the one ſhewing Hundreds of Diviſions, the other Diviſions of the Hundred; every laſt Diviſion in my ſmall one containing $\frac{1}{10}$ of an Inch, and that ſo preciſely, that, as I uſe it, there goes about 2 $\frac{1}{2}$ Diviſions to a Second. Yet I have taken Land-Angles ſeveral times to one Diviſion, tho' (for the Reaſon mention'd by M. *Auzout*) it be very hard to come to that Exactneſs in the Heavens, *viz.* the ſwift Motion of the Planets. Yet, to remedy that Fault, I have deviſed a *Reſt*, in which I find no ſmall Advantage, and not a little pleaſing thoſe Perſons who have ſeen it, being ſo eaſy to be made, and by the Obſerver manag'd without the help of another; which ſecond Convenience, my yet nameleſs Inſtrument hath in great Perfection, and is by reaſon of its Smallneſs and Shape, eaſily applicable to any Teleſcope.

A Deſcription of it, by Dr. Hook. N. 29. *p.* 542. Nov. An. 1667.

2. *a a a a*, is a ſmall oblong Braſs Box, ſerving both to contain the Skrews and its Sockets, or Female Skrews, and alſo to make all the ſeveral moveable Parts of the Inſtrument to move very true, ſmooth, and in a ſimple direct Motion. To one End hereof is ſkrewed on a round Plate of Braſs *b b b b*, about 3 Inches over; the extream Limb of whoſe Outſide is divided into 100 equal Parts, and numbred by 10, 20, and 30, *&c.* Through the Middle of this Plate, and the Middle of the Box *a a a*, is placed a very curiouſly wrought Skrew, of about the Bigneſs of a Gooſe-Quill, and of the Length of the Box, the Head of which is, by a fixed Ring or Shoulder on the Inſide, and a ſmall ſpringing Plate *d d*, on the Outſide, ſo adapted to the Plate that it is not in the leaſt ſubject to ſhake. The other End of this Skrew is by another little Skrew (whoſe ſmall Points fill the Center or Hole made in the End of the longer Skrew for this purpoſe) rendred ſo fix'd and ſteady in the Box, that there appears not the leaſt Danger of ſhaking. Upon the Head of this Skrew, without the ſpringing Plate, is put on a ſmall Index *e e*, and above that a Handle *m m*, to turn the Skrew round as often as there ſhall be Occaſion, without at all endangering the diſplacing of the Index, it being put on very ſtiff upon a cylindrical Part of the Head, and the Handle upon a Square. The Skrew hath that Third of it, which is next the Plate, bigger than the other two Thirds of it, by at leaſt as much as the Depth of the ſmall Skrew made on it: The Thread of the Skrew of the bigger Third is as ſmall again, as that of the Skrew of the other two Thirds. To the groſſer Skrew is adapted a Socket *f*, faſtned to a long Bar or Bolt *g g*, upon which is faſtned the moveable Sight *h*, ſo that every Turn of the Skrew promotes the Sight *h*, either a Thread nearer, or a Thread farther off from the fix'd Sight *i*. The Bar *g g* is made exactly equal, and fitted into two ſmall Staples *k k*, which will not admit of any ſhaking. There are 60 of theſe Threads, and anſwerable thereto are made 60 Diviſions on the Edge of the Bolt or Ruler *g g*, and a ſmall Index *l* fix'd to the Box *a a a a*, denotes how many Threads the Edges of the two Sights *h* and *i* are diſtant; and the Index *e e*, ſhews on the circular Plate what part of a Revolution there is more; every Revolution, as was ſaid before, being divided into 100 Parts. At the ſame time that the moveable Sight *h* is moved forwards or backwards, one or more Threads of the courſer Skrew, is the Plate *p p*, by the means of the Socket *q*,

Fig. 105.

Fig. 106.

to

to which it is ſkrewed, moved forward or backward, one or more Threads of the finer Skrew: So that this Plate being fix'd to the Teleſcope by the Skrews *r r*, ſo as the middle betwixt the Sights may lie in the Axis of the Glaſs, however the Skrew be turned, the midſt betwixt the Sights will always be in the Axis, and the Sights will equally either open from it, or ſhut towards it.

Fig. 107.

It is conceived by ſome ingenious Men, that it will be more convenient, inſtead of the Edges of the two Sights *h* and *i*, to employ two Sights *r* and *s*, fitted with the Hairs *t* and *v*, ſo that they may be conveniently uſed in the place of the ſolid Edges of the Sights *h* and *i*.

Fig. 108.

The Inſtrument is thus applied to the Teleſcope. The Tube A D is divided into three Lengths, of which (as in ordinary ones) B C is to lengthen or contract, as the Object requires: But A B is here added, that at A you may put ſuch Eye-Glaſſes as ſhall be thought moſt convenient, and to ſet them ſtill at the Diſtance moſt proper from the Indexes or Pointers, which here are ſuppoſed to be at B, which Length alters alſo in reſpect of divers Perſons Eyes. E is a Skrew, by which the great Tube can be fix'd ſo, as by the help of the Figures any ſmaller part of it can immediately be found, meaſuring only, or knowing the Diviſions on B C, the Diſtance of the Object-Glaſs from the Pointers. F is the angular Piece of Wood, that lies on the upper Skrew of the Reſt.

Fig. 109.

Ibid. p. 556.

This Reſt (by Dr. *Hook*'s Suggeſtion) may be rendred more convenient, if, inſtead of placing the Skrew Horizontal, it be ſo contrived, that it may be laid parallel to the Equinoctial, or to the Diurnal Motion of the Earth; for, by that Means, the ſame Thing may be performed by the ſingle Motion of one Skrew, which in the other Way cannot be done but by the turning of both Skrews; as will eaſily appear to thoſe that ſhall conſider it.

More ways to meaſure ſmall Diſtances intimated; by Dr. Hook. N. 25. *p.* 459. May, An. 1667

3. I have by me two or three ſeveral Ways of Meaſuring the Diameters of the Planets, whether Horizontal, Perpendicular, or Inclined, to the Exactneſs of a Second, by the help of a Teleſcope; as alſo of taking the Poſition and Diſtance of the ſmall fix'd Stars one from another, or from any of the leſs bright Planets, if the Diſtance be not above two or three Degrees.

Excellence of the Micrometer; by Mr. Flamſteed. N. 96. *p.* 6099.

4. Micrometro & Tubo Pedum 14, Planetarum frequenter Diametros & à Fixis Diſtantias, ad ſecundos fere Scrupulos, quod vix inexpertus credes, dimenſus ſum.

Plain Sights rejected; by Mr. Flamſteed. N. 89. *p.* 5119. N. 96. *p.* 6102.

VI. 1. Pluribus Argumentis evinci poteſt, *Tychonem* ſæpe cum in Locis, tum Latitudinibus, Fixis quibuſdam aſſignatis, duos treſve & interdum quatuor aut quinque totos Scrupulos à vero aberraſſe. Fixarum quidem Reſtitutionem ſuſcepiſſe Celeberrimum *Joannem Hevelium* audivimus, attamen quandoquidem Pinacidiis Vitrorum Caſſis fertur ipſum uti, dubium an multum ab ipſo Emendatiores Locos habituri ſimus quam reliquit *Tycho*, niſi ubi valde hallucinatus eſt.

Plain Sights pre-ferr'd to Tele-leſcopick; by M. Hevelius. N. 102. *p.* 27. Apr. An. 1674.

2. Percipio Veſtrates non omnes mihi adſtipulari in iſto Dioptrarum Negotio, de quibus in Machinæ meæ Cœleſtis *Organographia* tractavi. Verum etiamſi *Cl. Hookius* & *Cl. Flamſteedius,* aliique plane aliter ſentiant, experientia tamen quotidiana me edocuit, atque etiamnum docet, rem longe aliter ſe habere in Magnis illis Organis, Quadrantibus ſcil. Sextantibus & Octantibus, imprimis Quadrantibus Azimuthalibus, aliiſque Quadrantibus Regulis conſtructis, quæ nempe adeo procliviter commoveri & inverti (dum examinantur Dioptræ Teleſcopicæ) imo nullo modo poſſunt, ut quidem Inſtrumenta illa trium quatuorve Pedum Perpendiculo conſtructa. Res cumprimis in eo conſiſtit, quod nullam plane Obſervationem ſuſcipere poſſint ſuis Dioptris Teleſcopicis, niſi prius denuo eas Examinent, & Rectificent; in quo tamen Examine varia via idque jugiter, utut ſtudioſiſſime illud ſuſcipias, hallucinari datur. Ad hæc, in Quadrantibus Azimuthalibus, Octantibus & Sextantibus, qua ratione Examen illud omni tempore, commode, & ſine magno Temporis Diſpendio inſtitui poſſit, profecto nondum capio.

Video etiam aliquos (inter quos *Cl. Flamſteedius* invenitur) tuliſſe jam de noſtris Obſervationibus, qualibus qualibus, judicium, priuſquam alias viderunt, examinarunt, vel quicquam de iis cognoverunt. Nolo quidem vanus eſſe rerum mearum Jactator, nec unquam imaginatus mihi fui, me in omni iſto Negotio, Reſtitutionis ſcil. Fixarum rem acu omnino tetigiſſe, aut tangere pro mea tenuitate poſſe: Sed hoc mihi penitus imaginor, ſi quidem totum illud Negotium ſuſcepiſſem Dioptris Teleſcopicis, mihi non ſolum plurimos Annos Examinibus terendos, ſed & ſpe, ſine dubio, varia via (qua de re hic non eſt differendi locus) cadendum fuiſſe. Exinde gratulor mihi, me ad ſententiam illam necdum tranſiiſſe, meaque me Methodo omnia perfeciſſe quicquid præſtitum Dei Beneficio fuerit. Quando vero Obſervationes habebimus 20 & 30 Annorum ſpatio continuatas utrinque, nimirum tum quæ Dioptris Teleſcopicis, tum quæ ſolummodo noſtris de Cœlo depromptæ fuerint, res omnino clarior futura eſt. Interea ſuo quilibet Ingenio fruatur, remque ſua ratione pro Libitu, tentet.

Why Celeſtial Objects appear greater when nigh the Horizon than when higher elevated; Examined by Mr. Will. Molineux. N. 187. *p.* 314. Apr. An. 1587.

VII. 1. It is well known that the mean apparent Magnitude of the Moon is 30 min. 30 ſec. we will take it *Numero Rotundo* to be 30 min. at a Full Moon in the midſt of Winter, and when ſhe's in the Meridian, and at her greateſt Northern Latitude, and conſequently the utmoſt that ſhe can be elevated in our Horizon: 'Tis as well known alſo, that when ſhe is in this Poſture, being looked upon by the naked Eye, ſhe appears (that we may accommodate all to ſenſible Meaſures) to be *Magnitudinis Pedalis,* about a Foot broad. But the ſame Moon being looked upon juſt as ſhe riſes, ſhe appears to be three or four Foot broad, and yet if with an Inſtrument we take her Diameter, both in one Poſture and the other, we ſhall find that ſtill ſhe ſhall be but 30 min. That this Matter of Fact is true, beſides the Authority of many Authors, I can aſſert that I have accurately tried it my ſelf, and I have ſo found it. One of the ways I proceeded was thus: I took a very good Teleſcope of about 6 Foot long, in the inward Focus of whoſe Eye-Glaſs I apply'd a very fine Lattice made of the ſingle Hairs of a Man's Head; then

looking with this at the Moon, when ſhe was juſt riſen, and looked extraordinary big, I obſerved what Number of the Squares of the Lattice were occupied by her Body; and then obſerving her again, when more elevated and free from all extravagant Greatneſs, I ſtill found the ſame Square of the Lattice poſſeſſed by her. This Way is equivalent to that now more uſed, of taking her Diameter by Mr. *Townley*'s Micrometers; but I have alſo tried and found the ſame thing by an accurate Sextant, taking the Diſtance of the Moon's oppoſite Limbs.

The celebrated *Des Cartes* attributes this Appearance rather to a deceived Judgment, than to any natural Affection of the Organ or Medium of Senſe: For the Moon (ſays he) being nigh the Horizon, we have a better Opportunity and Advantage of making an Eſtimate of her, by comparing her with the various Objects that incur the Sight, in its Way towards her; ſo that tho' we imagine ſhe looks bigger, yet 'tis a meer Deceit; for we only think ſo, becauſe ſhe ſeems nigher the Tops of Trees, or Chimneys, or Houſes, or a Space of Ground, to which we can compare her, and eſtimate her thereby; but when we bring her to the Teſt of an Inſtrument, that cannot be deluded or impoſed upon by theſe Appearances, then we find our Eſtimate wrong, and our Senſes deceived. Theſe Thoughts, methinks, are much below the accuſtomed Accuracy of the noble *Des Cartes*; for certainly if it be ſo, I may at any time increaſe the apparent Bigneſs of the Moon, tho' in the Meridian; for it would be only by getting behind a Cluſter of Chimneys, a Ridge of a Hill, or the Tops of Houſes, and comparing her to them in that Poſture, as well as in the Horizon; beſides, if the Moon be looked at juſt as ſhe is riſing from an Horizon determined by a ſmooth Sea, and which has no more Variety of Objects to compare her to, than the pure Air, yet ſhe will ſeem big, as if look'd at over the rugged Top of an uneven Town, or rocky Country. Moreover, all Variety of adjoyning Objects may be taken off, by looking through an empty Tube, and yet the deluded Imagination is not at all helped thereby.

Fig. 110. The famous *Thomas Hobbs* gives this Solution. Let the Point G be the Center of the Earth, and F the Eye on the Surface of the Earth; on the ſame Center G let there be ſtruck the two Arches E H, determining the Atmoſphere, and A D to repreſent that blue Surface in which we imagine the Fixed Stars, and let F D be the Horizon: Divide the Arch A D into three equal Parts by the Lines B F, C F; it is manifeſt that the Angle A F B is greater than the Angle B F C, and this again is greater than the Angle CFD. Wherefore (ſays he) to make the Angle C F D equal to the Angle C F B, the Arch C D muſt be greater than the Arch C B; and conſequently, that the Moon may in the Horizon appear under the ſame Angle, as when elevated, ſhe muſt cover a greater Arch, and therefore ſeem greater; that is, the Moon in the Meridian appearing under the Angle B F C, that ſhe may appear under an equal Angle in the Horizon, as ſuppoſe C F D, 'tis neceſſary that the Arch C D ſhould be greater than C B; and conſequently, tho' ſhe appear to ſubtend a greater Arch when in the Horizon than when elevated, yet ſhe appears under the ſame Angle; and all this without Refraction.

The Geometry of this Figure is moſt certainly true and demonſtrable. At this I quarrel not; but it makes no more in our preſent Difficulty than if nothing had been ſaid: For he has made the Circle G F, repreſenting the Earth, very large in Proportion to the Circle A D; and then indeed taking the Point F in the Earth's Surface, and by Lines from thence dividing the Angle A F D into whatever equal Parts, the intercepted Arches A B, B C, C D, ſhall be unequal. But if he had conſidered, that the Earth is, as it were, a Point in reſpect of the Sphere of the fixed Stars, nay the very annual Orbit of the Earth is almoſt imperceptible, he would have found that the Lines F B, F C, F D, muſt be all conceived as drawn from the Point G, and then equal Angles will intercept equal Arches, and equal Arches equal Angles: And ſo it happens (at leaſt beyond the Poſſibility of the Diſcovery of Senſe) to the Eye on the Surface of the Earth; ſo that his drawing his Lines ſo far from G as F is, and to another concentrick Circle ſo nigh as A D, deceived him in this Point.

The famous *Gaſſendus* has written four large *Epiſtles* on this Subject, the Subſtance of all which is, That the Moon being nigh the Horizon, and looked at through a more foggy Air, caſts a weaker Light, and conſequently forces not the Eye ſo much as when brighter; and therefore the Pupil does more enlarge itſelf, thereby tranſmitting a larger Projection on the Retina. In this Opinion I find he is not alone; for this Diſquiſition being lately revived by a *French Abbe*, he therein follows the Sentiment of *Gaſſendus*, with this Addition, That this contracting and enlarging of the Pupil cauſeth a different Shape in the Eye; an open Pupil making the Cryſtalline flatter, and the Eye longer, and the narrower Pupil ſhortening the Eye, and making the Cryſtalline more convex: The firſt attends our looking at Objects which are remote, or which we think ſo; the latter accompanies the viewing Objects nigh at hand. Likewiſe an open Pupil and flat Cryſtaline attends Objects of a more ſedate Light, whilſt Objects of more forcible Rays require a greater Convexity and narrower Pupil. From theſe Poſitions, the *Abbe* endeavoured to give an Account of our *Phænomenon*, as follows: When the Moon is nigh the Horizon, by Compariſon with interpoſed Objects, we are apt to imagine her much farther from us than when more elevated; and therefore (ſays he) we order our Eyes as for viewing an Object farther from us; that is, we ſomething enlarge the Pupil, and thereby make the Cryſtalline more flat: moreover, the Duskineſs of the Moon in that Poſture does not ſo much ſtrain the Sight; and conſequently the Pupil will be more large, and the Cryſtalline more flat. Hence a larger Image ſhall be projected on the Fund of the Eye, and therefore the Moon ſhall appear larger. And this Diſpoſition of the Eye that magnifies her, magnifies alſo the Diviſions of our foremention'd Lattice, and conſequently ſhe by her Body ſhall poſſeſs no more of the Diviſions than when ſhe ſeems leſs. Theſe two foremention'd Accidents, *viz.* The Moon's imaginary Diſtance and Duskiſhneſs, gradually vaniſhing as ſhe riſes, a different Species is hereby introduced in the Eye, and conſequently ſhe ſeems gradually leſs and leſs, till again ſhe approaches nigh the Horizon. Theſe two Opinions of *Gaſſendus* and the *Abbe* being ſo near a-kin, I ſhall conſider them both together; and firſt I aſſert, That a wider or narrower Aperture of the Pupil increaſes not, neither diminiſhes the Projection on the Retina. I know, *Honoratus Faber* in his *Synopſis Optica* endeavours to prove the clear contrary to *Fig.* 111. this my Aſſertion, and that after this manner. A B is an Object, E F the greater Aperture of the Pupil, admitting the Projection K I on the Retina, whereas the leſſer Aperture C D, admits only the Projection G H; but G H is leſs than K I, wherefore a leſſer Aperture diminiſhes the Projection. I admire that any Man that undertook (as *Honoratus Faber*) to write of *Opticks* more accurately than all that went before him, ſhould be guilty of ſo very groſs an Errour; and I do more admire, that the celebrated *Gaſſendus*, and with him the noble *Hevelius*, ſhould be of the ſame Opinion: For tho' the aforeſaid Demonſtration hold moſt certainly true in direct Projections, as in a dark Room with a plain Hole; yet it will not hold in Projections made by Refraction, as it is in thoſe on the Retina in the Eye, by means of the Cryſtalline, *Fig.* 112. and other Coats and Humours of the Eye. For let A B be a remote Object, and E F the Cryſtalline at its large Aperture, projecting the Image I M on the Retina. Let then C D be the leſſer Aperture of the Pupil before the Cryſtalline: I ſay, the Image I M ſhall be projected as large as before; for the Cone of Rays E A F conſiſts partly of the Cone of Rays C A D, therefore where the former E A E is projected, the latter C A D, as being a part of the former, ſhall be projected alſo. So that no more is effected by this narrow Aperture, but that the Sides of the radiating Cones are intercepted, and conſequently the Point I ſhall be affected with leſs Light, but it ſhall ſtill be in the ſame Place: What is ſaid of that Cone and that Point, may be ſaid of all other Cones and other Points of the Object. From hence appears, *Firſt*, The Invalidity of the Account given of the Moon's Appearance by *Gaſſendus* from this Reaſon. *Secondly*, The Reaſon appears why a Teleſcope's leſſer or greater Aperture, makes no Difference in the Angle it receives: For imagine E F to be an Object-Glaſs of a Teleſcope, and 'tis plain. *Thirdly*, 'Tis evident why a greater or leſs Aperture on a Teleſcope ſhould make the Objects appear lighter or darker, for thereby more or leſs Rays are admitted to determine on the Projection of each Point. But all this by the By: And this is ſufficient for a Confutation of *Gaſſendus* and *Faber*. But our forementioned *Abbe* ſuperadds to a greater or leſſer Aperture of the Pupil, as a neceſſary Conſequent, a greater and leſſer Convexity of the Cryſtalline, as alſo a Lengthening and Shortening the Tube of the Eye. And this I muſt confeſs would do ſomething, if we find it true in our Caſe; and this let us try. *Firſt*, (ſays he) The Duskiſhneſs of the Moon nigh the Horizon admits the Pupil to enlarge itſelf, the Cryſtalline to flatten, and the Eye to lengthen: But what if we change our Object, and inſtead of the Moon take the Diſtance between ſome of the Fix'd Stars (as ſuppoſe thoſe of *Orion*'s *Girdle*) we ſhall find the ſame *Phænomenon* in them, and yet I hope neither he nor *Gaſſendus* will aſſert, that they at one time ſtrain the Eye more than at another, or that at any time their Fulgur ſtrains the Eye at all; if he do, let him take Stars of the leſſer Magnitudes, nay even thoſe that can but juſt be perceiv'd, and then he will be convinc'd: Or let him conſider whether this will hold in looking at the Sun through very dark Glaſſes, which render the Sight thereof as inoffenſive to the Eye, as that of a Green Field; but perhaps he will then ſay, that this other Reaſon holds, which is, *Secondly*, That the greater imaginary Diſtance at which we think the Moon near the Horizon, than when more elevated, makes us contemplate her as if really ſhe was ſo, *viz.* with ample Pupils, *&c.* But this I have ſufficiently overthrown in my Remarks againſt *Des Cartes:* Therefore I paſs it over, only ſubjoyning, that if there were any thing in this Surmiſe, methinks the horizontal Moon ſhould be fanſied nigher to us than farther from us; for if we are for trying natural Thoughts, let us take Children to determine the Matter, who are apt to think, that could they go to the Edge of that Space that bounds their Sight, they ſhould be able (as they call it) to touch the Sky; and conſequently the Moon ſeems then rather nigher to us than farther from us.

After I had writ thus far, I accidentally caſt my Eye upon *Riccioli*'s Treatiſe of *Refraction*, at the End of his Second Volume of *Almageſt*, *Lib.* 10. *Sect.* 6. *Cap.* 1. *Queſt.* 13. wherein he ſpeaks of our preſent Difficulty; but to my Wonder I find him aſſert, That he and Father *Grimaldi* had often taken the horizontal Sun and Moon's Diameters by a Sextant, when to the naked Eye they appeared very large (*Grimaldus* directing his Sight to the left Edge, and *Ricciolus* to the right) and that even by the Inſtrument they always found the Diameters greater than when more elevated, the Sun often ſubtending an Angle of almoſt a Degree, and frequently 45 Minutes, the Moon alſo 38 or 40 Minutes. This is downright contrary to the Matter of Fact which I have before alledged, and directly repugnant to the Matter of Fact aſſerted by the forementioned French *Abbe:* Whether of us be in the right, I leave to accurate Experiment to determine, and ſubmit the Whole to the Deciſion of the illuſtrious *Royal Society*. Only give me leave to add one Word againſt *Riccioli*, for had his Experiments been accurately proſecuted, he ſhould have tried them when the horizontal Moon had look'd ten times more large in Diameter than ordinary; and then if it be true, that even by an Inſtrument ſhe will be found proportionally broader than really ſhe ſhould, ſubtend an Angle of 300 Min. or 5 Deg. for very often I have ſeen the Moon when ſhe appeared ten times broader than ordinary, which the ſmall Addition of 8 or 10 Min. to her uſual Diameter will never cauſe.

This Phænomenon conſider'd, by Dr. Wallis. *Ibid. p.* 323.

2. I diſcourſed of this Appearance near 40 Years ago with Mr. *Foſter*, then *Profeſſor of Aſtronomy* in *Greſham*-College, who did then aſſure me (from his own Obſervation I ſuppoſe) that the apparent Magnitude taken by Inſtruments (however the Fancy may apprehend it) is not greater at the Horizon than when higher. Mr. *Caſwell* affirms the ſame thing; and I do not doubt but the thing is ſo: For though Refraction near the Horizon alters the Altitude of the thing ſeen; yet it cannot alter the Azimuth at all. For ſince this equally reſpects all Points of the Horizon; let the Refraction be what it will, the whole Horizon can be but a Circle: So that there is no room for the Breadth of a thing (as to the Angle at the Eye) to be made greater, whatever its Tallneſs may (the Refraction not equally affecting all Parts in the Circles of Altitude). Nor is there any Reaſon, why this ſhould

rather thruſt the other, than that the other thruſt this, out of Place. Whereas, in the Altitude, it is otherwiſe: For while what is near the Horizon is enlarged, that which is further off is thereby contracted; which as to the Azimuth, or Horizontal Poſition, cannot be.

Suppoſing then that the Sun's apparent Horizontal Diameter, taken by Inſtrument, is the ſame near the Horizon, as in a higher Poſition, I take its imaginary Greatneſs, which is fanſied near the Horizon, to be only a Deception of the Eye, or rather the Imagination from the Eye.

For ſure it is, that the Imagination doth not eſtimate the Greatneſs of the Object ſeen, only by the Angle which it makes at the Eye; but, by this compared with the ſuppoſed Diſtance. True it is, that *Cæteris paribus*, we judge that to be the greater Object, which makes at the Eye the greater Angle; but not ſo, if apprehended at different Diſtances.

For if through a Caſement (or leſſer Aperture) we ſee a Houſe at 100 Yards Diſtance; this Houſe (though ſeen under a leſs Angle) doth not to us ſeem leſs than the Caſement through which we ſee it, (or this greater than that, becauſe it makes at the Eye the greater Angle:) But the Imagination makes a comparative Eſtimate from the Angle and Diſtance jointly conſider'd.

So that of two things ſeen under the ſame or equal Angles, if to one of them there be ought which gives the Apprehenſion of a greater Diſtance, that to the Imagination will appear greater. Now, ſure it is, that one great Advantage for eſtimating the Diſtance of a thing ſeen, is from the Variety of intermediate Objects between the Eye and the thing ſeen. For then the Imagination muſt allow room for all theſe things.

Now when the Sun or Moon is near the Horizon, there is a Proſpect of Hills, and Valleys, and Plains, and Woods, and Rivers, and Variety of Fields and Incloſures, between it and us; which preſent to our Imagination a great Diſtance capable of receiving all theſe: Or if it ſo chance that (in ſome Poſition) theſe Intermediates are not actually ſeen; yet having been accuſtomed to ſee them, the Memory ſuggeſts to us a View as large as is the viſible Horizon.

But when the Sun or Moon is in a higher Poſition; we ſee nothing between us and them, (unleſs perhaps ſome Clouds) and therefore nothing to preſent to our Imagination ſo great a Diſtance as the other is.

And therefore, though both be ſeen under the ſame Angle, they do not appear (to the Imagination) of the ſame Bigneſs, becauſe not both fanſied at the ſame Diſtances: But that near the Horizon is judged bigger (becauſe ſuppoſed farther off) than the ſame when at a greater Altitude.

'Tis true, that as to ſmall and middling Diſtances (beſides this Eſtimate from Intermediates) the Eye hath a Means within itſelf to make ſome Eſtimate of the Diſtance. As, when we already know the Bigneſs of a thing ſeen, to which we have been accuſtomed; as, a Man, a Tree, a Houſe, or the like: If ſuch thing appear to us under a ſmall Angle, and indiſtinct, and faintly coloured, the Imagination doth allow ſuch Diſtance as to make ſuch a thing ſo to appear. And if this, through a Proſpective-Glaſs, be repreſented to us under a bigger Angle, and more diſtinct, it is accordingly apprehended as ſo much nearer. But the Caſe is otherwiſe, when we do not, by the known Bigneſs, judge the Diſtance; but, by the ſuppoſed Diſtance, judge of the Bigneſs, as in the Caſe before us. And accordingly, different Perſons, according to different fanſied Diſtances, judge very differently.

Again; In our two Eyes (when the Object is ſeen by both) there is yet another Means of eſtimating how far off it is. (And it is this by which we judge of Diſtances.) Namely, there are, from the ſame Object, two different viſual Cones, terminated at the two Eyes; whoſe two Axes contain, at the Object, different Angles, according to different Diſtances; an acuter Angle at a great Diſtance, and more obtuſe when nearer.

Now, that ſuch Object may be ſeen by both Eyes clearly; it is requiſite that the Eyes be put in ſuch a Poſition, as that the Sight of each Eye receive the reſpective Axis at Right Angles; which requires a different Poſition of the two Eyes, according to the different Diſtance of the Object; as will manifeſtly appear, if we look, with Attention, on a Finger (or other ſmall Object) at 2 or 3 Inches Diſtance from the Eye; and then upon another like Object at 3 or 4 Yards beyond it, (and this alternately ſeveral times). For 'twill be manifeſt, that while we look intently on the one, we do not ſee the other, (or but confuſedly) though both be juſt before us. And, as we change our View, from the one to the other, we manifeſtly feel a Motion of the Eyes (by their Muſcles) from one Poſture to another.

And according to the different Poſture in the Eyes, requiſite to a clear Viſion by both, we eſtimate the Diſtance of the Object from us.

And hence it is, that they who have loſt the Sight of one Eye, are at a great Diſadvantage, as to eſtimating Diſtances, from what they could do while they had the Uſe of both.

But now when the Diſtance grows ſo great, as that the Poſition of theſe viſual Axes become parallel, or ſo near to parallel as not to be diſtinguiſhable from it: This Advantage is loſt, and we can thenceforth only conclude, that it is far off; but not how far. Hence it is, that our View can make no Diſtinction of the Moon's Diſtance from that of the other Planets, or even of the fixed Stars: But they ſeem to us as equally remote from us; though we otherwiſe know their Diſtances from us to be vaſtly different, Becauſe the Parallax (as I may ſo call it) from the different Poſition of the two Eyes, is quite loſt, and undiſcernable in Diſtances much leſs than the leaſt of theſe.

So that, though as to ſmall Diſtances, we may make ſome Eſtimate from the known Magnitude of the Object; and as to middling Diſtances, from the Parallax (as I may call it) ariſing from the Interval of the two Eyes: Yet even this latter will hardly reach beyond, if ſo far as the viſible Horizon; and all beyond it is loſt. And therefore there being nothing left to aſſiſt the Fancy in eſtimating ſo great a Diſtance, but only the intermediate Objects: Where theſe Intermediates appear to the Eye, (as when the Sun or Moon are near the Horizon) the Diſtance is fanſied greater, than where they appear not (as when farther from it), and conſequently (tho' both under the ſame or equal Angles) that near the Horizon is fanſied the greater: And this I judge to be the true Reaſon of that Appearance.

An Experiment of the Refraction of the Air; by Mr. Lowthorp. N. 257. *p.* 329. Mar. An. 1699.

Fig. 113.

VIII. We took a Cylinder of Caſt-Braſs, A B C D, and cut one End of it C D, perpendicular to the Axis *a c x*, the other End A B, enclined to it at an Angle of about 27 deg. 30 min. and therefore the Perpendicular to this enclining Plain *p c*, and the Axis of the Cylinder *a c x*, comprehended an Angle *p c a*, of about 62 deg. 30 min. Theſe Ends were ground very true upon a Glaſs-Grinder's Braſs Tool, and each of them was compaſs'd about with a narrow Feril of thin Braſs *b b b b*. Into the upper Side of the Cylinder, at E, was ſoldered the Braſs-Pipe E F, and into the under Side, at G, the other Braſs-Pipe G H; the former of theſe Pipes being about 3 Inches long, and the latter 6 Inches. Upon the Plate *d d d*, were fix'd two other Plates, L L, perpendicular to it and parallel to each other. Each of theſe two Plates had an Arch of a Circle (whoſe Diameter was equal to that of the Cylinder) cut out of its upper Edge; ſo that when the Pipe G H was let thro' a Hole near the Middle of the Plate *d d d*, the Cylinder fell into the Arches; and being faſtned there with Solder, the Axis *a c x* laid parallel to the Plate *d d d*, and about an Inch and a half above it. The perpendicular End of the Cylinder, D C, was cloſed with an Object-Glaſs of a 7 ½ Foot Teleſcope *o o*, and the enclining End A B, with a well-poliſh'd flat Glaſs, *f f*; which was carefully choſen to tranſmit the Object diſtinct enough, notwithſtanding its Obliquity to the viſual Rays. The Ferils were filled with Cement round about the Edges of the Glaſſes, which laid flat, and every where touch'd the ſmooth Ends of the Cylinder, that they might firmly ſupport the Weight and Preſſure of the excluded Air.

Fig. 114.

Inſtead of a Ciſtern (as in the *Torricellian* Experiment) we made uſe of the inverted Syphon of Braſs M N O, ſolder'd to the Plate *g g g*. One of the Sides M N, ſtood perpendicular to the Plate *g g g*, and the other Side N O, enclined to it, and was ſupported near the upper End O, with a little Piece of Braſs *k k*.

Fig. 115.

We then placed the Cylinder upon a Table, which was well faſtned to a firm Floor: The Pipe G H, was let through a Hole in the Top of the Table; and the Plate *d d d*, was nailed down to it: The Tube of the Teleſcope *s s s*, with the Eye-Glaſs in it, was applied to the Object-Glaſs, and a Hair fix'd at *x*, the common Focus of both Glaſſes, in the Axis of the Cylinder continued to it. Upon the Floor (under the Cylinder) we nailed the Plate *g g g*, with the inverted Syphon upon it, and joyned M to H, by the Inſertion of the Glaſs-Tube T. The Joints were very carefully cloſed with Cement, and then covered over with Pieces of a Bladder wrap'd hard with ſtrong Thread. There was alſo a Bladder tied below each Joint at *m*, and when it was filled with Water it was tied about it at *n*; ſo that no Air could come to the Cement, to inſinuate itſelf through its Pores or Fiſſures, if any happen'd to be left uncloſed.

It will not (I hope) be thought more than neceſſary, that in this Account of the *Apparatus*, I have mention'd ſo many minute Circumſtances; for we found it difficult enough to exclude the Air, and almoſt impoſſible to diſcover the very little Holes through which ſo ſubtile a Fluid would freely enter, and poſſeſs the Spaces, deſerted by the *Subſiding Mercury*. But with all this Precaution, the Experiment ſucceeded at laſt, as I wiſh'd; after this manner:

We placed the Object *a* (which was a black Thread faſten'd in a liitle Frame over a Piece of white Paper) in the Axis of the Cylinder *x c a*: We fill'd the Pipes and Cylinder with *Mercury*; and having ſtopp'd the upper End of the Pipe at F, with the little Iron-Stopple K, and cloſed it, as the upper Part of the Tube and other Joints, we let the *Mercury* run out gently at O (into the Bladder *u*) till it remain'd ſuſpended at the uſual Height (as in the *Barometer*) leaving the upper Part of the Tube and the Cavity of the Cylinder between the Glaſſes *o o*, and *f f*, void of Air. We then ſaw the Object, which before appear'd in the Axis at *x*, raiſed conſiderably above it; and we reduced it to appear again at *x*, by removing it from *a* to *α*. The Axis therefore of the viſual Ray (which was alſo the Axis of the Cylinder) *x c a*, falling perpendicularly on the void Space, paſs'd through it without any Refraction: But emerging obliquely into the Air, it was refracted towards the Perpendicular *p c*, and received a new Direction to *α*. And therefore the Diſtance *a α*, ſubtended the Angle of Refraction *a c α*; all which we meaſur'd, and found as follows; *viz.*

	Inches	*Dec. Parts*
The Heighth of the Object above the Axis, or the unrefracted viſual Ray *a α* - - - - - - - -	000	425
The Diſtance of the Object from the refracting Plain *α c*, about 51 Feet, or - - - - - - -	612	000

	Deg.	*Min.*	*Sec.*
Therefore the Angle of Refraction *a c α*, was ——	00	02	23
The Angle of Emerſion *p c a*, (by the Conſtruction of the Cylinder) was - - - - - - - - -	62	30	00
Therefore the Angle of Incidence *p c α* = *p c a* + *a c α*, was - - - - - - - - - - - - -	62	27	37

And therefore *univerſally*, (according to the known Laws of Refraction)

The Sines of the Angles of Incidence being - - -	100000
The Sines of the Angles of Emerſion are - - - -	100036
And the refractive Power of the denſe Air - - - -	36

By the *refractive Power* of a pellucid Body, I mean that Property in it whereby the oblique Rays of Light are diverted from their direct Courſe, and which is meaſured by the proportional Differences (always obſerved) between the Sines of the Angles of Incidence and Emerſion.

This Property is not always proportional to the Denſity (at leaſt not to the Gravity) of the refracting Medium: For the refractive Power of Glaſs to that of Water is as 55 to 34, whereas its Gravity is as 87 to 34; that is, the *Squares* of their refractive Powers are (very near) as their reſpective Gravities. And there are ſome Fluids, which, tho' lighter than Water, yet have a greater Power of Refraction: Thus the refractive Power of Spirit of Wine (according to Dr. *Hook*'s Experiment, *Microgr. Obſ.* lviii. *p.* 220.) is to that of Water, as 36 to 23; and its Gravity *reciprocally* as 33 to 36, or 36$\frac{1}{2}$. But the refractive Powers of Air and Water ſeem to obſerve the *ſimple* Proportion of their Gravities *directly*, as I have compared them in the following Table. The Numbers there expreſſing the Refraction of Water are taken from the Mean of Nine * Experiments, made at ſo many ſeveral Angles of Incidence, *Jan.* 25. 164$\frac{0}{1}$. by Mr. *Gaſcoigne*, (the ingenious firſt Inventor of the Micrometer, and the Ways of meaſuring Angles by Teleſcopes) and thoſe of Air are produced by the preceding Experiment.

	Water.	*Air.*
The (aſſum'd) Sines of the Angles of Incidence on the Void from - - - - - - - - -	100000	100000
The Sines of the correſpondent Angles of Emerſion out of - - - - - - - - - - - -	134400	100036
The refractive Power of - - - - - - ——	34400	36
The ſpecifick Gravity (if as 900 to 1 at the Time of the Experiment) of - - - - -	34400	38
Or (if as 850 to 1) of - - - - - - - ——	34400	40

From hence it ſeems very probable, that their reſpective Denſities and refractive Powers are in a juſt *ſimple* Proportion. And if this ſhould be confirmed by ſucceeding Experiments, made at different Angles of Incidence, and with Cylinders continuing exhauſted through ſeveral Changes of the Air, it would be more than probable that the refractive Powers of the Atmoſphere are every where, and at all Heights above the Earth, proportional to its Denſities and Expanſions: And then it would be no difficult Matter to trace the Light through it, ſo as to terminate the Shadow of the Earth, and (together with proper Expedients for meaſuring the Quantity of Light illuminating an opake Body) to examine at what Diſtances the Moon muſt be from the Earth to ſuffer Eclipſes of the obſerved Duration.

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* *I am indebted for theſe Experiments to the Reverend and very* Accurate *Aſtronomer Mr.* Flamſteed; *who copied them, together with many other Obſervations and ſeveral Paſſages relating to them, from Mr.* Gaſcoigne's *Letter to Mr.* Crabtree: *They were happily preſerved, in the Time of our Civil War, by the late Sir* Jonas Moor, *and Mr.* Chr. Townley; *and they are now in the Hands of Mr.* Rich. Townley *of* Townley *in* Lancaſhire, *by whom they were imparted ſome Time ago to Mr.* Flamſteed.

IX. Give

To find the Parallax of the Fixed Stars; by *Dr.* Wallis, *to* *Mr.* William Molyneux. July, An. 1693. N. 202. *p.* 844.

IX. Give me leave to ſuggeſt a Speculation, which hath been in my Thoughts theſe Forty Years or more; but I have not had the Opportunity of reducing it to Practice: It is concerning the Parallax of the fixed Stars, as to the Earth's annual Orb.

Galileo complains of it a great While ſince (in his *Syſtema Coſmicum*) as a thing not attempted to be obſerved with ſuch Diligence as he could wiſh, and I doubt we have the ſame Cauſe of complaining ſtill. I know that Dr. *Hook* and Mr. *Flamſteed* have attempted ſomewhat that Way, but have deſiſted before they came to any thing of Certainty. What hath been done to that purpoſe Abroad I know not.

Galileo hath ſuggeſted divers Things conſiderable in order to it; as, the Times of Obſervation, the Stars to be obſerved, and the Manner of obſerving them, which yet I doubt is not practicable. That which occurred to my Thoughts upon theſe Conſiderations, was to this purpoſe: That ſome circumpolar Stars (nearer to the Pole of the Equator than is our *Zenith*, and not far from the Pole of the Zodiack) ſhould be made choice of for this purpoſe. And in caſe the meridional Altitude be diſcernably different at different Times, ſo will alſo be their utmoſt Eaſt and Weſt Azimuth, which may be better obſerved than their Riſing or Setting: And this will not be obnoxious to the Refraction, as is the meridional Altitude (for though the Refraction do affect the Altitude, yet not the Azimuth at all); and we may here have choice of Stars for the purpoſe; which, in Obſervations from the Bottom of a Well, we cannot have; being there confined to thoſe only which paſs very near our Zenith, tho' very ſmall Stars.

I would then take for granted, as a thing at leaſt very probable, that the fixed Stars are not all (as was wont to be ſuppoſed) at the ſame Diſtance from us, but the Diſtance of ſome vaſtly greater than of others; and conſequently, though as to the more remote, the Parallax may be undiſcernable, it may, perhaps, be diſcernable in thoſe that are nearer to us.

And thoſe we may reaſonably gueſs (though we are not ſure of it) to be neareſt to us, which to us do appear biggeſt and brighteſt, as are thoſe of the *Firſt* and *Second* Magnitude; and there are at leaſt of the *Second* Magnitude pretty many not far from the Pole of the Ecliptick, (as that in particular in the *Shoulder* of the *Leſſer Bear*:) And in caſe we fail in one, we may try again and again on ſome other; which may chance to be nearer to us than what we try firſt. And Stars of this Bigneſs may be diſcerned by a moderate Teleſcope even in the Day-time; eſpecially when we know juſt where to look for them.

The Manner of Obſervation, I conceive, may be thus. Having firſt pitched upon the Star we mean to obſerve, and having then conſidered (which is not hard to do) where ſuch Star is to be ſeen in its greateſt Eaſt or Weſt Azimuth; it may be then convenient to fix (very firm and ſteadily on ſome Tower, Steeple, or other high Edifice, in a convenient Situation) a good Teleſcopick Object-Glaſs, in ſuch Poſition as may be proper for viewing that Star. And at a due Diſtance from it near the Ground, build on purpoſe (if already there be not any) ſome little Stone-Wall, or like Place, on which to fix the Eye-Glaſs, ſo as to anſwer that Object-Glaſs: And having ſo adjuſted it, as through both to ſee that Star in its deſired Station (which may beſt be done while the Star is to be ſeen by Night in ſuch Situation, near the Time of one of the Solſtices) let it be there fixed ſo firmly, as not to be diſturbed (and the Place ſo ſecured, as that none come to diſorder it) and care be taken ſo to defend both the Glaſſes, as not to endangered by Wind and Weather. In which Contrivance, I am beholden to Mr. *John Caſwell*, M. A. of *Hart-Hall* in *Oxford*, for his Advice and Aſſiſtance, with whom I have many Years ſince communicated the whole Matter.

This Glaſs being once fixed (and a Micrometer fitted to it, ſo as to have its Threads perpendicular to the Horizon, to avoid any Inconvenience which might ariſe from Diverſity of Refraction, if any be) the Star may then be viewed from Time to Time (for the following Year or longer) to ſee if any Change of Azimuth can be obſerved.

This I thought fit to recommend to your Conſideration, who do ſo well underſtand Teleſcopes, and the Managery of them: But when I ſuggeſt (as a convenient Star for this purpoſe) the *Shoulder* of the *Leſſer Bear* (as being the neareſt to the Pole of the Zodiack of any Star that is of the Firſt or Second Magnitude) I do not confine you to that Star; but (without retracting that) ſuggeſt another; namely, the *Middle Star*, in the *Tail* of the *Great Bear*, which (though ſomewhat further from the Pole of the Zodiack) is a brighter Star than the other, and may be nearer to us.

But I do it principally upon this Conſideration; namely, That there is adhering to it a very ſmall Star, (which the *Arabs* call *Alcor*, of which they have a proverbial Saying, when they would deſcribe a ſharp-ſighted Man, That he can diſcern the *Rider* on the *Middle Horſe* of the *Wayn*: And of one who pretends to ſee ſmall Things, but overlooks much greater, *Vidit Alcor at non Lunam Plenam*:) Which *Hevelius* in his Obſervations finds to be diſtant from it about nine Minutes, and five or ten Seconds: So that beſides the Advantage of diſcovering the Parallax of the greater Star, if diſcernable; the Difference of the Parallax of that and of the leſſer Star (being both within the Reach of a Micrometer) may do our Work as well. For if that of the greater Star be diſcernable, but that of the leſſer be either not diſcernable or leſs diſcernable, their different Diſtances from each other at different Times of the Year may perhaps (without farther *Apparatus*) be diſcerned by a good Teleſcope of a competent Length, furniſhed with a Micrometer, if carefully preſerved from being diſordered in the Intervals of the Obſervations; and diſcover at once, both that there is a Parallax, and that the Fixed Stars are at different Diſtances from us; wherein, that I may not be miſtaken, my Meaning is not that the Inſtrument or Micrometer ſhould be removed for the obſerving of the Leſſer Star, but that (when the Azimuth of the greater Star is taken) by a Micrometer (conſiſting of divers fine Threads parallel and tranſverſe) may (at the ſame time) be obſerved the Diſtance of the two Stars, each from other, in that Poſition (both being at once within the reach of the Micrometer;) which Diſtance (the Inſtrument remaining unmoved) if it be found (at different Times of the Year) not to be the ſame, this will prove that there is a different Parallax of theſe two Stars.

This latter Part of the Obſervation (of their different Diſtances at different Times) I ſuggeſt, as more eaſily practicable, though not ſo nice as the former. For it may be done, I think, without any further *Apparatus* there than a good Teleſcope of ordinary Form, furniſhed with a Micrometer, (this being carefully kept unvaried during the Interval of theſe Obſervations.) And if this Part only of the Obſervation (without the other) be purſued; it matters not though the two Obſervations (near the two Solſtices) be, one at the Eaſtern, the other at the Weſtern, Azimuth (whereby both may be taken in the Night-time) for the Diſtance muſt (at both Azimuths) be the ſame. If, after obſerving the Azimuth of the greater Star, it be neceſſary to move the Micrometer for meaſuring its Diſtance from *Alcor*, that may be done another Night, (and it is not neceſſary to be done at one Obſervation) for that Diſtance cannot be diſcernably varied in a Night or two.

Concerning the Diſtance of the Fixed Stars; by Mr. Fr. Roberts. N. 209. p. 101.

X. Since the *Pythagorean Syſtem* of the World has been revived by *Copernicus*, (and now by all Mathematicians accepted for the true one) there ſeemed Ground to imagine, that the Diameter of the Earth's annual Courſe (which, according to our beſt Aſtronomers, is at leaſt 40000 times bigger than the Semidiameter of the Earth) might give a ſenſible Parallax to the Fixed Stars, and thereby determine their Diſtance. But there are ſome Conſiderations which make us ſuſpect that even this Baſis is not large enough for that purpoſe.

M. *Hugens* (who is very exact in his Aſtronomical Obſervations (tells us, He could never diſcover any viſible Magnitude in the Fixed Stars, though he uſed Glaſſes which magnified the apparent Diameter above 100 times.

Now, ſince in all likelihood the Fixed Stars are Suns, (perhaps of a different Magnitude) we may as a reaſonable Medium preſume they are generally about the Bigneſs of our Sun.

Let us then (for Example) ſuppoſe the *Dog-Star* to be ſo. The Diſtance from us to the Sun being about 100 times the Sun's Diameter, it is evident, that the Angle under which the *Dog-Star* is ſeen in Mr. *Hugen*'s Teleſcope, muſt be near the ſame with the Angle of its Parallax to the Sun's Diſtance, or Semidiameter of the Earth's annual Courſe; ſo that the Parallax to the whole Diameter, can be but double ſuch a Quanty, as even to Mr. *Hugens*'s nice Obſervation is altogether inſenſible.

The Diſtance therefore of the Fixed Stars ſeems hardly within the reach of any of our Methods to determine: But from what has been laid down, we may draw ſome Concluſions that will much illuſtrate the prodigious Vaſtneſs of it.

1. That the Diameter of the Earth's annual Orb (which contains at leaſt 160 Millions of Miles) is but as a Point in Compariſon of it; at leaſt it muſt be above 6000 times the Diſtance of the Sun: For if a Star ſhould appear through the aforeſaid Teleſcope half a Minute broad, (which is a pretty ſenſible Magnitude) the true apparent Diameter would not exceed 18‴, which is leſs than the 6000th Part of the apparent Diameter of the Sun, and conſequently the Sun's Diſtance not the 6000th Part of the Diſtance of the Star.

2. That could we advance towards the Stars 99 Parts of the whole Diſtance, and have only 100th Part remaining, the Stars would appear little bigger to us than they do here: For they would ſhew no otherwiſe than they do through a Teleſcope, which magnifies an hundred-fold.

3. That at leaſt 9 Parts in 10, of the Space between us and the Fixed Stars can receive no greater Light from the Sun, or any of the Stars, than what we have from the Stars in a clear Night.

4. That Light takes up more Time travelling from the Stars to us, than we in making a *Weſt-India* Voyage, (which is ordinarily performed in ſix Weeks:) That a Sound would not arrive to us from thence in 50000 Years, nor a Cannon-Bullet in a much longer Time. This is eaſily computed, by allowing (according to Mr. *Newton*) 10 Minutes for the Journey of Light from the Sun hither, and that a Sound moves above 1300 Foot in a Second.

The Places of the chiefeſt Fixed Stars, according to the beſt Antient Obſervers; *by Dr.* Edward Bernard, *to* D. Robert Huntington. N. 158. *p.* 567.

XI. Inter Codices tuos Arabicos in *Muſæo Mertonenſi* (numeras autem plus quadraginta Doctrinæ & Obſervationis Sideralis refertis) in *Tabulis Ilchanicis* ſtudio *Perſæ* præclari *Chonga Naſiredini Tuſii* factum offendi, ut præcipui Honoris Stellas Fixas, ſecundum variorum Aſtronomorum varia Obſervata, qua Latæ (ut loquuntur) quave illæ Longæ in Cælo deprehendebantur, brevis Pagina repreſentaret. Hoc ergo Canonion, partim ex tuo Peculio, ut vides, partim ab aliis, adauxi. Non equidem quaſi Rem Magnam mihi viderer adeò feciſſe; verum ut noſtri Homines ſenſum

ac Opinionem aliquam concipiant Aſtronomiæ Orientis, ubi Ars ea primum nata conſtat. Multa ſane commendant Aſtronomiam Orientalium; Felicitas quidem & Claritas Regionum, ubi Obſervatum; Machinarum Granditas & Accuratio, Quantas plerique noſtri credere nolunt Cœlo ipſos obvertiſſe; Contemplantium inſuper Numerus & Scribentium, decuplo major quam apud Græcos Latinoſque celebratur; Adde decuplo Plures Munificentiores, ac Potentiores Principes, qui viris boni Ingenii ſumtus & Arma Cœleſtia dederunt. Quid vero Aſtronomi Arabum in *Cl. Ptolomeo*, magno Conſtructore Artis Cœleſtis, injuria nulla reprehenderint; quam illi ſolicite Temporis Minutias, per Aquarum Guttulas, Immanibus Sciotheris, imo (mirabere) *Fili Penduli Vibrationibus*, jampridem diſtinxerint & menſurarint; quam etiam perite & accurate verſaverint in magno molimine Ingenii Humani, de Ambitu Intervalloque binorum Luminarium & noſtri Orbis, una Epiſtola narrare non debet.

Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

			Magnit.	*Cl. Ptolemeo* in Magna Syntaxi. A. C. 137.						
				Long.				Lat.		
				S.	°	′		°	′	
1	34	*Ultima* Fluvii	1	0	10	30	al. 50	53	30	m.
2	12	*Lucida* Cathedræ Caſſiopeiæ	3	0	7	50		51	40	b.
3	12	Perſei *Caput Meduſæum*. Algol.	2	0	29	40	al. 20	23	0	b.
4	14	*Oculus* Tauri *Auſtrinus*.	1	1	12	40	al. 20	5	10	m.
5	35	Orionis *Pes Siniſter*. Rigel.	1	1	20	50		31	30	m.
6	3	*Capella* Aurigæ.	1	1	25	0		22	30	b.
7	2	Orionis *Humerus Dexter*.	1	2	2	0		17	0	m.
8	1	Sirius. Alhabor.	1	2	17	40	al. 20	39	10	m.
9	2	Procyon. Algomeiſa.	1	2	29	30	al. 10	16	10	m.
10	8	*Cor* Leonis. Regulus	1	4	2	30	al. 10	0	10	b.
11	14	*Spica* Virginis.	1	5	26	40		2	0	m.
12	23	Arcturus.	1	5	27	0		31	30	b.
13	8	Antares. *Cor* Scorp.	2	7	12	40	al. 20	4	0	m.
14	1	*Caput* Ophiuchi.	3	7	24	50		36	0	b.
15	1	*Lucida* Lyræ.	1	8	17	20		62	0	b.
16	3	Aquilæ *Lucida*.	2	9	3	50		29	10	b.
17	5	*Lucida in Cauda* Cygni.	2	10	9	10		60	0	b.
18	3	*Lucid. in Crure Dext.* Pegaſi. Seat.	2	11	2	10		35	0	b.
19	2	*Ala* Pegaſi *Ultima*. Algenib.	2	11	12	10		12	30	b.
20	7	Perſei *Lucidum Latus*.	2	1	4	50		30	0	b.
21	27	*Cauda* Leonis.	1	4	24	30		11	50	b.
22	1	*Os* Piſcis Auſt. *in Aqua extrema*.	1	10	7	0		23	0	m.
23	44	Canopus.	1	2	17	10		75	0	m.

Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

	Ali Abolcaſimo. A.C. 938.					*Abdorahmano Sophio.* A.C. 964. *			*Ebnolalamo.* A. C. 980.				
	Long.			Latit.		Long.			Long.			Latit.	
	S.	°	′	°	′	S.	°	′	S.	°	′	°	′
1	0	11	50	53	30	0	12	53	0	16	42	53	30
2	0	20	20	51	45	0	20	32	0	24	9	51	45
3	1	12	17	22	45	1	12	22	1	15	1	22	45
4	1	24	35	5	15	1	25	22	1	28	40	5	15
5	2	2	1	31	4	2	2	32	2	5	55	31	20
6	2	6	49	22	50	2	7	42	2	10	50	22	50
7	2	13	16	16	45	2	14	42	2	17	21	16	45
8	2	29	30	39	20	3	0	22	3	3	40	39	20
9	3	10	40	16	0	3	11	12	3	15	45	16	0
10	4	14	40	0	15	4	15	12	4	18	45	0	15
11	6	8	23	2	6	6	9	22	6	12	33	2	6
12	6	8	50	31	12	6	9	42	6	12	55	31	12
13	7	24	35	4	24	7	25	22	7	28	40	4	24
14	8	6	53	36	0	8	7	32	8	11	2	36	0
15	9	0	40	61	45	9	0	2	9	4	45	61	45
16	9	15	53	29	12	9	16	32	9	20	58	29	14
17	10	20	4	59	36	10	21	52	10	24	9	59	36
18	11	14	0	31	10	11	14	52	11	18	54	31	10
19						11	24	52	11	24	52	12	30
20	1	16	15	30	8	1	17	32					
21	5	6	40	11	50	5	7	12					
22						10	18	12					
23						2	29	52					

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* Is cum *Ptolemæo* convenit κατὰ πλάτος.

Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

	Ex Canonibus Hacimicis Johanidæ Ægyptii. A. C. 996.					*Choagæ Naſirodino Tuſio in Tabulis Ilchanicis.* A. C. 1233.					*Ex Sultanicis Ologbeci.* A. C. 1437.				
	Long.			Latit.		Long.			Latit.		Long.			Latit.	
	S.	°	′	°	′	S.	°	′	°	′	S.	°	′	°	′
1	0	16	20	53	28	0	24	55	51	45	0	15	40	53	45
2	0	24	34	51	50	0	24	35	51	40	0	28	1	50	48
3	1	15	24	22	39	1	16	25	23	0	1	18	54	22	0
4	1	29	7	5	15	1	29	22	5	13	2	2	31	5	15
5	2	5	30	31	35	2	6	35	31	30	2	9	25	31	18
6	2	11	32	22	3	2	11	10	22	40	2	14	43	22	42
7	2	17	46	16	50	2	18	0	16	50	2	21	13	16	43
8	3	4	2	39	30	3	3	50	39	10	3	6	19	39	30
9	3	15	52	16	2	3	15	45	16	5	3	18	22	16	0
10	4	19	10	0	10	4	19	14	0	17	4	22	13	0	9
11	6	12	58	2	10	6	13	25	1	52	6	16	10	2	9
12	6	13	49	31	33	6	13	0	31	25	6	16	31	31	18
13	7	29	9	4	25	7	29	0	4	10	8	2	16	4	30
14	8	11	34	35	59	8	11	15	35	55	8	15	13	35	51
15	9	5	0	61	55	9	4	40	61	50	9	8	19	62	0
16	9	20	34	29	10	9	20	40	29	15	9	24	10	29	15
17	10	23	31	59	38	10	24	30	59	50	10	28	46	59	42
18	11	18	44	31	12	11	18	44	31	12	11	21	37	30	51
19						11	18	55	31	1	0	1	22	12	34
20						1	21	35	30	0	1	25	7	29	21
21						5	11	55	11	50	5	13	49	12	0
22						10	23	45	23	0	10	20	40	21	30
23						3	3	55	7	0			*		

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* Non vidit unquam Nepos *Timuri* Clarum *Canopum*; non cæteri hujus Canonis exceptis *Alexandrinis*.

Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

	Abdolgalio Segazio in Genethliacis. A.C. 1261.			*Ex Perſicis Chryſococcæ.* A. C. 1115.			*Ex Canonib. Perſa. Pemb.* A.C. 1346 *Melixæ* 338.			*Ex Cod. Arnoldino apud Gaſſendum pro* A. C. 1364.		
	Long.			Long.			Long.			Long.		
	S.	°	′	S.	°	′	S.	°	′	S.	°	′
1	0	6	45	0	15	10				0	18	20
2	0	25	2	0	22	50						
3	1	16	52	1	14	40		18	10	1	17	45
4	1	29	45	1	27	0	2	1	30	2	1	0
5	2	6	52	2	4	50		8	20	2	8	0
6	2	11	43	2	10	50		13	3	2	13	10
7	2	19	12	2	17	0		20	30	2	18	10
8	3	4	28	3	2	40				3	6	0
9	3	16	11	3	14	10		18	0	3	17	20
10	4	19	42	4	17	30		9	31	4	20	40
11	6	13	26	6	11	40		15	10	6	15	10
12	6	13	47	6	12	0		15	30	6	14	10
13	7	29	45	7	27	40	8	1	10	8	27	40
14	8	11	49									
15	9	5	38	9	2	20		5	0	9	5	30
16	9	20	56	9	18	50		22	20	9	22	0
17	10	26	15	10	24	10		10	20	9	27	18
18	11	18	58	11	17	10		20	40	11	20	20
19				11	27	10						
20	1	22	2	1	19	50						
21	5	11	38				5	13	2	5	12	40
22	10	23	53	10	22	00						
23	3	4	3	2	2	10						

Præcipuarum Fixarum Canon juxta Obſervationes Selectas.

	Ex Codicibus Savilii & Bodlei. quaſi pro A. C. 750.			*Mohammedi Tizinio.* A. C. 1533.				*Sabebodino Alepenſi ad Ilchanicas, pro* A. C. 1436.			
	Long.			Aſcenſ. Rect.		Declin.		Aſcenſ. Rect.		Declin.	
	S.	°	′	°	′	°	′	°	′	°	′
1	0	12	20	130	17	41	0	132	10	41	30
2				86	55	56	41	84	2	55	45
3	1	2	50	130	14	39	43	127	44	39	4
4	1	19	50	153	0	15	43	150	40	15	20
5				163	43	9	13	162	20	9	20
6	2	2	10	161	39	45	0	158	30	44	43
7	2	9	10	173	15	6	28	171	55	6	15
8	2	24	50	187	0	15	50	185	45	15	43
9	3	6	40	199	0	6	5	197	25	6	7
10	4	9	39	236	0	14	8	234	0	15	10
11	6	4	50	285	36	8	29	283	40	7	30
12	6	4	10	299	0	22	14	297	39	33	8
13	7	19	20	330	57	22	42	331	0	24	12
14				348	57	13	9	347	5	13	20
15	8	24		38	37	5	29	38	32	4	20
16	9	15	0	22	5	7	24	20	5	7	3
17				36	40	43	50	36	20	43	15
18				71	0	25	16	68	50	24	30
19				88	10	12	23	85	54	11	30
20				133	46	47	52	130	20	47	11
21	5	1	45	261	20	17	55	259	30	17	55
22				68	45	33	51	67	25	35	2
23				183	40	51	35	183	30	5	25

Præ-

Præcipuarum Fixarum Canon juxta Obſervationes Selectas.

	Olaodino Saterida in Tabulis Damaſcenis. A. C. 1480.							*Ex Tabulis Rodolphinis,* A. C. 1600. *ad Contemplationes Tychonis Brahei.*				
	Long.			Aſcenſ.		Declin.		Long.			Lat.	
	S.	°	′	°	′	°	′	S.	°	′	°	′
1	0	20	34	129	30	17	0	0	21	10	53	30
2	0	18	16	83	50	55	50	0	29	55½	51	14½
3	1	20	4	127	55	38	50	1	20	37	22	22
4	1	3	4	149	50	15	0	2	4	12½	5	31
5				167	50	5	10	2	11	37	31	11½
6	2	15	24	157	40	44	40	2	16	16	22	50½
7	2	22	24	170	10	6	10	2	15	23	16	36
8	4	4	4	184	20	15	45	3	8	35½	39	30
9	4	19	44	196	20	6	30	3	20	18½	15	57
10	4	24	44	233	30	15	5	4	24	17	0	26½
11	6	16	30	283	20	7	30	6	18	16	1	59
12	6	17	24	297	0	23	30	6	18	39½	31	2½
13	8	3	4	322	0	24	15	8	4	13	4	27
14								8	16	50	35	57
15	9	7	44	4	0	38	30	9	9	43	61	47½
16	9	24	14	20	0	7	5	9	26	9	29	21½
17				35	10	4	30	10	29	53½	59	56½
18	11	22	34	68	20	24	0	11	23	49½	31	7½
19	0	2	34	85	5	11	30	0	3	28½	12	35
20				127	20	1	40	1	26	17	30	5
21	5	14	54	259	5	18	10	5	16	3	12	18
22	10	24	24					10	28	11½	21	0
23				181	15	51	50					

Canon Præcipuarum è Stellis Fixis ſecundum Obſervata Majorum.

Ricciolo Bononienſi in Aſtronomia Reformata. A. C. 1660.

	Long.				Lat.			Aſcenſ.			Declin.		
	S.	°	′	″	°	′	″	°	′	″	°	′	″
1	0	22	0	14	53	30	0						
2	1	0	22	0	51	17	0	357	44	33	57	18	40
3	1	21	32	10	22	22	40	41	33	54	39	36	39
4	2	5	1	43	5	30	50	64	6	58	13	46	10
5	2	12	3	10	31	10	10	74	32	38	8	37	30
6	2	17	6	15	22	51	45	72	52	20	45	36	0
7	2	16	13	0	16	52	30	76	45	4	5	59	50
8	3	9	3	0	39	32	5	97	39	18	16	16	30
9	3	21	6	30	15	57	10	110	22	32	6	4	0
10	4	26	4	45	0	26	20	147	31	14	13	36	30
11	6	19	6	0	1	59	30	196	51	0	9	20	30
12	6	19	30	40	31	0	40	210	4	38	20	59	50
13	8	5	1	40	4	26	30	242	10	40	25	33	30
14	8	17	43	20	35	56	15	259	47	14	12	52	30
15	9	10	32	40	61	47	0	276	19	32	30	30	40
16	9	26	58	30	29	20	40	293	31	59	8	1	20
17	11	0	41	25	59	57	20	397	26	35	44	6	40
18	11	24	40	20	31	8	20	341	51	48	29	15	50
19	0	4	40	20	12	37	0	358	57	0	13	19	29
20	1	27	0	8	30	6	40	44	52	30	48	34	30
21	5	16	53	0	12	16	20	172	53	58	16	27	40
22	10	29	2	56	20	59	40	339	37	0	31	20	10
23													

Præcipuarum Fixarum Diſtantiæ juxta Obſervationes Selectas.

			°	′		°	′
1	*Ultima* Fluvii —						
2	*Luc. Cath.* Caſſiop.	à Seat ——	33	4	à *Cauda* Cygni -	33	$32\frac{1}{2}$
3	Perſei *caput Med.*	à *Latere* Perſei -	9	$23\frac{1}{6}$	à Capella — —	23	37
4	*Ocul.* Taur. *Auſtr.*	à Sirio ——	45	$58\frac{1}{3}$	*ab* Algomeiſa —	46	22
5	Orion. *Pes Siniſter*	*ab Oculo* Tauri -	26	33	à Capella — —	54	$15\frac{1}{4}$
6	*Capella* Aurigæ. —	*ab Oculo* Tauri -	36	$43\frac{1}{2}$	à Perſei *Latere* —	19	$11\frac{1}{12}$
7	Orion. *Humer. dex*	*ab Oculo* Tauri -	15	$47\frac{3}{5}$			
8	Sirius. —— ——	*ab* Algomeiſa —	25	42	à Rigel — —	23	40
9	Procyon. ——	à Regulo ——	37	20	à Rigel — —	38	$37\frac{1}{2}$
10	*Cor* Leonis. ——	à Spica ——	54	2	à *Cauda* Cygni -	24	$39\frac{1}{2}$
11	*Spica* Virginis. —	*ab* Arcturo —	33	2	*ab* Antare — —	45	52
12	Arcturus. ——	à Regulo —	59	49	*ab* Antare — —	56	$4\frac{1}{6}$
13	Antares. — —	à *Lucida* Aquilæ	60	$9\frac{1}{2}$	à *Cauda* Leonis -	24	$39\frac{2}{3}$
14	*Caput* Ophiuchi. -	à Spica ——	36	$14\frac{1}{2}$	à Lyræ *Lucida* -	29	$33\frac{2}{3}$
15	*Lucida* Lyræ. —	à Seat — —	55	$39\frac{2}{3}$	à Spica — —	9	$20\frac{1}{3}$
16	Aquilæ *Lucida.* —	à *Lucida* Lyræ-	34	9	à *Cauda* Cygni -	38	$4\frac{1}{3}$
17	*Luc. in Cauda* Cyg.	à *Lucida* Lyræ-	23	52	à Seat ——	32	$57\frac{1}{8}$
18	*Luc. in Crure dext.*	*ab* Algenib. —	20	37	à *Luc. Cath.* Caſſio	22	4
19	*Ala* Peg. *ult.* (Peg.	à *Latere* Perſei -	51	$45\frac{1}{6}$			
20	Perſei *Luc. Latus.*	*ab Oculo* Tauri -	36	$20\frac{1}{3}$			
21	*Cauda* Leonis. —	à Spica ——	35	$2\frac{1}{6}$			
22	*Os* Piſcis *Auſtralis.*						
23	Canopus. ——						

Octo Fixarum Nobilium Declinationes ex Curis Antiquis, Conſulto etiam Illuſtri *Braheo*.

		Ariſtyllo. A.*a.*C.300.	*Timocharidi.* A.*a.*C.295.	*Hipparcho.* A.*a.*C.128.	*Menelao.* A. C. 97.	*Ptolemeo.* A. C. 137.	*Tychoni.* A. C. 1600.
4	*Ocul.* Tauri.		8 45 b.	9 45 b.		11 0 b.	15 28 b.
6	Capella. —	20 0 b.		40 24 b.		41 10 b.	25 28⅔ b.
8	Sirius. —		15 20 m.	16 0 m.		16 45 m.	16 11 m.
10	Regulus.—		21 20 b.	20 40 b.		19 50 b.	13 53½ b.
11	Spica. —		1 24 b.	0 36 b.	0 40 m.	0 36 m.	9 1 m.
12	Arcturus.—		31 30 b.	31 0 b.		29 50 b.	21 18½ b.
13	Antares. —		18 20 m.	19 0 m.		20 25 m.	25 26 m.
16	Aqu. *Lucida*		5 48 b.	5 48 b.		5 50 b.	7 54 b.

Oculi *Tauri* Longitudino.

	Hermeti. — —	♓	25	17
	Hipparcho. —	♈	10	0
A. C. 140.	Ptolemæo. —	♈	12	40
A. C. 851.	Abomaſaro. —	♈	19	15
A. C. 831.	Thabeto. - - -	♈	21	17
	Arzacheli. - —	♈	23	20
	Albatanio. - -	♈	24	30
	R. Groſtetiō —	♈	28	40
A.C.1316.	W. Eveſhamio.—	♈	29	0

Spicæ Longitudo.

A.*a.*C.295.	Timocharidi	♍	22	20
A.*a.*C.283.	Timocharidi	♍	22	30
A.*a.*C.128.	Hipparcho. - -	♍	24	0
A. C. 97.	Menelao. — —	♍	25	45

Reguli Longitudo.

*A.a.*C.128.	Hipparcho. —	♋	29	50
A. C. 137.	Ptolemæo. - -	♌	2	30
A. C. 879.	Albatanio. - -	♌	14	0

Spatio itaque annorum 742. grad. 11. 50′. aut potius gr. 11. 30′. propter *Reguli* Long. *Ptolemaicam* ♌ 2°. 30′. non 2°. 10′. ſicut olim legerat Aſtronomus *Raccenſis.*

Addit Fixarum Longitudinibus Ptolemaicis.

A. C. 1303.	Alphonſus. —	17	8
A. C. 1320.	Wymundus. —	15	52
A. C. 1440.	Walterus *Vigorn.*	19	5

Prima *Arietis* diſtat ab Æquinoctio verno. 25°. 40′.

Hæc omnia Walterus Vigornienſis, *Cod.* 13. inter Digbianos.

Progrediuntur autem Stellæ Fixæ Gradum unum Annis Solaribus.

1. *Hipparcho, Ptolemæo, Theoni, Proclo* & *Alfergano* —— —— 100
2. *Timocharidi Alexandrino,* qui obſervarat *Spicam Cœleſtem* Annis *Nabon,* 454 & 466. *Abdorahmano Salchio,* & de *D. Petavio,* $72\frac{1}{2}$: ſive $\frac{60 \times 12}{10}$: & 50″. quovis Anno.
3. *Johannida Ægyptio,* Canonum *Hacimiticorum* Conditori —— —— $70\frac{3}{4}$
4. *Jabia* f. *Abomanſori,* aliiſque Probatæ, quam vocarunt, Aſtronomiæ Auctoribus: nec non *Naſirodino Tuſio, Cothodino Siraſio, Ologbeco* Mogolorum Domino, *Xacholgio, Abolphetacbo, Abeneſdre, Maimonide,* & pleriſque juniorum —— —— —— —— 70. & 51″. 26‴.
5. *Chryſococca* in Perſicis, & Aſtron. Anglicis. *An. Chr.* 1300.- 68. & 52″. 23‴.
6. Aſtronomis pleriſque Arabum ſub *Mamone* Principe —— $66\frac{2}{3}$
7. *Abdorahmano Sophio, Bahodino Chorcio, Alphonſo* Regi, *Albatanio* ex *Racca,* (quæ eſt *Callinicos Meſopotamiæ*) *Abdolgalila Segazio, Levi* & *Zacuto* Judæis, & Obſervatorum *Maragenſium* nonnullis —— 66. & 53″. 33‴.
8. *Copernico, Meſtlino,* aliiſque fide illorum —— fere 71. & 50″. 12‴. 5⁗.
9. Nonnullis apud *Chorcium Arabem* —— —— —— —— 54″.
10. *Tychoni Braheo, Keplero, Bullialdo,* ex obliquitate Zod. — $23\frac{1}{2}$°. $70\frac{7}{12}$. & 51′.
11. *Longomontano* - - - - - - - - - - - - $72\frac{1}{2}$. & 49″. 54‴.
12. *Gaſſendo* - - - - - - - - - - - - - $70\frac{1}{3}$. & 51″. 19‴. 24⁗.
13. *Ricciolo* in Aſtr. Reform. ex obliq. Zod. 23°. 30′. 20″. 71. $19\frac{1}{2}$d. 50″. 40‴.
14. *Nobis,* & Ægyptiorum *Hierophantis* — 71. $9\frac{2}{3}$. *menſ.* & 50″. $9\frac{1}{4}$‴. fere.

XII. Tubum habeo Pedum $13\frac{1}{2}$, Lentibus Convexis, & Micrometore exactiſſimo *Townleniano* Inſtructum, quocum Noctibus ſerenioribus Menſium *Octobris* & *Novembris,* nuper elapſorum, Minutas frequenter Pleiadum Stellularum Intercapedines dimenſus ſum, idque adeo auſpicato, ut nunquam 20″. imo per-raro 10″. inter ſe diſſiderent repetitæ Obſervationes; à prægreſſis etiam Defuncti *D. Gaſcognii,* & nuperis Generoſiſſimi *Townleii* (quantorum Hominum!) eadem ratione peractis Obſervationibus, confirmatæ; quæ quidem Diſtantiæ limatiſſimæ ſic ſe habent.

The Pleiades obſerved, in 1671. *by Mr.* Flamſteed. N. 79. p. 3061, 3062.

Stellæ.	*Diſtantiæ.*			*Stellæ.*	*Diſtantiæ.*		
	Mihi		Muto		Mihi		Muto
	′	″	′		′	″	′
a b	35	40		a d	18	30	22
a e	27	40	31	e d	26	10	30
e b	20	00	32	d i	16	25	
c b	21	45	22	a i	18	18	
e c	10	00	11	f i	29	04	
e g	14	40		f a	23	00	27
c g	11	55		h a	23	20	
b d	22	04	24	f a	04	45	04

Addit *Vinc. Mutus* in Epiſtola ad Doctiſſimum *Ricciolum*, (cujus meminit in Appendice ad *Alm. No. Tom.* 1. *Pag.* 747.) *Occidentalem Lucidiorem tranſiiſſe Meridianum in eadem omnino Altitudine ac Lucidam Pleiadum.* Qua fretus Adverſione & Obſervatis Diſtantiis, Loca Stellis aſſignavi infra ſcripta; *Media* prius *Lucida*, eodem omnino Loco & Latitudine donata, quæ *Authori Carolino* arrideat; cæteris etiam abinde diſpoſitis; quas tamen omnes, propriam ſi hac in re ſententiam ſequi licuiſſet, tres vel ſaltem duos Scrupulos Primos Promotiores, nec non & Latiores ab Ecliptica proponerem: Ineunte *Ann.* 1672. conſtitutæ,

Stellæ.		Long. ♉			Lat. Bor.			mag.
Pleiadum.		°	′	″	°	′	″	
Occidentalis Lucidior ———	b	24	45	15	4	08	51	5
Intra hanc & Borealiorem Teleſcopicam	g	24	46	47	4	19	21	8
Occidentalis Borealior ———	c	24	54	48	4	28	19	6
Suprema in Quadrilatero ———	e	25	01	24	4	20	39	5
Infima Auſtralis oppoſita ———	d	25	02	18	3	53	59	6
Media & Lucida ———	a	25	19	48	4	00	00	3
Quæ in Cuſpide ad Ortum ———	f	25	41	29	3	52	19	5
Orientalium ſuperior Teleſcopica ———	h	25	42	55	3	56	51	7
Teleſcopica alia ———	i	25	14	04	3	42	37	9

XIII. 1. Inter *Canem Majorem* & *Navem* nuper deprehendi *Nebuloſam* viſu Pulcherrimam, ſi magnis Teleſcopiis inſpiciatur, ex Stellis Confertiſſimis compoſitam, quæ Cœlum mediat cum *Cane Minori.*

A Nebuloſe Star, by M. Caſſini. N. 123. *p.* 565.

2. Cœlos infra Procyonem perluſtrans, *Nebuloſam* offendi, Latam, & Stellulis Confertiſſimam. Hanc eandem credo quam *Cl. Caſſinus* obſervavit.

By Mr. Flamſteed. *Ibid. p.* 567.

XIV. *Ann.* 1664. I diſcovered the firſt Star in the Head of *Aries* to be a double Star, made of two conſiderable Stars, ſo near as not to be diſcovered two, but by a Glaſs of 6 or 8 Foot long.

The firſt of Aries *a Double Star; by Dr.* Hook. Phil. Coll N. 4. *p.* 108. *Marg.*

XV. 1. Deſunt in Cœlo duæ Stellæ Secundæ Magnitudinis in Puppi *Navis* ejuſque Tranſtris, *Bayero* β & γ, prope *Canem Majorem*, à me & aliis occaſione præſertim *Cometæ. Ann.* 1664. Obſervatæ & recognitæ. Earum Diſparitionem cui Anno debeam, non novi, hoc indubium, quod à die 10 *April.* 1668. ne Veſtigium quidem illarum adeſſe amplius obſervo; cæteris circa eas, etiam Quartæ & Quintæ Magnitudinis, immotis. Plura de aliarum Stellarum Mutationibus, pluſquam Centenis, at non tanti ponderis, annotavi.

Changes amongſt the Fix'd Stars; by S. Montanari. N. 73. *p.* 2202. *Marg.*

2. M. *Caſſini* hath diſcovered many New Stars; *viz.* One of the Fourth Magnitude, and two of the Fifth, in *Caſſiopeia.* He hath diſcovered two others towards the beginning of *Eridanus*, where we were ſure they were not yet about the end of the Year 1664. conſidering that this Place of the Heavens, where paſſed the then appearing *Comet*, was diligently beheld by many, who perceived divers other ſmall Stars, without obſerving thoſe two. The ſame hath alſo obſerved, towards the *Arctick* Pole, four of the Fifth or Sixth Magnitude.

By M. Caſſini. N. 73. *p.* 2201.

He hath alſo obſerved, That the Star which *Bayerus* puts near that which he marketh in the Figure of *Urſa Minor*, appears no more; That that which is marked A, in the Figure of *Andromeda*, is alſo diſappear'd; That in lieu of that which is marked υ, at the *Knee* of the ſame Figure, there are two others more Northward; and that, that which is ξ, is very much diminiſh'd; the Star, which *Tycho* placeth at the Extremity of *Andromeda*'s *Chain*, and calls it of the Fourth Magnitude, is now ſo ſmall that one can ſcarce ſee it; and that which is in his *Catalogue* the 20th of the Conſtellation of *Piſces*, is now no more ſeen.

3. On the 24th of *Sept.* (*St. N.*) 1666. I have obſerv'd that New Star in *Pectore Cygni*, (which from the Year 1662. until this Time, hath been almoſt altogether hid) not only with my naked Eye, like a Star of the Sixth or Seventh Magnitude, but alſo with a very great Sextant. It is ſtill in the very ſame Place of the Heavens, where it was from *Ann.* 1661. to almoſt 1662. For its Diſtance from *Scheat Pegaſi* hath been by me found 35°. 51'. 20''. and from *Marcab*, 43°. 10'. 50''. which Diſtances are altogether equal to thoſe which I obſerved *Ann.* 1658. the Firſt of *November*. For the Diſtance from *Scheat* at that Time was 35°. 51'. 20''. and from *Marcab*, 43°. 10'. 25''.

The New Star in Pectore Cygni; *by M.* Hevelius. N. 19. *p.* 349. N. 21. *p.* 392.

where that former from *Scheat* exactly anſwers to the Recent; and that from *Marcab*, 'tis true, differs in a very few Seconds, but that Diſparity is of no moment, ſince it only proceeded from thence, that this New Star is not yet ſo diſtinctly to be ſeen as at that Time, when it was of the Third Magnitude. It is therefore certain, that it is the ſelf-ſame Star, which *Kepler* did firſt ſee *Ann.* 1601. and continued till *Ann.* 1662. He that will obſerve this Star, muſt take care leſt he miſtake the Three more Southern ones of the Sixth Magnitude; the higheſt of which is diſtant from *Scheat Pegaſi*, 36°. 25′ 45″. the Middlemoſt from the ſame 37°. 25′. 20″. and the Loweſt 38°. 4′. 30″.

N. 134. *p.* 855. Ann. 1662. *Novemb.* 28. Stella illa nova in *Pectore Cygni* quæ aliquamdiu ab *Ann.* 1662. plane dilituit, Cœlo ſereno quaſi Reviviſcere videbatur.

Ann. 1666. *Septemb.* 21. Nudis Oculis (etiam *Luna* ſplendente) apparuit. *Sept.* 24. Minor erat illis tribus præcedentibus in *Collo* & vix 6 Magn. videbatur.

N. 65. *p.* 2089. Ann. 1670. *Aug.* 26. Senſim Creſcere videtur quanquam necdum Major eſt Stellis 6 Magn. *Sept.* 3. Adhuc Creſcere videbatur; 8. Paulo adhuc Creſcere deprehendimus; & *Octob.* 13. Satis clare apparuerit.

N. 134. *p.* 855. Ann. 1671. *April.* 29. Vix major adhuc apparuit quam Anno Preterito; ſi quidem Stellis 6 Magn. æquabatur.

Ann. 1671. *Jun.* 26. Major fere videbatur.

Ann. 1672. *Mart.* 29. Adhuc Creſcere videbatur.

Ann. 1675. *Jul.* 22. Apparuit inſtar 6 Mag.

N. 154. *p.* 854. Ann. 1677. Nondum ad priorem Magnitudinem (Tertii videlicet Honoris) atque Claritatem & Splendorem (qua Magnitudine *Ann.* 1657, 1658, & 1659. apparuit) pervenit: ſiquidem non niſi inſtar 6 Magn. adhuc fulget.

Ph. Col. N. 5. *p.* 162. Ann. 1681. *Aug.* 18. *Nova* in Collo *Cygni*, Nudis Oculis ob ejus tenuitatem parvitatemque haud quidem conſpecta, ſed Teleſcopio tamen deprehenſa eſt.

The New Star, ſub Capite Cygni. N. 65. *p.* 2092. N. 73. *p.* 2198. 4. 1. Don *Anthelme*, a *Carthuſian* at *Dyon*, on the 20th of *June*, *Ann.* 1670. diſcover'd a Star of the Third Magnitude beneath the *Head* of *Cygnus*, ſituated in the Section of the two ſtreight Lines, one of which goeth from *Lyra* to the neareſt of the *Quadrangle* in the *Dolphin*, and the other from the *Eagle* to the Star, which is on the Top of the *Upper Wing* of *Cygnus*. He ſent the News of this Diſcovery to M. *L' Abbe Mariotte*, one of the *Royal Academy*, who communicated it to the reſt. They all agree, 'tis a *New* Star, though M. *B*...... oppoſed it at firſt, affirming it to be in *Bayerus*'s Tables; but they prove that Star in *Bayerus* to be another; giving for Diſtinguiſhment theſe Meaſures.

	°	′	″
The Bright Star *ad Roſtrum Cygni*, its *Aſcenſio Recta*	289	22	00
Declinatio Borealis	27	19	20
But this *New* Star's *Aſcenſio Recta*, is —— — ——	293	33	00
Declinatio Borealis	26	33	20

Longi

	°	′	″
Longitudo ♒	1	55	
Latitudo	47	28	10
Its *Diſtance* from that *ad Roſtrum Cygni* towards *Jaculum*	3	47	30
From the Tail of *Cygnus* ———	20	54	30
And from the *Lucida Lyræ* ———	18	39	40
It came to the Meridian after the Star *in Roſtro Cyni* ——		16	44
And before the *Lucida Aquilæ* ———		0	27

In the Beginning of *July*, this Star was obſerved to decreaſe: *July* 11. it ſcarce appear'd of the Fourth Magnitude.

Aug. 10. it was of the Fifth, and continued to decreaſe till it wholly diſappear'd.

Anno 1671. *March* 17. D. *Anthelme* ſpied it again of the Fourth Magnitude.

April 4. M. *Caſſini* found it greater than the two Stars of the Third Magnitude that are below in the Conſtellation of *Lyra*, and a little ſmaller than that in the Book of *Cygnus*, but more radiant.

April 9. he found it a little diminiſh'd, and almoſt equal to the greateſt of the two Stars that are below in *Lyra*.

The 12th, it was equal to the leaſt of theſe two Stars.

The 15th, he perceived that it increaſed, and found it equal the ſecond time to the greateſt of theſe two Stars.

From the 16th unto the 27th, it appeared of different Magnitudes, being ſometimes equal to the biggeſt of theſe two Stars, ſometimes equal to the leaſt, and now and then between both.

But the 27th and 28th, it was become as big as the Star in the *Swan's Beak*.

The 30th, it appeared a little clearer; and the firſt Six Days in *May* it was greater.

The 15th of *May*, it was ſeen ſmaller than the ſame Star.

The 16th, it was in Bigneſs between the two Stars that are below in *Lyra*: And ever ſince ſhe hath ſtill diminiſhed.

Thus this Star hath been twice in her greateſt Splendour; firſt on the 4th of *April*, and the ſecond time in the Beginning of *May*.

By M. Hevelius. N. 65. *p*. 2087.

Hiſce te inviſere volui, quo vos de Obſervatione quadam notatu digna certiores facerem, mentemque ſimul meam ea de re vobis exponerem: De *Nova*; *viz*. illa Fixa 3 Mag. fere, Circa & Infra *Caput Cygni*, inter informes conſpicua, cujus Longitudo 1°. 52′. 26″. ♒. & Latitudo 47°. 25′. 22″. Bor. modo exiſtit; ut Obſervationes, Die 25 *Julii*, Anno 1670. à me habitæ, luculenter oſtendunt. *Novam* autem Stellam hanc ipſam omnino eſſe, & in Cœlo ad *Ann*. 1660. penitus inconſpicuam fuiſſe, non eſt quod quiſquam dubitet. Accidit enim ut *Annis* 1659, 1660, & 1661. pleraſque Stellas illa omnes, in Aſteriſmo *Cygni* apparentes ſumma diligentia, debitis Organis Dimenſus fuerim, atque ita omnes illas, etiam circa Collum, & Caput De-

Deprehenſibiles notaverim, earumque Diſtantias à diverſis Fixis ceperim; nullam autem Stellam 3 Mag. eo loco ubi jam dicta *Nova* notatur, tum deprehendi; quam tamen optime, ſi adfuiſſet, conſpexiſſem. Sic ut primo hinc Certus ſim, Ann. 1660, & 1661. hanc Stellam nondum extitiſſe viſibilem. Deinde clare etiam patet ex *Bayeri Uranometria*, hanc modo dictam *Novam* Stellam neque Ann. 1603. Apparuiſſe, & per conſequens neque *Tychoni*, multo minus *Hipparcho*. Si quidem & *Bayerus* Fixam tantæ Magnitudinis deprehendiſſet, cum haud procul ab illa aliam 6 Mag. depinxerit: prout in Aſteriſmo ejus *Cygni* videre eſt. At, inquies, forte ea ipſa eſt, quam tu *Novam* dicis, quippe cum *Bayerus* Congruis Organis Stellas illas haud Obſervaverit, fieri facile potuit, ut à vero ejus loco ad Gradum, vel paulo plus, aberravit. Sed, crede, id factum non eſt, quandoquidem Stella illa Parvula adhuc eodem loco, ubi *Bayerus* fere eam depoſuit, commoratur, nec Major eſt Stella 6 Mag. ut eodem tempore ei videbatur. Diſtat enim, ut ipſemet nuper deprehendi, ab Ore *Pegaſi* 32°. 39′. 00″. Dextro Genu *Pegaſi* 39°. 23′. 45″. hinc provenit ejus Long. 00°. 06′. 28″. ♒. & Lat. 46°. 11′. 14″. Bor. ad Annum ſcilicet Currentem 1670. Complet. *Julio*. At *Nova* Elongatur ab Ore *Pegaſi* 32°. 31′. 35″. & à Dextro Genu *Pegaſi* 38°. 18′. 50″. ex quibus Diſtantiis Long. 1°. 52′. 26″. ♒. & Lat. 47°. 25′. 22″. Bor. Elicitur. Adeo ut hæc *Nova* plane ſit diverſa ab illa 6 Mag. à *Bayero* notata; (quanquam hæ duæ nondum ad duos gradus ab invicem removentur.) Atque ex Dictis manifeſtum fit, hanc *Novam* nec Ann. 1603. nec Ann. 1670. inter cæteras emicuiſſe Stellas.

Cum primitus à me obſervabatur, quoad Magnitudinem & Splendorem, Stellæ in Pectore *Aquilæ* æquabatur, niſi quod aliquanto Obtuſioris fuerit Luminis; quo ad Situm, reſpectu reliquarum Stellarum, in Linea Recta cum illa in Ancona Alæ Superioris *Cygni*, & illa in Humero *Aquilæ*, nec non cum *Lucida Lyræ*, & illa in Rhombo *Delphini*, Mediarum Borealiori conſiſtebat. Triangulum vero Æquilaterum cum illa in *Capite* & *Roſtro Cygni* conſtituebat.

N. 66. p. 2028. N. 134. p. 855. Mirum vero in modum M. *Sept.* decrevit, adeo ut 14 *Oct.* nulla ratione amplius Sectante obſervari à me potuerit; licet omnem adhibuerim diligentiam.

N. 73. p. 2197. N. 134. p. 856. Ann. 1671. *Apr.* 29. Denuo Obſervavi. Excedit illam in *Roſtro Cygni*, nec non eam quæ eſt in Ancone Inferioris Alæ *Cygni*, fereque illi quæ eſt in Pectore *Cygni* æquatur, niſi quod Lumine paulo Obtuſiori & Rubicundiori modo Luceat. Qua vero Die primum rurſus illuxerit, affirmare adeo certo non poſſum. Certus interim ſum, ad Menſ. *Dec. Jan.* imo *Feb.* haud Conſpicuam fuiſſe. Etenim poſt 14 *Oct.* quo videri deſiit, memini me eam ſæpius quæſiviſſe eo in Loco, ſed nuſquam Apparuiſſe. Id circo quantum colligere datur, vix ante initium *Mar.* quin ſine dubio adhuc tardius, iterum Prodiit. *Apr.* 30. eam à Reliquis quibuſdam Fixis ſum dimenſus. Diſtat à Cauda *Cygni* 20°. 55′. 20″. ab Ancone Alæ Superioris *Cygni*, 17°. 47′. 50″; à Capite vero *Serpentarii* 34°. 19′. 40″; ſic ut eodem plane Loco adhuc perſiſtat, ubi antea fuerat.

N. 144. p. 856. *Maii* 17. Aliquanto Minor videbatur *Roſtro Cygni*, & illa in Humero *Aquilæ*, tum etiam Lumine Obtuſior; Major tamen illa in Cuſpide *Sagittæ*, & æqualis fere illi Seq; in Jugo *Lyræ*.

Maii 25. Minor videbatur quam Die 29 *Apr.* qua primum viſa fuit; ſic ut decreſcere videretur. Minor jam erat Roſtro *Cygni*, nec non illa in Ancone Alæ Auſtr. etiam Minor illis in Jugo *Lyræ*, & Humero *Aquilæ*; vix Major apparuit Minori duarum in Pede *Cygni*, & illa in Pectore *Aquilæ*.

Jun. 26. Minor apparuit illa in Collo *Cygni*; ſicut notabiliter decreverit.

Jul. 3. Minor fere illa in Collo *Cygni*; & 18, vix Stellis 5 Mag. æquiparari videbatur.

Aug. 2. Vix 6 Magn. apparuit imo Minor quam reliquæ omnes circa Caput & Collum *Cygni* exiſtentes; per intervalla tantummodo micabat.

Sept. 11. Haud amplius conſpecta.

Ann. 1672. *Mar.* 6. I obſerv'd it again; but it can hardly be ſeen with the naked Eye.

N. 81. *p.* 4018.

Mar. 29. Vix 6 Magn. apparuit; à quo tempore neutiquam amplius (*viz.* ad *Ann.* 1677.) in Conſpectum venit, utut ſæpius illam diligenter Quæſiverim.

N. 134. *p.* 857, 854.

5. *Ann.* 1667. in *Jan.* The *Nebuloſa* in *Andromeda*'s Girdle (which may well enough be ſeen by the bare Eye) appeared much obſcurer than the Year before. In the Months of *February* and *March* I did not ſee it.

The Nebuloſa *in the Girdle of* Andromeda; *by M.* Bullialdus. N. 25. *p.* 459.

6. 1. *Ann.* 1667. *Jan.* 20. The *New Star* in the *Neck* of the *Whale*, did approach to the Bigneſs of a Star of the Sixth Magnitude, and grew bigger afterwards.

The New Star *in* Collo Ceti, *by M.* Bullialdus. *Ibid.*

Feb. 12. I ſaw it at laſt of the Fourth Magnitude.

Feb. 24. It was equal to the Stars of the Third Magnitude, ſhining very bright.

Feb. 26 and 27. It appeared yet to encreaſe.

2. *Ann.* 1667. In the Beginning of *January* this Star did not appear.

*By M.*Hevelius. N. 25. *p.* 460. N. 134. *p.* 855.

Jan. 23. I found a little Star of the Sixth or Seventh Magnitude about the ſame Place where the ſaid *New Star* uſes to appear. But it then ſeemed to me not the genuine *New Star*, but another; to wit, preceding the *New*, whoſe Longitude in *Ann.* 1660. was defined by me, ♈. 25°. 43′. 3″. and the Latitude 14°. 41′. 32″..

Feb. 2. It appeared very bright, and that, when the Moon ſhone, of the Bigneſs of that in the *Mouth* of the *Whale*, or *Nodo Lini*; from which time I always obſerv'd it to grow bigger.

Mar. 13. I did ſtill find it extreamly bright, but could not by my naked Eye, becauſe of the vivid Crepuſcle, and the low Sight of the Star, accurately determine its Magnitude.

Ann. 1668. *Octob.* 26. *Nova* in Collo *Ceti* primum viſa; ſed inſtar minutiſſimæ Stellulæ.

N. 132. *p.* 855.

Nov. 7. *Nova* in Collo *Ceti* Medium fere in ore æquabat.

Ann. 1669. *Jan.* 28. Minor erat illa in Ore.

Sept. 26. Inſtar 6 Magn. apparuit.

Oct. 16. Illa in *Ore* Major erat, & Clarior.

Oct. 27. Lucidam *Mandib.* æquabat.

Nov. 19. Major illa in *Ore* & Minor. *Mandib.*

Anno 1670. *Aug.* 27. Maximo gaudebat Lumine, æquales fere Stellis 2 Mag. *Mandib. Ceti.*

Sept. 3. Admodum Fulgida extitit; & 8 æqualis *Mandib. Ceti.*

N. 66. *p.* 2028.

Ad medium uſque Menſis *Oct. Mandib. Ceti* æqualis fere exiſtit Magnitudine, & Claritate eam propemodum ſuperavit; adeo ut hoc Anno ſecundæ fuerit Magn. ac Major quam præcedentibus Annis, excepto Anno 1660. quo Major etiam *Mandibula Ceti* à me fuit deprehenſa. Aliis temporibus non memini eam tertiæ Magnitudinis Stellas ſuperaſſe, Certum igitur eſt, ipſam non eandem ſemper præ ſe ferræ Magnitudinem nec Claritatem, utut in maximo ſuo exiſtat Incremento.

N. 134 *p.* 856.

Dec. 5. Adeo decreverat, ut vix Stellæ 6 Magn. æquaretur.

Anno 1671. *Aug.* 14. Æquabatur Stellæ ad *Genam*, imo fere Major paulo videbatur.

Sept. 12. Æquabatur illi in *Ore* 4 Magn.

Oct. 30. Vix 6 Magn. apparuit.

Nov. 3. Non amplius apparuit.

Anno 1672. *Aug.* 9. Clariſſimis fulgebat Radiis, Major erat illa in *Ore*, & Minor *Mandibula.*

Sept. 17. Minor illa ad *Genam*, vix 4 imo 5 Magn. & 25. vix 6 Magn.

Ibid. p. 854.

A Menſe circiter *Octob.* ad *Decemb.* 23. Anno 1676. ne ſemel quidem Prodiiſſe, utut ſemper omni ſtudio Vigiles Oculos ad eam, quoties Obſervationibus operam ſerenis noctibus dedi, direxerim.

Ibid. p. 858.

Anno 1676. *Dec.* 10. Bene memini me *Novam* hanc in *Collo Ceti* haud vidiſſe, licet ea in Cœli parte plurimas Stellulas obſervaverim.

Dec. 23. *Novam* hanc in *Collo Ceti* Cœlo admodum ſereno clariſſime vidimus; & quidem tanta Claritate & Magnitudine fulgentem, ut *Mandibulam Ceti* non ſolum æquaret, ſed Magnitudine & Claritate vinceret.

Dec. 31. Fere Major *Mandib. h. e.* 2 Magn.

Anno 1677. *Jan.* 1. Clariſſime rurſus Affulgebat, Major fere *Mandib. Ceti*, Major quoque quam extrema *Alæ* & *Marcab. Pegaſi*, Colore & Lumine fere æqualis *Mandib.* Memini tamen me olim obſervaſſe, quando Secundæ exiſtebat Magnitud. eam paulo Albicantiorem & Splendiorem.

Phil. Col. N. 5. *p.* 162.

Anno 1681. *Aug.* 18. *Nova* Stella in Collo *Ceti*, hac nocte Luna etſi Plena & Splendente, major erat ea in Ore *Ceti*, ſed nondum æquabat Lucidam *Mandibulæ.*

By M. Caſſini. N. 123. *p.* 565.

3. Anno 1676. *Mar.* Inſpecta mihi eſt Stella *Nova* in *Ore Ceti*, quæ Annos aliquot latuit, Solaribus Radiis tempore Maximæ Fulſionis immerſa; nunc vero Stellas 3 Mag. facile ſuperat.

By Mr. Flamſteed. *Ibid. p.* 567.

4. *Novam* in *Pectore Ceti* ſæpius ante Octo Menſes vidi, nec Minorem quam innuit Cl. *Caſſinus.*

A New Star in Eridanus; *by M.* Caſſini. N. 35. *p.* 683.

7. *March* 10. 1668. Not far from the Star in *Eridanus*, which is called the 14th by *Bayerus*, there appeared a Star equal to the brighteſt of the Fourth Magnitude, almoſt in the ſame Place, where was obſerved the Comet of *Ann.* 1664. *Dec.* 31. which Star was not then ſeen, nor at other times elſewhere, nor is deſcribed in any Catalogue, on any Globe or Map, that I can learn; which therefore I deem to be a *New* one, that is, of *New Appearance.*

8. The *Comet*, *Anno* 1672. had (on the 1ſt of *April*, *N. S.*) paſs'd 45′. beyond the moſt Northern Star of the *Head* of *Taurus*, and was diſtant 1°. 43′. from the Star that was neareſt to that towards the South. M. *Caſſini* having conſider'd theſe two Stars, obſerv'd that the Second is not leſs bright than the Firſt, and yet that *Bayerus* hath not marked it; and that at firſt Sight it ſeems that *Tycho* hath left it out in his Catalogue: For he puts four Stars in the Place he calls *in Quadrilatero Cervicis*, and he ſpeaks not of this which is the fifth, and maketh, with the other four, an irregular Pentagone. This Omiſſion of *Bayerus*, and the Denomination which *Tycho* uſeth to denote theſe Stars, which ſuits not with the Number nor the Configuration that now appears, do adminiſter Cauſe to doubt, whether the Star in Queſtion be not one of thoſe that appear from Time to Time.

A New Star in Taurus; *by* M. Caſſini. N. 82. *p.* 4046.

XVI, 1. Supponit Cl. *Caſſinus*, ad Planetam in Ellipſi moventem extendi ab utroque Foco duas Rectas, quarum altera ſit Medii, altera autem Veri Motus Linea. Conſtructio porro talis eſt:

To find the Aphelia *of the Planets directly*; *by M.* Caſſini. *Conſidered by Mr.* Nic. Mercator. N. 57. *p.* 1168. Mar. An. 1670.

Fig. 117.

L eſt Centrum Concentrici ABCDE.
BLD eſt Diameter.
BA, BC, BP, ſunt Intervalla Apparentia.
DE, DF, DQ, ſunt Intervalla Mediorum Motuum.
BE, BF, BQ; item DA, DC, DP, ſunt Lineæ Rectæ.
BE ſecat DA in H; BF ſecat DC in G; BQ ſecat DP in R.
RHG eſt Linea Recta.

BI eſt Perpendicularis ad RHG.
I eſt Centrum Ellipſeos.
LI eſt Excentricitatis.
IO = LI.
O eſt Focus, circa quem ordinatur Medius motus; L, circa quem Verus.
IM = IN = LB.
M, eſt Apogeon; N, Perigeon; BLM, Anomalia Vera.

Demonſtratio. 1. Illuſtriſſimus ac Reverendiſſ. *Sethus Wardus*, Epiſcopus *Sarisburienſis*, in Examine Aſtronomiæ Philolaicæ, docuit Methodum, ex data Anomalia Media Planetarum, inveſtigandi Verum; quæ eſt hujuſmodi.

Fig. 158.

C, eſt Centrum Ellipſeos AEP: F, focus circa quem ordinatur Medius Motus. S, focus circa quem ordinatur Verus Motus. A, Apogeon. P, Perigeon. E, Erro ſive Planeta. AFE, Anomalia Media. ASE, Anomalia Vera. FET, Linea Recta; ET = SE. ST, eſt Linea Recta.

In Triangulo SFT, dantur, 1. SF, diſtantia focorum: 2. FT = FE + ES = AP. 3. AFT, Angulus Externus, ſive Anomalia Media, æqualis Summæ Angulorum, FST & T. Ergo inveniri poteſt FSE, ſive Anomalia Vera, æqualis Differentiæ Angulorum, FST & T. Nimirum.

Ut Semi-ſumma Laterum FT *&* FS, *ad Semi-differentiam eorundem; Ita Tangens Semi-ſummæ Angulorum* FST *&* T, *ad Tangentem Semi-differentiæ eorundem.*

Sed Semi-ſumma laterum FT & FS invenitur, ſubſtituendo pro FT æqualem AP, cujus Semis eſt AC, qui additus CS, ſemiſſi ipſius FS, facit Semi-ſummam AS, Diſtantiam Planetæ Maximam.

Tum ſi ex Semi-ſumma A S, auferatur Latus Minus F S, reſtant Semi-differentia Laterum F A, æqualis P S, diſtantiæ Planetæ minimæ: ut ſi Regula ex Anomalia Media data inveniendi veram.

Ut A S, *Diſtantia Planetæ Maxima, ad* P S, *Diſtantiam Minimam*; *Ita Tangens dimidiæ Anomaliæ Mediæ, ad Tangentem dimidiæ Anomaliæ Veræ.*

Corollar. 1. Si continuetur S E uſque ad V, ita E V ſit = ipſi F E, & tota S V = Axi A P, erit Trianguli, F S V, Angulus V, Semis Proſtaphæreſeos F E S, ideoque æqualis Semi-differentiæ Angulorum Anomaliæ Mediæ & Veræ, *h. e.* ipſorum A F E, & A S E; & Externus A F V = Semi-ſummæ eorundem, A F E & A S E, Angulorum, ablata *ſcil.* Semi-differentia, V F E, ex Majori A F E; unde oriuntur duæ Analogiæ,

1. *Ut Sinus Semi-ſummæ Anomaliæ Mediæ & Veræ,* A F V, *ad Sinum Semi-differentiæ eorundem* V; *ita* S V, (= *Axi tranſverſo* A P) *ad* S F, *Diſtantiam Focorum.*

2. *Ut Sinus Semi-ſummæ Anomaliæ Mediæ & Veræ* A F V, *ad Sinum Anomaliæ Veræ,* F S V; *ita* S V, (*vex Axis* A P) *ad* F V, *Subtenſam Anomaliæ Veræ: Ita quoque Semiaxis,* A C, *ad Semi-ſubtenſam* V X, *vel* F X.

Corollar. Si in eodem Triangulo, F S V, ex Subtenſæ F V, Puncto medio X, erigatur Perpendicularis X E; ſecabit illa S V in duas partes, quarum altera V E = eſt lineæ Medii Motus F E, altera vero S E, eſt ipſa linea Veri Motus.

Fig. 191.

2. Sit *a* Centrum Concentrici *c h f i.*
c a d, Diameter, eademque Linea Apſidum.
c h, Arcus Anomaliæ Veræ, cui reſpondet.
d i, Arcus Anomaliæ Mediæ. Itaque

c d h, eſt Angulus dimidiæ Anomaliæ Veræ, &
d c i, Angulus dimidiæ Anomaliæ Mediæ.
c i, & *d h,* ſunt Lineæ Rectæ, ſecantes ſe mutuo in *g.*

Ab Interſectionis Puncto *g,* demittatur ad *c d,* Perpendicularis *g b.* Erit igitur,

$db : bg ::$ Radius : Tang. bdg, vel cdh.

Et $cb : bg ::$ Radius : Tang. bcg, vel dci.

Ergo $db \times$ Tang. $cdh = bg \times$ Rad. $= cb \times$ Tang. dci.

Quare $db : cb ::$ Tang. dci : Tang. cdh; hoc eſt, db erit ad cb, ut Tangens dimidiæ Anomaliæ Mediæ & Tangentem dimidiæ Anomaliæ Veræ; adeoque (per Regulam ſupra expoſitam) ut diſtantia Planetæ Maxima, ad diſtantiam Minimam. Quamobrem *d b* = erit diſtantiæ Planetæ Maximæ; & *c b,* Minimæ; & *a b,* Excentricitati.

Cumque idem eodem modo Demonſtretur de cæteris omnibus Interſectionum Punctis, nimirum Perpendiculares ab ipſis ad *c d* Lineam incidere in Punctum *b*; oportet, ut Recta, jungens ipſas Interſectiones, congruat Perpendiculari, *b g f.*

3. Ducta Diametro hak, fiat Arcus $kl = id$, & ducantur kc, & hl, Secantes ſe mutuo in p. Ab h, in bgf, demittatur Perpendicularis, hr, eademque Parallela Apſidum Lineæ cd; erit Angulus rhs, Semi-differentia Arcuum Anomaliæ Veræ ch, & mediæ di. Tum ab eodem h Puncto ducatur Recta hk, faciens cum kh, Angulum = Angulo rhs, & occurrens Lineæ Apſidum in β, erit Trianguli $a\beta h$, Angulus βah, Menſura Arcus ch, ſive Anomaliæ Veræ, & βha, ſemi-differentia Anomaliæ Veræ & Mediæ (ex Conſtructione); & Externus $c\beta h$, (æqualis duobus Internis & Oppoſitis βah, & βha, adeoque compoſitus ex Anomalia Vera & Semi-differentia ejus à Media) erit Semi-ſumma Anomaliæ Veræ & Mediæ. Ergo per *Corollarii* primi *Analogiam* priorem; *Ut ſinus* $c\beta h$, *ad ſinum* βha; *Ita Radius* ah, *ad Excentricitatem* $a\beta$. Sed ſupra Demonſtravimus quoque ab æqualem Excentricitati. Ergo Punctum β congruit Puncto b.

Tum ex b excitetur ipſi hb, Perpendicularis bt; Aio, hanc continuatam Incidere in Punctum Interſectionis p. Nam Triangula, rhs, & bht, ſunt Similia, ex Conſtructione; quemadmodum & Triangulum hpk, ſimile eſt Triangulo hgi, cum eidem Peripheriæ ch, inſiſtentes Anguli pkl, & gih, ſint æquales, nec non æqualibus Peripheriis kl, & id, inſiſtentes Anguli phk, & ghi, æquales; quare & Tertius hpk, æqualis eſt Tertio hgi. Et ex æqualibus phk, & ghi, ablatis æqualibus bht, & rhs, reſtant æquales phb, & ghr. Unde ſic Arguo; $srh = tbh$, & $rhs = bht$, Ergo $hsr = htb$; ergo & Complementa horum ad Semicirculum ſunt æqualia, nimirum $rsi = btk$, & $sig = tkp$, Ergo & $igs = kpt$, quibus ablatis ex æqualibus igh, & kph, reſtat $hgs = hpt$; & $ghr = phb$; Ergo & $hrg = hbp$. Sed hrg, eſt Rectus; Ergo & hbp, Rectus eſt, Cum vero, & hbt Rectus ſit, ex Conſtructione, erit tb inde Rectum ipſi bp. Cumque idem eodem modo Demonſtretur de quavis alia Interſectione linearum ab h, & k, ad Congruentia Anomaliæ Veræ & Mediæ Functa ductarum; patet, non modo Rectam, jungentem Interſectiones, tranſituram per b punctum; ſed & hb, lineam perpendicularem fore ad eandem Jungentem. Q. E. D.

Corollarium. Si à quovis puncto Anomaliæ Veræ, puta h ad reſpondens punctum Anomaliæ Mediæ i, ducatur Recta hi; excitata è Centro Excentrici b, ipſi cbd, Perpendicularis bf, ſecabit ipſam hi in s, ea ratione quam linea Medii Motus obtinet ad lineam Veri Motus.

Nam per *Corollarii* Primi *Analogiam Poſteriorem*, hb, eſt Semi-ſubtenſa; ergo per *Coroll.* 2. Perpendicularis erecta ex b, nimirum bt, ſecat Diametrum hk, in t, ea ratione quam linea Medii Motus obtinet ad lineam Veri Motus. Ergo & rs, (ſive bf,) ſecat hi, lineam eadem ratione in s; propter Demonſtratam modo Figurarum $tbhkphb$, & $srhighr$, Similitudinem.

Cæterum ex laudata ſuperius Reverendiſſ. *Wardi* Methodo inveniendi primam inequalitatem, non eſt difficile, alium adhuc modum inveſtigandi Apogea & Excentricitates, non minus Directum & Geometricum, & Obſervationes quotvis admittentem, producere; quem & paucis exponam. Plures modos invenient Aſtrophili in Reverendiſſ. Viri *Aſtronomia Geometrica*, ad quam eos remitto. Interim

Fig. 120. Sint l & d, duco Foci Ellipſeos; t & u, duo Puncta Veri Motus Planetæ; Arcus Ellipſeos tu, ex l ſpectatur ſub Angulo tlu, & ex d, ſub Angulo tdu; item Diſtantia Focorum ld, ex t ſpectatur ſub Angulo dtl, & ex u, ſub Angulo dul: Aio differentiam Angulorum tlu, tdu, æqualem eſſe differentiæ Angulorum dtl, & dul.

Cum enim Trianguli lux, tres Anguli ſimul ſumpti æquales ſint Trianguli dtx, tribus Angulis ſimul ſumptis; ſi auferantur utrinque æquales lxu, & dxt, reliquorum duorum ſumma, $ulx + lux$, erit = ſummæ reliquorum $tdx + dtx$, & ab his æqualibus Summis ſi auferantur inæquales, *V. g.* ulx, ex priori, & tdx, ex poſteriori; reliquorum lux, & dtx, differentia = eſt differentiæ ablatorum ulx, & tdx; quod erat propoſitum.

Centro l. Intervallò Axis Tranſverſi mn, deſcribatur Circulus abc, cujus Arcus ab, rurſus ex l ſpectatur ſub Angulo alb, & ex d, ſub Angulo adb, item diſtantia focorum ld, ex a ſpectatur ſub Angulo lad, & ex b, ſub Angulo lbd. Ergo rurſus differentia Angulorum alb, & adb, = eſt differentiæ Angulorum lad, & lbd. Sed per *Coroll.* 1. Angulus lad, Semis eſt Anguli lud, & Angulus lbd, Semis Anguli ltd. Ergo horum Angulorum lad, & lbd, differentia = eſt Semi-differentiæ Angulorum lud, & ltd; ergo & Angulorum alb, & adb, differentia = eſt Semi-differentiæ Angulorum ult, & udt, quorum prior eſt Intervallum Apparens duarum Obſervationum, poſterior autem, Intervallum Motus Medii. Data igitur horum intervallorum differentia, datur quoque hujus (differentiæ) Semis, nimirum differentia Angulorum alb, & adb. Sed alb, idem eſt cum ult, dato; ergo datur quoque abd, Angulus, ſub quo Peripheria ab, ſpectatur ex d.

Simili modo oſtendetur, differentiam Angulorum tly, & tdy, æqualem eſſe Summæ Angulorum ltd, & lyd; nec non differentiam Angulorum ble, & bdc = eſſe Summæ Angulorum lbd, & lcd. Cumque lbd, ſemis ſit ipſius ltd, & lcd, ſemis ipſius lyd; erit ſane Summa ipſorum lbd, & lcd, = ſemi-ſummæ Angulorum ltd, & lyd; hoc eſt, differentia Angulorum blc, & bdc, = erit ſemi-differentiæ Angulorum tly, & tdy, quorum prior eſt Intervallum Apparens duarum Obſervationum, poſterior autem, Intervallum Motus Medii. Quare, data horum Intervallorum differentia, datur quoque hujus Semis nimirum differentia Angulorum blc, & bdc. Sed blc, idem eſt cum tly, dato; ergo datur quoque bdc, Angulus, ſub quo Peripheria bc, ſpectatur ex d.

Unde liquet, & datis Intervallis Obſervationum Mediis & Apparentibus dari Angulos, ſub quibus ex d ſpectantur Circuli abc, Peripheriæ quotvis, interceptæ a lineis Veri Motus. Ergo per *Herigoni Theor. Plan. lib.* 1. *cap.* 3. *Prop.* 12. *Schol.* 1. totidem Circuli Segmenta deſcribi poſſunt, capacia Angulorum ſub quibus iſti Arcus conſpiciuntur ex d, quæ ſegmenta omnia ſe mutuo interſecabunt in d. Poſſunt igitur & hac Methodo inveniri Apogea

& Excentricitates Planetarum, delineatione Geometrica, adhibitis Obſervationibus quotvis; nec difficilius eſt, Circulos ducere, quam Lineas Rectas.

Sed ut demus id, quod verum eſt, Clariſſimi *Caſſini* Delineationem Geometricam non nihil expeditiorem eſſe; verendum eſt interim ne, ſi ἀκρίβειαν Aſtronomis expetitam ſectemur, Diagrammata requirat Enormis Magnitudinis, adeoque operoſior evadat, quam ipſe Calculus. Ad hunc autem accedentes, utramque Methodum æquipollere deprehendemus. Ne quis vero Apogei & Excentricitatis utraque Methodo inventæ à vero diſcrepantiam cenſeat Errori Calculi imputandam: reſtat ut Hypotheſin Excutiamus.

Et Ellipticæ quidem Orbitæ inventio, ſine controverſia *Keplero* debetur; ſed quibus Accelerationis & Retardationis gradibus incedant Planetæ, definire, non minus pertinet ad integrandam Hypotheſin, quam ipſius Orbitæ determinatio. Quanquam autem ex Cl. *Caſſini* (vel Interpretis ejus) ſermone id nuſquam apparet; attamen ex Conſtructione Problematis, & ejus Analyſi, manifeſtum eſt, eum ſupponere, Planetem ex Foco ſuperiori videri prorſus æquabili Motu incedere. Fuit ſane, cum idem exiſtimaret *Keplerus*, quod ejus ſcripta evolventibus liquere poteſt. Sed cum id Obſervationibus nequaquam Congruere animadverteret, mutavit ſententiam, & Lineam Veri Motus Planetæ æqualibus temporibus æquales Areas Ellipticas verrere profeſſus eſt: Punctum autem, ex quo Planeta exacte æquabili Motu procedere videtur, nullum omnino extare in hoc Univerſo, niſi id libratile ſtatuere libeat. Nulli interim puncto propius æquabilem videri inceſſum Planetæ, quam ipſi Foco ſuperiori Ellipſeos. Neque inventus fuit hactenus, qui Areas *Kepleri* Phænomenis ſatisfacere poſſe negaret; ſed cum eas Calculo directo exhibere nec ipſe, nec poſt eum quiſquam potuerit, cauſati ſunt nonnulli *Keplerum*, nimis indulgentem cauſis, Phyſicis à Geometria diverſum abiiſſe; quaſi cauſæ Phyſicæ repugnent Geometria, aut minus Geometricum ſit Problema; quod, nulla injecta Phyſicarum Cauſarum mentione ſic proponitur: *Data Area Trilinei, inter Lineas Apſidum, & veri Motus, nec non Peripheriam Ellipticam intercepti, invenire Angulum ad Solem.* Habent igitur à *Keplero* reſponſum, qui illi ἀγεωμετρείαν objiciunt; nim. *Eant ipſi & Schema ſolvant.*

Quamvis autem Religio fuerit *Keplero*, ab Hypotheſi, quam naturalem eſſe plane perſuaſum habeat, recedere; quidni liberum foret allis periculum facere, num via quævis alia detur, inequalitatem Planetarum Primam directo Calculo inveſtigandi? Ideoque Vir Cl. *Iſm. Bullialdus* aggreſſus eſt Ratiocinio Geometrico indagare, qua ſemita, & quibus intenſionis ac remiſſionis gradibus conveniret Planetas ferri, ut ab æquabili inceſſus Norma, Aſtronomis ante *Keplerum* aſſumpta, ad eam quam ſpectamus Inæqualitatem perduceremur. Perennant Illuſt. Viri monumenta, unde omnem hujus Inventi rationem hauriri licet Aſtrophilis. Amplexus eandem Reverendiſſ. *Seth. Wardus*, primum oſtendit, paria facere cum linea æquabilis Motus circa alterum Ellipſeos Umbilicum gyrata; deinde & Calculi Directi Methodo ornavit ea, quam paulo ante recitavimus: Ita ut nihil amplius deſiderari poſſet, quam ut Urania felicibus Cœptis annueret. Cujus quidem Nomine ſuſcipere auſus fuit Illuſtriſſ. *Comes Paganus*, edito biennio poſt ejuſdem fere tenoris ſcripto adeo veram eſſe Hypotheſin, ut deprehenſam circa Octantes diſcrepantiam, Aſtronomorum Inſcitiæ tributam mallet. At Cl. *Bullialdus*, audiendum potius ipſam Aſtronomiam ratus, Obſervatorum ore loquentem, ſecundis Curis, adhibita prioribus Inventis Limitatione quadam, diſcrepantiam illam exterminavit. Unde porro intelligitur, Hypotheſin illam, cui Cl. *Caſſinus* Inveſtigationem Apogeorum & Excentricitatum ſuperſtruit, tantundem fere deficere à vero quantum Cl. *Bullialdi* Limitatio pollet, atque ab illo defectu pullulare Calculi à Cœlo diſſenſum.

By Mr. Edm. Halley. Aug. An. 1676. N. 128. *p.* 683.

2. Motus Terræ Annuus per Eclipticam, Opticam Inæqualitatem inducit Motibus cæterorum planetarum; Aſtronomis *Copernicanis* nomine Parallaxeos Orbis notiſſimam; quam quidem inæqualitatem, ex Obſervationibus non multa opera datam, Methodi ſequentis baſin firmiſſimam conſtituo; ubi præter Obſervata nihil aliud ſupponitur, quam quod Orbes Planetarum ſint Ellipſes, quodque Sol in Foco, omnium Orbibus communi, ſit conſtitutus & denique quod Tempora Periodica ſingulorum ita innoteſcant, ut non ſentiatur error aliquis, ſaltem in duabus vel tribus Revolutionibus: His conceſſis, Motus Terræ, pro cæteris Planetis neceſſario requiſitus, primo aggrediendus eſt.

Fig. 121.

Sit S, Sol; A B C D E, orbis Terræ; P. Planeta Mars, (qui in hanc rem plurimis de cauſis longe præferendus eſt; & primo obſervetur Verum Tempus & Locus, quo Mars opponitur Soli; Tunc enim Sol & Terra coincidunt in Lineam Rectam cum Marte; vel (quod fere ſemper accidit ſi habuerit Latitudinem, cum puncto, ubi Perpendicularis à Marte demiſſa in planum Eclipticæ incidit. Sic in Schemate, S, A, & P, ſunt in Linea Recta; deinde poſt 687 dies, Mars revertitur ad idem punctum P, ubi in priori Obſervatione Soli opponebatur; Terra vero cum non revertatur ad A, niſi poſt 730½ dies, in B, Solem reſpicit in Linea S B, Martem vero in Linea B P, & Obſervatis Longitudinibus Solis & Martis, omnes Anguli Trianguli, P B S, dantur, & ſuppoſita P S 100000, in iiſdem partibus invenitur Longitudo Lineæ S B; pari ratione poſt alteram Martis Periodum, Terra exiſtente in C, invenitur Linea S C, nec abſimiliter Linea S D, S E, S F, differentiæque Obſervatorum Locorum Solis, ſunt Anguli ad Solem A S B, B S C, C S D, D S E: Sic tandem ventum eſt ad hoc Problema Geometricum; *Datus tribus lineis, in uno Ellipſeos Foco coeuntibus, tam Longitudine quam poſitione, invenire Longitudinem Tranſverſæ Diametri, cum Diſtantia Focorum:* Cujus Reſolutio extenditur etiam ad Reliquos Planetas, ſi poſt Theoriam Motus Terræ cognitam, ſcrutemur (ſecundum Methodum propoſitam à Reverendiſſ. Epiſcopo *Sarisburienſi* in *Aſtronomia* ejus *Geometrica, Lib.* 2. *Part.* 2. *Cap.* 5.) tres Diſtantias Planetæ alicujus à Sole in poſitionibus ſuis. Quoniam vero Rev. *Epiſcopus* ſupponit Planetam ita ferri in Orbe ſuo, ut æqualibus temporibus æquales Angulos ad Focum alterum Ellipſeos abſolvat, & ei Calculum ſuum ſuperſtruit, non incongruum videtur, oſtendere, quomodo id ipſum fieri poſſet abſque iſta ſuppoſitione, quam Obſervatio nos rejiciendam monet.

Sit S, Sol; A L B K, Orbis Terræ; P. Planeta, vel punctum in Plano Eclipticæ, ubi Perpendicularis, à Planeta demiſſa, incidit; A B, Linea Apſidum Orbis Terræ: Obſerventur primo Planetæ, in P, Longitudo & Latitudo, ſimulque Solis Longitudo a Terra in K; & poſt Periodum ejuſdem Planetæ. Terra exiſtente in L, Obſerventur denuo poſitiones Planetæ, Soliſque, ut prius: jam ex Obſervatis Longitudinibus Solis & Aphelii Terræ, Anguli A S K, A S L, dantur, & conſequenter Latera, S K, S L. Jam in Triangulo K S L, dantur Latera K S, L S, & Angulos K S L, quæruntur Latus K L, & Anguli S K L, S L K: Deinde Triangulo K L P, dantur K L, K P L differentia Obſervatarum Longitudinum Planetæ, & P K L Differentia Angulorum S K L, ultimo inventi, & S K P, Elongationis Planetæ à Sole in prima Obſervatione, quæritur L P: Tum in Triangulo L S P, Latera L S, L P, & Angulus P L S, Elongatio Planetæ à ſole in ſecunda Obſervatione, dantur; Latus S P, & Angulus L S P, requiruntur quibus Inventis, ut S P ad L P ita Tangens Latitudinis Obſervatæ ex L, ad Tangentem Inclinationis ſive Latitudinis ad Solem; & ut Co-ſinus Inclinationis ad Radium, ita S P Curtata Diſtantia, ad Veram Diſtantiam Planetæ à Sole: Sic tandem invenimus Poſitionem & Longitudinem deſideratam. Jam reſtat ut oſtendam, quomodo ex Datis tribus Diſtantiis à Sole cum Angulis interceptis invenienda ſit Media Diſtantia cum Excentricitate Elipſeos. *Fig.* 122.

Sit S, Sol, & S A, S B, S C, tres Diſtantia in debita Poſitione, ductiſque A B, B C, ſit A B, Diſtantia Focorum Hyperbolæ, & S A – S B = E H, Tranſverſa Diameter; quibus poſitis, deſcribatur Lina iſta Hyperbolica, cujus Focus Interior eſt punctum A, extremitas Lineæ Longioris S A: pari modo ſint B C, Foci Alterius Hyperbolæ, cujus Diameter S B – S C = K L; ex quibus deſcribatur Linea Hyperbolica Focum habens Interiorem in puncto B: Dico has duas Hyperbolas ſic deſcriptas ſeſe interſecare in puncto F, qui eſt alter Ellipſeos quæſitæ Focus, ductaque Linea F A, F B, vel F C, S A, + F A, S B + F B, vel S C + F C æquabitur Tranſverſæ Diametro, & S F eſt Diſtantia Focorum: quibus poſitis deſcriptio Ellipſeos facillima eſt. Cum vero hujus Conſtructionis ratio non omnibus ita facile percipiatur, non abs re erit, illuſtratrionem ejus aliquam afferre; ideo dico quod ex notiſſima Ellipſeos proprietate S B + F B = S A + F A, & tranſpoſitis Æquationis partibus F B – F A = S A = S B, ita ut etiam ſi F B & F A nos lateant, earum tamen differentia æqualis ſit S A – S B, hoc eſt, E H, cumque ſit ex natura Hyperbolæ, ut habeat quaſvis duas lineas à ſuis Focis ad quodvis punctum in ſua Curva conſtanter Differentes quantitate Tranſverſæ Diametri; conſtat punctum F eſſe alicubi in Curva Hyperbolæ, cujus Diemeter Tranſverſæ æquatur S A – S B, & Foci, A B; Pari modo Demonſtrari proteſt punctum F eſſe in Hyperbola cujus Diameter eſt S B – S C, & Foci B, C. Ergo neceſſe eſt, ut ſit in Interſectione duarum iſtarum Hyperbolarum, quæ, cum ſeſe interſecent in unico ſolum puncto, clare oſtendunt ubi ſit Focus alter Ellipſeos quæſitæ. *Fig.* 123.

Jam ut id ipſum Analytice expediatur, puta Factum, ſitque F B = *a*, S *a* − S B = F B − F A = *b*, A B = *c*, S B − S C = F C − F B = *d*, B C = *f*, ſitque Sinus Anguli A B C = S, Co-ſinus ejuſdem = *s*.

Tum ut *c* ad *b*, ita $2a - b$ ad $\frac{2ab - bb}{c}$; & $\frac{2ab - bb + cc}{2c}$ = B D, per 36. 3. *Eucl.* & ut *f*, ad *d*, ita $2a + d$, ad $\frac{2ad + dd}{f}$ & $\frac{ff - 2ad - dd}{2f}$ = B G, per eandem; & ut minuatur labor calculi, ſit $\frac{cc - bb}{2c} = g$, & $\frac{b}{c} = h$, ſimiliter ſit $\frac{ff - dd}{2f} = k$, & $\frac{d}{f} = l$, tunc B D = $g + ha$, & B G = $k - la$; & quoniam in omni Triangulo.

$$\left\{\begin{array}{l}\text{Obtuſangulo}\\ \text{Acutangulo}\end{array}\right\}\text{Quadratum baſis æquatur}\left\{\begin{array}{l}\text{Summæ}\\ \text{Differentiæ}\end{array}\right\}$$

Quadratorum Laterum, & Dupli Rectanguli Laterum in Co-ſinum Anguli comprehenſi ducti, erit $gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$ æqualis quadrato D G: Sed D G æqualis eſt Sinui Anguli D F G, vel D B G, in *a*, id eſt F B, ducto, (eſt enim Quadrilaterum F B D G, Circulo, cujus Diameter eſt F B, Inſcriptum;) ideo $SSaa = gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$; quæ æquatio facile reſolvitur, cum non excedat Quadraticam Affectam, ſemperque componitur ex iſtis Quadratis & Rectangulis; ſigna tamen + & − ob diverſam trium linearum conſtitutionem multa cautione ſunt Rectangulis adhibenda.

The Obliquity of the Ecliptick from the Obſervations of the Ancients; *by Dr.* Ed. Bernard. Sept. An. 1684. N. 163. *p.* 721

XVII. 1. Obliquitatem Zodiaci repetit *Eratoſthenes*, ante Natum *Chriſtum*, Ann. 230. Grad. 23. atque inſuper 51′. 19″. 31‴. 5″″.

Diſtantia enim Tropicorum ipſi fuit $\frac{11}{83}$. Circuli Meridiani ſive 47° $\frac{59}{x}$. *Ptol.* Μεγ. Συντ. p. 18. 21. Quare λόξωσις *Eratoſthenica* minor erat *Ptolemaico* tantum $\frac{1}{81}$ unius Minuti Secundi, re ſane contemnenda.

Eratoſthenes apud *Cleomedem*, *Ricciolo* eruente, (ſupra Grad. 23.) 46′. 00″.

Eratoſthenes a *Ricciolo* quaſi correctus, 31′. 5″.

Hipparchus, (ante *Chriſt.* 140.) *Eratoſthenicam* retinuit. *Ptol.* Συντ. Μεγ. p. 18. & p. 60.

Theonis, ὡς ἀκριβῶς εἰλημμένη. 51′. 19″. 31‴. 5″″.

Tabulæ tamen *Chovareſmicæ*, conditæ poſt *Chriſt.* 830. exhibent Canonicam λόξωσιν *Alexandrinorum*, juxta MS. Lat. D. *Hattoni*, 51′.

Pytheas Maſſilienſis, ante *Chriſt.* 324. *Ricciolo* 52′. 41″.

Ariſtarchus, ante *Chriſt.* 280. Illuſtri *Savilio* ſupputante, 51′. 20″.

Ariſtarchus, ex ratiocinio *Riccioli*, 30′, 00″.

Strabo Geographus, p. 93. poſt *Chriſt*. 30. $\frac{4}{60}$. Circuli, ſive præter Gradus 23, adhuc unius, ſive 60′.

Nec aliter *Geminus* (tempore *Chriſti*) cap. IV. Element. Aſtron. Et *Tatius* c. 26. atque *Proclus* de Sphæra. Indeque ſive Aſtrologi, apud *Noddamum* Arabem, *Abrahamum Abeneſdram*, &c.

Noddamus Aſtronomus, qui floruit circa Ann. Dom. 1200. notat Λόξωσιν neque obſervatam unquam Majorem Gr. 2,. neque Minorem 23° 33′. continuo tamen decreviſſe.

Cl. Ptolemæus, poſt. *Chriſt*. 140. ſæpius expertus, & Crico ſuo & Plinthide, ſemper reperit proxime eandem cum *Eratoſthenica*, 51′. 20″.

Diſtantia enim Tropicorum verſabatur inter 47$\frac{2}{3}$. & 47$\frac{3}{4}$. Sed elegit pro *Selidio* ſuo 47. 42′. 40″. Συντ. Μεγ. p. 18, 20, 21. & p. 27. capit pene Medium, μοιρ. κγ. να′. κ″. ἔγγιστα. Nec aliter in Hypotheſibus Planetarum. *Theo* vero in Canonibus προχείροις facilitatis cauſa præteriit Minuta Secunda. Fallitur autem *Ricciolus*, dum ex Climate *Rhodi* colligit Λοξώσεως modum pro *Ptolemæo* 230. 30′.

Pappus Alexandrinus (poſt *Chriſt*. 390.) l. 6. Theor. 35′. *Ricciolo* 30′.

Pappus, *Fr. Commandino* colligente 50′. 00″. 00‴.

Theo, (poſt *Chriſt*. 370.) p. 88. accuratius 51′. 20″. 00‴.

Alibi numero rotundo, ut p. 57. & paſſim in Canonibus ſuis προχείροις nondum vulgatis 51′, 00″. 00‴.

Almamon Princeps *Ann. Chriſti* 825. *Hegira* 210. 23°. 35′. *Grav*. p. 44. ex *Ebn-Shatir* Damaſceno MS. *Seld*. adſiſtentibus ei plurimis Aſtronomis. Ita enim refert *Abneſdras* MS. Lat. in Archivis *Digbeanis*. Inſuper *Aſtronomus Incertus* in Arch. *Seld*. affirmat *Jahia Ebn Albimanſur* cum multis aliis Philoſophis, temdore *Almamonis* τὴν Λόξωσιν Experimento بالتيلش الهمدن deprehendiſſe, 23°. 35′.

Idem tradit de obſervatis *Almamonis* Doctiſſimus *Al Noddam* in Commentariis ſuis ad Aſtronomica *Hoſcin Niſaburienſis*. Imo addit ille eodem Ævo ſæpius obſervaſſe *Beni Muſa* modum eundem 23°. 35′ *Bagdadi* in Campis. MS. Arab. *Coll. S. Joan. Oxon*. Hunc etiam placuiſſe pleriſque ſequentium Aſtronomorum. Sane in eo quieſcit *Alferganus* Aſtron. ſuæ c. 3.

Mohammed Ebn. Gaber Al. Batanius, (*Al. Bategnius*,) *Racca*; *Ricciolo*, A. D. 880 Ill. *Savilio*, 890. *Gravio* p. 44. 882. *Hegira*, 269. Obiit ille *Hegira*, 317. A. D. 929. *Abolfaragi* Hiſt. p. 191. 35′. 00″.

Al Batanius hac in re ſuas τηρήσεις præferre non dubitat *Ptolemæi* dictis, c. 4. aitque ſe adjutum longiſſima *Alhidada*, ſeu Regula Parallactica ad formam *Ptolemaicarum* cum cura & aſſiduitate reperiſſe apud *Raccam* Tropicorum Diſtantiam, 47° 10′. (hoc eſt 59°. 36′. minus 12°. 26′) atque adeo Latitudinem *Raccæ* 35°. quam tamen *Ulocbegus* ſtatuit 36°. 10′. *Schickardus* apud *Curtium*, (p. 33.) & *Ricciolus*, 36°.

Thabet Ebn Corra (*Ricciolo*, A. D. 1210. rectius 901, *Hegira* 289.) reperit λόξωσιν 33′ 30″.

Abul Hoſein Ebn Suphi, 35′. 00″.

Abul Waffi Albuziani, & *Abn Hamed Saganienſis*, Vir Ingenioſiſſimus, (A.D. 987. *Heg.* 377.) *Bagdadi* repererunt λόξωσιν tantum non 35′.

Ita & auctor زيج دالكجيل Perſa in Arch. *Seld.* 35′.

Tabulæ itidem Perſicæ *Chryſococcæ*, 35′.

Al Batrunius Riban, (A. A. 995. *Hegira* 385. *Abolfaragius* hunc ponit ad *Hegira* 463. ſeu A. D. 1070.) uſus Quadrante, cui Radius xv. cubitorum *Grav.* p. 44, ex Cod. Arab. *Birunii.* 35′.

Verum *Abu Joafer Alchazan*, cum Socio ſuo *Abufaldo Harwanenſi* apud *Edeſſam*, & iſtius Ævi alii A.D. 970.) obſervarunt τὴν λόξωσιν ad 23°. 35′. plane non acceſſiſſe, ſed paulo fuiſſe Minorem.

Almæon F. *Almanſoris* (A. D. 1140 *Ricc.*) 30′. 30″. at ille *Clavio* & *Mæſtlino* 33′.

Iſmael Abulfeda Princeps *Hamæ*, (A. D. 1311. *Hegira* 711.) in Tabulis ſuis MS. Arab. *Coll. S. Joan.* retinet forte ab *Almamonis* auctoritatem 35′. 00″.

Prophatius Judæus (A. D. 1300. *Ricc.* 1303. *Mæſtlino* apud *Curtium*, p.40. 230 annis poſt *Arzachelem*, inquit *Copernicus*) & *Ricciolo*, & MS. Coll. *Merton.* 32′. 00″.

Abu Mahmud Al Chogandi A. D. 992. *Hegira* 382. tempore *Fecroddaula*, Sextante cujus Radius erat Cubitorum XL. limbuſque in minuta ſecunda diſtinctus, invenerat λόξωσιν Minorem quam unquam captaverat aliquis Majorum ſuorum, nimirum 32′. 21″.

Hinc *Noddamus* Aſtronomus adfirmat (MS. *Coll. Joan.*) Solis Declinationem Maximam vix unquam Minorem fuiſſe repertam 23°. 33′.

Arzachel Hiſpanus, (*Gravio* p. 44. A D. 1089. *Hegira* 482. *Ricciolo* 1070. *Mæſtlino* apud *Curtium*, p. 35. 1075. *Copernico*, l. 3. c. 6. Annis 190. poſt *Al Batanium*) propoſuit λόξωσιν 23°. 33′ 30″. Ita MS. *Coll. Mert. Oxon.* ubi dicitur differentia 17′. 30″. intercedere inter λόξωσιν *Ptolemæi* & *Arzachelis*.

Apud *Maragam* Nobiliſſimus Perſa *Chojah Naſiroddinus Tuſenſis*, A. D. 1269. *Hegira* 668. (at *Gravio*, p. 44. 1261. *Hegira* 660.) accuratiſſime obſervavit τὴν λόξωσιν 23° 30′. 00″.

Hæc eſt minima ex Maximis Solis Declinationibus, quæ ad hunc uſque diem reperta fuit, ait Doctiſſ. Commentator ad Aſtronomica *Hoſein Niſaburienſis*

Ebn Shatir Damaſcenus, MS. *Seld.* A. D. 1363. ait ſe emendaſſe λόξωσιν, non neglecta Solis Parallaxi, quæ Horizontalis capta eſt 2′. 59″. Huic Solis Max. Declin. 23°. 31′. 00″.

Olocbegus Princeps, A. D. 1437. *Hegira* 841. cum *Aly Cuſhgio* aliiſque Aſtronomis, uſus ſumma cura, & maximis Inſtrumentis (vide *Gravinm*, p.44.) reperit λόξωσιν 23°. 30′ 17″. Ita MSS. *Coll. D. Joan.* & *Bibliotheca Savilianæ*, nam MS. *Seld.* exhibet 23°. 30′. 27″.

Rabbi Moyſes Ben Maimon Judæorum Doctiſſimus ait in *Jad. de Conſecratione Calendarum*, c. ult. ſect. 4. Maximam Zodiaci Obliquitatem fuiſſe, A. D. 1174. 23° 30′. ἔγγιςα.

Scias vix dimidiam partem Aſtronomorum Orientalium quorum Scripta Academiæ *Oxon*. Bibliothecis ſervantur à me conſultam fuiſſe. Ex hiſce autem Obſervatis aliiſque quæ mecum adhuc cis vulgus ſervo, unam eandemque ſuſpicor fuiſſe à primordio Mundi λόξεον Zodiacam. Æva enim recentia, quod vides, melioribus Organis errorum exceſſumque veteris Aſtronimæ probe correxerunt.

2. Whether the Poles and Axis of the Earth be really fix'd in the Globe, or ſubject to be transferr'd from place to place, is an old Enquiry, though now lately revived by Mr. *Hook* in his ingenious Eſſays upon the great Mutations and Cataſtrophes which in all appearance have happened to the Earth's Surface. A neceſſary Conſequence of ſuch a Tranſlation of the Poles would be the Change of the Latitudes of Places, which would Encreaſe in thoſe Regions towards which the Poles approach, and Decreaſe in thoſe from which they recede: and under the Meridian 90 Degrees removed from that in which the Poles ſhift, the Latitudes continuing the ſame, the Meridian Line would only alter; but not two Places conſiderably differing in Longitude can be ſuppoſed, wherein, if there be any ſenſible Motion of the Poles, it ſhall not be perceived by the Alteration of the Latitude of one or both of them.

The Obliquity of the Ecliptick, and Elevation of the Pole continue unaltered; by . . . .
Nov. An. 1687.
N. 190. *p.* 403.

The accurate M. *Wurtzelbaur*, has lately furniſhed us with the Means of examining this Hypotheſis by Obſervation, having ſent us the Meridian Altitude of the Sun taken at *Nurenburgh* about the two Solſtices in the Year 1686. *Jun.* 10. He found the Meridian Altitude of the Sun 64°. 2′. 20″. and the next day, 64° 2′. 25″. and on *Decemb.* 14. (three days after the Solſtice, wherein the Sun was got two Minutes higher) he found the Meridian Altitude 17°. 9′. 10″. wherefore the Solſtitial Altitude was 17°. 7′. 10″. Theſe Heights were taken by an Inſtrument of 6 Foot Radius of Braſs; and the Skill and Diligence of the Obſerver is not to be doubted.

To compare with theſe, I find among *Bernard Walther*'s Obſervations made in the ſame City of *Nurenburgh* two hundred Years before, *viz.* In the Year 1487. that the Meridian Altitude of the Sun in the Summer Solſtice was obſerved by the parallactick Inſtrument of *Ptolemy*, whereby the Chord of the Sun's Diſtance from the Zenith was obſerved 44890 parts of 100000 Radius; the ſame being obſerved by the Concurrence of the Obſervations of ſeveral Years both before and after. The Arch anſwering to this Chord gives the Sun's Diſtance from the Zenith 25 . 56′. 30″ and conſequently the Meridian Altitude, its Complement to a Quadrant 64°. 3′. 30″. Again, The ſame Year 1487. the Chord of the Meridian Diſtance of the Sun from the Zenith, on the day of the Winter Solſtice, was found 118790, confirmed likewiſe by many ſubſequent Obſervations; the Arch anſwering to this Chord is 72°. 52′. 40″. and its Complement 17°. 7′. 20″. the Meridian Heighth of the Sun in the Winter Solſtice.

Hence it appears, That the Solſticial Heights were very nearly the ſame at *Nurenburg* 200 Years ago as now they are, that of the Summer Solſtice being but one Minute differing, the other only 10′. both which may poſſibly ariſe from the Defects of the Inſtruments of theſe Obſervers, being made with Plain Sights: But what I ſhall neceſſarily conclude from hence is, That if there be ſuch a Motion of the Poles, it is either very ſlow, or elſe nearly at Right Angles to the Meridian of *Nurenburg*; in which latter caſe, the Latitudes of Places about *Tunking*, *Siam*, *Malacca*, and *Java*, on the one ſide, and in our *American* Plantations of *New-England*, *Virginia*, *Jamaica*, &c. on the other, ought to change faſteſt; but I have never yet heard of any ſuch thing obſerved by any of our Navigators; whence, if there be ſuch a Change of the Earth's Poles, it muſt neceſſarily require a long time to become ſenſible.

Beſides, from theſe Obſervations, it appears, That the Obliquity of the Ecliptick has continued unaltered for theſe 200 Years laſt paſt; that is to ſay, that the Angle which the Earth's Axis makes with the Plain of the Ecliptick or Orb wherein ſhe moves annually round the Sun, has been without ſenſible Change in all that time; which will be very hard to conceive, if we allow a Tranſlation of the Earth's Poles; for the Direction of the Axis being perfectly at Liberty, it muſt be purely caſual, if it ſo hit, that after ſuch Change, it make the ſame Angle with the Ecliptick as before.

A farther Argument of this Slowneſs of the Change of the Poles, is the Latitude of *Alexandria*, the Habitation of thoſe famous Aſtronomers of Antiquity, *Eratoſthenes*, *Timocharis*, *Hipparchus*, and *Ptolemy*, and for that Reaſon it may be concluded, that this, of all the Latitudes the Ancients have left us, ought to be one of the moſt Correct. This by *Ptolemy* is ſaid to be 30°, 58′. North, (which he uſes in all his Computations in his *Almegiſt*, and ſeems derived from the proportion of the Gnomon to its Equinoctial Shadow, as 5 to 3. but in his *Geography*, 31°. juſt. In the Year 1638. the Curious and Ingenious Mr. *Graves*, when he went to viſit the *Egyptian Pyramids*, of which he has given ſo good an Account, did, with a ſufficient Inſtrument, obſerve the Latitude of *Alexandria*, and found it 31°. 4′. or 6 Minutes more than it is reputed by *Ptolemy*, and before him by *Eratoſthenes*; ſo that in about 2000 Years the Latitude of *Alexandria* has altered only a few Minutes, and ſo few, that the Accuracy of the Obſervations of the Ancients may well be queſtioned: but both being granted, this Motion will amount to no more than a Degree in 20000 Years.

This is ſaid not with intent to invalidate what Mr. *Hook* hath from ſo good Grounds advanced, *viz.* That the Ball of the Earth, at leaſt the Fluids thereof, being neceſſarily of the Figure of a *Spheroides Prolatus*, or flat Oval, whoſe ſhorteſt Diameter is the Axis, and greateſt Circle the Equinoctial; if the Poles be ſuppoſed changed, the Equinoctial will be ſo too; and conſequently the Water muſt riſe and cover thoſe Parts from which the Poles recede, and fall off, and leave bare thoſe Places towards which the Poles approach. By this Means it may be accounted for, how ſuch ſtrange Marine things are found on the Tops of Hills, and ſo deep under Ground; and ſcarce any other way. But from theſe, and the like Obſervations, it will follow, That if theſe Inundations are produced by any regular Motion of the Poles, it would require a prodigious number of Ages to effect the Changes we may be certain have been. Beſides, if the Acceſs and Receſs of the Sea were after ſuch a gradual manner, as when produced by ſuch an eaſie Tranſlation of the Poles, as can by Obſervation be admitted, thoſe Inundations could never be fatal to the Inhabitants, for that they would always give notice of their coming, ſo that the People might provide for their Safety. But the *Holy Scriptures*, and Pagan Tradition, do unanimouſly agree, That the laſt great *Deluge* was brought to paſs in a few Days, with no previous Notice; ſo that the Account we have thereof could not, by this *Hypotheſis* be made out, without the Suppoſition of a great and ſudden Alteration in the Poles of the Earth's Diurnal Revolution; for which, whether we ſhould have recourſe to the intelligent Powers that firſt impreſs'd this whirling Motion on the Ball, or leave it to be performed naturally, by the caſual Shock of ſome tranſient Body, ſuch as a Comet, or the like, whereby the former Axis might be loſt, and a New Revolution produced, differing both in Time and Poſition from the Old, I ſhall not undertake to diſpute; ſuch a Suppoſition would include likewiſe a Change of the Length of the Year, and Eccentricity of the Earth's Orb; for which yet we have no ſort of Authority.

A ſuppoſed Alteration of the Meridian Line, by . . . .
n. 255. p. 285.
June An. 1699.

1. As I was wondering how an ordinary Mathematician could miſs ſo eaſie a thing as the drawing a true Meridian, ſo far as in the inſtance of the old Meridian in the Church of St. *Petronio* in *Bononia*, which is found by M. *Caſſini* to vary 8 or 9 Degrees from the true Meridian of the Place; and in that of the Meridian of *Uraniburge*, which is found by M. *Picart*, and others, to vary 18′; I hit upon this Thought, that Meridians muſt needs vary. For you know, that (taking it for granted that the Earth moves, *&c.*) beſides the Diurnal and Annual Revolutions, there muſt be alſo a third to account for that ſlow Motion of the Fixed Stars, upon the Poles of the Ecliptick, in about 25000 Years; which is ſolved by the Direction of the Earth's Axis from one Point to another of the polar Circle. And that Direction being nothing but a certain Wabble in the Earth's Motion, muſt needs make the Noon-ſhade of a Perpendicular not lie always in the ſame Line.

Conſider'd by Dr. Wallis.
Ibid p. 286.

2. This being a new Suggeſtion, deſerves to be conſider'd: For it is not probable that ſo careful a Man as *Tycho*, and thoſe concerned in the Church of St. *Petronio*, ſhould be ſo much miſtaken in the Meridian Line. But if there be ought of this Nature, it muſt ariſe from a Change of the Terreſtrial Poles (here on Earth: of the Earth's Diurnal Motion; (not of their pointing to this or that of the Fixed Stars :). For if the Poles of this Diurnal Motion remain fixed to the ſame Place on the Earth, the Meridians (which paſs thro' theſe Poles) muſt remain the ſame.

The Parallax of the Sun, by Mr. Flamſteed.
n. 89. p. 1519.
Nov. An. 1672.

XVIII. I have had the good hap to meaſure the Diſtances of *Mars* from two Stars the ſame Night; whereby I find, that his Parallax was very ſmall, certainly not 30″: So that I believe the Sun's Parallax is not more than 10″.

n. 96. p. 6100.

To find the Sun's Ingreſs into the Tropical Signs, by Mr. Edmund Halley N. 215. p. 12. Feb. An. 1695.

XIX. It may perhaps paſs for a Paradox, if I ſhould aſſert, That is is an eaſier Matter to be aſſured of the Moments of the Tropicks, or of the Times of the Sun's entrance into *Cancer* and *Capricorn*, than it is to obſerve the true Times of the Equinoctials or Ingreſs into *Aries* and *Libra*. But I here deſign to ſhew a Method to find the Moment of the *Tropicks* capable of all the exactneſs the moſt Accurate can deſire; and that without any conſideration of the Parallax of the Sun, of the Refractions of the Air, of the greateſt Obliquity of the Eclíptick, or Latitude of the Place: All which are required to aſcertain the Times of the Equinoctial, from Obſervation; and which being faultily aſſumed, have occaſioned an Error of near three Hours in the Times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*, the Vernal being ſo much later, and the Autumnal ſo much earlier than by the *Calculus* of theſe famous Authors.

Now before we proceed, it will be neceſſary to premiſe the following *Lemmata*, ſerving to demonſtrate this Method; *viz.*

1. That the Motion of the Sun in the Ecliptick, about the Time of the *Tropicks*, is ſo nearly Equable, that the Difference from Equality is not ſenſible, from 5 days before the Tropick to 5 Days after, by reaſon of the nearneſs of the *Apogæon* of the Sun to the Tropick of *Cancer*.

2. That for 5 Deg. before and after the Tropicks, the Differences whereby the Sun falls ſhort of the Tropicks, are as the *Verſed Sines* of the Sun's Diſtance in Longitude from the Tropicks, which *Verſed Sines* in Arches under 5 Degrees, are beyond the utmoſt Nicety of Senſe, as the Squares of thoſe Arches. From theſe two follow a third,

3. That for 5 Days before and after the Tropicks, the Declination of the Sun falls ſhort of the utmoſt Tropical Declination, by Spaces which are in *Duplicate* Proportion, or as the Squares of the Times by which the Sun is wanting of, or paſt, the Moment of the Tropick.

Hence it is evident, That if the Shadows of the Sun, either in the Meridian, or any other Azimuth, be carefully obſerved about the time of the Tropicks, the Spaces whereby the tropical Shade falls ſhort of, or exceeds thoſe at other times, are always proportionable to the Squares of the Intervals of Time between thoſe Obſervations and the true Time of the Tropick, and conſequently if the Line, on which the Limits of the Shade is taken, be made the Axis, and the correſpondent Times from the Tropick *Expounded* by Lines, be erected on their reſpective Points in the Axis as Ordinates, the extremites of thoſe Lines ſhall touch the Curve of a *Parabola*. Thus *a*, *b*, *c*, *e*, being

Fig. 124.

ſuppoſed Points obſerved, the Lines *a* B, *b* C, *c* A, *e* F, are reſpectively proportional to the Times of each Obſervation before or after the tropical Moment in *Cancer*.

This premiſed, we ſhall be able to bring the Problem of finding the true Time of the Tropick by three Obſervations, to this Geometrical one. *Having three Points in a* Parabola A, B, C, or A, F, C, *given, together with the Direction of the Axis, to find the Diſtance of thoſe Points from the Axis.*

Of this there are two Caſes; the one, when the time of the ſecond Obſervation B is preciſely in the middle between A and C: In this Caſe, putting t for the whole Time between A and C, we ſhall have Ac, the Interval of the remoteſt Obſervation A, from the Tropick, by the following Analogy.

As $2ac - bc$ to $2ac - \frac{1}{2}bc$; ſo is $\frac{1}{2}t$, or A E to Ac, the Time of the remoteſt Obſervation A, from the Tropick.

But the other Caſe, when the middle Obſervation is not exactly in the middle between the other two Times, as at F, is ſomething more operoſe, and the whole Time from A to C, being put $= t$, and from A to F, $= s$, $ce = c$, and $bc = b$, the *Theorem* will ſtand thus, $\frac{ttc - bss}{2tc - 2bs} =$ Ac, the Time ſought.

To illuſtrate this Method of Calculation, it may perhaps be requiſite to give you one or two Examples.

An. 1500. *Bernard Walter* in the Month of *June* at *Nuremberg*, obſerv'd the Chord of the Diſtance of the Sun from the Zenith, by a large *Parallactick* Inſtrument of *Ptolemy*, as follows;

June 2.	45467	and	*June* 8.	44975.
June 9.	44934		*June* 12.	44883.
June 16.	44990		*June* 16.	44990.

In both theſe Caſes the Middle Term is exactly in the middle between the Extreams, and therefore in the former three, $ac = 533$, $bc = 477$, and t, the Time between, being 14 Days, by the firſt Rule, the Time of the Tropick will be found by this Proportion; as 589 to 827$\frac{1}{2}$, So $\frac{1}{2}t$, or 7 Days to 9 Days, 20^h. 2'. Whence the Tropick, *An.* 1500. is concluded to have fallen *June* 11 20^h. 2'. In the latter three ac is $= 107$, and $bc = 15$, and the whole interval of Time is 8 Days = to t; whence, as 199 to 206$\frac{1}{2}$ ſo is 4 Days to 4^d. 3^h. 37'; which taken from the 16th Day at Noon, leaves 11^d. 20^h. 23'. for the Time of the Tropick, agreeing with the former to the third part of an Hour.

Again, *An.* 1636. *Gaſſendus* at *Marſeilles*, obſerved the Summer Solſtice by a Gnomon of 55 Foot high, in order to determine the Proportion of the Gnomon to the Solſtitial Shade, and he hath left us theſe Obſervations, which may ſerve for an Example for the Second Rule.

June 19	St. N. ſhadow	31766	Parts, whereof the Gnomon was 89428
20		31753	
21		31751	
22		31759	

Theſe being divided into two Setts of three Obſervations, each, *viz.* The 19th, 20th, and 22d, and the 19th, 21ſt, and 22d, we ſhall have in the firſt three, $c = 13$, and $b = 7$, $t = 3$ Days, $s = 1$; and in the ſecond, $c = 15$, and $b = 7$, $t = 3$, and $s = 2$. Whence, according to the Rule, the 19th Day at Noon, the Sun wanted of the Tropick a Time proportionate to one Day, as $ttc - ssb$ to $2tc - 2bs$, that is, as 100 to 64 in the firſt Sett, or 105 to 62 in the ſecond Sett; that is, $1^d.\ 17^h.\ 15'$ in the firſt, or $1^d.\ 17^h.\ 25'$. in the ſecond Sett: So that we may conclude the Moment of the Tropick to have been *June* $10^d.\ 17^h.\ 20'$. in the Meridian of *Marſeilles.*

Now that theſe two Tropical Times thus obtained, will be found to confirm each others Exactneſs from their near Agreement, appears by the Interval of Time between them, *viz.* $1^d.\ 2^h.\ 30'$ leſs than 136 *Julian* Years, whereof $1^d.\ 1^h.\ 8'$. ariſes from the defect of the length of the Tropical Year from the *Julian*, and the reſt from the Progreſſion of the Sun's *Apogaeon* in that Time; ſo that no two Obſervations made by the ſame Obſerver in the ſame Place can better anſwer each other, and that without any the leaſt Artifice or Force in the managament of them.

What were the Methods uſed by the Ancients to conclude the Hour of the Tropicks, *Ptolemy* has no where delivered; but it were to have been wiſhed that they had been aware of this, that ſo we might have been more certain of the Moments of the Tropicks we have received from them, which would have been of ſingular Uſe to determine the Queſtion, Whether the Sun's *Apogaeon* be fix'd in the Starry Heaven; or if it move, What is the true Motion thereof? It is certain, that if we take the Account of *Ptolemy*, the Tropick ſaid to be obſerved by *Euctemon* and *Meton*, *Junii* 27 *Mane*, *Ann.* 432 *Ante Chriſtum*, can no ways be reconciled, without ſuppoſing the Obſervation made the next Day, or *June* 28. in the Morning. And *Ptolemy*'s own Tropick, obſerved in the third Year of *Antoninus*, *An. Chriſt.* 140. was certainly on the 23d, and not on the 24th Day of *June*, as will appear to thoſe that ſhall duly conſider and compare them with the length of the Year deduced from the diligent and concordant Obſervations of theſe two great Aſtronomical Genii, *Hipparchus* and *Albatani*; eſtabliſhed and confirmed by the Concurrence of all the Modern Accuracy. For theſe Obſervations give the Length of the Tropical Year, ſuch as to anticipate the *Julian* Account only one Day in 300 Years; but we are now ſecure, that the ſaid Period of the Sun's Revolution does anticipate very nearly 3 Days in 400 Years; ſo that the Tables of *Ptolemy*, founded on that Suppoſition, do Err about a whole Day in the Sun's Place, for every 240 Years. Which principal Error in ſo Fundamental a Point, does vitiate the whole Superſtructure of the *Almageſt*, and ſerves to *convict its Author of want of Diligence, or Fidelity, or both.*

But to return to our Method, the great Advantage we have hereby, is, That any very high Building ſerves for an Inſtrument, or the top of any high Tower or Steeple, or even any high Wall whatſoever, that may be ſufficient to intercept the Sun, and caſt a true Shade, and make the Spaces large and fair, though the Heighth and Diſtance of the Building, and Poſition of the Plain, upon which you receive the Shade, and of the Line on which you meaſure the Spaces, be not exactly known. But it is convenient that the Plain on which you take the Shade be not far from Perpendicular to the Sun, at leaſt not very Oblique, and that the Wall which caſts the Shade, be ſtraight and ſmooth at the top, and its Direction nearly Eaſt and Weſt. The principal Objection is, That the *Penumbra*, or Partile Shade of the Sun is in its Extreams very difficult to diſtinguiſh from the true Shade, which will render this Obſervation hard to determine nicely. But if the Sun be tranſmitted through a Teleſcope, after the manner uſed to take his *Species* in a *Solar Eclipſe*, and the upper half of the Object-Glaſs be cut off by a Paper paſted thereon, and the exact upper Limb of the Sun be ſeen juſt emerging out of, or rather continging the Species of the Wall, (the Poſition of the Teleſcope being regulated by a fine Hair extended in the Focus of the Eye-Glaſs) I am aſſured, that the limit of the Shade may be obtained to the utmoſt Exactneſs. I ſhall only further advertiſe, That the Winter Tropick by this Method may be more certainly obtained than the Summer's, by reaſon that the ſame Gnomon does afford a much larger Radius for this manner of Obſervation.

The Solar Numbers corrected by Mr. J. Flamſteed. n. 110. *p.* 220. Jan. Ann. 1675.

XX. I have found it neceſſary to make new Solar Numbers, becauſe in my old, I have neglected to apply Refractions in all the Altitudes above 30 Degrees; wherein yet Reaſon and ſome little Experience hath ſhewed me, they are not inſenſible. I found S. *Caſſini*'s Obſervations, which I took from *Ricciolus* his *Aſtronomia Reformata*, much more Accurate than *Tycho*'s, and therefore ſought out Numbers that might anſwer them. The *Apogæum* I found it neceſſary to promote 44 Minutes; ſo that *Anno Ineunte* 1655. it might be in ♋ 7 . 30 . 00″. and to make the greateſt Equation only 1°. 54′. 13″. whereby I found the *Phænomena* would be anſwered much more accurately than I expected, and as near, all things conſidered, as I could deſire.

But ſtill I was uncertain, whether the Refractions in the ſaid *Caſſini*'s Tables were juſt Meaſures or not, and I had no Conveniencies for making Trial. At laſt I thought on this Expedient, which fully ſatisfied me; *viz.*

I conſidered, That if ſome of thoſe Obſervations of the Diſtances of ♀ from the ☉ by Day, and from the Stars in the Night preceding or following, were skilfully Examined, they might ſhew me the true Quantity of the Equations of the Sun's Orb, or rather the Difference of his Mean and Equal Motion. I turned over his *Progymnaſmata*, and pitched on two: The firſt made *Ann.* 1585, *March* 5. 4^h. 42′ and 7^h. 12 *poſt Meridiem*: whereby I found the ☉ at 4^h. 42′. was 94°. 47′. in Antecedence of the *Lucida Calcis* ♊. The ſecond made *Ann.* 1585, *Septemb.* 15. 5^h. 15′. and 6^h. 55′. *Mane.* Where-from, (applying and conſidering the Refractions

fractions in both) I ſound the Sun at 6^h. 55′. to be 71°. 30′. in Conſequence of the *Lower Head* of ♊. The Difference of Longitude betwixt theſe two Stars is 17′ 59′: And therefore now the Sun in Conſequence of the *Lucida Calcis* ♊. 92°. 29′. So that the Sun's apparent Motion betwixt the Year 1582. *Mar*. 5. 4^h. 42′. and the Year 1585. *Septemb*. 15. 6^h. 55′. *Manè*, (beſides the whole Revolutions) was 187°. 16′. but the Mean Motion is 191°. 2′. greater than the Apparent by 3°. 46′. which parted in proportion to the Equation of the Earth's Motion, collected for thoſe Times from my New Tables, gives the greateſt Equation of the Orb, 1°. 54′. 15″. conſenting to my wonder, without any wreſting of the Obſervations with that, which I deduced from *Caſſini*'s Correct Meridional Altitudes.

The Sun's Motion, by the Tables which I now uſe, grounded on this Equation, is leſs than *Tycbo*'s by no leſs than 9′. That great Equation made him commit no ſmall Errors, and put him upon ſtrange Shifts to hide and ſolve them. So that all his Obſervations of the Planets in their Oppoſitions to the Sun, are to be Corrected, before we attempt to repreſent them by Numbers: For his Errors in the Sun's Place made him Err ſometimes 5 or 6 Hours in the time of the Oppoſition; which muſt be Reformed.

The Equality of Natural Days; by ...... *Profeſſor of Mathematicks at* Seville. N. 118. *p.* 426. Oct. An. 1676.

XXI. Non parvum adhuc eſt Diſſidium inter Aſtronomos, quanta ſit in aquando Tempore Proſthaphæreſis, ita ut *Longomontanus* fateatur, nullam in Aſtronomico Pulvere majorem Difficultatem ſe inveniſſe; quod cum notarem, animadverti in quibuſdam Cœli Obſervationibus à me factis, quid ex illis inveniret; &, cum mihi eſſet Horologium Rotatile Pendulum, admodum exactum, linea Meridiana artificialiter Conſtituta, examinabam Solis in Meridianum Ingreſſum, ſingulis diebus, cum quo ad amuſſim Horologium meum congruebat, & ſi diſcrepabat aliquando, rariſſime, duobus minutis diſcriminabat, quod, cum opus erat emendabam. Quare per triennium continuando & quotidie Solem in Meridiano obſervando, cum licebat (quod in hac Regionis parte ſæpe ſæpius fit) inveni tandem nullam Diem Naturalem Longiorem Revolutionem, in Uno vel Alio Anni Tempore, alia Die habuiſſe, unde intrepide dico omnes Dies Naturales æquales eſſe, & ſi adhuc aliqua Differentiola intercedit, non eſſe Senſibilem. Hæc volui notum facere, ut Aſtronomos hoc Scrupulo liberarem, quod tam multos torſit & indies torquet, quanquam *Tychonica* Æquatio propter Eclipticæ Obliquitatem non ſit rejicienda.

Refuted; by Mr. Flamſteed. *Ibid*, p. 430.

2. Dies quomodo Æquales eſſe poſſint, & tamen Æquatio *Tychonica* admitti, vix me capere fateor. Ob inæquales etenim æqualium Eclipticæ partium Rectas Aſcenſiones, dies unus Æquinoctialis Tropico uno Brevior erit Scrupulis Horæ ſecundis 40; & Dies 14 Tropici, totidem Æquinoctialibus. Longiores ſunt ſexta Horæ parte, ſeu Scrupulis primis 10. Hanc autem Differentiam Majorem credo, quam ut eam in Obſervationibus ſuis non perciperet *Profeſſor Hiſpalenſis*, proindeque ipſum in examinandis iis *Tychonicam* Temporis Æquationem adhibuiſſe autumem.

Sed & ponamus, Æquales eſſe Primi Mobilis Revolutiones (quod nulli, qui *Ptolemaicam Hypotheſin* admittant, unquam iverunt inficias) neceſſario conſequitur, nec Æquationem Temporis, ad inæquali Solis in Orbita ſua Inceſſu enatam, rejiciendam eſſe: Eteniam cum Apogeus quotidie promoveatur tantum 57′. 10″. Perigeus vero 61′. 15″; Apogeus equidem citus 16″, (ſeu Tempore abſumpto dum primum Mobile revolvit 4′. 5″.) a Meridie in Meridianum Diei ſequentis recurret, quam Perigeus: Attamen quandoquidem progenita ex hac cauſa Æquatio tardius admittit Diurnum Incrementum, Scil. 8″. quotidie, ad ſummum cum velociſſima, & vix Diebus 15 ad duorum Scrupulorum Quantitatem excreſcit, et, cujus ille meminit, duorum Scrupulorum Emendatione, in Horologio ſuo ablatam credo: de qua videat propterea Vir Doctiſſimus.

Demum vero, ſi in *Copernicanam Hypotheſin* fit pronior, quam in *Ptolemaicam*, in ea etiam, ſuppoſitis Terræ Iſochronis Revolutionibus, eædem conſequuntur Æquationes. Fateor equidem, amoveri poſſe, & in contrarium trahi, ab Inæquali inceſſu Solis in Orbita ſua provenientem Temporis Æquationem, ſi inæquales Terræ vel primi Mobilis (perinde enim eſt utrum horum ſtatuerimus: Revolutiones ſupponamus: Sed ſi Temporis Naturam bene perpendat, facile intelliget, impoſſibile eſſe, omnem ejus Inæqualitatem removeri.

XXII.
An Equation Table; by M. Caſſini. N. 214. *p.* 248.

Tabula *Æquationis* Dierum cum Solis Loco adeunda.

	♈		♉		♊		♋		♌		♍	
G.	′ Sub.	″	′ Add.	″	′ Add.	″	Sub.	″	′ Sub.	″	′ Sub.	″
0	7	45	1	11	4	3	0	59	5	43	2	8
1	7	26	1	24	4	0	1	15	5	45	1	53
2	7	7	1	37	3	56	1	29	5	46	1	37
3	6	48	1	49	3	51	1	42	5	47	1	21
4	6	29	2	1	3	45	1	54	5	48	1	5
5	6	10	2	12	3	39	2	6	5	48	0	48
6	5	51	2	23	3	32	2	19	5	48	0	30
7	5	31	2	33	3	25	2	32	5	46	0	12
8	5	11	2	43	3	17	2	44	5	44	0 Add.	7
9	4	51	2	53	3	9	2	56	5	40	0	26
10	4	31	3	3	3	0	3	8	5	36	0	35
11	4	11	3	13	2	51	3	20	5	31	1	3
12	3	52	3	22	2	41	3	32	5	25	1	21
13	3	33	3	30	2	31	3	43	5	19	1	40
14	3	14	3	37	2	21	3	54	5	13	1	59
15	2	55	3	43	2	10	4	4	5	6	2	19
16	2	37	3	48	2	0	4	14	4	58	2	40
17	2	19	3	53	1	49	4	26	4	49	3	1
18	2	1	3	57	1	37	4	34	4	39	3	22
19	1	43	4	1	1	25	4	43	4	30	3	44
20	1	26	3	5	1	13	4	51	4	20	4	6
21	1	9	4	8	1	1	4	59	4	9	4	29
22	0	52	4	10	0	49	5	6	3	57	4	51
23	0	35	4	12	0	37	5	13	3	45	5	13
24	0	19	4	13	0	24	5	19	3	32	5	35
25	0	3	4	11	0	10	5	24	3	19	5	57
26	0 Add.	12	4	9	0 Sub.	3	5	29	3	5	6	19
27	0	27	4	8	0	16	5	33	2	51	6	41
28	0	42	4	6	0	29	5	37	2	37	7	2
29	0	57	4	5	0	44	5	40	2	23	7	23
30	1	11	4	3	0	59	5	43	2	8	7	44

Tabula

Tabula *Æquationis* Dierum cum Solis Loco adeunda.

	♎		♏		♐		♑		♒		♓	
G.	′ Add. ″		′ Add. ″		′ Add. ″		′ Add. ″		′ Sub. ″		′ Sub. ″	
0	7	44	15	34	13	25	0	59	11	48	14	36
1	8	5	15	42	13	7	0	27	12	4	14	29
2	8	25	15	48	12	48	0 Sub.	5	12	19	14	21
3	8	45	15	53	12	29	0	35	12	35	14	13
4	9	5	15	57	12	10	1	4	12	50	14	4
5	9	25	16	1	11	50	1	33	13	5	13	55
6	9	44	16	5	11	30	2	3	13	19	13	46
7	10	3	16	7	11	10	2	32	13	32	13	37
8	10	22	16	8	10	49	3	1	13	44	13	27
9	10	41	16	9	10	28	3	29	13	55	13	17
10	11	00	16	9	10	6	3	57	14	5	13	7
11	11	19	16	9	9	42	4	25	14	14	12	56
12	11	38	16	8	9	17	4	53	14	22	12	44
13	11	57	16	7	8	51	5	20	14	29	12	32
14	12	15	16	5	8	25	5	48	14	35	12	19
15	12	33	16	1	7	58	6	15	14	40	12	6
16	12	50	15	56	7	31	6	42	14	45	11	52
17	13	7	15	50	7	5	7	9	14	50	11	37
18	13	22	15	44	6	38	7	34	14	54	11	21
19	13	36	15	37	6	12	7	58	14	56	11	4
20	13	49	15	30	5	45	8	21	14	58	10	46
21	14	2	15	22	5	19	8	45	14	59	10	28
22	14	14	15	13	4	52	9	8	15	00	10	10
23	14	26	15	3	4	26	9	31	15	00	9	52
24	14	37	14	52	3	58	9	53	15	00	9	34
25	14	47	14	40	3	30	10	13	14	58	9	16
26	14	57	14	27	3	1	10	32	14	55	8	58
27	15	7	14	13	2	31	10	51	14	51	8	40
28	15	16	13	58	2	1	11	10	14	47	8	22
29	15	25	13	42	1	30	11	29	14	42	8	4
30	15	34	13	25	0	59	11	48	14	26	7	45

Spots Obſerv'd in the Sun; by Mr. Boyle. N. 74. *p.* 2216.

XXIII. *Ann.* 1660, *April* 27. About 8 of the Clock in the Morning there appear'd a Spot in the lower Limb of the Sun, a little towards the South of its Æquator, which was entred about $\frac{1}{4}$. of the Diameter of the Sun itſelf, being about $\frac{1}{160}$ in its ſhorteſt Diameter, of that of the Sun, its longeſt, about $\frac{1}{10}$ of the ſame. It diſappeared upon Wedneſday Morning, *May* 9. though we ſaw it the day before about 10 in the Morning, to be near about the ſame Diſtance from the Weſtward Limb, a little South alſo of its Æquator, that it firſt appeared to be from the Eaſtward Limb, a little South alſo of its Æquator. It ſeem'd to move faſter in the middle of the Sun than towards the Limb. It was a very dark Spot, almoſt of a Quadrangular Form, and was encloſed round with a kind of duskiſh Cloud.

We firſt obſerved this very ſame Spot both for Figure, Colour, and Bulk, to be Re-entred the Sun *May* 25, when it appear'd to be in a part of the ſame Line it had formerly traced; and was Entred about $\frac{4}{23}$ of its Diameter about 7 a Clock in the Afternoon. At the ſame time there appear'd another Spot, which was juſt Entered, and appeared to be Entered not above $\frac{1}{12}$ part of the Sun's Diameter. It appeared to be longeſt towards the North and South, and ſhorteſt towards the Eaſt and Weſt. There ſeem'd to be diſpers'd about it divers ſmall Clouds here and there.

Spots Obſerv'd in the Sun; by M. Picard. N. 74. *p.* 2238. N. 75. *p.* 2253. *By M.* Caſſini. N. 74. 2238. N. 75. 2250.

XXIV. 1. *Ann.* 1671. M. *Picard*, at Sea near the *Texel*, obſerv'd a Spot in the Sun from *Aug.* 3. *St. N.* to the 19*th*. It appear'd at firſt, like the Tail of a *Scorpion*, but on the 19*th* day reſembling a Melon-Seed.

2. *Aug.* 11. (*St. N.*) 1671. About 6 a Clock a Night, M. *Caſſini*, with a three-Foot Glaſs, remark'd in the Sun's Disk, two Spots very dark, diſtant from its apparent Center about the third part of his Semi-Diameter. They were in the Southern part of the Sun, and their Elongation from the *Parallel of the Æquator* paſſing through the Center of the Sun, was about $\frac{1}{6}$ part of his Diameter. The Time which lapſed between the Tranſite of the Sun's Center and that of the firſt of theſe Spots was 22″. or 23″. the Semidiameter of the Sun then paſſing in 66″. The firſt of theſe Spots, being looked upon with a Teleſcope of 17 Foot long, appear'd with a ſomewhat Oval Figure; the other was Oblong and a little curved, like the *Hebrew* Letter *Jod*; and both together were ſurrounded by a *Corolla*, or Coronet, made up of little dark Points, which conformed itſelf to the Figure of the Spots, conſidered as they were joyned together.

Aug. 12. He perceived that they were nearer his Center. The Time between the paſſage of the Sun's Center, and that of the interior edge of the *Coronet* which encompaſs'd them both, was then of 16″. At 7 a Clock it was but of 15″. and the Southern Limb of the *Coronet* touched the Parallel paſſing through the Sun's Center. The firſt Spot was compoſed of two others almoſt round, and conjoined. The ſecond repreſented the ſhape of a *Scorpion*. The third was round. And they were all three environed with a *Coronet*, which was compoſed, as we ſaid above, of abundance of little obſcure Pricks. This *Coronet* appear'd to be clearer than the reſt of the Sun, when look'd upon with the ſhort Glaſs, and darker when ſeen with the long. Without it there were other Points, but very black ones; *viz.* Five near the round Spot on the South-ſide and another near the *Scorpion*'s Tail on the North-ſide.

At 8 a Clock 48′. the Figure of the *Scorpion* was ſeen divided into ſeveral pieces, as if the Tail and Arms had been cut off. The Northern Point appear'd no more, there remainining none but thoſe on the South-ſide; and the length of the encloſure of all the *Spots*, comprehended between the Extremities, was of 1′. 15″. and the Breadth of 30″.

The ſame 12*th* day at 6 in the Evening, he found no great Change in the firſt *Spot*. The other two were ſevered into 5 diſtinct ones, compaſs'd about with a *Coronet*, together with 5 black Points, which ſtood in a ſtreight row, and after another manner than they did in the Morning. From 6 at Night unto 7, the time when the paſſage of the*Sun*'s Center, and that of the *Coronet*'s Limb, was found to be at one time of 8″. and another time of 7½. The Diſtance of the *Spots* unto the Parallel paſſing through the Sun's Center, was near the ſame on the North-ſide with what it had been obſerved to be in the Morning on the South-ſide.

Aug, 13. Between the Riſing of the Sun, and half an Hour paſt 6 in the Morning, the edge of the *Coronet* was turned to a Point on the South-ſide, and was diſtant from the *Æquator* on the North-ſide, half a Minute; and there was but a Second of Time from the paſſage of the Sun's Center unto the paſſage of the ſame anterior edge of the *Coronet*.

At 8 a Clock, 30′. the fore-edge was in the ſame Horary Circle with the Center of the Sun: ſo that in one day and an half, theſe Spots have run thro' very near the third part of the Sun's apparent Semidiameter, which giveth an Arch of 19 Degrees and an half of the Circumference of the Sun's Body; and conſequently their *Diurnal* Motion about the Sun's Axe hath been of 13 Degrees; and the time of their *Periodical* Revolution, as far as we could conjecture in ſo little a time, muſt be about 27 days and an half.

Aug. 14. At 6 in the Morning, there paſs'd 15″. of Time between the paſſage of the Anterior Limb of the *Crown*, and the paſſage of the *Sun*'s Center through the ſame Horary Circle: And then the Southern Limb of the *Crown* was a Minute and an half diſtant toward the North, from the Parallel of the *Æquator*, paſſing through the ſame Center of the Sun. The Figure of the firſt Spot was almoſt the ſame with that of the day before. The ſecond had taken the Form of an Heart, the Point of which was turned to the North-ſide, and its baſe between the South and the Eaſt. Three other ſmall Spots, diſpoſed Triangle-wiſe, ſtood over the ſaid Baſe, and were accompanied with two others upon a Line turned Southward. And they were all encompaſſed by a Crown running out into a Point on the South-ſide; and on the North-ſide, Eaſtward, it had an *Appendix*.

Aug. 15. At 6 in the Morning, there paſſed 27″ between the paſſage of the Anterior Limb of the Crown, and that of the Sun's Center through the ſame Horary Circle. The Southern Limb of the ſame Crown was two Minutes and an half diſtant from the Parallel of the *Æquator*, paſſing through

the

the Center of the *Sun*, whoſe Diameter paſs'd in 2′. 9″. through the ſame Horary Circle. The firſt *Spot* had a little changed the Figure; the ſecond was Quadrangular, longer from Eaſt to Weſt, than from North to South: It appeared bigger than ordinary, and had withal on its ſides, within the compaſs of the *Crown*, three other ſmall *Spots*. There were alſo ſeen four more without the ſaid *Crown* on the South-ſide.

Aug. 16. At 6 in the Morning, there was 27″. between the paſſage of the *Sun*'s Anterior Limb, and the paſſage of the Anterior Limb of the *Crown* through the ſame Horary Circle; and 38″. between the paſſage of the Anterior Limb of the *Crown*, unto the paſſage of the *Sun*'s Center. The Southern Limb of the *Crown* was 3½′ off from the Parallel of the *Æquator*, paſſing thro' the Center of the *Sun*, towards the North. And the Obſervation having been made yet more exactly at half-an hour paſt 7 of the ſame Morning, this Diſtance was found of 3′. 33″. The Figure of the firſt *Spot* in the beginning of the Obſervation, differ'd not much from that of the precedent day; but afterwards it was ſeen divided into two. The ſecond, which likewiſe ſeemed to be the ſame in the beginning, was afterwards divided into three, accompanied with black and dark Points without the *Crown* on the South-ſide. The ſame day at 6 a Clock, and 15 . at night, the Figures of theſe *Spots* were much changed. There were 5 *Spots* encloſed in the *Crown*; The two foremoſt were part of that which had been ſeen in the Morning as one; the two others following thoſe two firſt, were part of the ſecond in the Morning; and without there were 5 Points on the South-ſide, and two more a little further to the North, which Points were ranged as in another Area made up of other Points ſo ſmall that they could ſcarce be perceived.

Aug. 17. in the Morning, immediately after the Riſing of the *Sun*, there appeared three very dark *Spots*, which form'd in a manner theſe Letters *F n J*, poſited from Eaſt to Weſt, and included in their wonted *Crown*, which ſtretched out, as 'twere two Arms, or two Handles, one to the South, and the other to the North. There paſs'd 18″. between the paſſage of the foremoſt Limb of the *Sun*, and that of the foremoſt Limb of the *Crown*, and 47½″. between the paſſage of the Anterior Limb of the *Crown*, unto the paſſage of the *Sun*'s Center. The Southern Limb of the ſame *Crown* was diſtant 11′. 17′. from the *Parallel* that touched the *Sun* on the North-ſide, and 4′. 38′. from the *Parallel* that paſs'd through the Center.

Aug. 18. at 7 in the Morning, the *Spots* which appear'd through ſome Clouds, had almoſt the ſame ſhape with thoſe of the day before, only with this difference, that they were a little cloſer together, drawing from Eaſt to Weſt. There lapſed 13″. between the paſſage of the Anterior Limb of the *Sun*, and that of the Anterior Limb of the *Spot*, through the ſame Horary Circle, and 52½″. of the foremoſt ~~Limb of the~~ *Spot* unto the paſſage of the Center. The Southern Limb of the *Spot* was 9′. 13″. diſtant from the *Parallel* that touched the Northern Limb of the *Sun*, and 6 , 41″. from the *Parallel* that paſs'd through his Center. At 5 a Clock and 55′. at night of the ſame day, there lapſed 11″. between the paſſage of the Anterior Limb of the *Sun* through the ſame Horary Circle, and the paſſage of the Anterior

Limb of the *Crown*, and from thence unto the Paſſage of the *Sun*'s Center $54\frac{1}{2}''$. The Limb of the *Crown* next the *Parallel* paſſing through the Center of the *Sun*, was diſtant from the ſame *Parallel* 7'. 40''.

Aug. 19. from 4 to 5 in the Evening, the *Spot* appear'd Oblong near the *Sun*'s Circumference, from which it was diſtant about the breadth of the ſame *Spot*.

Aug. 20. in the Morning, which was not the full ſeventh from the day that they were arriv'd to the middle of the Disk, they diſappeared.

The apparent Velocity nigh the Center was ſuch, that if it had continued the ſame, the *Spots* would have arrived almoſt in 4 days to the Limb of the Disk; but in this Hypotheſis, that the *Spots* were adherent to the *Sun*'s Surface, or at leaſt very nigh to it, this apparent Velocity was to leſſen according as they ſhould remove from the Center, as hath come to paſs in effect. The Diminution of the length of the Miſty *Crown* was in a manner proportionable to the Diminution of the apparent Velocity; ſince that, when this Crown was in the middle, and in a Situation, wherein its true Figure could be beſt ſeen, it appear'd Oblong, and of the Form of an human Ear, its greateſt Diameter reſpecting Eaſt and Weſt; but being nigh the Limb, this ſame Diameter ſeemed to ſhorten; and having appeared greateſt in its firſt Situation, it appear'd leaſt in this, becauſe it was almoſt in a Circle that paſs'd through the Center of the Sun, whoſe equal Arches are by ſo much the more Oblique, by how much they approach more to the Limb of his Disk, and conſequently appear leſs, according to the Rules of *Opticks*; mean time, the Diameter, that was turned from South to North, apparently kept the ſame bigneſs it had near the Center, becauſe it was in a Circle almoſt Parallel to the Horizon of the *Sun*, which formed the Repreſentation of its Limb, and whoſe equal Arches, (by the ſame *Optical* Reaſons) do not appear contracted.

3. Several curious Obſervers at *London*, have ſeen one of theſe *Spots* recurr'd to the Sun's Eaſtern Limb, about *Aug.* 25. *St. N.* as M. *Caſſini* predicted they ſhould return. *By ſeveral at* London. N. 75. *p.* 5253.

Aug. 30. 1671. I ſaw a large *Spot* in the Center of the *Sun*'s Face about Noon. *By* Dr. Hook: N. 77. *p.* 2295.

Sept. 1. At 3 a Clock, I ſaw the ſame *Spot* moved about a quarter of the Diameter of the *Sun* Weſtward: it conſiſted of one greater and two leſſer black *Spots*, with a *Dusky Cloud* incompaſſing them: The Diameter of the whole Phænomenon was about $\frac{1}{7}$. of the Diameter of the *Sun*, and it was diſtant from the next adjoyning Limb $\frac{1}{2}$ (that is exactly one quarter) of the Diameter of the *Sun*.

5. Maculæ Solares Obſervatæ fuere nobis *Hamburgi* à 26. *Aug.* *St. V.* (primo fere die, quo iterum Apparere cœperunt) ad uſque 5. *Sept.* quo ad Limbum quam proxime acceſſere. *By Mr.* Hen. Siterus. N. 78. *p.* 3033.

XXV. *Ann.* 1676. *Jun.* 28. St. N. Habemus in ſole ſatis Ingentem *Maculam*, quæ *Solem* ipſum mediavit h. 4. poſt Meridiem, cum Latitudine Auſtrali $4'\frac{1}{4}$; ejus Diſtantiam à Polo Auſtrali *Solis* ex pluribus Obſervationibus ſupputavi *gr.* 78 '. Si ſatis habuerit Conſiſtentiæ ad abſolvendum Circulum expectanda Reſtitutio ejus ad Medium, Diei 25. *Jul.* veſpere, cum majore Latitudine Auſtrali. *Spots Obſerv'd in the Sun by M.* Caſſini. N. 127. *p.* 665.

XXVI. 1.

Spots Obſerv'd in the Sun; by Mr. Flamſteed, *and* Mr. Halley. N. 128. p. 687.

Jul. St. V.	Grenovici, A. D. 1676. *Tempus Obſervationum.*	*Longit. à Solis Centro.*	*Latit. Auſt.*	Oxonii, A.D. 1676. *Tempus Obſervationum.*	*Lon.*	*Lat. Auſt.*
	h. ′	′ ″	′ ″	h. ′	′ ″	″
25				6 46 p. m. Con.	13 40	2 8
27	10 3 a. m. Con.	9 34	3 25			
28	4 51 —	5 40	2 50			
29	10 31 a. m. —	3 05	3 27	6 21 a. m. —	3 55	3 22
	3 54 p. m. —	2 25	3 10			
30	9 15 a.m Ant.	0 37	3 33	7 20 a. m. —	0 00	3 30
31				7 40 a. m. —	3 36	3 28
Aug. 1.	9 25⅚ a. m. —	6 48	4 09	7 3 a. m. —	6 54	3 50
				5 6 p. m. —	8 7	3 53
2	8 8 a. m. —	9 49	3 55	7 16 a. m. —	9 57	3 40
3	9 36 a. m. —	82 28	3 27	5 9 p. m. —	13 15	3 56
	4 16 p. m. —	12 55	3 58	6 2 p. m. —	13 25	3 26
4	7 38 a. m. —	14 02	4 4	7 33 a. m. —	14 7	3 14
				4 54 p. m. —	14 43	3 23

Mr. *Halley* ſaith, That he ſaw a Spot again on the fifth day, 8h. 30′. *mane*, very near the Limb of the Sun, ſo that it appeared only as a fine Line; but by reaſon of its fineneſs and the too great Height of the Sun, he could not take any meaſures to determine its Place and Latitude by; and that while the Spot continued one, as it was *July* 25. he meaſured to the middle of it; as alſo when the Pieces were divided, but not far disjoined: Afterwards, when they were ſeparated conſiderably, he obſerved the middle of the bigger Spot, which was to the South, apparently, I ſuppoſe; but really North: for ſo only his Obſervations will agree with thoſe of Mr. *Flamſteed* exactly.

Hence it ſeems very evident, (ſaith Mr. *Flamſteed*) that the Spot's way was not inclined to the *Ecliptick* ſix or ſeven Degrees, as *Scheiner* and ſome others make it, but much leſs by the joint Conſent of the Obſervations of both our Obſervers. Mr. *Halley* adds, That conſidering the Motion of the Spot croſs the Sun's Diſk, as both their Obſervations give it, it appears that the Latitude was not ſo great at its Entrance into the Sun as in the middle of him. And by Mr. *Flamſteed*'s Obſervation it was greateſt on the firſt of *Auguſt*, and then again inclining towards the Ecliptick. If you grant this, it will follow, infers Mr. *Flamſteed*) that the Sun's Axis was inclined to the Plain of the *Orbis magnus*; but the quantity of this Inclination muſt not be very great. The *Nodes* of the Sun's Equinox and Ecliptick he gueſſes to be not far from the beginning of *Cancer* and *Capricorn*; and that from *Cancer* to *Capricorn* the Earth is North of the Sun's *Æquator*; from *Capricorn* to *Cancer*, South of the ſame: And the Period of the Sun's Revolution in reſpect of the fixed Stars 25 Days, 9½ hours ſufficiently exact. Of which things theſe two Obſervators ſay, they might have been more certain, had not the *Spot* in its paſſage broken into ſo many parts, and thoſe often varied their Poſitions to each other.

2. *Solarem Maculam* hic obſervavimus à die 6 *Aug.* ad 14 *St. N.* Collatione que Obſervationum didicimus, eam medium Itineris ſui in ſolis Diſco Apparente tenuiſſe circa Mediam Noctem poſt 8. diem *Aug.* in diſtantia Apparenti trium minutorum à Centro Auſtrum verſus. In plures diſtracta partes eſt, quæ invicem Boream & Auſtrum verſus indies ſatis manifeſto intervallo disjungebantur, adeo ut, præter Motum Communem circa Solis Axem, ſingulæ partes Proprium inter ſe directum habuerit. Hanc Porro *Maculam* diverſam eſſe ſentio ab ea quam præcedenti Menſe *Junio* obſervaveramus. Illa quippe cum medium Itineris ſui in Diſco Solis Apparente tenuerit die 28 ejuſdem Menſis ad eundem proxime ſitum reverſa eſſet (ſi fuiſſet ſuperſtes die 25 *Julii* Nocte ſequente, ut deducitur tum ex ejus Velocitate, Tempore ſuæ Apparitionis obſervata, tum etiam ex Curſu aliarum *macularum*, quæ periodum ſuam circa Solem à nobis videntur abſolvere Spatio dierum 27 cum triente, vel 27 cum ſemiſſe. Ejus præterea ſemita diverſa eſt à præcedenti; prior quippe paulo remotior fuit ab Æquatore *macularum* quam poſterior. Hæc porro, ſi ſatis habuerit Conſiſtentiæ, ad Medium *Solem* rediit die 5 *Septembris* mane.

XXXVII. *Ann.* 1684. *Apr.* 25. About an Hour before Noon I diſcover'd a large Spot entered within the Sun's Diſk a little diſtant from his following Limb. Theſe Appearances however frequent in the days of *Scheiner* and *Galileo*, have been ſo rare of late, that this is the only one I have ſeen in his Face ſince *December* 1676. By the obſerved Meridional Diſtances of it, and the Sun's Southern Limb from the *Vertex* at Noon, I found it to have 3′. 40″. more North Declination than the Sun's Center, and at 3h. 35′. after Noon, I meaſuted its Diſtance from his next Limb 40″.

Spots Obſerved in the Sun; by Mr. Flamſteed. N. 157. p. 535.

Next Morning, *April* 26. I ſaw it more Remote from his Limb, and by the Obſervations then made at 8 h. *mane*: determined its Longitude from the Sun's Axis 66½ Deg. and its Declination from the Solar Æquator 9½ Deg. South. Whence ſuppoſing the Revolution of any point of the Sun to the ſame fixed Star to be performed in 25 days 6 hours, the Angle of his *Equator* and our *Ecliptick* 7 Deg. and the Longitude of his Northern Pole ♍ 16 Deg. I deſigned the Line of its Way or Trace over the Sun, and the Points in it where the Spot would appear every Morning after at the ſame hour, till its Egreſs on the 8th of *May*, which I found altogether confirmed by ſuch Obſervations as I made till then; ſo that I had no Reaſon to doubt of the Theory.

When the *Spot* was near the middle of the Sun, it appeared very broad, and almoſt ſquare, the *Nucleus* of the ſame Figure about 40″. Diameter; but when it was near the Limb much narrower, and almoſt Oval: It ſeemed to have Conſiſtence enough to endure a ſecond Return; if it ſhall, it will enter the Viſible Disk of the *Sun* on the 21ſt of *May* in the Evening, and in its paſſage over him deſcribe a Line nearly ſtreight, with greater Latitude from the *Eclipticк*.

To find in what Proportion the Planets are Enlightned by the Sun; by M. Auzout. N. 4. *p.* 68.

XXVIII. One of the Means uſed by M. *Auzout* to Enlighten an Object in what Proportion one pleaſeth, is by ſome great Object-Glaſs, by him called a *Planetary* one, becauſe that by it he ſhews the difference of Light, which all the Planets receive from the *Sun*, by making uſe of ſeveral Apertures, proportionate to their Diſtances from the Sun, provided that for every 9 Foot Draught, or thereabout, an Inch of Aperture be given for the Earth. Doing this, one ſees (ſaith he) that the Light which *Mercury* receives, is far enough from being able to burn Bodies, and yet that the ſame Light is great enough in *Saturn* to ſee clear there, ſeeing that (to him) it appears greater in *Saturn* than it doth upon the Earth, when it is over-caſt with Clouds, which (he adds) would ſcarce be believed if by means of the Glaſs it did not ſenſibly appear ſo.

The Æquinoxes; by M. Wortzelbaur. N. 264. *p.* 623.

XXIX. The *Æquinoxes* of this Year 1699, according to the Obſervations of M. *Wortzelbaur* at *Nurenberg*, happened *March* 9. 20h. 35′. 27″. and *Sept.* 12. 10h. 22′. 42″. which by his Tables ought to have been *March* 9. 20h. 40′ 30″. and *Sept.* 12. 10h. 32′. 52″.

To Obſerve Solar Eclipſes; by Mr. Flamſteed. N. 55. *p.* 1104.

XXX. For the well obſerving of Solar Eclipſes, caſt the Species of the Sun through a good Teleſcope of a competent Length, on an extended Paper, placed behind the Eye-Glaſs ſo far, as that the ſaid Species may appear at leaſt 6 Inches over; then divide both his *Periphery* into 360 Degrees for the better Obſerving the Inclination of the *Cuſps* of each Phaſis, and his Diameter into Digits and their Parts, by Concentrick Circles, for meaſuring the Quantities of the obſcured Parts.

An Eclipſe of the Sun, Ann. 1666. June 22. *at* London; *by Mr.* Willoughby, *Dr.* Pope, *Dr.* Hook, and *Mr.* Phillips. N. 15. *p.* 295.

XXXI. 1. The *Eclipſe* began at 5h. 43′.

					h.	′
It was darkned	$\frac{3}{11}$ Diam.—at	6	00	5 Dig.— at	7	6
	4 Digits. at	6	7	4 Dig.— at	7	13
	5 Dig.— at	6	13	3 Dig.— at	7	20
	6 Dig.— at	6	21	2 Dig.— at	7	26
	7 Dig.— at	6	39½	1 Dig.— at	7	32
	6 Dig.— at	6	57	0 Dig.— at	7	37

Its *Duration* hence appears to have been one hour and 54′. Its greateſt *Obſcurity* ſomewhat more than 7 Digits. About the middle, between the Perpendicular and Weſtward Horizontal Radius of the Sun, viewing it thro'

Mr. *Boyle*'s 60 Foot Teleſcope, there was perceived a little of the Limb of the Moon without the Disk of the Sun; which ſeemed to ſome of the Obſervers to come from ſome ſhining Atmoſphere about the Body either of the Sun or Moon. They affirm to have obſerved the Figure of this Eclipſe, and meaſured the Digits, by caſting the Figure through a 5 Foot Teleſcope on an extended Paper, fix'd at a certain diſtance from the Eye-Glaſs, and having a round Figure; all whoſe Diameters were divided by 6 concentrick Circles, into 12 Digits.

At Paris; *by M.* Payen. *Ibid. p.* 296.

2. The *Eclipſe* began at $5^h. 44'. 52''$ *mane*. It ended at $7^h. 43'. 6''$. So that its whole duration was $1^h. 58'. 14''$. The greateſt Obſcuration was 7 *Dig.* 50′. but it ſeemed to have been greater by 3′. which Mr. *Payen* imputes to a particular Motion of Libration of the Sun's Globe, which entertained that Luminary in the ſame Phaſis for the ſpace of 8′. and ſome Seconds, as if it had been ſtopped in the midſt of its Courſe; rather than to a Tremulous Motion of the Atmoſphere, as *Scheiner* would have it. The apparent Diameters were almoſt Equal: for in the Phaſis of 6 *Digits*, the Circumference of the Moon's Disk paſſed thro' the Center of that of the Sun, ſo as that two Lines drawn through the two Horns of the Sun, made with the common Semidiameter two Equilateral Triangles.

The Beginning and Middle of the Eclipſe happened to be in the North-Eaſtern Hemiſphere, and the End in the South-Eaſtern. The firſt Contact (as 'twere) of the two Disks, was obſerved in the ſuperior Limb of the Sun's Disk in reſpect to the Vertical Line, and in the Inferior in reſpect to the Eclíptick. But the Middle and the End were ſeen in the Superior Limb, in reſpect both to the Vertical and the Ecliptick: And what to M. *Payen* ſeems extraordinary, both the Beginning and the End of this Eclipſe happen'd to be in the Oriental Part of the Sun's Disk.

At Madrid; *by the Earl of* Sandwich. *Ibid. p.* 296.

3. The *Eclipſe* began about 5 a Clock in the Morning; at $5^h. 15'$. The Sun's Altitude was $6^{\circ}. 55'$.

The Middle of it was at 6^h 2′. The Sun's Altitude $15^{\circ}. 5'$.

The End was exactly at $7^h. 5'$. The Sun's Altitude $25^{\circ}. 24'$.

The Duration 2^h 4′.

Thirty ſeven parts of the Sun's Diameter remained Light, and 63 were Darkned.

At Dantzick; *by M.*Hevelius, N. 19. *p.* 1347.

4. In this *Eclipſe* it is chiefly obſervable, That the Semidiameter of the Moon, from the very beginning to about 5 or 6 Digits of the Increaſing Phaſis, was almoſt equal to the Semidiameter of the Sun: But, after the greateſt Obſcuration, when I again contemplated the Moon's Semidiameter, I found it 8″. or 9″. bigger than that of the Sun; ſo that the Semidiameter of the Moon was not always, during the *Eclipſe*, conſtant to itſelf. Of this Variation, the Excellent *Iſmael Bullialdus* hath alſo obſerv'd ſomething at *Paris*. For he hath written to me, That in this ſame *Eclipſe*, the Semidiameter of the Sun to the Semidiameter of the Moon was, as 16. 9″. to 16′. 22″. but that in another Phaſis of 6 Digits, the Semidiameters appeared equal.

Ibid.
p. 349.
N. 21.
p. 370.

Ordo Phaſium.	Quantitas Phaſium.	Tem. Æſt. Sec. Horol. Ambulat.	Temp. ex Sciother.	Altitud. ☾.	Tempus Correct.	Animadvertenda.
		h. ′ ″	h. ′ ″	° ′	h. ′ ″	
		5 51 11	5 51 00	17 45	5 53 12	Quod Sciatericum cum Correcto Tempore non omnino convenit, non niſi Lineæ Meridianæ imputandum.
		5 57 05	5 57 00	18 37	5 59 28	
		6 00 00	6 00 00	18 55	6 1 28	
	Initium.	6 55 30			6 57 30	Initium circa 79 gr. à puncto Zenith Occaſum verſus contigit.
1	$0\frac{3}{8}$ Dig.	8 57 30			6 59 30	
2	$0\frac{3}{4}$	7 0 23	7 0 0		7 2 23	
3	$1\frac{5}{8}$	7 2 30	7 2 0		7 4 30	
4	$1\frac{1}{2}$ dig.	7 4 50	7 5 fere.		7 6 50	
5	$1\frac{3}{8}$ fere.	7 10 57	7 10		7 12 57	
6	$3\frac{3}{8}$	7 14 59	7 15		7 16 59	
7	$3\frac{3}{4}$	7 17 50	7 18 fere.		7 19 50	
8	4 dig.	7 21 35	7 21		7 23 35	
9	$4\frac{2}{3}$	7 23 43	7 23 fere.		7 25 43	Hucuſque Semidiameter Lunæ æqualis extitit Solari.
10	$5\frac{1}{4}$	7 27 53	7 28		7 29 53	
11	6	7 31 50	7 32		7 33 50	
12	$6\frac{3}{4}$	7 36 55	7 37		7 38 55	
13	$6\frac{7}{8}$ paul.plus	7 38 5	7 38		7 40 0	
14	$7\frac{1}{8}$	5 39 45	7 39		7 41 45	
15	$7\frac{1}{4}$ paul plus	7 42 30	7 42		7 44 30	
16	$7\frac{1}{2}$	7 44 6	7 44		7 46 6	
17	$7\frac{2}{3}$	7 46 0	7 46		7 48 0	
18	8 fere.	7 48 25	7 48 fere.		7 50 25	
19	$1\frac{1}{5}$	7 51 15	7 51		7 53 15	
20	$8\frac{1}{4}$ paul.plus	7 53 37	7 52		7 55 37	
21	$8\frac{3}{4}$	7 55 45	7 56 fere.		7 57 45	
22	$8\frac{3}{8}$ pau.min.	7 59 5	7 59		8 1 5	Mox. Obſcuratio extitit. Dig. 8. 25′. hora 8. 2′.
23	$8\frac{1}{5}$	8 6 30	8 6		8 8 30	

Ordo Phaſium.	*Quantitas Phaſium.*	*Tem. Æſt. Sec. Horol. Ambulat.*	*Temp. ex Sciother.*	*Altit. ☉.*	*Tempus Correct.*	*Animadvertenda.*
		h. ′ ″	h. ′ ″	° ′	h. ′ ″	
24	7¼	8 11 25	8 12		8 13 25	Hic Semidiameter Lunæ ad 8″. vel 9″. major apparuit
25	7¼ fere.	8 17 30	8 18		8 19 30	
26	7 fere.	8 19 41	8 19		8 21 40	
27	5⅞	8 28 8	8 28		8 30 8	
28	5½ fere.	8 30 14	8 30		8 22 14	
29	4¾	8 36 25	8 36		8 38 25	
30	3⅝	8 43 19	8 43		8 45 19	
31	3¼	8 46 12	8 46 fere.		8 48 12	
32	3	8 47 32	8 47		8 49 32	
33	2¾	8 50 57	8 50		8 52 57	
34	2½ fere.	8 54 15	8 54		8 56 15	
35	1⅜	8 58 24	8 58		9 0 24	
36	1⅛	8 59 35	8 59		9 1 35	
37	0⅙	9 1 38	9 1		9 3 38	
38	0½	9 3 20	9 3		9 5 20	
39	Finis.	9 6 53	9 6		9 8 53	Punctum Finis diſtitit à Verticali ad Ortuum 143 gr.
		9 23 6		47 33	9 25 28	
		9 24 16		47 42	9 26 45	
		9 28 29		48 10	9 30 42	
		9 30 36		48 28	9 33 12	

XXXII. *An Eclipſe of the Sun*, Jun. 23. (*St. N.*) 1675. *at* Dantzick; *by M.* Hevelius. N. 127. *p.* 660.

Ordo Phaſium.	*Animadvertenda.*	*Alt. Solis.*			*Temp. Cor.*		
		h.	′	″	h.	′	″
	Solis Centrum in Horizonte.				3	21	30
	Nihil in Sole.				4	42	0
	Initium.				4	44	0
1	$6\frac{1}{2}$ Digiti fere Obſcurati erant.				5	32	0
2	$6\frac{2}{3}$ Digit.				5	34	0
3	Maxima fere obſcurat. 6 Digit. 42′.				5	38	50
4	$6\frac{1}{2}$ Digit.				5	43	30
5	$6\frac{1}{4}$ Digit				5	47	30
6	6 Digit.				5	49	30
7	$5\frac{1}{2}$ Digit. Diamet. Lunæ, 14′.37″. Data nempe				5	55	0
8	$5\frac{1}{4}$ Digit. Semidiametro Sol. 15′.				5	57	0
9	5 Fere Digit.				5	59	30
10	$4\frac{5}{8}$ Digit.				6	2	0
11	$4\frac{1}{2}$ Digit.				6	4	15
12	4 Digit.				6	6	30
13	$3\frac{5}{8}$ Digit.				6	9	15
14	$3\frac{1}{4}$ Digit.				6	14	35
15	$2\frac{5}{8}$ Digit.				6	18	10
16	$2\frac{3}{8}$ Digit.				6	20	0
17	2 Digit.				6	22	0
18	$1\frac{1}{2}$ Digit.				6	24	0
19	$1\frac{1}{4}$ Digit.				6	25	25
20	1 Digit.				6	27	10
21	$\frac{1}{2}$ Digit.				6	30	30
	Finis.	°.	′		6	33	30
	Altitudo ☉.	25	20		6	41	22
	Altitudo ☉.	28	5		7	0	14

An Eclipſe of the Sun, June 1. 1676. *at* Weſtminſter; *by Mr.* Francis Smethwick. N. 126. *p.* 637.

XXXIII.	1.	Initium Defectionis.	7^h.	50′.
		Finis.	9	$54\frac{3}{4}$.
		Totius Eclipſis Duratio.	2	$4\frac{3}{4}$.

Tempus Obſervatum fuit cum Horologio Oſcillatorio, Correcto per Obſervatione. Tubus adhibitus fuit bonæ notæ, pedum $7\frac{1}{2}$.

2. *At* Wapping *by Mr.* Colſon. *Ibid.*

Temp. juxta Horol. Oſcil. h.	′	″	*Phaſes.*	*Solis Alt.* °	′	*Temp. Correct. ex Altit.* h.	′	″	
7	34	50		22	46	7	36	0	
7	37	14		33	10	7	38	40	
7	39	10		33	30	7	40	48	
7	50	40	$0\frac{1}{4}$ Digit.			7	51	51	Tubo Optico æſtim.
8	8	34	$1\frac{1}{4}$ Digit.			8	9	45	Tub. Optico menſur.
8	17	25	$2\frac{1}{2}$ Digit.			8	18	36	
8	27	10	$3\frac{1}{6}$ Digit.			8	28	21	
9	39		$1\frac{1}{2}$ Digit.			9	40		Tubo æſtim.
9	43		$1\frac{1}{4}$ Digit.			9	44		
9	48		$0\frac{1}{4}$ Digit.			9	49		
9	54	25	Non Finita.			9	55	36	
9	55	55	Finita.			9	57	6	
4	26	5		32	10	4	26	56	
4	28	58		31	53	4	29	52	
4	31	2		31	31	4	32	16	

At Greenwich; *by Mr.* Flamſteed. N. 127. *p.* 662.

Hiſce Obſervationibus peragendis Socium acciveram Amicum meum *Ed. Halleium.* Tubos præparaveram duos, alterum Digitos $196\frac{1}{2}$. longum, quocum & *Micrometro Townleiano*, Ego ipſe octo Phaſium priorum cepi Menſuras, alterum Digitorum duntaxat $103\frac{1}{2}$, quocum & *Micrometro* meo, iis adſcriptas Menſuras *Halleius* cepit: in duabus tamen ultimis Animadverſionibus, Ego Minori Tubo & *Micrometro* meo (in hunc uſum altero accomodatiore) Diſtantiam cepi *Azimutharum*, per Solis Limbum Lucidum, & Cuſpidem proximam *Eclipſis* decidentium; *Halleio* interea Partes Lucidas & Cuſpidum Diſtantiam majori Tubo dimetiente. Paulo ante Initium advenerat Nobiliſſimus Præſes Regiæ Societatis Dom. Vice-Comes *Brouncker*, qui Menſuram Diametri Solaris, Tubo Longiori captam, ſuo Judicio probavit. Hora 7. 45′. Sol primum per Nubes apparuit. Obſervata deinde ſic ſe habuerunt.

Phaſium

Fig. 125.

Phaſium Ordo.	Hor. Horol. Oſcillatorii.	Correcta.	*Longiori Tubo.*	*Breviori.*
	h ′ ″	′ ″		
	7 46 00	7 45 00	Nulla Eclipſis.	
1	7 54 50	7 53 50	Solis Eluctati è Nubibus Magno Dexter Eclipſatus apparuit. ′ ″	
2	7 58 24	7 57 24		
3	8 04 12	8 03 12	I C —— 2040 = 10 10	
4	8 13 40	8 12 40	I C —— 2773 = 13 56	′ ″
5	8 18 37	8 17 37	I C —— 3580 = 17 52	PL – 3198 = 26 18
6	1 21 06	8 21 06	PL —— 4975 = 24 50	I C – 2334 = 29 13
			Sol. Diamet. 6360 = 31 43	PL – 2989 = 24 35 3850 = 31 40
7	8 28 01	8 27 01		PL – 2888 = 23 57
8	8 29 01	8 28 01	PL —— 4565 = 21 46	
9	8 35 12	8 34 12	PL —— 4478 = 22 18	AZ – 2310 = 19 00
10	8 40 20	8 39 20	I C —— 4417 = 22 00	AZ – 2070 = 17 02
	10 02 00	10 01 00	Sol deinceps ſub Nubibus receptus latuit uſque ad Emergentis Limbus per Nubes Defectu Liber apparuit; Prodibat Clarius, & nihil in ejus Limbo deficere compertus.	
	10 04 00	10 03 00		

Pro Correctione Horologii, acceperam pridie *Eclipſis Maii* 31. *Mane*.

Hor. Horologii.		*Alt.* ☉.	*Temp. inde ſup.*	*Error.*
h. ′ ″		° ′	h. ′ ″	′ ″
7 07 12	Altitudinem Limbi Solis Inferioris.	27 47	7 06 09	1 03
7 10 16	ejuſdum Limbi.	28 16	7 09 19	0 57
	Iterum *Jun.* 1. *p. m.*			
5 32 00	Altitudinem Limbi Solis Inferioris.	22 06	5 31 06	3 56
5 35 23	Limbi Superioris	22 06	5 34 34	0 49
5 45 17	Inferioris.	20 6	5 44 18	0 59
	Denique *Jun.* 2. *m*			
8 9 44	Altitudinem Limbi Solis Inferioris.	37 34	8 08 45	0 59
8 13 36	——	38 09	8 12 34	1 02
8 15 44	——	38 28		
8 17 51	——	38 47		
8 20 1	——	39 07	8 18 49	1 11

Unde liquet & Motus conſtantiam ſervaſſe Horologium, & in *Eclipſi* debite fuiſſe correctum.

Hora

4.
At Townley; *by Mr.* Rich. Townley. *Ibid.* 633.

Hora Horol. Oſcillatorii.	*Correct. per Lin. Merid.*	*Menſura.*	*Phaſium.*
h. ′ ″	h. ′ ″		′ ″ ′ ″
8 06 45	8 8 27	I C 1190	10 09 forſan 1109 = 14 50
8 11 00	8 12 42	P L 1935	26 15
8 18 00	8 19 42	I C 1405	19 04
8 21 00	8 22 42	P L 1805	24 30
8 26 14	8 27 56	I C 1504	20 47
8 34 00	8 35 42	P L 1711	23 13
8 42 15	8 43 57	I C 1551	21 03 accurate.
8 46 30	8 48 12	P L 1702	23 20 vel — 1720 = 23 15
8 51 45	8 53 27	I C 1553	20 04 accurate.
9 00 00	9 01 42	P L 1801	24 33
9 12 34	9 14 16	I C 1357	18 25
9 30 55	9 32 37	I C 872	11 50
9 41 15	9 42 57		

Fig. 125.

Præciſe. Deſiit Eclipſis, quantum per Aeris Vibrationem potui dicernere. Exitus Locus adeo Vertici vicinus erat, ut in quam ab ea partem inclinaret, bene non potuerim definire; etiamſi hora 9 29′. per Horologium Cuſpides Horizonti apparerent Parallelæ.

Solis Diameter hora 9. 10′. erat 2334; ſatis ut putavi præciſe.

Deinde, accedente ſole ad Meridiem per Lineam longam Meridianam, Horologium juſto tardius inventum fuit ſcrupulis 1′. 42″. Magno autem Æquinoctiali Sciaterico, quo medias minoreſve Scrupuli horarii partes poſſum diſtinguere, Horologium toto hoc mane tardius duntaxat 45″. Correctioni tamen per Lineam Meridianam quam Sciaterico fidendum puto.

5.
At Wingfield; *near* Derby; *by Mr.* Imman. Halton. *Ibid. p.* 664.

7h. 50′. Nihil ſub Sole.
7 50$\frac{1}{2}$ Initium accurate.
7 52 Notabilis defectus.
9 00 Digiti 3 $\frac{1}{2}$
9 11 Digiti 3 $\frac{1}{10}$.
9 21 Digiti 2 $\frac{15}{100}$.
9 47$\frac{1}{2}$ Non Finita; Imminente Fine.

At Paris; *by M.* Caſſini.

6. Cum Sol è nubibus emergeret, Altitudinem graduum 48 accedens, ad eum direxi Quadrantem, quem ad hanc Altitudinem immotum tenui. Ex quo, *Solis* Margo ſuperior *a*, tetigit Filum Horizontale *c d*, in Foco Teleſcopii, ad adventum Centri *b*, fluxere ſecundæ horariæ 104 = *ab* vel *br*; A tranſitu Centri *b* ad tranſitum Marginis Lunæ Superioris *o*, Secundæ 11 = *bs*; A tranſitu Centri *b*, ad cornu Superioris occidentalis *e*, fluxere Secundæ 25$\frac{1}{2}$ = *eh*; A tranſitu Centri *b*, ad tranſitum Cornu Inferioris & Orientalis *i*, Secundæ 93 = *ik*; Hinc determinatur Linea Cornuum *ie*, (ſecluſa variatione) ejuſque Inclinatio ad Horizontem *lk*; & Punctum *p* concurſus Tangentis Lunam cum Secante *iep*, & Tangens ipſa *po*, Media proportionalis inter *pi*, *pe*: & Anguli *noe*, *toi*, hinc Angulus *ioe*; & Triangulum *ioe*, Lunari Circumferentia inſcriptum.

Fig. 126.

Ex iis, aliiſque ex *Aſtronomia* datis, deduxi
Initium eſſe debuiſſe *Pariſiis* — 7ʰ. 55′
Finem vero ——————— 10 12′. vel circiter.

7. *At* Dantzick; *by M.* Hevelius. *Ib. p.* 666.

Temp. juxta Sciatericum & Hor. Oſcil.	*Altitudines ☉.*	*Tempus ex Altitud. Sol. correct.*	*Ordo Phaſium.*	*Magnitudo Phaſium.*	*Animadvertenda.*
h ′ ″	° ′ ″	h ′ ″			
7 58 10	36 17 0	7 58 18			
8 1 30	36 41 0	8 1 6			
8 3 30	37 3 0	8 3 39			
8 50 30		8 50 0			Sol omnino Purus apparuit.
9 21 30		9 21 0			Nihil adhuc in Sole.
9 22 30		9 22 0	1	Digit.	Initium Eclipſeos.
9 24 10		9 23 40	2	$\frac{1}{3}$ fere.	
9 24 55		9 24 25	3	$\frac{1}{2}$ Digit.	
9 27 28		9 27 0	4	$\frac{3}{4}$ Digit.	
9 29 40		9 29 10	5	1 Digit.	
9 33 25		9 33 0	6	1 $\frac{1}{4}$ Digit.	
9 36 30		9 36 5	7	1$\frac{5}{8}$ fere.	
9 39 35		9 39 10	8	2 Digit.	
9 45 49		9 45 25	9	2$\frac{1}{2}$ Digit.	
9 54 22		9 54 0	10	3$\frac{1}{8}$ Digit.	
10 3 44		10 3 22	11	4$\frac{1}{2}$ Digit.	
10 8 30		10 8 10	12	4$\frac{3}{4}$ Digit.	
10 18 17		10 18 0	13	4$\frac{1}{5}$ fere.	
10 22 42		10 22 22	14	4$\frac{1}{2}$ pau.plus	
10 26 19		10 26 0	15	4$\frac{2}{3}$ fere.	
10 35 24		10 35 6	16	4. 22′.	
10 38 53		10 38 38	17	4$\frac{1}{3}$ fere.	Hora 10. 31′. 0″. fere, Max.
10 47 34		10 47 20	18	4 Dig. fere.	incidit Obſcuratio.
10 53 49		10 53 30	19	3$\frac{5}{8}$ Digit.	
10 58 17		10 58 8	20	3$\frac{1}{3}$ Digit.	
11 5 27		11 5 20	21	2$\frac{7}{8}$ Digit.	
11 8 50		11 8 44	22	2$\frac{1}{4}$ Digit.	
11 22 13		11 22 8	23	1$\frac{3}{4}$ fere.	
11 29 14		11 29 10	24	1 $\frac{1}{10}$	
11 35 25		11 35 20	25	$\frac{1}{2}$	
11 36 59		11 36 55	26	$\frac{1}{4}$ paul.plus.	
11 39 15		11 39 15			Nond. Sol omnino Purus exſtitit
11 39 40		11 39 40			Finis Eclipſeos.
4 18 10	33 13 0	11 18 19			
4 20 0	32 25 0	11 20 36			

	Ex Calculo Rudolph.	*Ex Obſervat.*	*Diff.*	*Temp.*
	° ′ ″	° ′ ″	′ ″	h. ′ ″
Semid. ☉	0 15 0			
Semid. ☾	0 15 3	0 13 53	1 10	10 0 0
		0 14 0	1 3	10 24 0
		0 14 50	0 13	11 0 0
		0 15 0	0 0	Ultimo
	h. ′ ″	h. ′ ″		
Duratio.	1 50 58	2 17 40		

8. Diebus præcedentibus, locum aptiſſimum elegimus in quo Aëre Puro frueremur, videlicet *Conventum RR. PP. Carmelitarum Diſcalceatorum*, qui reſpectu Civitatis *Aven.* ad Ortum vergit & mœnia ſtringens Aëre, Fumo & Vaporibus urbanis libero gaudet; in medio Horti Cameram Obſcuram Tapetibus conſtruximus, & in eâ Inſtrumenta ad obſervationem neceſſaria ritè collocavimus.

At Avignion; *by M.* Gallet. n. 141. *p.* 1020.

Tuboſpicillum aptavimus Lente Oculari Concavâ, & Objectivâ Convexâ inſtructum, duplicem habens motum firmo Suſtentaculo, Verticalem ſcil. & Horizontalem, affixam Tabellam immobilem firmatis cochleis ſecum circumducens, Oculari Vitro ſemper parallelam chartâ candidiſſima indutam, in qua Solarem Speciem, diſtantiâ Tuboſpicilli determinatam deſcripſimus, hujus Diametrum Circulis Concentricis in duodecim Digitos diviſimus, & quemlibet Digitorum in partes Sexageſimas.

Loco Quadrantis, qui pluribus indiget cautionibus, & nimium obnoxius eſt vacillationibus, Gnomonem ad captandas umbras Solis in partes 400, optimè diviſum diſpoſuimus, ita ut liberè moveretur Situm Verticalem ope perpendiculi conſervans. Tandem Horologium Rotatile, minuta prima & ſecunda indicans, motu Penduli cum Cycloide præparavimus.

Ipſa die Eclipſis undecimâ *Junii*, horâ unâ circiter poſt ortum Solis, uſque ad Initium & Finem Eclipſis, Speciem ejus Lucidam in Charta, ſine intermiſſione recepimus, & quilibet ex nobis Inſtrumento ſibi deſtinato ſemper invigilavit; Dominus de *Beauchamps* Muſarum *Avenionenſium Mæcenas* Ampliſſimus. Ego quoque cum illo, Tuboſpicillo; Dominus de *S. Florent*, Viſus perſpicaciſſimi, Gnomoni; Dominus *Montonier*, Horologio, cum Domino *Marii* Preſbytero, in Mathematicis & præſertim in Horologiis verſatiſſimo.

Statim ac ſenſibiliter cœpit Umbra Diſcum inire, quantitatem partium Obſcuratarum, Umbram in partibus Gnomonis, & Horam horologii notavi è directo primæ Phaſis, & ita collegi Phaſes 39, contentas in ſequenti Tabella.

Num. Phaſ.	Digiti Obſcur.		Umbra Gnomon. in partibus qualium Gnomon. continet. 400	Altitudo Solis Appaparens.			Altitudo Solis Vera.			Hora Horologii Penduli.			Hora correcta per Altitudinem Solis.			
	D.	′		°	′	″	°	′	″	h.	′	″	h.	′	″	
1	0	27	561	35	29	23	35	28	48	7	50	31	7	50	34	
2	1	0	536	36	44	0	36	43	28	7	57	25	7	57	28	
3	1	30	520	37	34	7	37	33	37	8	2	3	8	2	7	
4	3	0	478	39	55	23	39	54	57	8	15	14	8	15	13	
5	3	25	466	40	38	30	48	38	6	8	19	0	8	19	14	
6	4	30	438	42	24	14	42	23	53	8	29	19	8	29	6	
7	4	40	434	42	39	58	42	39	37	8	30	59	8	30	34	
8	5	0	424	43	19	53	43	19	32	8	34	34	8	34	18	
9	5	30	412	44	9	12	44	8	52	8	39	19	8	38	56	
10	6	0	394	45	10	57	45	10	39	8	44	54	8	44	44	Cornu Verticalia.
11	6	40	375	46	35	50	46	35	33	8	53	19	8	52	45	
12	6	50	371	46	54	15	46	54	0	8	54	54	8	54	31	
13	7	0	366	47	17	30	47	17	14	8	56	44	8	56	44	
14	7	20	350	48	30	37	48	30	23	9	3	44	9	3	44	Maxima Obſcuratio.
15	7	8	339	49	29	10	49	28	58	9	9	14	9	9	15	
16	7	0								9	11	0				
17	6	35	325	50	39	22	50	39	10	9	15	54	9	16	12	
18	6	25	321	51	0	12	51	0	0	9	18	14	9	18	11	
19	5	25	296	53	10	29	53	10	19	9	31	5	9	31	1	
20	5	0	286	53	55	14	53	55	5	9	35	44	9	35	30	
21	4	40	283	54	27	6	54	26	57	9	38	39	9	38	43	
22	4	35								9	42	3				Cornua Parallela Horizonti.
23	4	0								9	47	19				
24	3	53	266	56	3	35	56	03	27	9	48	45	9	48	36	
25	3	35	262	56	28	29	56	28	22	9	51	29	9	51	11	
26	3	30	262	56	37	32	56	37	25	9	52	11	9	51	59	
27	3	26	260	56	43	34	56	43	27	9	52	34	9	52	45	
28	3	6	254	57	15	59	57	15	53	9	56	5	9	56	10	
29	3	0								9	57	40				
30	2	48	249	57	50	48	57	50	42	9	59	34	9	59	53	
31	2	35	246	58	9	32	58	9	26	10	1	34	10	1	53	
32	2	25	243	58	26	11	58	26	5	10	3	0	10	3	41	
33	2	0								10	6	46				
34	1	50	236	59	12	33	59	12	28	10	8	56	10	8	47	
35	1	0	226	60	16	59	60	16	55	10	15	51	10	16	0	
36	0	40	220	60	56	21	60	56	16	10	20	57	10	20	31	
37	0	30	217	61	16	11	61	16	6	10	22	54	10	22	50	
38	0	20	214	61	36	12	61	36	8	10	25	0	10	25	12	Cornu occidentale verticale cum Centro Solis.
39	Finis.		209	62	6	23	62	6	19	10	28	41	10	28	50	

Proportio Diametrorum apparuit æqualis in Eclipſi 6 Digitorum, tunc enim Cornua Solis Verticalia diſtabant à Verticali Solis hinc inde gradibus circiter 30. Unde patet Centrum Lunæ tunc reperiri Peripheria Solis, & Lineam Diacentron eſſe æqualem Semidiametro Solis. Verùm poſt Medium Eclipſis, mutationem aliquam in Diametro Umbræ deprehendimus; apparuit enim Umbra paululum magis Convexa, & ideo Semidiameter brevior, ſed ferè inſenſibiliter.

XXXIV.
An Eclipſe of the Sun, July 2d. 1684. *at* Greenwich; *by Mr.* Flamſteed. n.162. p.691.

Temp. per Horolog. Oſcillator.			*Temp. inde ab Obſcrv. correcta.*			*Obſervationes.*	*Partes Obſcur. quarum 32 æquales erant Diam. Solis.*	*Partes libera Micrometro.*	
h.	′	″	h.	′	″			′	″
2	12	30	2	03	45	Sol integer; tunc nubes —			
2	21	2	2	12	17	Sol def. perexiguum ima Ora. —			
2	21	25	2	12	40	Partes obſcuratæ ſupra Scenam. —	0½		
2	32	20	2	23	35	— — —	4		
2	46	40	2	37	55	— — —		22	19
2	51	37	2	42	52	— — —	11		
3	5	23	2	56	38	— — —	15		
3	9	5	3	00	20	Centrum — —	16		
3	10	4	3	1	19	— — —		15	07
3	11	47	3	3	2	— — —		14	36
3	12	15	3	3	30	— — —	17		
3	13	23	3	4	38	Cornua Horizontalia —			
3	17	43	3	8	58	— — —	18		
3	21	48	3	13	3	— — —		12	30
3	23	0	3	14	15	— — — —	19		
3	25	40	3	16	55	— — — —		12	11
3	29	51	3	20	6	— — — — —	19½		
3	33	54	3	25	9	— — — — —	19½		
3	40	8	3	31	23	— — — — —	19		
3	43	16	3	34	31	— — — — —	18½		
3	45	35	3	36	50	— — — — —	18		
3	49	15	3	40	30	Cuſpis inferior ad Nadir Solis —			
3	50	2	3	41	17	— — — — —	17		
3	54	20	3	45	35	Centrum — — —	16		
3	56	45	3	48	00	— — — — —	15		
4	2	58	3	54	13	— — — — —	13		
4	6	0	3	57	15	— — — — — ferè	12		
4	8	45	4	00	00	— — — — —	11		
4	16	32	4	7	47	— — — — —	8		
4	17	44	4	9	00	Cuſpides Verticales diſt. 84 gr.—			
4	24	8	4	15	23	Inter Cuſpides — — —		17	32
4	24	20	4	15	35	— — — — —	5		
4	27	14	4	18	29	Inter Cuſpides — — —		15	35
4	29	36	4	20	51	— — — — —	2½		
4	30	32	4	21	47	Inter Cuſpides — — —		12	22
4	33	06	4	24	21	— — — — —	1		
4	36	22	4	27	37	Finis accuratè — — —			

Deinde ad Errorem *Horologii* inveſtigandum.

Hor. Horol.			*Diſtantiæ à Vertice.*				*Hor. inde ſup.*			*Error. Hor.*	
h.	′	″		°	′	″	h.	′	″	′	″
4	43	38	Limbi Solis	60	11	00	4	34	56	8	42
4	45	48	Inferior.	60	31	20	4	37	7	8	41
4	46	56		60	41	50	4	38	15	8	43
4	49	3		61	1	40	4	40	22	8	41

At Paris; *by* M. Bullialdus. n. 162. *p.* 693.

2. Alt. ☉. 50°. Initium Elapſum erat, Sole Nubibus tecto, & Digitus circiter deficiebat.

Alt. ☉. 41°. 15′. Paulo amplius quam Digiti 7. Attigit Digitos, 8.

Alt. ☉. 29°. 30′. Finis.

At the Obſervatory; *by* M. Caſſini. n. 162. *p.* 693. n. 163. *p.* 715.

3. The Beginning of the Eclipſe could not be ſeen, but was deduced from the following Phaſes. The Apparent Diameter of the Moon appeared leſs than that of the Sun. It was judged that the Dilatation of the Sun's Light, might make the Moon's Diameter ſeem leſs.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
	h.	′	″		h.	′	″
Begin.	2	25	30	7½ Digits	3	35	00
or	2	25	55	7 Digits	3	55	50
1 Digit	2	32	50	6 Digits	4	4	10
2 Digits	2	40	00	5 Digits	4	12	25
3 Digits	2	47	40	4 Digits	4	19	15
4 Digits	2	54	10	3 Digits	4	25	50
5 Digits	3	2	00	2 Digits	4	32	15
6 Digits	3	10	5	1 Digits	4	37	40
7 Digits	3	20	10	End.	4	43	23

4. By M. de la Hire *and* Pothenot. *Ibid. p.* 716.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
	h.	′	″		h.	′	″
Beginning.	2	25	24	7 Digits	3	53	34
1 Digit	2	33	2	6 Digits	4	3	53
2 Digits	2	40	30	5 Digits	4	11	3
3 Digits	2	47	47	4 Digits	4	17	42
4 Digits	2	54	41	3 Digits	4	25	14
5 Digits	3	2	41	2 Digits	4	31	56
6 Digits	3	12	6	1 Digit	4	38	11
7 Digits	3	20	54	End.	4	43	27
7 Dig. 5′	3	36	27				

The Beginning was deduced from many Obſervations made ſoon after it. The Moon's Diameter appear'd then not to be more than about 30′; though by the Obſervations of her Diameter ſome Days before and after, it was judged to be 31′. 30″. But the Extremities of the Horns, on which depended the Exactneſs of the Determination, appear'd a little Blunted.

5. *At the College of* Lewis the Great; *by* R. P. Fontenay. *Ibid. p.* 717.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
Leſs than	h.	′	″		h.	′	″
$\frac{1}{2}$ Digit	2	29	30	7 Digits	3	51	20
$1\frac{1}{2}$ Digit	2	37	40	6 Digits	4	2	25
2 Digits	2	40	25	5 Digits	4	10	50
3 Digits	2	48	34	3 Digits	4	24	31
4 Digits	2	54	30	2 Digits	4	29	54
5 Digits	3	3	00	$0\frac{1}{3}$ ferè	4	41	00
6 Digits	3	12	40				
7 Digits	3	22	18				
$7\frac{3}{4}$ Digits	3	38	+				

6. The Beginning was at $2^{h}. 54'. 30''$. The End at $5^{h}. 9'. 9''$. The Greatneſs of the Eclipſe $8\frac{1}{2}$ Digits.

At Aix; *by* M. Gautier. *Ib. p.* 718.

7. *At* Lyons; *by* R. P. Paul Hoſte. *Ib.*

Phaſes.	*Time.*					
	By Fixt Stars.			*By the Sun.*		
	h.	′	″	h.	′	″
1 Digit	2	45	3	2	50	3
$8\frac{1}{2}$ Digits	3	53	52	3	58	52
1 Digit	4	59	20	5	4	20

At $3^{h}. 26'. 14''$. (by the *Stars*) The Diameter of the *Sun* and *Moon* 30′. 58″. but at $4^{h}. 20'. 34''$. The Diameter of the *Sun* 30′. 58″. of the *Moon*, 30′. 5″.

8. *At the Bay* de Roſes; *by M.* Chaſſelles. *Ib. p.* 719.

Phaſes.	*Times.*		
	h.	′	″
The Beginning of the Eclipſe	2	40	00
The Edge of the Moon at the Sun's Center	3	25	00
The Horns Horizontal	3	40	00
The Horns Vertical	4	15	00
The End of the Eclipſe	5	1	30

The Greatneſs of the Eclipſe about $\frac{3}{4}$ of the Sun's Diameter, at which time *Venus* might be ſeen without Pain.

At Honſieur; *by* M. de Glos. *Ibid.* 9. The Beginning was at 2h. 15'. 2''. The End, at 4h. 34'. 35''. The Greatneſs more than 8 Digits, but leſs than 9.

At Pau; *by* R. P. Richaud. *Ib.* 10. At 1¾h. The Ecliſpe was not Begun. At 3¼h. at 10 Digits. The End, at 4¾h.

11. *At* Avignon; *by* R. P. Bonſa. *ibid.*

Phaſes.	*Time.*	*Phaſes.*	*Time.*
	h. ' ''		h. ' ''
The Beginning.	2 43 27	Horns Vert.	4 24 32
1 Digit	2 51 58	1½ Dig.	5 1 16
9 Digits	4 2 00	The End.	5 4 37

The Sun's Diameter 31'. 38''. The Moon's 30'. 6''.

At Oxford; *by* Dr. Ed. Bernard. n. 164. p. 747. 12. Phaſes Quadraginta hujus *Defectus* captavit definivitq; bona manus *Walliſii.* Tempus etiam juſtum æquumque ejuſdem Deliquii Altitudinibus aliquot Solaribus comprobarunt Clariſſimi Viri *D. Caſvellus* & *D. Rookius*, ubi Horologiis & Oſcillis noſtris fuerat peccatum.

Digiti Obſcuri, cum ſuis decimis.	*Tempora Oſcillatoria à Cœlo Correcta.*	
	h. ' ''	
	2 03 00	*Initium* Eclipſis.
0 6	2 07 44	
1 0	2 10 44	
1 3	2 13 19	
1 6	2 15 44	
2 2	2 21 34	
2 6	2 23 44	
2 9	2 25 39	
3 5	2 29 54	
4 0	2 33 04	
4 4	2 36 34	
4 6	2 40 04	
5 0	2 43 14	
5 3	2 48 19	
5 6	2 50 09	Medium quaſi *Solem* fecit τὶ λελαμπρόν.
6 5	2 57 30	
6 7	2 59 14	
7 4	3 08 24	Obſcuritas Maxima.
7 1	3 11 09	
6 8	3 29 19	
6 7	3 31 39	αὖθις ἐπὶ κέντρον τὸ σκότος.
6 0	3 37 24	
5 7	3 39 29	

		h.	'	''	
5	5	3	44	29	
5	0	3	47	59	
4	4	3	50	34	
3	9	3	53	54	
2	9	4	2	49	
2	5	4	4	39	
2	2	4	6	19	
2	0	4	7	54	
1	4	4	13	9	
1	0	4	15	4	
0	7	4	16	49	
0	3	4	19	39	
		4	21	14	Finis Eclipſis.

At Lisbon. *Ib. p.* 749.

13. Mr. *Jacobs* at *Lisbon* Noted.
The Beginning of the Eclipſe at 1h. 30'. exactly.
The Ending at ———— ———— 4 12.

In Ireland. *ib.*

14. Mr. *Aſh* and Mr. *Molyneux*, toward the Middle of the Eclipſe, having a ſhort View of the Sun; they Judged that about 8 Digits were covered: at the Ending alſo having a faint View thereof; they aſſigned its End, at 3h. 56'. p. m.

The ſame Eclipſe was obſerved by one Mr. *Osburn*, nigh *Tredagh*. *Initium* 1h. 37'. 30''. *Finis* 3h. 56'. 20''.

At Bononia; *by* S. Domin. Gulielmini. n. 203. *p.* 858.

15. Deſumebant Solis à Vertice Diſtantias D. D. *Jo. Ludovicus Donellus*, & *Nocolaus Ignatius Joanettus*; Phaſibus determinandis tres aderamus, D. D. *Jo. Galeatius Manzius*, *Hercules Vanottus*, & *Ego*. Obſervationes ſcribebat D. *Gregorius Maliſardus*; Horas vero in Horologio notabat D. *Bartolomeus Ferrarius*.

Hora Horologii.			*Hora eruta ex Diſtantiis obſervatis.*			*Diſtantia Centri Solis à Vertice.*		*Phaſes.*		
h.	'	''	h.	'	''	°	''		'	
3	34							Dig. 2	30	
3	37	30						Dig. 3		
3	48	14	3	47	47	51	31	Dig. 4	20	dubia ubique.
3	52	00						Dig. 5		optima.
3	57	00	3	56	2	52	58	Dig. 5	30	
4	2	00	4	2	29	54	10	Dig. 6		
4	4	45	4	5	59	54	44	Dig. 6	30	optima.
4	9	20	4	9	1	55	17	Dig. 7		exacta.
4	14	40	4	15	43	56	28	Dig. 7		
4	19	00						Dig 7	20	dubia.

h.	'	''	h.	'	''	°	'	''		'	
4	27	44	4	28	42	58		39	Dig. 7	20	
4	32	35							Dig. 7		ſatis exacta.
4	34	20							Dig. 6	45	
4	44	30							Dig. 6	30	
4	47	15							Dig. 6	30	
4	51	30							Dig. 6		optima.
4	54	38	4	53	53	63		16	Dig. 5	30	exacta.
4	58	2	4	57	38	63		56	Dig. 5		exacta.
5	1	55	5	1	2	64		32	Dig. 4	30	ſatis exacta.
5	4	40	5	5	28	65		19	Dig. 4		Diligens ubique.
5	7	0	5	6	23	65		28	Dig. 3	30	exacta.
5	10	30	5	9	9	65		58	Dig. 3		exacta.
5	12	35	5	13	28	66		43	Dig. 2	30	ſatis exacta.
5	16	15	5	16	14	67		13	Dig. 2.		exacta.
5	19	50	5	20	24	67		57	Dig. 1.	30	exacta.
5	22	30	5	23	21	68		28	Dig. 1.		exacta.
5	25	0	5	25	14	68		48	Dig. 0	30	diligens.
5	27	40	5	28	7	69		18	Dig. 0		accuratiſſima.

Notabile fuit; cum Quantitas Eclipſis fuerit Dig. 7. 20'. quod non modicam Aëris Offuſcationem debebat inducere, (ut aliàs multoties in conſimilibus Defectibus obſervatum eſt,) nihilominus tamen vix ſenſibiliter conſuetum in Sole Libero Aëris ſtatum mutatum fuiſſe; unde plurimis Solem non reſpicientibus orta ſuſpicio, aut Solem non defeciſſe, aut minimum quidem; cujus quidem rei non alia mihi videtur aſſignanda cauſa, quam ingens vis Nubium à Sole maximè illuminatarum, quæ non multum ab eo diſtabant; ab his enim Solis Radius per Reflexionem & Refractionem multiplicatus certe Intenſior redditus deficientem aliundè Splendorem potuit compenſare.

An Eclipſe of the Sun, May 1. 1687. *at* Oxford. n. 187. *p.* 329. *In divers other Places*. n. 189. *p*. 370.

XXXV. 1. Dr. *Wallis* writes from *Oxford* that this Eclipſe of the Sun was obſerved there about $\frac{1}{2}$ a Digit; between one and two a Clock Afternoon.

2. Hæc *Eclipſis*, etiamſi contemnendæ Quantitatis fuerit, ac nudis oculis non omnino percipi potuerit, tamen ad accuratam determinationem Parallaxis & Latitudinis Lunæ maximè idonea videtur.

Londini, ſeorſim obſervationibus *Hockio* & *Halleio*; Initii Momentum, Cœlo licet puriſſimo, ob Obliquam Incidentiam Lunæ, debite Definire non licuit. Sed 1h. 16'. jam cœpta erat Eclipſis ſatis notabiliter: circa 1h. 40'. prope Medium Eclipſis, Chorda partis Eclipſatæ, ſive inter Cornua, inventa eſt 9'. 30'. cui reſpondet Arcus 36°, in Diametro vero non niſi 1'. 30''. Finis, conſenſu utriuſque Obſervatoris, contigit accurate 2h. 3'. 0''.

Grenovici in *Obſervatorio Regio*, *Flamſteedius* eadem de cauſa, Initium non vidit: Finem vero determinavit 2h. 4'. 15''. Medio Eclipſis ſive Maxima Obſcuratione, Chorda Partis Eclipſatæ erat 9'. 54''.

Apud *Totteridge* prope *Londinum* verſus Corum, Finem vidit D. *Haines*, R. S. S. ad 2h. 2'. Quantitatem vero Maximam dimidii Digiti ab Auſtro.

In Inſula *Barbada*, ad Oppidum *Bridge-Town*, Finem habuit D. *Frank* 1′. 30″. ante quam Solis Altitudo fuit 31°. 47′. ad ortum, hoc eſt 7^{h}. 56′. 45″. Quantitatem Maximam æſtimatione definivit duorum Digitorum ab Auſtro.

Norimbergæ eandem Eclipſin obſervavit J. P. *Wurtzelbaur*. Initium quidem accurate 1^{h}. 58½′. circa Medium, ſc. ad 2^{h}. 36½′. Quantitatem Maximam duorum Dig. præciſe; Finem vero ad 3^{h}. 18′. 33″.

Ulmæ Sueviæ, Obſervavit *Honoldus* Initium ad 1^{h}. 48′. Quantitatem Maximam 2⅓ Dig. Finem vero ad 3^{h}. 16′.

Lipſiæ, obſervatore *Kirchio*, Eclipſis jam ſatis notabilis ad 2^{h}. 20′. 10″. ad 2^{h}. 47⅓′. Digiti 1⅓ circiter. Finis vero Incidit præciſe in 3^{h}. 15′.

Uratiſlaviæ Sileſiæ denique obſervavit D. G. *Schultzius* Maximam Obſcurationem paulo citius quam 3^{h}. 12′. fuiſſe 1½ Dig. Finem vero 3^{h}. 37′.

An Eclipſe of the Sun, Sept. 3. 1699. *at* Oxford; *by Dr.* D. Gregory. N. 256. *p.* 336.

XXXVI. 1. I did not ſee the Beginning of the late Eclipſe, but the End happen'd here, preciſely 24′. 9″. after 10 a-Clock in the Morning, apparent Time: The greateſt Obſcuration, which was 10 Digits and a Quarter, was about 7 Min. after 9.

2. *At* Nuremberg; *by M.* Worzelbaur. N. 265. *p.* 619.

Phaſes.	*Quantities eclipſed.*		*Times by a Pendulum.*		
	Dig.	′	h.	′	″
Initium.			8	57	14
1	0	52	9	3	26
2	1	32	9	8	23
3	2	28	9	14	14
4	3	19	9	19	40
5	4	8	9	24	57
6	5	15	9	31	57
7	5	50	9	35	2
8	6	26	9	38	43
9	6	53	9	40	36
10	7	20	9	43	47
11	7	56	9	50	39
12	8	30	9	55	9
13	9	23	10	1	44
14	9	53	10	5	46
15	10	24	10	10	34
16	10	38	10	14	37
17	10	45	10	17	54
18	10	45	10	22	29
19	10	12	10	27	31
20	9	49	10	30	10
21	9	21	10	33	11
22	8	52	10	35	53
23	8	30	10	38	46
24	7	38	10	42	42
25	7	14	10	40	7
26	6	33	10	49	42
27	6	6	10	53	22
28	5	27	10	56	37
29	5	9	11	0	00
30	4	33	11	4	24
31	3	57	11	8	16
32	3	13	11	13	3
33	2	41	11	18	3
34	2	11	11	21	37
35	1	32	11	25	38
36	1	2	11	28	27
Finis.	0	00	11	33	56

From the 8th to the 12th Phaſis, the opake Limb of the Moon on the South-ſide was a little rough; but about the Northern Horn, to near a 4th part of the Segment, it was more ſmooth: But when the Horns of the Eclipſe were almoſt parallel to the Horizon, before and after the 15th Phaſis, the Extremity of the gibbous Limb of the Moon looking downward, was ſomewhat enlightned, and of a kind of a Saffron Colour; but though the Sky was free from Clouds, yet no Stars were viſible. Nor was even *Venus* itſelf viſible in the open Air, unleſs by ſome more ſharp-ſighted than ordinary.

Amongſt many round Plates cut out of thick Paper of divers Magnitudes, differing from one another $5''$s about the firſt Phaſis, and after, none agreed to the Limb of the Moon but that which was cut to a Radius or Semidiameter of $15'. 30''$. (taking the Radius or Semidiameter of that of the Sun to be $16'. 4''$.) and that gradually ſo ſwelled or augmented, that larger Plates were neceſſary to be made uſe of; and about the 36th Phaſis, none leſs than one deſcribed of a Radius of $16'. 5''$. would agree with, or equal the Appearance; and conſequently, that the Diameter of the Moon, about the End of the Eclipſe, did equalize, if not exceed, that of the Sun. Beſides, in the 27th Phaſis (when the obſcure Part was 6 Dig. $6'$.) the Body of the Moon did obſcure more than two 3ds of the Sun's Limb; which is an Argument that its Semidiameter at that Time was equal to that of the Sun.

By Otbers, ib. *p.* 623.

3. This Eclipſe by the Obſervations of Mr. *Godfred Tuber* at *Ciza*, began at 9^h, and ended at $11^h. 35'$. and increas'd to 11 Dig. by the Obſervations of Mr. *Jacob Honold* at *Harvelſing* near *Ulm* of *Suevia*, it began at $8^h. 55'$. and ended at $11^h. 31'$. and its greateſt Defect was 10 Dig. And by Obſervations at *Leipſick*, it began at $9^h. 11'$. and ended at $12^h. 38'. 30''$. The greateſt Obſcurity was 11 Dig. $20'$. which laſted from $10^h. 16'. 45''$. for $6'$. Ten Digits being obſcured, the Sky (being otherwiſe very clear) began to appear of a more livid or wan Complexion, and more ſad than it uſually looks with a clear Sky when the Sun is ſet, or below the Horizon. The Cocks alſo, which had hitherto crowed very frequently, as if ſilenc'd, going to Rooſt, left off Crowing, and did not renew it, till, by the Recovery of the Sun's Light, they had recover'd their former Gaiety and Mirth: However, we cannot learn that any Star, beſides that of *Venus*, was diſcover'd, by thoſe which were Spectators of it, in the open Air.

Changes likely to be diſcover'd in the Moon; by M. Auzout. N. 7. *p.* 120. Dec. An. 1665.

XXXVII. I ſometimes think that the Earth muſt appear, to the ſuppoſed Inhabitants of the Moon, to have a different Face in the ſeveral Seaſons of the Year; and to have another Appearance in Winter, when there is almoſt nothing Green in a very great Part of the Earth; when there are Countries all cover'd with Snow, others all cover'd with Water, others all obſcur'd with Clouds, and that for many Weeks together; another in Spring, when the Foreſts and Fields are Green; another in Summer, when the whole Fields are Yellow, *&c.* Methinks, I ſay, that theſe Changes are conſiderable enough in the Force of the Reflections of Light to be obſerv'd, ſince we ſee ſo many Differences of Lights in the Moon. We have Rivers conſiderable enough to be ſeen, and they enter far enough into the Land, and have Breadth capable to be obſerved. There are Fluxes in certain Places, that reach into large Countries, enough to make there ſome apparent Change, and in ſome of our Seas there float ſometimes ſuch bulky Maſſes of Ice, that are far greater than the Objects, which we are aſſured we can ſee in the Moon. Again, we cut down whole Foreſts, and drain Marſhes, of an Extent large enough to cauſe a notable Alteration: And Men have made ſuch Works, as have produced Changes great enough to be perceived. In many places alſo are *Vulcans* that ſeem big enough to be diſtinguiſh'd, eſpecially in the Shadow. And when Fire lights upon Foreſts of great Extent, or upon Towns, it can hardly be doubted, but theſe luminous Objects would appear either in an Eclipſe of the Earth, or when ſuch parts of the Earth are not illuminated by Sun: But yet, I know no Man, who hath yet obſerved ſuch things in the Moon; and one may be rationally aſſured, that no *Vulcans* are there, or that none of them burn at this time. This it is, which all curious Men, that have good Teleſcopes, ought well to attend; and I doubt not, but if we had a very particular Map of the Moon, as I had deſign'd to make one, with a *Topography*, as it were, of all the conſiderable Places therein, that we or our Poſterity would find ſome Change in her. And if the Maps of the Moon of *Hevelius*, *Divini*, and *Riccioli*, are exact, I can ſay that I have ſeen there ſome Places conſiderable enough, where they put Parts that are clear; whereas I there ſee dark ones. 'Tis true, that if there be Seas in the Moon, it can hardly fall out otherwiſe, than it doth upon our Earth, where Alluviums are made in ſome Places, and the Sea gains upon the Land in others; I ſay, if thoſe Spots we ſee in the Moon are Seas, as I muſt believe them to be, whereas I have many Reaſons, that make me doubt, whether they be ſo. And I have ſometimes thought, whether it might not be, that all the Seas of the Moon, if there muſt be Seas, were on the ſide of the other Hemiſphere, and that for this Cauſe it might be, that the Moon turns not upon its Axis, as our Earth, wherein the Lands and Seas are as it were ballanced: That thence alſo may proceed the Non-appearance of any Clouds rais'd there, or of any Vapours conſiderable enough to be ſeen as there are rais'd upon this Earth; and that this Abſence of Vapours is, perhaps, the Cauſe that no Crepuſcle is there, as it ſeems there is none, my ſelf at leaſt not having been hitherto able to diſcern any Mark thereof: For, methinks, it is not to be doubted, but that the reputed Citizens of the Moon might ſee our Corpuſcle, ſince we ſee that the ſame is without Compariſon ſtronger than the Light afforded us by the Moon, even when ſhe is Full; for a little after Sun-ſet, when we receive no more the firſt Light of the Sun, the Sky is far clearer than it is in the faireſt Night of the Full Moon. Mean while, ſince we ſee in the Moon, when ſhe is Encreaſing or Decreaſing, the Light ſhe receives from the Earth; we cannot doubt, but that the People of the Moon ſhould likewiſe ſee in the Earth that Light wherewith the Moon illuminates it, with, perhaps, the Difference there is betwixt their Bigneſs. Much rather therefore ſhould they ſee the Light of the Crepuſcle, being, as we have ſaid, incomparably greater. In the mean time, we ſee not any faint Light beyond the Section of the Light, which is every where almoſt equally ſtrong, and we there diſtinguiſh nothing at all, not ſo much as that cleareſt part, which is call'd *Ariſtarchus* or *Porphyrites*, as I have often try'd; although one may there ſee the Light, which the Earth ſends thither, which is ſometimes ſo ſtrong, that in the Moon's Decreaſe, I have often diſtinctly ſeen all the Parts of the Moon that were not enlightned by the Sun, together with the Difference of the clear Parts and the Spots, ſo far as to be able to diſcern them all. The Shadows alſo of all the Cavities of the Moon ſeem to be ſtronger, than they would be, if there were a ſecond Light. For although afar off the Shadows of our Bodies, environed with Light, ſeem to us almoſt dark, yet they do not ſo appear, ſo much as the Shadows of the Moon do; and thoſe that are upon the Edge of the Section, ſhould not appear in the like Manner. But I will determine nothing of any of theſe Things.

To find the Parallax of the Moon; by
N. 9. p. 151.
Feb. An. 1666.

XXXVIII. At certain times agreed on by two Obſervators, making uſe of Teleſcopes large, good, and well-fitted for this Purpoſe, by a Meaſuring-Rod placed within the Eye-Glaſs at a convenient Diſtance, that it may be diſtinctly ſeen, and ſerve for meaſuring ſmall Diſtances by Minutes and Seconds, (which is eaſy enough in large Teleſcopes) Let each of ſuch Obſervators thus furniſhed, obſerve the viſible Way of the Moon among the fix'd Stars (by taking her exact Diſtance from any fix'd Star, that lies in or very near her Way, together with the exact Time of her ſo appearing) and the then apparent Diameter of her Disk; continuing theſe Obſervations every time for two or three Hours; that ſo, if poſſible, two exact Obſervations of her apparent place among the fix'd Stars being made at two places thus diſtant in Latitude, and as near as may be under the ſame Meridian, by theſe Obſervations concurring at the ſame time, her true and exact Diſtance may be hence collected, not only for that time, but at all other times, by any ſingle Obſervator's viewing her with a Teleſcope, and meaſuring exactly her apparent Diameter. It were likewiſe deſirable, that as often as there happens any conſiderable Eclipſe of the Sun, that this alſo might be obſerv'd by them, noting therein the exact Meaſure of the greateſt Obſcuration compared with the then apparent Diameter of his Disk. For by this means, after the Diſtance of the Moon hath been exactly found, the Diſtance of the Sun will eaſily be deduced.

As for the Time fitteſt for making Obſervations of the Moon, that will be when ſhe is about a Quarter, or ſomewhat leſs illuminated, becauſe then her Light is not ſo bright, but that with a good Teleſcope ſhe may be obſerv'd to paſs cloſe by, and ſometimes over ſeveral fix'd Stars, which is about four or five Days before or after the Change: Or elſe at any other time, when the Moon paſſes near or over ſome of the bigger ſort of fix'd Stars, ſuch as of the Firſt and Second Magnitude; which may be eaſily calculated and foreſeen: Or beſt of all, when there is any total Eclipſe of the Moon; for then the ſmalleſt Teleſcopical Stars may be ſeen cloſe adjoyning to the very Body of the Moon.

A Method for obſerving Lunar Eclipſes; by Mr. Rook.
N. 22. p. 388.
Feb. An. 1666.

XXXIX. 1. Eclipſes of the Moon are obſerved for two principal Ends: One Aſtronomical, that by comparing Obſervations with Calculations, the Theory of the Moon's Motion may be perfected, and the Tables thereof reformed; the other Geographical, that by comparing among themſelves the Obſervations of the ſame Ecliptick Phaſes, made in divers Places, the Difference of Meridians or Longitudes of thoſe Places may be diſcerned.

The Knowledge of the Eclipſe's Quantity and Duration, the Shadow's Curvity and Inclination, *&c.* conduce only to the former of theſe Ends. The exact Time of the Beginning, Middle, and End of the Eclipſes, as alſo in total Ones, the Beginning and End of total Darkneſs, is uſeful for both of them.

But becauſe in Obſervations made by the bare Eye, theſe times conſiderably differ from thoſe with a Teleſcope; and becauſe the Beginning of Eclipſes, and the End of total Darkneſs, are ſcarce to be obſerved exactly, even with Glaſſes (none being able clearly to diſtinguiſh between the true Shadow and Penumbra, unleſs he hath ſeen for ſome time before, the Line ſeparating them, paſs along upon the Surface of the Moon:) And laſtly, becauſe in ſmall partial Eclipſes, the Beginning and End, and in total Ones of ſmall Continuance in the Shadow, the Beginning and End of total Darkneſs are unfit for nice Obſervations, by reaſon of the ſlow Change of Appearances, which the oblique Motion of the Shadow then cauſeth. For theſe Reaſons I ſhall propound a Method peculiarly deſign'd for the Accompliſhment of the Geographical End in obſerving Lunar Eclipſes, free (as far as is poſſible) from all the mention'd Inconveniencies.

For, *Firſt*, It ſhall not be practicable without a Teleſcope. *Secondly*, The Obſerver ſhall always have Opportunity, before his principal Obſervation, to note the Diſtinction between the true Shadow and the Penumbra. And, *Thirdly*, It ſhall be applicable to thoſe Seaſons of the Eclipſe, when there is the ſuddeneſt Alteration in the Appearances. To ſatisfy all which Intents,

Let there be of the eminenteſt Spots, diſpers'd over all Quarters of the Moon's Surface, a ſelect Number generally agreed on, to be conſtantly made uſe of, to this purpoſe, in all Parts of the World. As for Example, Thoſe which M. *Hevelius* calleth, *M. Sinai, M. Ætna, M. Porphyrites, M. Serorum, Inſ. Besbicus, Inſ. Creta, Palus Mæotis, Palus Mareotis, Lacus Niger Major.*

Let in each Eclipſe, not all, but (for Inſtance) three of theſe Spots, which then lie neareſt to the Ecliptick, be exactly obſerv'd, when they are firſt touch'd by the true Shadow; and again, when they are juſt compleatly entred into it; and (if you pleaſe) alſo in the Decreaſe of the Eclipſe, when they are firſt fully clear from the true Shadow. For the accurate Determinations of which Moments of Time (that being in this Buſineſs of main Importance) let there be taken Altitudes of remarkable fix'd Stars; on this ſide of the Line, of ſuch as lie between the *Æquator* and *Tropick of Cancer*; but beyond the Line, of ſuch as are ſituate towards the other Tropick, and in all Places of ſuch as at the Time of Obſervation are about four Hours diſtant from the Meridian.

By M. Ja. Caſſini. N. 236. *p.* 15.

Vid. infra. §. LIV. 2.

Fig. 127.

2. Eclipſis *Lunæ*, Die 29 *Oct. An.* 1697. Obſervata eſt *Roterodami* per Teleſcopium quatuor fere pedum Pariſienſium Oculari Convexo, in cujus Foco erant Fila Quatuor, ſeſe in Axe interſecantia ad Angulos Rectos & Semirectos, ad Phaſes dimetiendas, Macularumque Lunarium ſitum determinandum hoc Teleſcopium impoſitum erat Fulcro habenti Axem ſitu parallelo Axi mundi conſtitutum, ut poſtquam ad Lunam directum eſſet ad unius Phaſis Obſervationem, poſſet ad alias Phaſes obſervandas per Lunæ Semitam ad Occaſum revolvi. Ita autem primo dirigebatur ad Lunam, ut eo immoto permanente Lunæ Limbus Borealis ſuo Motu ad Occaſum raderet unum ex his Filis, quod ideo Parallelum dicimus, licet ob Motum Lunæ in Declinationem Motui Lunæ ad Occaſum multo celeriori commixtum non nihil ab Æquatore declinaret dum Lunæ diſcus in reliqua tria Fila ſucceſſive incideret. Horum trium Filorum intermedium Angulos Rectos cum Parallelo efficiens, Rectum Perpendiculare & Verticale appellamus. Reliqua duo Obliqua, quorum Primum dicimus in quod prius Luna incidit, Secundum Obliquum in quod Luna incidit poſterius. Initio Eclipſis,

quando

Fig. 128.

quando Lunæ Punctum Borealiſſimum nondum in Umbra erat Immerſum, illud Filo aptavimus Parallelo. Deinde poſtquam tale Punctum Umbræ Immerſum eſt, eidem Filo aptavimus Auſtraliſſimum Lunæ Punctum. Unde factum eſt ut quod Filum initio fuerat Primum in aliarum Phaſium determinatione fuerit Poſtremum,& Primum evaſerit quod Poſtremum fuerat initio. Cum autem Lunæ Limbus Filum Parallelum percurreret, Lunæ Centrum intelligebatur deſcribere Lunarem Semitam huic Filo Parallelam, quæ ab aliis tribus Filis ſecabatur. Portiones autem hujus Semitæ ſupponuntur proportionales Temporibus, quibus ipſas Lunæ Centrum percurrit, inæqualitas enim Motus Proprii univerſali Motui immixti exiguo Tempore imperceptibilis eſt. Cum igitur Lunæ Limbus Parallelum percurreret, obſervabatur beneficio Horologii Pendulo inſtructi, & diebus præcedentibus ad Solem conformati, Tempus adventus Lunæ Macularum aliquot & Lunarium Cornuum ad hæc tria Fila, & deprehenſum eſt dictæ Eclipſis tempore Lunæ Diſcum tranſire per Filum Rectum, 2′. 24″. per Fila vero Obliqua 3′. 24″. ideoque Semidiametrum Lunæ tranſire per Rectum 1′. 12″. per Obliqua vero 1′. 42″. Differentia utriuſque Tranſitus exiſtente 30″. Hinc obſervato uno Appulſu Lunæ ad quodvis horum Filorum, vel uno Egreſſu, dantur omnes alii ad reliqua Fila. Semidiameter Lunæ AB, jacens in Lunari Semita ABCDEF, pertranſit per ejus punctum quodlibet dum Centrum A percurrit ſpatium ſibi æquale AB, ut alia Semidiameter AK, Angulum Rectum efficiens cum alia recta Linea NCK ad punctum K, in quo proinde Lunam continget in K, ab ejus Semita declinans Angulo KCA, tranſit per ipſum Filum CK, dum Centrum Lunæ percurrit AC, Hypothenuſam Trianguli Rectanguli AKC: eſtque Tempus Tranſitus Semidiametri AB, per Filum perpendiculare Lunam contigens in B, ad Tranſitum Semidiametri AK, per Filum Obliquum NCK, ut AB, vel AK, Sinus Anguli ACK, ad AC, Sinum Anguli Recti, ſive Radium. Filo igitur NCK, faciente cum ſemita Lunæ Angulum KCA Semirectum, & Angulus KAC, in Triangulo Rectangulo, Semirectus erit, ideoq; Latera CK, KA, æqualia, erit Tranſitus Rectus ſecundum AB, ad Tranſitum Semidiametri AK, per Filum Obliquum NK, ut Sinus Anguli Semirecti, ad Sinum Anguli Recti, ut 707 ad 1000, ſive ut 72″. ad 102″. vel 1′. 42″. fere, ut obſervabatur, Lunaris Centri Semita exiſtente AH, Lunæ Semidiametro ipſi perpendiculari AM, ducta MNO, Parallela ipſi AH, ipſa congruet Filo quod Lunæ Limbus motu ſuo ad Occidentem radet, quod ſecabitur ab Obliquis NCK, NGI, & à Recto NEP, in puncto N, qua tranſit Axis Teleſcopii; facietque cum his Filis Lunaris Orbita duo Triangula Rectangula NEC, NEG, quæ ſupponuntur habere Angulos Semirectos ad puncta N, C, G: Sunt ergo Similia & Æqualia, habentque Latera CE, EG, EN, Æqualia Semidiametro Lunæ AM. Si hinc inde ab Interſectionibus, C, & G, accipiantur in Filis ipſi Semidiametro æquales CK, CS, GI, GR, & in orbita CA, CF, GD, GH, æquales CN, & jungantur AK, FS, DR, HI, erunt ipſæ omnes æquales inter ſe, efficientque ad Fila Angulos Rectos ad K, S, R, I. Quare Centro Lunæ exiſtente in A, Luna Tanget Primum Obliquum in K, & poſtquam Centrum Lunæ venit ab A in C, ejus Semidiameter congruet Lineæ CE, ideoque Luna tanget Filum Rectum in E. Poſtquam autem Centrum Lunæ venerit ab A in D, tanget Secundum

Obliquum in R. Eſt autem AD, æqualis Diametro Lunæ, nam cum GD ſit æqualis CA, addendo DC, habebitur AD æqualis GC, qui quidem eſt Diametro Lunæ æqualis. Sed cum GD ſit æqualis CF, ſi ab his auferantur æquales GE, EC, erit FE æqualis ED, & erit DF dupla; tantumque erit à Contactu primo Secundi Obliqui in R, ad Contactum ultimum Primi Obliqui in S; & poſtquam Centrum Lunæ progreſſum fuerit in G ad diſtantiam Semidiametri unius EG, Luna continget ultimo Filum Rectum in E. Luna Centro progreſſo à G in H, ipſa tanget ultimo Secundum Obliquum in I. Suppoſito igitur tranſitu Recto Lunæ fieri 2′. 24″. ut obſervatum eſt. Et,

	′	″	*Diff. Contact.* ′	″
Poſito Centro in A, & contactu Primi Obliqui in K - -	0	0	1	42
Centrum Lunæ erit in C, & continget 1°. Rectum in E -	1	42	0	42
Centrum perveniet in D, & continget 1°. 2 Obliquum in R	2	24	0	30
Lunæ Centrum erit in E, Filo intermedio Perpendiculari -	2	54	0	30
Centrum perveniet in F, & continget ultimo 1 Obliquum in S	3	24	0	42
Centrum erit in G, & continget Ultimo Rectum in E —	4	6	1	42
Erit tandem in H, & continget ultimo 2 Obliquum in I	5	48		

Hinc Calculo correſpondebant ut plurimum Obſervationes in hac Eclipſi intra Secundum unum. Sufficiebat igitur in una Phaſi obſervare duos in his Tranſitibus in reliquis Phaſibus unum, ut reliqui omnes innoteſcerent. Quod ad Lunares Maculas attinet; Comparatur Tranſitus Marginis præcedentis Lunæ & Maculæ per Filum Rectum ad habendam Differentiam quam dicimus Longitudinem Maculæ à Magine præcedenti: & Tranſitus Rectus Maculæ comparatur cum Obliquo ad habendam Differentiam quæ æqualis eſt diſtantiæ viæ Maculæ à Semita Puncti Borealiſſimi vel Auſtraliſſimi radentis Filum Parallelum. Cum enim via Maculæ ABC, Parallela ſit viæ Marginis DEF, eoſdem cum eiſdem Filis Angulos facit Semirectos ad A & C, Rectos ad B, unde Angulus ad A, æqualis eſt Angulo ad C, & Latus BA, æquale Lateri BE, Latitudini Maculæ B à Filo FED. Data autem Longitudine & Latitudine Maculæ datur ejus Situs in Luna. Deſcripto quippe circa ipſam Quadrato cujus Latus AB, intelligatur congruere Filo Parallelo, & ſit diviſum in tot æquales partes quot Secundis Luna per filum Rectum tranſit, Latera vero AC, BD; ipſi Filo perpendicularia ſint in totidem ſimiliter partes æquales diviſa. Sumpta in Parallelis Longitudine AE, CF, & Ducta FE, & in Perpendicularibus Latitudine AG, BH, quam æqualem dicimus viæ interceptæ inter Rectum & Obliquum, determinatur Situs Maculæ M, in communi harum rectarum Interſectione. *Fig.* 129. *Fig.* 130.

Quod ſpectat ad Lunæ Cornua in Eclipſi, ipſa determinari poſſunt ſola Longitudine, modo ſciatur quo in Semicirculo Auſtrali vel Boreali ſint. Ut Cornu I, per Longitudinem AE, vel CF, recta quippe FE, Lunæ Marginem ſecat in duobus punctis L, & I, quorum unum eſt in Semicirculo Boreali, alterum in Auſtrali. Poteſt etiam determinari ſola Latitudine AK, vel BM, modo ſciatur quo in Semicirculo Orientali vel Occidentali ſit Punctum I. Ex Lineis autem Longitudinis & Latitudinis illa exactius Situm Cornu determinat, quæ propior eſt Centro; ut hic punctum I, exactius determinatur Longitudine quam

Fig. 131.

Latitudine; è contra Punctum O exactius Latitudine quam Longitudine, idque ob minorem Obliquitatem Lineæ rectæ ad Circumferentiam, qua efficitur ut exigua variatio Diſtantiæ magis ſit in Circumferentia ſenſibilis. Alia ratione per Obliquos Tranſitus determinatur Situs Macularum & Cornuum Lunæ, ſi Linea A D, Parallela Semitæ Lunari P Q, ipſius Marginem tangenti, fiat Diameter quadrati Lunæ Circumſcripti quæ dividatur in tot æquales partes quot Secundis Luna per Filium Obliquum pertranſit, ut in hac Eclipſi in partes 204. Hujus Quadrati duo Latera A C, B D, Primum Obliquum repræſentabunt, utpote illi Parallela, reliqua A B, C D, Secundum Obliquum, ſumpta autem Differentia inter Tranſitum Marginis Præcedentis Lunæ & Maculæ M, per Obliquum, in Secundis horariis ab Angulo præcedente ab A in T, & ducta per T, Recta E F, Parallela Lateri, A C, & ſimiliter ſumpta ab eodem Angulo A, differentia inter tranſitum Marginis præcedentis K, & Maculæ M, per Secundum Obliquum A B, ut A V per punctum V, ducatur Recta G V H, Parallela Lateri A B, repræſentabit Secundum Obliquum ſecans Priorem in Puncto M, ibique Situm Maculæ determinabit. Eadem ratione determinabitur Situs Cornu E, per Differentiam ipſius Tranſitus & Marginis per Primum Obliquum ſumptam in Diagonali ut A T, Situs Cornu H, per differentiam ipſius Tranſitus per Secundum Obliquum A B, ut AV, & ducta per V, Recta G H, Parallela Lateri A D, modo ſciatur ſitne Cornu in Semicirculo Præcedente aut Sequente.

An Eclipſe of the Moon. 1665. *Obſerved at* Dantzick; *by M.* Hevelius. N. 19. *p.* 348.

XL. The Tables did not indicate an *Eclipſe* of the *Moon*, *July* 27. (*N. S.*) 1665. but tho' the Sky here was very clear, yet the Moon was not at all obſcured by the true Shadow, but entred only a little into the Penumbra, wherein it continued 50′. The Beginning of its touching the Penumbra did then almoſt happen, when *Aquila* was elevated 36°. 18′.

An Eclipſe of the Moon, June 6. A. 1666; *by M.* Hevelius. *Ibid. p.* 348. N. 21. *p.* 371.

XLI. In the *Eclipſe* of *June* 16, (*N. S.*) 1666, the firſt *Phaſis* of 1 *Dig.* 45′. appear'd in the Moon's Altitude of 2°. 30′. when the greateſt Obſcuration was already paſt. The End fell out 9^{h}. 27′. about 120°. from the Zenith Weſtward.

An Eclipſe of the Moon, Sept. 19. A. 1670: *by M.* Hevelius. N. 61. *p.* 2023.

XLII. Die 29 *Sept.* (*St. N.*) 1670. mane Initium hujus *Eclipſis* incidit 2^{h}. 22′. quanquam id ipſum vix omnino accurate obſervari potuerit, ob Umbram Terræ Dilutiſſimam: Siquidem, durante Eclipſi, tota Umbra adeo Tenuis erat atque Diluta, ut omnes præcipuas Maculas per eam, meo viginti pedum Tubo, quin & brevioribus, optime conſpicere potuerim.

Maxima ejus Obſcuratio incidit 3^{h}. 50′. Finis vero 5^{h}. 21′. Tota itaque Duratio fuit 2^{h}. 59′. & Quantitas vix amplius 9 Digitorum. Circa Medium hujus Eclipſis 3^{h}. 40′. Stellulam quandam Incognitam, ac ſolo Tubo conſpicuam, à Luna circa Lacum Nigrum Majorem tectam, clariſſime conſpexi; ſed exire eam non deprehendi. Deinde finita Eclipſi, jucundiſſimum quoque obſervatu erat, bina Luminaria ſimul ſupra Horizontem videre. Nam priuſquam Luna occideret, Sol oriebatur. Cætera notatu digna ex particulari hoc Typo deprehendis.

Ordo

Ord. Phaſium.	*Phaſes Lunæ Teleſcopio obſervatæ.*	*Altitudines Obſervatæ.*	*Tempora ex Altitudinibus Correct.*	*Animadvertendæ.*
			h ′ ″	
		Caudæ *Cygni* 50°. 54′.	12 8 34	
		Caudæ *Cygni* 50 25	12 11 32	
		Pollucis ♊. 26 39	1 28 56	
		Pollucis ♊. 27 16	1 33 17	
	Penumb. Init.		2 10 0	
	Initium Eclipſ.		2 22 0	Cœpit *Eclipſis* in part. ☽æ ſup. circa Sin. *Hyperb.*
1	$1\frac{3}{8}$ Dig.		2 31 0	Umbra Sin. *Apollinem*, & M. *Porphyr.* Stringebat.
2	$2\frac{1}{4}$		2 36 0	*Lacus Niger Maj.* tegi incipieb.
3	$3\frac{1}{4}$		2 41 40	Umbra ad *Inſ. Corſicam* pervenerat.
4	$3\frac{3}{4}$ paul. plus.		2 45 50	
5	$4\frac{1}{2}$		2 49 0	*Pal. Mæotis* & M. *Ætna* plane tecta.
6	5 Dig. fere.		2 53 0	
7	$5\frac{3}{4}$		2 57 0	*Inſ. Besbic.* & *Inſ. Macra* tecta.
8	6 Dig.		3 3 0	
9	$6\frac{1}{4}$		3 5 0	
10	$6\frac{1}{2}$		3 8 30	*Inſ. Melos* tegi incipiebat.
11	$7\frac{1}{8}$		3 11 0	
12	$7\frac{5}{8}$		3 16 0	*Inſ. Rhodus* tecta.
13	$7\frac{7}{8}$		3 20 0	
14	$8\frac{1}{4}$		3 25 30	
15	$8\frac{1}{2}$		3 29 0	*Inſ. Major Caſpii* tegebatur.
16	$8\frac{3}{4}$		3 33 40	
17	$8\frac{7}{8}$		3 37 50	
			3 40 0	

18	9 Dig.		3	47	0	*Max.Obſcuratio* per *J.Leto.* & *M. Aber.* ſe extend.
19	9 Paulo min.		3	51	10	*Pal. Maræotis* ſub Umbræ Sectione Subſiſteb.
20	$8\frac{3}{4}$		3	57	0	
21	$8\frac{1}{2}$		4	1	40	
22	$8\frac{1}{4}$ fere.		4	6	0	
		Polluc. ♊. 49°. 50′.	4	10	28	
23	8 paul. plus.		4	11	0	*Inſ. Melos* & *Rhodus* umbra exiverant.
24	$7\frac{3}{4}$		4	17	20	
25	$7\frac{3}{8}$		4	22	25	*M. Porphyrites* illuſtrari incipiebat.
26	$6\frac{3}{4}$		4	27	35	Umbra *M. Ætnam* jam reliquerat,
27	$6\frac{1}{4}$ fere.		4	33	40	
28	$5\frac{3}{4}$		4	37	37	
29	$4\frac{3}{4}$		4	44	46	*M. Argentar.* & *Inſ. Besbic.* illuſtrabantur.
30	4 Dig.		4	50	56	
31	$3\frac{3}{8}$		4	54	0	
32	$2\frac{3}{4}$		4	57	0	
33	$2\frac{1}{4}$ fere		5	1	23	Umbra totam fere *I. Caſpiam* Maj. Tranſierat.
34	$1\frac{5}{8}$		5	4	45	
35	1 Dig.		5	7	43	
		Cordis ♌ 25°. 48′.	5	10	50	
	Finis *Eclipſ.*		5	21	25	Finis circa *Riphæi Montes* contingit.
	Penumb. Finis		5	24	0	
		Sirii 18°. 54′.	5	28	32	

An Eclipſe of the Moon, Sept. 8. 1671. *At Ecton; by Mr.* Palmer. n. 76. *p.* 2272.

XLIII. 1. *Sept.* 8. 1671. Circa Horam ſextam veſpert. Luna aſcendebat totaliter Obſcurata *Ectoniæ* in Comitatu *Northamptoniano.*

Cœpit Emergere ex Umbra, Centro *Lunæ* Alto 9°. 35′, ſive $7^h. 18'$.

Finis, *Arcturo* Alto 16°. 30′. ſive $8^h. 16'. 20''$. Unde computatur Medium Eclipſis fuiſſe $6^h. 28'. 16''$.

At London; *by Mr.* Street, *ib.*

	°	′	h.	′
2. The Emerſion: Alt. of the upper Edge of the ☽.	10	30	7	21
The End of the Eclipſe: Alt. of *Arcturus.*	16	20	8	$16\frac{2}{3}$

By Dr. Hook. n. 77. *p.* 2296.

3. $7^h. 27'\frac{1}{2}$, I firſt obſerved the Moon Eclipſed when it began to be enlightned, the Total Darkneſs being already paſt. The ſhadow paſſed through the middle of the Spot called by *Hevelius*, *M. Porphyrius.*

$7^h.49'$. The Shadow paſſed through the middle of *M. Sinai*, through the middle of the Eaſtermoſt of the three Lakes called *Mare Adriaticum*, and juſt touched the Ridge of the *Appennine* Mountains.

$7^h.54'$. It paſſed the middle of the *I. Besbicus* in the *Propontis*.

$8^h.0'\frac{1}{2}$. It paſſed through the *Streights* of the *Pontus Euxinus*, at the Promontories *Acherusia* and *Ariſtes*.

$8^h.6'\frac{1}{2}$. It touched the *Palus Mæotis*, which *Palus Mæotis* was then diſtant from the Limb of the Moon, next adjacent, one third part of its ſhorter Diameter or Breadth.

$8^h.17'$. The Shadow went off the Body of the Moon upon the innermoſt Limb-line of *Hevelius's* large Chart of the Moon at the 29 Diviſion, juſt without the *I. Major* of the *Caſpian* Sea. The duskiſh *Penumbra* left not the Limb of the Moon quite without ſome kind of Darkneſs till $8^h.29'$; at which time I found that that ſide of the Moon which the ſhadow laſt left, was full as light and clear as the other.

About four or five Minutes after the Shadow was gone off, I perceived a faint Repreſentation of Colours upon that part of the Body of the Moon, which was moſt affected with the *Penumbra*, ſomewhat reſembling the Colours of a faint *Halo* about the Moon; this grew fainter and fainter, and after a few Minutes was no more Viſible. It did not ſeem to be cauſed by any Clouds or Exhalations in the Air, the Sky near the Moon being very clear, and the ſaid Colours not appearing any where, but upon the dusky part of its Phaſes. Poſſibly it might be cauſed by the Refraction of the Light from the Sun through the Atmoſphere about the Earth.

	h	'	"	°	'	"
4. Obſervatus eſt *Finis*, alto *Arcturo* ad occaſum				13	41	
Ablata Refractione, alto				13	37	45
Hinc datur *Finis* Obſervatus	8	29	16			
Finis *Penumbræ* alto *Arcturo*				13	0	
Correct.				12	56	
Unde datur	8	33	40			
Obſervata eſt *Mæotis* tota extra Umbram	8	24	16			
Alto *Arcturo*				14	26	0
Correct.				14	23	55

At Paris; *by M.* Bullialdus. n. 76. *p.* 2273.

Exivit *Luna* ex Umbra é regione *Petræ Sogdianæ*, *Hevelii*.

5. Hora 8. 30'. Per Nubes Dehiſcentes, ſatis tamen craſſas, *Lunam* ſub obſcure animadvertimus, & quidem tanto Lumine jam imbutum, ut dixiſſes Eclipſin jam eſſe præteritam. Exinde certum erat, Totalem Obſcurationem jam minimum eſſe eo tempore præteritam, imo aliquanto adhuc citius: Siquidem Lunam Rurſus adeſſe nos omnes per Nubes illas ſatis dilucide deprehendimus; Sic ut Eclipſis in Cœlo ultra dimidiam horam citius ingruerit, quam *Kepleri* Calculus id indicaverit. $8^h.34'$. Minimum ad integrum Digitum animadverti jam extra Umbram ſeſe *Lunam* extricaſſe; & denuo $9^h.41'$. ad $1\frac{1}{2}$. Dig. Lumen *Lunæ* jam excreviſſe, quantum id dijudicare circiter dabatur.

At Dantzick; *by M.* Hevelius. n. 78. *p.* 3028.

At Hamburgh; *by Dr.* Fogelius. *Ib. p.* 3033. 6. Æque nos hic ac *Hevelius*, *Gedani*, Calculum *Rudolphinum* in nupera Eclipſi aberraſſe deprehendimus. Emerſiſſe enim jam *Lunam* ex Umbra Terræ ante horam nonam etiam hic vidimus.

An Eclipſe of the Moon, Jan. 1. 1674-5. *at* London; *by Dr.* Hook. n. 111. p. 237.

	h.	″
XLIV. 1. *Initium* veræ Umbræ	5	22
Immerſio	6	19
Emerſio	7	58
Finis veræ Umbræ	8	58

The Penumbra was ſeen to continue near half an Hour before it wholly quitted the Body of the Moon.

At Derby; *By Mr.* Flamſteed. *Ib.* 2. Mr. *Flamſteed* obſerved the Beginning of the Entrance of the true Shadow h. 5. and 19′.

At Paris; *by M.* Bullialdus. *Ib. p.* 238.

		°	′	h.	′	″
3.	*Initium* veræ Umbræ Alta *Capella*	52	26	5	32	29
	Immerſ. Alta *Capella*	62	8	6	33	3
	Emerſ. Alt. Cap. *Pollucis*	43	46	8	9	30
	Fin. veræ Umbræ, Alt. *Sirio*	20	47	9	10	0

At Paris; *by M.* Caſſini, *M.* Picard, *and M.* Roemer. *Ib. p.* 238. *n.* 112. *p.* 257. 4. At 5^{h}. 12′. in the Evening, in the *Royal Obſervatory*, they began to perceive, that the Oriental part of the Moon, by little and little loſt its Light; ſo that at 5^{h}. 25′. they ſaw a manifeſt *Penumbra*; then at 5^{h}. 32′. 50″. the Limb over againſt the Spot called *Hevelius* grew ſo dark, that they all agreed, that this was the true *Beginning* of the *Eclipſe*. At 8^{h}. 7′. one of the Obſervers believed the *Emerſion*, another at 8^{h}. 8′. and the third at 8^{h}. 9′. 30″. but afterwards conſidering the *Emerſion* of the firſt Spots, they all eſteemed it at 8^{h}. 8′. At 7^{h}. 21. the Southern Limb of the Moon was come cloſe to a Teleſcopick Star, at 8^{h}. 9′. 20″. another Star yet leſs than the former, came out of the darkeſt Side, almoſt over againſt the Spot *Langrenus*. At 9^{h}. 9′. 40″. all the three Obſervers agreed, that the Moon then came out of the Shadow. The Diameter of the Moon being meaſured before the Eclipſe was of 32′. 15″.

The Times were noted by great *Pendulum Watches*, that had been adjuſted by the Sun the ſame Day, and that were afterwards verified the next Day: Beſides that, before the Eclipſe at 4^{h}. 45′. 1″. by the Watches, the Star *Capella* was 45 Degrees high towards the Eaſt.

Time,

Time.			*Phaſes.*
h.	′	″	
5	32	50	*Beginning* over againſt the Spot *Hevelius*.
5	36	00	The firſt Spot of *Grimaldi*. *Palus Maræotis*.
5	36	30	The ſecond Limb of *Grimaldi*.
5	45	00	The middle of *Ariſtarchus*. *Mons Porphyrites*.
5	46	00	*Merſennus*.
5	48	30	*Herigone*.
5	53	00	*Heraclides*.
5	53	15	The firſt Limb of *Copernicus*. *Ætna*.
5	54	15	The middle of *Copernicus*.
5	54	40	*Pitheas*, or *Hiera Inſula*.
5	55	05	The ſecond Limb of *Copernicus*.
5	57	40	The firſt Limb of *Timocharis*. *Corſica*.
5	59	35	The firſt Limb of the *Sinus Medius Æſtuum*. *Adriatick Sea*.
6	01	30	The middle of the *Sinus Medius*.
6	02	40	The firſt Limb of *Tycho*, or *Sinai*: and the firſt Limb of *Plato*, or the *Lacus Niger Major*.
6	03	50	The ſecond Limb of *Plato*, and the middle of *Tycho*.
6	04	30	The *Center* of the Disk.
6	09	00	The middle of *Manilius*, or *Mons Besbicus*.
6	12	00	The middle of *Menelaus*, or *Byſantium*.
6	13	45	*Dionyſius Areop*. or *Mons Amanus*.
6	14	30	*Plinius*.
6	15	45	*Vitruvius*.
6	20	35	*Endymion*, or *Lac. Hyperbor. ſuperior*.
6	21	00	*Promont. Heraclium*.
6	24	50	Betwixt *Alcuin* and *Taruntius*.
6	26	00	The firſt Limb of the *Caſpian Sea*, *Mare Criſium*. *Palus Mæotis*.
6	28	15	The middle of the *Caſpian Sea*.
6	29	40	The other Limb of the *Caſpian Sea*.
6	30	5	The firſt Limb of *Langrenus*, or *Inſula Maj*.
6	3	5	The middle of *Langrenus*.
6	35	46	*Total Immerſion*, betwixt *Langrenus* and the *Caſpian Sea*.
8	8	00	Firſt *Emerſion*, towards *Grimaldus*.
8	12	35	The firſt Limb of *Grimaldus*.
8	14	00	The ſecond Limb of *Grimaldus*.
8	20	20	*Merſennus*.
8	24	5	*Herigone*.
8	24	35	The middle of *Ariſtarchus* & the middle betwixt *Herigone* & *Morin*.
8	26	30	The middle of *Kepler*, or *Loca Paludoſa*.
8	28	30	The firſt Limb of *Tycho*.
8	29	50	The ſecond Limb of *Tycho*.

Time.

Time.			*Phaſes.*
h.	′	″	
8	34	5	The middle of *Copernicus.*
8	35	35	The ſecond Limb of *Copernicus.*
8	36	10	*Pitheas.*
8	36	30	*Heraclides.*
8	40	0	The firſt Limb of *Timocharis.*
8	42	35	The firſt Limb of *Plato.*
8	43	45	The ſecond Limb of *Plato.*
8	49	30	The middle of *Manilius.*
8	52	10	*Menelaus* and *Dionyſ. Areopag.*
8	55	0	*Poſſidonius.*
8	56	6	*Vitruvius.*
8	59	30	*Endymion.*
9	6	20	The firſt Limb of the *Caſpian Sea.*
9	7	10	The middle of the *Caſpian Sea.*
9	8	40	The other Limb of the *Caſpian Sea.*
9	9	40	The *End*, between the *Caſpian Sea* and *Langrenus.*

At Dantzick; *by M.* Hevelius. n. 113. *p.* 289.

5. Vigilantes oculos, per totam Eclipſeos Durationem tubo 20 Pedum, aliiſque præſtantioribus, ad quatuor Fixas (neglectis cæteris minoribus, quas optime etiam conſpiciebam) inter quas Luna eo tempore verſabatur, direxi. Ab *a* Stellula vix quatuor minutis Limbo ſuo Inferiori in 6, diſtabat; tres vero reliquas, utpote *b*, *c*, & *d*, Luna corpore ſuo omnino texit. Ex omnibus autem his quatuor inſignioribus Stellulis, non niſi unica *c* ab Aſtronomis hactenus obſervata, Globiſque adſcripta eſt; nominatur *Informium inter* ♊ & ♋ *Suprema a tergo Pollucis*; cujus curſus cum ingreſſu, via itnieraria, atque egreſſu imprimis probe notandus. Quippe ex hujus generis obſervationibus, multo proclivius datur Motum Lunæ redintegrare, ejuſque Nodos Latitudinemque reſtaurare, quam, meo quali judicio, ex nudis *Solis Eclipſibus*. Stellula *b*, ad montem *Eoum* circiter tecta eſt, & *d* ad ipſum Limbum Lunæ Inferiorem; illa per *Sinum Sirbonis*, *I. Rhodum*, & S. *Athenienſem*; hæc vero per *Deſertum Mingui* tranſiit.

Temp. ſec. Hor Oſcil. ex altitudinidus Cor.	*Ord. Phaſium.*	*Altitudines Fixarum. Nec non Animadverſiones quædam notatu digna.*	*Per quas Maculas tranſiverint Umbræ Sectiones.*
h. ′ ″			
6 22 18		Altitudo *Caudæ Cygni.* 39°. 3′. 0″.	
6 25 4		Eadem Altitudo. 38°. 41′. 0″.	
6 35 0		Penumbram *Lunæ* ſubiit.	
6 41 50		*Initium* Eclipſeos.	*Incepit* circa 5°. a puncto Nadir Ortum verſus.
6 43 45	1	Paludem *Maræotidem* attigit.	Tempore *Initii*, *Sinus Sigaricus*, *Inſ. Besbica* & *Inſ. Melos* in Linea Recta extiterunt.
6 44 55	2	*Palus Maræotis* omnino obſcur.	
6 49 0	3		Sectio Umbræ per *M. Pentadactylum* & *Eoum* tranſiit.
6 52 30	4	*Mons Porphyrites* tectus.	Per *Loca Paludoſa Montemque Cataract.*
6 57 35	5		Ad *M. Baronium*, per *M. Petri*, *Athor*, & *M. Troicum*.
7 2 15	6	*M. Ætnam* tegere incepit.	Ad *Sin. Apollinis*, *Inſ. Ficariam*, ad radices *Montis Ætnæ*, *Inſ Didymæ*, *Lacumq; Meridionalem.*
7 2 55	7	*M. Ætna* omnino tectus.	Ad *Inſ. Sardiniam*, per *Inſ. Hieram*, *Inſulamque Cretam.*
7 7 40	8		Per *M. Atlanticos Majores*, *Inſ. Vulcaniam*, *Rhodam*, *Montemque Amnæ.*
7 10 40	9	*M. Sinai* tectus.	Per *Inſ. Ophiuſam*, *Inſ. Cyprum* & M. *Sinai.*
7 14 0	10	*Lacus Niger Major* tectus.	Ad L. *Nigrum Majorem*, per M *Sipylum*, *Libanum*, Montemq; *Seir.*
7 16 40	11		Per L. *Nigr. Minorem*, *Inſ. Besbicam*, M. *Olympum* & *Didymum*.

7 20 20	12	Fixa *c*, diſtitit a Limbo ☽ ortum verſus 30′. fere.	Sectio Umbræ tranſiit per M. *Carpathos*, *Byzantium* & *Taurum.*
7 25 0	13		Per Lacum *Boryſthenem*, *Inſ. Apolloniam*, M. *Moſchum* & *Sogdianos.*
7 28 30	14		Per Montes *Macrocemnios*, *Promont. Arietis*, *Herculeum*, per Sinum extremum *Ponti*, Montemque *Parapæmiſum.*
7 32 0	15		Per *Paludes Hyperboreos*, *I. Corocondametis*, Montemque *Caucaſum.*
7 34 10	16		Per *Montes Ripheos*, Palud. *Mæotidem*, Sinum Inferiorem Maris *Caſpii.*
7 42 44		*Totalis Immerſio* contigit circa 50°. Limbi a puncto Zenith occaſum verſus.	
7 55 30		Stellula *a*, in ipſa ☌ cum ☽; Diſtitit a Limbo ejus inferiori 4 circit. Minut.	
8 0 50		Stellula *b*, ad M. *Eoum* Tecta.	
8 35 20		Stellula *c*, ad *Lacum Merid.* ſubiit.	
8 51 20		Stellula *d*, ad ipſum Limbum Inferiorem occultata.	
9 9 10		Stellula *c*, rurſus emerſit, ſub *Monte* ſc. *Nevoſo*; ſicut ☽ ad 3′. 20″. ſubingreſſa fuerit. Spectaculum ſane erat Jucundiſſimum, hæcce omnia dilucidiſſima, & quidem ſub ipſa fere Maxima Obſcuratione, & ☌ æ ☽ deprehendere.	
9 12 13		*Emerſio* Lunæ ex umbra contigit ad 30°. fere Limbi ☽ a puncto Nadir Ortum verſus.	
9 20 20	17		Umbra inceſſit per Pal. *Mareotidem Fontes Amaros* & M. *Eoum.*
9 25 0	18		Per M. *Audum*, *Ajacem* & *Troicum.*
9 30 20	19		Per M. *Pentadactylum*, *Carpathum*, S. *Sirbonis*, & M. *Lion.*

9 32 40	20	Mons *Porphyrites* exiit ex Umbra.		Per M. *Porphyrit. Caſſium* & *Didymum.*
9 36 5	21	M. *Sinai* illuminari cœpit.		Per M. *Baronium*, ad *I. Siciliam*, per *Cretam* ad radices *Sinai.*
9 42 15	22	M. *Ætna* ſub ipſa ſectione Umbræ.		Per *Promontorium Apollinis*, M. *Ætnam*, per *Rhodum* Monteſque *Seir.*
9 47 10	23			Per *Inſ. Vulcaniam*, M. *Maſicytum Cragum* & *Antiliban.*
9 51 0	24			Ad *L. Nigrum Majorem*, S. *Paſtanum*, M. *Didymum*, & *Montes Coibacaronos.*
9 56 50	25	*Inſula Besbica* rurſus prodiit in Lucem.		Per S. *Nigrum Minorem*, ad I. *Besbicam*, per M. *Uxerios.*
9 59 35	26			Per *L. Salmiderſam*, M. *Horminiſum*, *Moſchum* & *Lacum Thoſpitis.*
10 5 0	27			Per *Pontum*, *I. Cyaneam*, S. *Athenienſem*, & M. *Caucaſum.*
10 7 0	28			Per *L. Boryſthenem*, ad *I. Apolloniam*, per *Heracleum* ac M. *Tanconem.*
10 19 35		Umbra nondum plane exiverat.		
10 20 0		*Finis* Eclipſeos.		
10 23 0		*Penumbra.*		
10 52 58		Altitudo ♂.	37°. 12′	
10 58 35		Alt. *Lucidæ* ♈.	28 52	
11 11 33		Alt. *Capellæ.*	70 11	

6. Obſervatio hujus Eclipſis fuit admodum exacta; nam Sole prope Horizontem exiſtente, per hujus Loci Altitudinem Minutorum tempora facile innotuerunt: Cætera meo Horologio (quod ne unum quidem Minutum ea die diſcrepabat) notavi, ita ut nulli dubio in obſervatione detur locus.

At Seville; *by* S..... *Profeſſor of Mathematiques.* N. 118. p. 428.

	h.	′	
Initium veræ Umbræ	4	56	
Immerſio	6	1	
Emerſio	7	33	
Finis	8	39	& aliquantulum productior.

XLV. 1. *An Eclipſe of the Moon,* Jun. 27. 1675. *at* London; *by Mr* Flamſteed *and Mr.* Halley. N. 116. *p.* 371. N. 118. *p.* 432.

	Hora Horologii Oſcillatorii.			*Phaſes.*	
	h.	′	″		
1	1	1	00	Lunæ Diameter 3190 = 31′. 40″	Multoties repetitæ menſuræ tum à me tum ab adjutore *Edm. Halley.*
3	1	11	00	Rurſus Lunæ Di. 3191 = 31 46	
5	1	37	50	Nullum Penumbræ veſtigium: jamque Luna nubes ſubtervolantes ſubiit, ſub quibus latuit uſque	
6	1	46	40	Dum per earum hiatus Penumbra denſa, vel forſan ipſum Initium apparuit; ſed certus eſſe non poteram.	
7	1	51	40	E nudibus elevatæ Limbus, notabili ſatis defectu laborare viſus eſt, obſcurato ſextante vel minimum octante Peripheriæ.	
8	1	55	15	*Pentadactylus* tectus.	
9	2	2	20	*Porphyrites* tectus.	
10	2	5	30	*Sinæ* Limbus primus.	
11	2	6	00	*Ætnæ* Limbus proximus.	
12	2	8	40	Partes reſiduæ illuminatæ 2071 = 20′. 38″.	
13	2	12	00	Fixa exigua Teleſcopica, minori tubo non viſibilis, in majori apparuit dimidium fere capacitatis ejuſdem, vel 15′. à Limbo apparenter inferiori diſtans.	
14	2	17	45	Partes illuminatæ rēſidūæ, 1655 = 16′. 27″.	
15	2	23	5	*Besbici* Limbus prior.	
16	2	26	3	*Horminius* tectus.	
17	2	29	00	*Æmus* tectus.	
18	2	30	45	Partes illuſtratæ reſiduæ 1047 = 10′. 24″.	
19	2	35	00	Partes lucidæ reſiduæ 865 = 8 38; jamque tubo longiori fixa alia exigua apparuit, Maculæ *Caſpiæ* Longitudinem à limbo Lunæ, Latitudinem ejus à Linea per cuſpides ducta dextram verſus, diſtans.	
20	2	37	15	Umbra tegebat occidentale Littus *Ponti.*	
21	2	39	30	Ipſa tetigit Limbum primum *Corocondometis.*	
22	2	45	00	Paludem *Mæotidem* tetigit.	
23	2	50	45	*Mæotis* tota tecta.	
24	2	56	10	Dubium ad aliquid veri Luminis ſupererat.	
25	2	56	55	*Immerſio:* Certe enim Lux primaria Lunam penitus deſeruerat, ſcil. è regione paulo ſupra Montes Ripheos & circa gradum Limbi *Heveliani* 330. 2^{h}. 57′. 30″. Limbus Coloris Cineritii per tubum apparuit.	

Aer à ſeptima obſervatione ad *Immerſionem* ſereniſſimus extitit; & Lucula quædam Albicans per totum defectus tempus Cuſpides obſcuratæ Lunæ viſa eſt inſidere. quæ eam etiam poſt Immerſionem à parte, quæ ultimum in Umbram inciderat, reddebat conſpicuam. Tenuis admodum erat Eclipſis hujus Penumbra, nec major quam *Sinæ* aut *Ætnæ* Latitudo. Palus *Mæotis* Lata apparuit admodum, & quam maxime fere potuit à Limbo Lunæ remota. *Maræotis* è contra compreſſa admodum, nec pluſquam dimidium Longitudinis ipſius à Limbo Lunæ diſtans.

At Paris; *by* M. Bullialdus. *Ibid.* 372.

	°	′	h.	′
2. *Initium* veræ Umbræ alta *Capella* ad ortum	18	45	1	55
Umbra attigit *Paludem Mæot.* alta *Lyra* ad occ.	50	51	2	55
Immerſio Totalis alta *Lyra* ad occ.	48	50	3	6

3. *By* M. Caſſini, M. Picart, *and* M. Roemer. N. 117. *p.* 388.

Temp.			*Tranſitus Umbra.*
h.	′	″	
1	56	45	*Initium* Infra *Grimaldum.*
1	57	20	Per primum Limbum *Grimaldi.*
1	58	50	Secundum Limbum *Grimaldi.*
2	1	45	*Galileum.*
2	2	15	Primum Limbum *Merſenni.*
2	4	15	Initium *Gaſſendi.*
2	4	40	Medium *Gaſſendi.*
2	5	30	Alterum Limbum *Gaſſendi.*
2	6	15	*Herigonum* & *Seleucum.*
2	7	45	*Morinum.*
2	8	18	Medium *Kepleri.*
2	11	35	*Ariſtarchum* & *Bullialdum.*
2	12	40	*Ariſtarchus* diſparet.
2	16	25	Initium *Tychonis.*
2	16	40	Initium *Copernici.*
2	17	25	Medium *Tychonis* & *Copernici.*
2	18	12	Alterum Limbum *Tychonis.*
2	21	45	*Pytheam* & primum trium *Sinuum Mediorum.*
2	24	0	Medium Secundi *Sinus Medii*
2	24	53	*Heraclidem* vel *Virginem.*
2	26	10	Primum Limbum *Timocharis.*
2	26	40	Medium *Timocharis.*
2	31	40	*Promontorium* inter *Verginem* & *Platonem.*
2	32	20	*Abulfedam.*
2	34	15	Initium *Manilii.*
2	36	15	*Dionyſium Areopagitam.*
2	38	10	Littus Maris *Tranquillitatis.*
2	38	45	Primum Limbum *Menelai* & *Platonis.*
2	39	20	*Fracaſtorem.*
2	39	53	Medium *Platonis.*

Temp.			*Tranſitus Umbræ.*
h.	′	″	
2	41	15	Per Alterum Limbum *Platonis.*
2	44	15	Promontor. inter *Conſorinum* & *Bedam.*
2	50	20	Primum Limbum *Paludis Somnii.*
2	50	45	Cornua erant verticalia.
2	53	55	Initium *Langreni.*
2	55	20	Primum Limbum *Maris Caſpii.*
3	1	10	Alterum Limbum *Maris Caſpii.*
3	1	45	*Endymionem.*
3	3	15	*Meſſalam.*
3	7	45	Finis ſive *Totalis Immerſio*, ſupra *Mare Caſpium.*

Mare Caſpium tunc diſtabat à Limbo Occidentali circiter ¼ ſuæ Latitudinis. Poſt *Immerſionem Totalem* dignoſcebatur adhuc totum Corpus Lunæ.

XLVI. 1. *An Eclipſe of the Moon* Decemb. 22. 1675. *at* Greenwich; *by Mr.* Flamſteed. N. 121. p. 435.

Hor. cor. Horol.			*Phaſes.*
h.	′	″	
2	29	30	Inter Cuſpides 2085 = 17′. 16″.
2	55	45	*Hamum* fere tetigit.
3	0	30	*Hamum* certe tetigerat.
3	11	30	Cuſpis dexter à *Mareotide* 1235 = 10′. 14″.
3	35	0	Partes Lucidæ circiter 2800 = 23. 11. vel paulo forſan amplius; difficile enim erat admodum, Umbræ veræ Terminos, per Aerem Vaporibus fœdatum, definire.
3	42	30	Umbra prope *Macram.*
3	52	45	Inter Cuſpides circiter 2288 = 18′. 57″.
4	7	15	*Finis:* Limbus enim apparuit, & nihil videbatur in rotunditate Lunæ deſiderari.
4	8	0	Limbus admodum dilucide per Tubum conſpectus.
4	15	30	*Penumbra*, quæ nudis Oculis Eclipſin referebat.
4	19	30	☽æ capta diameter 3757 = 31′. 5″. Sed vix ſatis certa; quam tamen haud multum à veritate abeſſe putem.
4	23	0	Etiamnum, & poſtea, Limbus ab *Eclipſi* derelictus Obſcurior videbatur ac alter.

Corſica à Limbo ☽æ Remoto diſtantia 2732 = 22′ 37″.
Limbus ejus proximus à proximo *Lunæ* 1045 = 8 39.

Sinæ Limbus Remotior à *Lunæ* Proximo	599 =	4′. 58″. bona.
Lacus Nigri Majoris Medium à Limbo proximo	452 =	3 45

Notavi præterea, quod Umbra ſemper longe diſtinctior apparuit ad Cornua, quam alicubi in Facie Lunæ: In prima Obſervatione, vel paulo ante, Cornua fuere Horizonti Parallela.

Tunc etiam *Porphyrites*, & *Lacus Niger Major*, æqualiter ex Umbra extitere, Longitudinem ſcil. circiter *Mareotidis*.

Nunquam tamen *Porphyritem* ſuperavit in hac Eclipſi; alte vero illum in Penumbram merſum vidi.

In ſumma Eclipſi ad *Corſicam* fere Umbra pertigerit; nunquam tamen eam extinctam vidi, ſed alte adeo in Penumbra immerſam, ut ægre eam potuerim diſcernere.

Nec unquam Umbra vera *Inſulam Macram* pervagabatur, ſed Penumbra duntaxat denſa, per quam difficile erat ipſam percipere.

4^h^. 5′½. Limbum videre non potui; nec 4^h^. 6′½; ſed 4^h^. 7′. videre me putabam Limbi Lucem ſed languidiſſimam, & ægre admodum; 4^h^. 7′. 15″. certior factus ſum ex Umbris Emerſiſſe, nec aliquid in ejus rotunditate deſiderari: Ergo tunc *Finem* obſervatum ſtatuo.

Exibat Umbra juxta *Lacum Hyperboreum Superiorem* manente Penumbra, quæ Eclipſin nudis Oculis exhibebat uſque 4^h^. 15′½; ſed Limbus ab Eclipſi derelictus Limbi oppoſiti Claritudinem recuperavit non niſi 4^h^. 28′. vel ſerius.

Tempora Phaſium Correcta, ab Altitudinibus *Arcturi* & *Lucidæ Coronæ*, Quadrante Teleſcopico, pedum trium & amplius Radio, captis; quibus, clare aliquando in altera Cœli cardine emicantibus, captandis incubui, quoties Lunam ſubiere Nubes.

2. *Initium* Eclipſis accidit antequam ad Inſtrumenta veni: Quod tamen *Londini* in Vico *Wintonienſi* obſervavit *Edmundus Halleius*, cum Lunæ Limbus ſuperior a Vertice diſtaret

At London; *by Mr.* Edm. Halley. *Ib. p.* 498.

	°	′		h	′
ſuperior a Vertice diſtaret	39	51	unde horam ſupputavit	2	16
alto eodem	41	1	Cornua Horizonti Parallela	2	25
	54	12		3	58

3. *Joh. Coelſonus* in Vico *Wappingenſi*, ad *Anachoreſin*, (Angl. *the Hermitage*) Limbum Lunæ Deficientem aliquantulum vidit 2^h^. 17′¾. At 4^h^. 9′. 25″. ex Umbra vera ipſum exiiſſe comperit, denſa duntaxat Penumbra remanente.

By Mr. Colſon, *Ib.*

4. In hac Eclipſi Duo præcipua à nobis exacte determinata ſunt, *Medium* ſc. Eclipſis tempus, ejuſque Magnitudo. *Medium* deductum eſt non ſolum ex comparatione *Initii*, & *Finis*, ſed etiam Duarum æqualium Phaſium, determinatu facillimarum, quando *Scil.* diſtantia Cornuum æqualis erat Lunæ ſemidiametro, ante Eclipſin captæ, 15′. 28″. Scilicet cum *Initium* Eclipſis exiſtimatum fuerit 2^h^. 24′. 35″.

At Paris; *by* M. Caſſini. N. 123. *p.* 561.

	h	′	″
Finis vero *Totalis*, relicta Penumbra ſimili ac fuerit in determinatione *Initii*	4	15	25
Duratio totius Eclipſis provenit	1	50	50
Dimidia	0	55	25
Et Eclipſis *Medium*	3	20	0
Sexta vero circumferentiæ pars abſciſſa eſt	2	38	5

	h.	′	″
Atque iterum	4	2	25
Intervallum	1	24	20
Dimidium		42	10
Hinc *Medium* Eclipſis	3	20	15

Intra quartam minuti partem priori determinationi conveniens.

In Situ Umbræ & Eclipſis Magnitudine D. *Flamſtedio* plane convenimus. Ab utriſque quippe Noſtrum annotatum eſt, Umbram nunquam ſuperaſſe *Porphyritem*, licet is alte in *Penumbram* fuerit Immerſus.

Porphyriti proximus eſt Mons parvus albicans, quem tunc *Ariſtarchi Comitem* appellavimus, eo quod ab ipſo ſeu *Porphyrite* vix diſtet ſui Diametro. Is Monticulus Immerſus eſt in Umbram 2^{h}. 51′. 15″ ; Emerſit autem 3^{h}. 8′. 25″. totoque tempore interjecto fuit Umbra *Porphyriti* proxima.

Uterque pariter annotavimus, in ſumma Eclipſi Umbram ad *Corſicam* fere pertigiſſe, nunquam tamen ab ea fuiſſe tectam, ſed relictum exiguum intervallum, cujus termini diſtantia à Lunari Margine proximo capta eſt 8′. 17″, cum *Flamſtedius* Inſulæ ipſius paulo remotioris diſtantiam ab eodem Limbo invenerit 8′. 39″. *Inſulam* quoque ſeu potius *Peninſulam Macram* utrique Umbræ diutiſſime adjacentem conſpeximus; nos id fieri cœpiſſe notavimus 3^{h}. 28′. 15″. & per horæ quadrantem in eadem diſtantia perſeveraſſe.

5.
By M. Bullialdus. N. 125. *p.* 610.

Capellæ *diſtant à Vertice.*		*Phaſes.*	*Temp. m.*		
°	′		h.	′	″
39	36	*Penumbra* tenuis.	2	6	12
40	42	*Penumbra* craſſior.	2	12	7
42	30	*Initium* ſenſibile è reg. *Sinus Hyperb.* circa gr. 70.	2	23	32
42	50	Digiti fere $\frac{1}{8}$.	2	25	48
44	25	Umbra attigit *Atlantem Minorem.*	2	36	11
47	28	Umbra paulo ſupra *Baronium*, ſupra *Liguſtinum*, occupaverat *Macr. M.*	2	56	27
48	56	U. attigere fere *Catenam Mundi.*	3	6	22
49	54	U. attig. *Montunial.*	3	12	54
50	30	Attigit *Sinum Perontic.M. Pyram* & Med.*Palud.Hyp.*	3	17	41
52	7	Occupavit *Sinum Sagaricum* & *Peronticum*, atque *Promont. Lunæ.*	3	29	1
54	52	*Leucopetra* extra Umbram.	3	48	21

55 40

55	40	*Sinus Peronticus* extra Umbram.	3	54	6
55	48	*Sinus Sagaricus* extra Umbram.	3	55	12
56	17	*Sinus Cercinites* fere Emerſerat.	3	58	29
57	16	Pars ſub Umb. æqualis fere Latitudini *Paludis Maotidis*.	4	5	38
58	30	*Finis* verſus è reg. *Mont. Macrocem* circa g. 355.	4	13	59
59	6	*Penumbra.*			

Corſicam non attigit Umbra, neque *Lacum Thraſymenum*, propterea Eclipſis non exceſſit 3 Dig. 30′. vel minus etiam. *Initium* uno ſcrupulo primo, vel 45″, anteceſſit adnotatum, ita ut ſtatui exactius poſſit 2h. 22′. 30″. Hinc tota *Duratio* ſatis præciſe 1h. 51′. 24″. Quare *Maxima Obſcuratio* contigit 3h. 18′. 14″.

6.
At Strasburg; *by M.* Richelt. *Ib.*

Alto Arcturo.		*Tempor. m.*			*Phaſes.*
°	′	h.	′	″	
30	30	2	48	48	*Initium.*
36	0	3	20	8	Umbra per M. *Porphyritem* & *Promont. Lunæ.*
39	50	3	45	44	Umbra ſtrinxit *Lacum Thraſymen. Mont. Baronium,* & *Sinum Cercinitem.*
44	15	4	13	20	Umbra tranſiit per *Prom. Lunæ* & *M. Cimmerium.*
46	25	4	27	36	Umbra tetigit *Lacum Nigrum Minorem* & *M. Carpathum.*
48	30	4	41	44	Deſiit in regione *Hyperborea* media ad *Mare Hyperboreum.*
Tota itaque *Duratio* 1h 52′. 56″.					

At Dantzick; *by* M. Hevelius. N. 124. *p.* 589.

7. In hac Eclipſi probe notandum eſt, quod omnes Sectiones nunquam *Montem Porphyritem* omnino texerint, ſed ille per totam Durationem, etiam in ipſa Maxima Obſcuratione, in ipſo Umbræ Limbo conſpicuus perſtiterit.

Temp.

Temp. ex Altit. Cor.			*Altitudines Fixarum.*			*Ordo Phasium.*	*Per quas Maculas tranſiverint Umbra Sectiones.*
h.	′	″	°	′			
2	28	38	26	12	Dext. Humeri *Orionis.*		
2	33	20	25	35	Dext. Hum. *Orionis.*		
2	36	40			Initium *Penumbr.*		Ad *Montem Baronium.*
3	8	10					*Penumbra* denſior.
3	16	35					Denſa *Penumbra.*
3	24	20					Denſiſſima *Penumbra.*
3	30	0			*Initium* Eclipſeos.	1	Ad *Sinum Apollinis.*
3	36	25				2	Per *M. Alabaſt.* & *Sin. Hyperb.*
3	42	5				3	Ad *Sinum Apoll.*
3	46	30				4	Per *M. Baron.* & *L. Nigr. Majorem.*
3	52	10				5	Per *Inſ. Ophiuſam.*
3	59	15				6	Ad *M. Porph.* & *L. Nig. Min.*
4	7	45			Stellula diſtabat in Limb. 36′. vel 40′.	7	Per *M. Porph.* & *M. Serr.*
4	11	45				8	Per *Porph. Pr.* ☽ æ, & *M. Carp.*
4	18	5				9	Per *Inſ. Corſ. M. Arg.* & *M. Macroceran.*
4	26	0				10	Per *M. Porphyrit.* & *Lac. Traſimenum.*
4	56	20				11	Ad *Sinum Apollinis, M. Chriſti,* & *Inſ. Macr.*
5	55	28	32	33	Lucidæ *Lyræ.*		
5	57	33	32	50	Lucidæ *Lyræ.*		
5	59	0					*Penumbra.*
6	8	0					*Penumbra* pene evanuit quoad conjicere potuimus ob nubes.

XLVII.
An Eclipſe of the Moon, Oct. 29. (*St. n.*) 1678. *At* Paris; *by* M. Caſſini. N. 141. *p.* 1015.

Phaſes Lunæ & Macularum Secundum Denominationem Riccioli.	In *Obſervatorio Regio.*						In *Collegio Claromontano.*		
	Infra.			*Supra.*					
	h.	′	″	h.	′	″	h.	′	″
Incipit Umbra	6	43	30	6	43	40	6	43	54
Grimaldi Limbus Sequens	6	45	0				6	45	29
Galileus	6	46	0	6	46	0			
Finis *Gallilei*	6	47	0						

	h.	′	″	h.	′	″	h.	′	″
Merſennus	6	48	20						
Chorda Eclipſis Dig. 6.				6	49	00			
Gaſſendi Initium	6	50	50	6	50	50			
Gaſſendi Medium				6	51	30	6	51	37
Schikardi Initium	6	51	43						
II. Digiti Ecliptici				6	52	00			
Ariſtarchi Initium	6	52	50						
Ariſtarchi Medium				6	53	10	3	56	7
Ariſtarchi & *Morini* Finis	6	54	00						
Capuanus ſive Oculus *Draconis*	6	56	00						
Digiti III.				6	56	30			
Chorda 9 Digitorum				6	57	00			
Initium *Terræ Pruinæ* & *Copernici*	6	58	54						
Copernici Initium				6	59	10			
Copernici Medium	7	00	00	7	00	00	6	59	30
Copernici Finis	7	00	55	7	00	55			
Pithea Initium	7	1	50	7	1	40			
Pithea Finis	7	2	30						
Caput Virginis	7	2	45	7	2	40	7	3	18
Harpalus	7	2	55						
Tychonis Initium	7	4	20	7	4	20			
Tychonis Medium	7	5	00				7	5	48
Tychonis Finis	7	5	55						
Eratoſthenes	7	6	20						
Digiti V.				7	6	50			
Promontorium inter *Virginem* & *Platonem*	7	7	00						
Inſula in ultimo *Sinuum Mediorum*	7	7	30						
Clara ſequens *Tychonem*	7	8	31						
Digiti VI.				7	11	20			
Tymocharis	7	11	48						
Platonis Initium				7	13	20	7	13	29
Platonis Medium				7	13	40			
Platonis Finis & Initium *Manilii*	7	14	00						
Platonis Finis				7	14	40			
Manilius				7	14	50	7	15	4
Finis *Manilii*	7	15	12						
Dionyſii Initium	7	17	15						
Dionyſius				7	17	25			
Menelaus				7	18	10	7	17	59
Dionyſii Finis & *Menelai* Initium	7	18	28				7	18	11
Plinius Incipit	7	20	56						
Plinius				7	21	10	7	21	51
Picolomineus ſeu *Clara* ſupra *Annulum*	7	21	30						
Initium *Fracaſtorii* ſeu *Annuli*	7	23	00	7	23	5			
Initium *Poſſidonii*	7	23	55				7	24	38
Finis *Fracaſtorii*	7	24	25						

	h.	′	″	h.	″	″	h.	′	″
Clara ante Angulum *Promontorii Acuti*				7	24	35			
Angulus *Promontorii Acuti*	7	25	10						
Digiti IX.				7	26	10			
Palus Somni	7	28	00						
Initium *Endymionis*	7	29	20						
Initium *Taurentii*	7	30	30						
Angulus Cornuum cum Parallelo 77° 55′				7	30	40	7	31	16
Hermetis Initium	7	31	5						
Finis *Taurentii* ſive Capitis *Serpentis*	7	31	40						
Hermetis Finis	7	32	00						
Proclus	7	32	20						
Limbus *Maris Caſpii*	7	32	29	7	33	3	7	33	10
Macula Inferior	7	33	5						
Initium *Langreni*	7	34	24						
Meſſala	7	34	40						
Finis *Langreni*	7	35	40						
Peninſula in *Caſpia*	7	37	50						
Finis *Caſpiæ*	7	37	50	7	37	50	7	38	18
Macula Oblonga *f.*	7	39	16						
Finis	7	40	41	7	41	00	7	41	28
Initium Emerſionis	9	21	30	9	21	30	9	21	5
Grimaldus	9	22	44				9	22	10
Grimaldi Finis	9	23	40	9	23	40			
Gallileus	9	24	35						
Mare Humorum	9	28	15						
Schikardi Medium	9	29	00						
Ariſtarchi Initium	9	29	44				9	29	41
Ariſtarchi Medium				9	30	20			
Ariſtarchus Totus	9	30	25						
Gaſſendus Totus	9	31	00						
Kepleri Initium				9	31	40	9	31	6
Keplerus Totus	9	32	24						
Morinus	9	30	00						
Finis Inſulæ *Kepleri*	9	34	22						
Capuanus Totus	9	34	40						
Initium *Terræ Pruinæ*	9	35	43						
Virgo	9	37	00						
Aurea inter *Pitheam* & *Keplerum*				9	37	20			
Harpalus & *Clara* ante *Copernicum*	9	37	34						
Digiti III.				9	37	40			
Bullialdus	9	38	30						
Initium *Copernici*				9	39	00	9	38	36
Tychonis Initium	9	40	00				9	40	22
Copernicus Totus	9	40	18						
Pitheas				9	40	30			

	h.	′	″	h.	′	″	h.	′	″
Tychonis Medium	9	41	00	9	41	10			
Tychonis Finis				9	42	20			
Digiti IV.				9	42	40			
Timocharus Totus	9	45	30						
Plato Incipit				9	45	30	9	45	00
Digiti V.				9	47	00			
Plato Totus	9	47	00	9	47	30			
Inſula ſub *Sinu Medio*	9	50	00						
Finis *Sinus Medii*	9	50	50						
Archimedes	9	51	37						
Digiti VI.				9	52	00			
Manilius				9	54	00	9	53	52
Fretum	9	54	22						
Abifeldea Initium	9	54	40						
Abifeldea Finis	9	55	30						
Ariſtoteles & *Eudoxus* Toti	9	56	00						
Digiti VII.				9	56	20			
Dionyſii Initium	9	57	12						
Menelaus	9	57	50	9	57	40	9	57	16
Dionyſii Medium				9	58	30			
Menelaus Totus	9	59	00						
Dionyſius Totus	10	00	00						
Digiti VIII.	10	1	30						
Oſtium *Lacus Mortis*				10	2	00			
Plinius				10	2	15	10	1	28
Promontorium ſupra *Dionyſium*	10	3	00						
Plinii Finis. *Poſſidonius*	10	3	40						
Promontorium Acutum				10	5	30			
Hermes	10	6	00						
Digiti IX.				10	6	15			
Chorda Umbræ 8 Digitorum				10	9	30			
Meſſala	10	11	00						
Initium *Maris Caſpii*	10	13	30	10	12	10	10	11	30
Langrenus Totus	10	16	00						
Chorda 6 Digitorum				10	14	00			
Caſpii Prior Finis				10	16	50	10	16	44
Caſpii Alter Finis	10	17	33	10	17	25			
Albedo in *Mari Caſpio*	10	18	15						
Finis Totalis	10	20	00	10	20	10	10	20	42

XLVIII. 1. *An Eclipſe of the Moon*, Aug. 19. *m.* 1681. *at* Greenwich; *by Mr.* Flamſteed. *Ph. Coll. n.* 3. *p.* 67.

Temp. per. Hor. Oſcil.			*Temp. inde per Obſervat. Corr.*			*Obſervationes.*
h.	′	″	h.	′	″	
1	5	00	1	00	00	Lunæ *Diameter* 6191 = 30′. 53″.
1	46	30	1	41	30	*Penumbra* notabilis.
1	53	40	1	48	40	*Penumbra* denſa.
1	55	40	1	50	40	*Initium.*
1	58	00	1	53	00	Umbra ad *Montem Climacem.*
2	3	20	1	58	20	*Sinus Sirbonis* Incipit.
2	5	50	2	00	50	*Mareotis* Incipit.
2	7	30	2	2	30	*Mons Cataractes* Incipit.
2	8	16	2	3	16	*Mareotis* Tota tecta.
2	10	50	2	5	50	*Sina* Incipit.
2	12	56	2	7	56	*Sina* Tota tecta.
2	18	00	2	13	00	*Audus.*
2	27	24	2	22	24	*Circinna.*
2	31	00	2	26	00	——— *Abulfeda* Ricciolo.
2	32	30	2	27	30	*Pentadactylus* Incipit.
2	33	32	2	28	32	*Ætna* Limbus primus.
2	34	26	2	29	26	*Pentadactylus* Totus.
2	35	6	2	30	6	*Ætnæ* Medium.
2	36	00	2	31	00	*Ætna* Tota tecta.
2	40	00	2	35	00	Umbra per Med. *Corphyriten.*
2	44	50	2	39	50	*Hiera Inſula* & *Ficaria* ſimul ad Umbram.
2	46	10	2	41	10	Umbra ad *Horminium.*
2	50	10	2	45	10	*Mons Hercules* Incipit.
2	51	10	2	46	10	M.. *Hercules* intra.
2	52	10	2	47	10	Umbra per Med. *Besbici.*
2	54	00	2	49	00	per Med. *Inſulæ Majoris.*
2	55	30	2	50	30	per Med. *Byzantium.*
2	57	40	2	52	40	per Med. *Inſulæ Cyaneæ.*
3	1	50	2	56	50	ad Limbum primum *Corocondometis.*
3	4	30	2	59	30	ad Limbum primum *Mæotidis.*
3	13	00	3	8	00	*Macra* Incipit obſcurari : dub.
3	14	00	3	9	00	*Mæotis* Tota tecta.
3	19	12	3	14	12	Partes Lucidæ reſiduæ erant 1092 = 5′. 27″.

Digits

2. *At* Paris; *by* M. Caſſini. *Ibid. p.* 66.

Digits Eclipſed.	Phaſes.	Time.
d. ' ''		h. ' ''
	The *Beginning* at	1 58 30
	The arrival of the Shadow at *Grimaldus.*	2 1 30
	At *Tycho.*	2 8 00
	At the *Center* of the *Moon.*	2 37 30
	At the middle of *Copernicus.*	2 39 30
	At *Ariſtarchus.*	2 40 00
8 16 00	At the Middle of *Manilius.*	2 54 30
	At *Plinius.*	3 1 00
9 2 00	At the Lower part of the *Caſpian Sea.*	3 7 30
9 31 20	At the Upper ſide of the *Caſpian Sea.*	3 18 30
10 1 00	The *Greateſt Obſcurity.*	3 35 45
4 46 00	The *Return* of the Shadow To *Ariſtarchus.*	3 47 00
	To the *Center* of the *Moon*	4 29 00
	To *Tycho.*	4 41 30
	The *End* of the *Eclipſe.*	5 13 00
	The *Whole Duration.*	3 14 30

At 5h. 13'. 56''. the *Apparent Altitude* of the upper Edge of the Sun, was 17'. 10''. and that of the *Moon*, 1°. 11'. 30''.

3. *At* Dantzick; *by M.* Hevelius, n. 5. *p.* 160.

Temp. juxta Hor.	*Phaſes.*	*Altitudines.*	*Temp. Correct.*
h. ' ''		° '	h. ' ''
1 1 03	Altitudo *Palilicii.*	24 14	1 4 48
2 4 30	Eadem Altitudo.	32 58	2 6 32
2 9 00	Rurſus capta.	33 30	2 10 32
2 45 00	Veſtigium *Penumbra.*		2 46 30
2 53 00	*Penumbra* paulo Denſior.		2 54 00
2 59 00	Adhuc Denſior.		3 00 00
3 1 00	Denſiſſima.		3 2 00
3 2 00	*Initium* Eclipſeos.		3 3 00
3 6 00	*Prima* Phaſis.		3 6 30
3 12 00	*Secunda* Phaſis.		3 12 30
3 14 00	*Tertia* Phaſis.		3 14 30
3 20 00	*Quarta* Phaſis.		3 21 00
3 22 30	*Mons Sinai* Totus tectus.		3 23 00
3 29 00	*Sexta* Phaſis.		3 30 00
3 43 00	Altitudo *Pollucis.*	30 00	3 43 00
5 2 00	Centrum *Solis* Oritur.		

Color hujus *Eclipſeos* erat Cinericius ſive Fuliginеus. Tempore Obſervationis *Mons Porphyrites* & M. *Ætna* fere in eodem Perpendiculo exiſtebant.

Phaſis I. Per Montem *Eoum* tranſibat.
II. Ad *Paludes Arabiæ* incedebat.
III. Inferiorem Ripam *Paludis Mareotidis*, & Extremitatem *Sinus Sirbonis* tangebat.
IV. Per Inſulam *Letoam* incedebat ad ipſum M. *Sinai* uſque, ſed dictus Mons Totus adhuc erat conſpicuus.
V. *Sinum Syrticum* tranſire videbatur, ſic ut totum M. *Sinai* Umbra jam tegeret.
VI. Supra *Sinum Syrticum*, ultra Inſulam *Cretam*, & per *Mare Mortuum* incedebat.

An Eclipſe of the Moon, Feb. 11. 1682. *by Mr*. Flamſteed. n. 145. *p*. 89.

XLIX. 1. Ann. 168$\frac{1}{2}$. *Feb*. 11. H. 8. 1′. Tubo pedum 16 *Lunæ* cepi Diametrum 6702 = 33′. 25″. deinde diſtantiam Limbi ejus Proximi à Limbo Proximo *Mareotidis* 145 = 00′. 43″. ſed ejuſdem Limbi Maculæ à Limbo Lunæ remotiore 6575 = 32′. 48″. Hujus etiam Tubi ope Tempora, cùm Obſcuratio ad Centrum Lunæ pertigerit, & cùm Radius ejus Arcus in Peripheria Deficientes vel Reſtitutos ſubtenderit, obtinui; è quibus medium derivari poteſt, forſan non minus accuratè, quam ab obſervatis *Initio* & *Fine*, *Immerſione* & *Emerſione*, collatis.

Phaſes.	*Tempora Phaſium per Horologia Oſcillatoria correcta.*								
	Grenovici in *Obſervatorio.*						*Londini.*		
	Mihi.			D. *Hallcio.*			D. *Hayneſio.*		
	h.	′	″	h.	′	″	h.	′	″
Fumus ad Oram Inferiorem. Nudis Oculis	8	48	38						
Umbra.	9	4	8						
Denſa *Penumbra.*	9	11	44						
Initium.	9	12	32	9	13	04	9	12	18
Medium Paludis *Mareotidis* tectum.	9	14	02				fortè citiús.		
Tota Palus tecta.				9	14	39	9	13	48
Sexta Pars *Peripheriæ Obſcurata*	9	18	10						
Circinna intra Umbram	9	20	10						
Porphyrites Medius.				9	21	1			
Umbra ſtringit *Syrbonem.*				9	21	9			
Limbus 1^us^. Montis *Cataractes* vel *Gaſſendi*	9	21	32						
Oculus Draconis.	9	27	58						
Legebat Limbus primos *Cretæ* & *Ætnæ.*	9	28	22	9	28	44	9	28	19
M. *Ætna* Totus tectus. incipit *Hiera.*	9	29	48	9	30	16	9	30	12
Hiera tot.				9	30	44			
Initium *Corſicæ.*	9	33	46						
Medium *Corſicæ.*				9	34	48			
Sinæ Mons incipit.	9	36	38	9	37	10	9	36	20
Sinæ Medium.	9	37	22						
Totus *Sinæ* tectus	9	38	04	9	38	10	9	38	20
Centrum Lunæ ſive *Digiti* VI.	9	38	48						
Incipit *Lacus Niger Major.*	9	39	58	9	40	36			
Medium *Lacus Nigri Majoris.*							9	40	48
Totus tegitur Lacus.				9	41	40			
Initium *Besbici.*				9	43	48			
Medius *Besbicus.*	9	43	42				9	44	13
Totus *Besbicus* & ſtringit *Pont. Euxinum.*				9	44	36			
Incipit *Byzantium.*	9	46	58						
Incipit *Horminius.*	9	47	22						
Carpathes.	9	47	50						
Mons Serrorum.	9	48	30						
Apollonia.	9	50	24	9	51	21			
Macra.	9	52	44	9	53	33			
Mons Hercules.	9	54	26						
Macra tota tecta				9	54	53			
Mons Hercules Totus				9	55	33			
Corocondometis Palus.	9	59	10						
Per Medium *Lacum Hyperbor. Superiorem.*				9	59	53			
Corocondometes tota tecta.	10	00	38						
Umbra per Montem *Coracem.*				10	1	17			
Stringit *Mæotida.*	10	2	20	10	2	38			

Per.

	h.	′	″	h.	′	″	h.	′	″
Per *Lacum Hyperboreum Inferiorem.* —				10	3	53			
Per Medium *Inſulæ Majoris.* —	10	5	14						
Tota ferè *Mæotis tecta* —	10	6	54	10	8	00	10	6	48
Sexta pars Peripheriæ Lucida reſidua. —	10	7	28						
Immerſio. — —	10	10	14	10	10	11	10	9	48
Emerſio — — —	11	47	38	11	47	9	11	47	48
Sexta pars Peripheriæ Illuminata, cum Limbo 1°. *Mareot.* —	11	50	02	11	50	7			
Medium *Maræotidis.* —	11	50	42				11	50	48
Tota *Maræotis* Illuminata —	11	51	6	11	50	58			
Umbra per Medium *Porphyritidis* —	11	52	50						
Totus *Porphyrites* tectus. —	11	53	34				11	52	54
Circinna tota. —	11	58	18						
Stringit *Sinum Syrbonis.* —				11	58	59			
Cataractes Syrbonis, vel *Gaſſendus* totus.	12	1	00						
Hiera incipit. —				12	3	14			
Ætna incipit emergere. —	12	3	20						
Per Medium *Lacus Nigri Majoris.*							12	3	48
Per Medium *Ætnæ.* —				12	4	19			45
Ætna totus retectus. —	12	5	18	12	5	4	12	5	
Creta Illuminatur. — —	12	7	34						43
Sina incipit. — —	12	13	38				12	13	
Sinæ Medium. — —	12	14	28	12	14	12			
Sina totus. — —	12	15	34	12	14	55			
Medium *Besbici.* — —	12	17	34				12	17	48
Centrum Lunæ, ſi *Digiti* VI *retecti.* —	12	18	28						
Per Medium *Lacum Hyperbor. Superiorem*				12	19	5			
Umbra per Medium *Byzantii.* —							12	20	18
Byzantium extra. — —	12	20	50	12	20	25			
Macra tota. — — —				12	22	36			
Inſula Apollonia tota, & *Horminius.* —	12	24	24	12	24	16			
Lacus Hyperboreus Inferior. — —				12	26	6			
Mons *Hercules.* — — —				12	30	56	12	31	18
Medium *Corocondometidis* ad Umbram.	12	31	38						
Mons *Corax.* — — — —				12	32	37			
Mæotis incipit illuſtrari. — —	12	33	18				12	32	18
Mæotis tota retecta. — —				12	37	57	12	38	3
Sexta pars Peripheriæ Obſcura reſidua. —	12	39	6						
Media *Inſulæ Majoris Caſpii.* —				12	41	22	12	42	18
Tota *Inſula* retecta. — —	12	42	18						
Dubius *Finis.* — — —	12	45	10				12	44	48
Finis certè. — — —	12	45	38	12	44	35	12	45	18

Ingruente *Eclipſi,* Diſtantias *Lunæ* à Fixis Sextante Ceperant Ds. *Halleius* cum *Tho. Fabro* ſequentes;

Tempora per Hor. Correct.			*Obſervationes.*	*Diſtantiæ.*		
h.	'	"		°	'	"
8	53	07	Limbi *Lunæ* Remotioris à *Regulo* ——	8	05	20
8	55	07	repet. ——	8	06	20
9	2	34	Limbi *Lunæ* Rem. à *Lucid.* in *Lumbis* Leonis	14	58	15
9	4	18	repet. ——	14	57	55
9	8	42	Limbi *Lunæ* Remot. à *Regulo* iterum —	8	12	50
			Luna in Umbram penitus merſa, Ds. *Halleius* mecum.			
10	19	21	Limbi *Lunæ* Proximi à *Regulo* ——	8	13	30
11	38	06	Remoti à *Regulo* —	9	23	50
11	41	48	repet. ——	9	25	35
			Et finita *Eclipſi,*			
12	52	00	Limbi Remotioris à *Regulo* iterum ——	9	58	20
12	55	10	denuo. ——	10	00	00

* * *

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At *Paris* and
Copenhagen.
n. 146. *p.* 145.

Phaſes.	*Pariſiis.*		*Hafniæ.*	
	In *Obſervatorio Regio.*		In *Collegio. Claramont.*	AD. *Roemer,* reductione Meridianorum facta Subtr. 41′. 40″.
	A D. *Caſſino.*	ADD. *Picard* & *De'la Hire.*	A R. *P. de Fonteney.*	
	h. ′ ″	h. ′ ″	h. ′ ″	h. ′ ″
Initium	9 20 55	9 21 58	9 21 25	9 21 50
Ariſtarchus Medius	9 29 57	9 29 33	9 29 28	9 29 50
Medium *Copernici*	9 38 45	9 38 25	9 38 40	9 38 48
Initium *Tychonis*	9 45 52	9 45 48	9 45 48	9 46 20
Finis *Tychonis*	9 47 40	9 47 36	9 47 20	9 47 50
Initium *Platonis*	9 49 47	9 49 08	9 49 25	9 49 50
Centrum *Lunæ*		9 51 10		9 50 20
Centrum ex Comparatione *Initii* & *Finis*	9 50 24	9 50 44	9 50 10	9 50 30
Manilius Medius	9 53 16	9 53 30	9 51 18	9 53 20
Menelaus	9 56 43	9 56 20	9 56 14	9 56 20
Dionyſius	9 57 09	9 57 40		9 57 20
Promontorium Acutum	10 04 31	10 04 15	10 04 12	10 04 20
Initium *Caſpiæ*	10 11 40	10 11 25	10 11 20	10 11 50
Finis *Caſpiæ*	10 16 27	10 16 30	10 15 50	10 16 20
Luna penitus Immerſa	10 19 53	10 19 30	10 18 55	10 19 50
Initium *Emerſ. Grimaldum* inter & *Galileum*	11 57 51	11 56 00	11 58 00	11 55 20
Ariſtarchus Medius	12 02 00	12 01 55	12 03 09	12 01 50
Medium *Platonis*	12 12 06	12 12 10	12 13 26	12 12 20
Medium *Copernici*	12 14 32	12 14 30	12 14 29	
Medium *Tychonis*	12 23 35	12 23 55	12 24 21	12 23 20
Centrum Lunæ		12 25 50		12 26 20
Centrum ex Comparatione *Emerſ.* Limborum	12 26 09	12 25 15	12 26 08	12 24 50
Manilius Medius	12 26 22		12 26 20	12 26 20
Menelaus Medius	12 29 08	12 29 20	12 29 22	12 29 05
Dionyſius	12 32 30	12 33 15	12 32 20	12 32 50
Promontorium Acutum	12 39 45	12 39 50		12 39 50
Initium *Caſpiæ*	12 41 53	12 41 30	12 41 11	
Finis *Caſpiæ*	12 47 15	12 46 30	12 48 20	12 47 20
Finis Eclipſis	12 54 27	12 54 30	12 54 17	12 54 20

Tempus

3. At *Dantzick*; by M. *Hevel*. *Ib*. p. 146.

Tempus ſecund. Horol. Ambul.	*Or. Phaſ.*	*Digiti Eclipt.*	*Altitudines Stellarum.*		Per quas *Maculas* tranſiverint Umbræ *Sectiones*, & quæ inſuper Notata fuerint.	*Tempus* ex Alt. *Correct.*
h. ′ ″				° ′		h. ′ ″
8 41 18			*Palilicii*	39 53		8 40 46
8 44 9			*Pal.*	39 27		8 44 20
8 48 14			*Pal.*	39 0		8 47 59
10 8 28					Initium *Penumbræ* Dilutiſſima.	10 9 0
10 12 50					*Penumbra* Denſior.	10 13 20
10 19 5					*Penumbra* adhuc Craſſior.	10 19 36
10 23 50					*Penumbra* Denſiſſima.	10 24 24
10 24 30					*Initium* Eclip. circa 150° Limbi in 95° ſc. à Puncto ſuper. Lineæ perpend. Nonageſimi Ortum verſus contigit.	10 25 5
10 29 45	1	$1\frac{1}{4}$ *Dig.*			Incedebat per M. *Germanician.* ad *Mare Syrtic.* & Mont. *Acabe.*	10 30 30
10 32 13	2	2 *ferè.*			Per M. *Porphyritem, loca Paludoſa*, Inſ. *Cercinnæ, Sinum Syrtic.* per *Mont. Sacr.* & M. *Caſpium.*	10 32 50
10 36 30	3	$2\frac{1}{2}$ *ferè.*			Ad M. *Baronium*, Inſ. *Æthuſam*, per medium *Sinum Syrbonis* & M. *Pharan.*	10 37 10
10 43 25	4	$3\frac{5}{8}$			Per *Sinum Apollinis*, M. *Ætn.* Inſ. Leto. ad Inſ. Didymam.	10 44 10
10 47 25	5	$4\frac{5}{8}$ *Dig.*			Ad Inſ. *Majorc. Vulcaniam, Lemnos, Carpath.* & M. *Horeb.*	10 48 15
10 52 20	6	$5\frac{3}{4}$			Ad *Lacum Nigrum Majorem*, per M. *Argentarium, Sipylum, Maſicytam*; *Tabor, Sinai* & Deſertum *Raphidim.*	10 53 10
10 55 45	7	$6\frac{1}{2}$			Per *Scopulos Hyperboreos*, Inſ. *Besbycam*, M. *Olympum, Didymum*, & M. *Antilibanum.*	10 56 40
11 0 33	8	$7\frac{3}{4}$ *Dig.*			Ad Inſ. *Cyaneam*, M. *Hormin.* M. *Uxii* & Montes *Coibacaranos.*	11 1 30
11 4 30	9	$8\frac{1}{2}$			Per M. *Ambonum* ad Inſ. *Apolloniam* & *Sinum Athenienſ.* per M. *Moſch. Uxii*, & *Coibacaranos.*	11 5 30
11 9 59	10	$9\frac{1}{4}$			Per *Lacum Hyperbor. Superior. Pal. Byces*, per Mont. *Hercul.* & Sinum extremum *Ponti.*	11 10 55

Per

h. ′ ″					h. ′ ″
11 13 30	11	10 *Dig.*		Per *Lacum Hyperboreum Inferiorem*, M. *Cimmerium*, & ad Sinum Inferiorem *Maris Caſpii.*	11 14 27
11 16 30	12	10½ *ferè*		Per M. *Riphæos*, Pal. *Mæotidem*, Inſ. *Alopeciam*, ad Inſ. Majorem *Caſpii*, per M. *Neroſum.*	11 17 17
11 19 10	13	11¼		Ad Ripam Paludis *Mæotidis* & Mont. *Hippoci.*	12 20 10
11 22 0	14	11½		Per M. *Alaunum*, & *Lacum Occident. Minorem.*	11 23 0
11 26 30		12 *Dig.*		*Totalis Obſcuratio* circa 297° Limbi in 94° ſc. à Puncto Super. Lineæ perpendicularis Nonageſimi Ortum verſus deprehenſa.	11 27 30
11 35 36			*Pollucis* 52° 13′		11 37 4
11 37 35			*Pollucis* 52 0		11 38 42
11 38 50			*Pollucis* 51 41		11 39 35
12 58 20				*Recuperatio Luminis* circa 118° *Limbi* in 88° ſc. à Puncto Superiori Lineæ perpendicularis Nonageſimi Occaſum verſus extitit.	1 0 8
1 0 40	15	½ *Dig.*		Ad M. *Alabaſtrinum*, M. *Pentadactylum*, M. *Audum*, & Pal. *Mareotidem.*	1 2 31
1 3 35	16	1¼		Per M. *Porphyritem*, ad *Mare Syrticum*, & per M. *Eoum.*	1 5 32
1 8 10	17	2 *ferè*		Per M. *Baronium*, *loca paludoſa*, Inſ. *Cercinna*, inter *Mare Syrticum* & *Ægyptiacum*, ad S. *Syrbonis.*	1 10 32
1 11 48	18	2½ *ferè*		Per *Sinum Apollinis*, Inſ. *Taraciniam*, Inſ. *Æthuſam*, Sinumque *Syrbonis.*	1 13 54
1 16 20	19	3½		Ad Inſ. *Majorcam*, per M. *Ætnam*, M. *Neptunum*, *Jachyntum*, Inſ. *Letoam*, Inſ. *Didymam* & M. *Lyon.*	1 18 33
1 20 50	20	4½ *Dig.*		Ad Inſ. *Corſicam*, per Inſ. *Vulcaniam*, & *Carpathos*, ad *Mare Mortuum*, per Deſertum *Sin.*	1 23 6

Ad

h.	′	″					h.	′	″
1	24	39	21	$5\frac{1}{4}$ *Dig.*		Ad M. *Argentarium, Mare Pamphilium*, Inſ. *Cyprum*, per M. *Horminium.*	1	27	0
1	26	25	22	6 *ferè*		Per Mont. *Apenninum*, ad Lacum *Traſimenum*, ad Mont. *Sipulum* per M. *Infer. Libanum.*	1	28	40
1	28	32	23	$6\frac{1}{2}$		Ad M. *Carpathos*, per Inſ. *Besbicam*, M. *Olympum, Didymum* & M. *Dalangueros.*	1	31	0
1	31	12	24	$7\frac{1}{2}$ *Dig.*		Per M. *Perce, Byzantium*, ad M. *Horminium*, & per M. *Antitaurum.*	1	33	44
1	35	59	25	$8\frac{1}{4}$		Per *Sinum Circinitem*, Inſ. *Macram*, Inſ. *Apolloniam*, Medium Montem *Moſchum*, atque *Montes Sogdianos.*	1	38	37
1	39	33	26	9 *Dig.*		Per *Lac. Hyperboreum Inferiorem*, Pal. *Byces*, M. *Strobilum*, per Sin. Extremum *Ponti* ac M. *Paropamiſum.*	1	42	15
1	41	45	27	$9\frac{3}{8}$ *Dig.*		Inter Pal. *Byces* & *Lacum Corocondametis*, per M. *Herculis* & M. *Caucaſum* Inferiorem.	1	44	30
1	44	19	28	10		Per M. *Cimmerium*, M. *Tancon*, & M. *Neroſum.*	1	47	9
1	47	20	29	$10\frac{1}{2}$		Per Pal. *Mæotidem*, Inſ. *Minorem* Maris *Caſpii, Montemque Neroſum* Superiorem.	1	50	14
1	51	44	30	11 *paulò plus.*		Per M. *Alaunum*, M. *Sanctum*; *Monteſque Hippoci.*	1	54	42
1	56	12		12 *Dig.*		*Finis*, circa 294° *Limbi*, in 97 *ſc.* gradu à Puncto ſuperiori *Lineæ* perpendicularis Nonageſimi Occaſum verſus contigit.	1	59	17
1	57	10				Denſiſſima *Penumbra.*	2	0	17
2	1	15				Satis adhuc Craſſa.	2	4	27
2	2	30				*Paulo* Dilutior.	2	5	45
2	10	20				*Penumbra* Dilutiſſima.	2	13	42
2	12	30				Finis *Penumbræ.*	2	15	0
2	15	39			*Alt. Pol.* 29° 12		2	19	7
2	17	35			*Ead. Alt.* 28 46		2	22	8

Ingruente *Eclipſi* Umbra erat valde diluta, Limbuſque ejus quaſi Anfractuoſus, & minime terminatus, ſic ut difficulter admodum ab Initio Phaſes determinari potuerint, nec accuratè diſtingui per quas Maculas Umbra tranſibat, ſucceſſu tamen temporis creſcente *Eclipſi*, diſtinctiùs omnia deprehendebantur. Color ab *Initio* videbatur ſatis Triſtis, Obſcurus, & Fuliginoſus, ac ſi *Eclipſis*, eadem ratione, circa *Maximam Obſcurationem*, ut illa Anno 1642. menſ. *April.* adeò ſeſe Obumbratam ſiſtere vellet, quo vix conſpiceretur ; ſed res planè aliter cecidit, ſiquidem Luna cum jam omnino eſſet Eclipſata, Totus tamen ejus Diſcus ſatis clarè in oculos incurrebat : Color namque ejus tum omninò Rubidus ſive Sanguineus aut Rubiginoſus erat, qui eouſque perſeverabat, donec Luna ad medietatem Lumen ſuum recuperaſſet, atque tum rurſus ſatis Obſcura & Fuliginioſa apparuit.

At *Lisbon*; by Mr. *Jacobs*. Ib. *p.* 151.

4. The Beginning of this *Eclipſe* was obſerved at *Lisbon* by Mr. *Jacobs* at 8.. 31′. *p. m.*

An Eclipſe of the Moon, *June* 17. *m.* 1684. at *Greenwich*; by Mr. *Flamſteed.* n. 162. *p.* 689.

L. Difficillima fuerit parvæ hujus *Eclipſis* obſervatio, propter obliquam Lunæ in Umbram Terræ incidentiam, Umbræque ipſius Tenuitatem, per quam Limbum Lunæ, media etiam Eclipſi, ſatis diſtinctè cernere potuimus. Partes diametri tunc ab Umbra vera deficientes, propter confuſos ejus terminos accuratè definire non licuit. Diſtantiam ergo cœpi inter Cuſpides malè definitos, circa Medium Defectus, è qua dictæ partes facile deduci poſſint, & Erroris minore periculo.

Tempora per Horolog. Oſcillator.			*Tempora Vera ab Obſ. Correct.*			*Obſervationes.*		
h.	′	″	h.	′	″			
1	40	40	1	31	8	*Eclipſis* Minæ	′	″
2	4	30	2	54	58	*Lunæ* Diameter Tubo ped. 16. erat, 1605=	31	57
2	6	40	2	57	8	—— —— —— rep. 6430=	32	4
2	12	0	2	2	20	*Penumbra* denſa, forſan *Initium* ——		
2	16	00	2	6	28	Umbra Limbum ſupra *Sinam* temeraverat		
2	18	00	2	8	28	*Umbra* certè intra diſcum		
2	21	36	2	12	4	Chorda *Peripheriæ* Obſcuratæ — 1670=	8	28
2	26	00	2	16	28	—— —— —— rep. 2010=	10	1
2	30	00	2	26	28	—— —— —— iter. 2290=	11	25
2	42	00	2	32	28	Decrevit *Eclipſis* ſenſibiliter ——		
2	43	00	2	33	28	Inter Cuſpides Obſcur. iter. 1895=	9	20
2	50	00	2	40	28	*Finis*; ſed Dubius Mihi		
2	54	00	2	44	28	*Finita* certè, Miniſtro *Fabro* Conſentiente.		
2	57	00	2	47	28	*Penumbra* Denſa.		
3	11	00	3	1	28	Etiam adhuc.		
3	20	00	3	10	28	*Limbus Auſtrinus* haud adhuc *Limbi Borei* Claritudinem recuperaverat. Sed lux ejus hebetior quam in *Limbo Boreo*, ut in prima Obſervatione apparuit.		

An Eclipſe of the Moon, November 30. (ſt. n) 1685. at *Dantzick*; by M. *Hevelius.* n. 178. *p.* 1256.

LI. Artis profectò Laboriſque haud exigui erat Tubos, etſi breviores *ſc.* 5, 6 & 7, pedum, adeò firmiter continue ad *Lunam* retinere, ut Penumbram rectè diſcernere potuerimus, Phaſeſque omnes exactè deſignare, per quas nimirum Maculas tranſirent, vel quas omni tempore attingerent. Attamen pro viribus, quouſque ſevera Tempeſtas atque tranſeuntes Nubeculæ permittebant, rem peregi. Notandum imprimis habes, quòd Denſiſſima *Penumbra* genuinum *Initium Eclipſeos* præceſſerit, ita ut vix ac ne vix verum *Initium* diſcernere quiverim. Colorem quod attinet, hunc in hac *Eclipſi* maxime notandum habes; quippe talem diverſitatem raro admodum in ipſo Colore deprehendi: modo enim erat Ferrugineus vel Muſtelinus; Ingruenti vero *Totali Obſcuratione*, Limbus Lunæ circumcirca erat Sublividus, ex parte Subluſtris, & Rubicundus; verum in Lunæ Medio, quaſi ſatis Denſo, & Obſcura Nubecula conſpiciebatur, ut vix Maculas rectè in Lunæ diſtinguere potuerimus; quæ Nigricantior Umbra paulatim ſucceſſive verſus Dextram, & Paludem *Mæotidem* promovebatur, ſic ut circa Initium *Recuperationis Luminis* tota genuina Umbra admodum Obſcura, & Nigricans appareret, atque circa ultimam annotatam Phaſin reliqua pars adhuc Obſcurata Lunæ, ſive ejus Limbus, neutiquam deprehendi vel minimum potuerit.

Ord. Ph.	*Tempus ſec. Horolog. ex Alt. Correct.*	*Notanda.*	*Altitudines* ☉, ☽, & *Fixarum.*	Per quas *Maculas* tranſiverint *Umbræ Sectiones.*
	Hor. ′ ″		° ′ ″	
		Altitudo ☉ Meridiana.	12: 39 0	
	3 19 0	Altitudo ☉	1 54 0	
	4 28 0	Altitudo ☽	9 0 6	
	5 2 30	Altitudo *Capellæ.*	31 0 0	
	5 8 0	Altitudo *Capellæ.*	31 2 0	Dub.
	9 32 25	*Penumbra.*		
	9 42 35	*Penumbra* Denſior.		
	9 48 5	*Penumbra* Denſiſſima.		
	9 50 10	*Init.* quantum collig. dabatur.		
1	9 53 20	Pal. *Mareotis* jam tota erat tecta.		Ad M. *Andum*, & Pal. *Mareotidem.*
2	9 57 0			Per *Germanicianum* ad *Mare Syrticum*, & M. *Acabe.*
3	10 0 15	M. *Porphyrit.* nond. planè tect. erat.		Ad M. *Porphyritem*, per *Mare Syrticum*, Montemque *Ajacem.*

Per

	h.	′	″			
4	10	3	5			Per M. *Petri*, inter Inſ. *Circinnam* & Inſ. *Tarracinniam*, per *Sinum Syrbonis*, & *Paludes Arabiæ*.
5	10	6	25			Ad Inſ. *Ficariam*, *Maltam*, M. *Athos*, per M. *Pharan* & M. *Troicum*.
6	10	10	25			Ad *Sinum Apollinis*, per Inſ. *Erroris*, per M. *Ætnam*, Inſ. *Letoam* & *Didymam*.
7	10	14	55	Inſula *Melos* jam tota tecta erat.		Supra *Sin. Apollinis*, ad *Lacum Herculeum*, Inſ. *Siciliæ*, per M. *Parthenium* & *Taigetum*, ad Inſ. *Melos* & Mont. *Lion*.
8	10	19	55			Ad Inſ. *Minorcam*, per Sinum *Paſtanum*, M. *Micalo*, Inſ. *Cyprum*, M. *Hor*. ad M. *Sinai* & Deſertum *Raphidim*.
9	10	24	5			Ad M. *Liguſtinum*, *Panyeum*, inter M. *Sipulum* & *Didymum*, per M. *Libanum* & *Seir*.
10	10	27	10			Supra M. *Nigrum Minorem*, per M. *Appenninum*, Inſ. *Besbicam*, M. *Cimeum*, Montemq; *Calchaſtan*.
11	10	31	27			Ad *Montes Serrorum*, & *Carpathes*, ad *Byzantium*, per M. *Horminium Taurum*, & M. *Delangueros*.
12	10	36	0			Per *Montes Macrocemnios*, Inſ. *Cyaneam*, M. *Amanum* & *Antitaurum*.
13	10	41	20			Per Inſ. *Macram*, per *Erichtenios Scopulos*, *Sin. Athenienſem*, & Pal. *Arceſam*.
14	10	45	35			Per *Lac. Hyperbor. Superiorem*. Pal. *Byces*, Inſ. *Acam*, M. *Herculis*, Sin. Extremum *Ponti*, Montemque *Parapomiſum*.
15	10	48	40			Ad *Lacum Hyperbor. Inferiorem*, M. *Immerium*, per M. *Coracem*, *Taucan*, Sin. Inferiorem *Maris Caſpii*.
16	10	52	10			Per *Montes Ripheos*, Pal. *Mœotidem*, Inſ. *Majorem Caſpii*, & M. *Neroſum*.

	h.	′	″					
	10	56	35	*Luna* nond. plenè *Obſcurata.*				
	10	58	25	*Totalis Immerſio.*				
	h.	′	″		°	′	″	
	11	20	45	Altitud. *Procyonis.*	31	8	0	
	11	25	45	Eadem Altitudo.	31	38	0	
	11	36	0	Obſcura illa Umbra vel Nubec. planè in Med. Diſci Lun. conſiſtebat.				
	12	43	0	*Init. Emerſionis* ſeu Recuper. Luminis.				
17	12	46	40	Palus *Mareotis* jam penitus tecta.				Ad Mont. *Audum*, haud procul à Palude *Mareotide.*
18	12	51	0					Per Mare *Eoum*, Mare *Syrticum*, Montemque *Caſium.*
19	12	57	0					Per M. *Atlantem Minorem*, inter Inſ. *Circinam* & Inſ. *Taraciniam*, per *Mare Ægyptiacum*, *Sin. Sirbonis*, Montemq; *Pharan*
20	1	5	0					Ad Inſ. *Majorc. Sardin.* & *Hier.* per M. *Ætnam*, Inſ. *Melos*, ad Mont. *Annæ* & *Hajalon.*

Initium Eclipſeos paulo ſupra Paludem *Mareotidem* extitit.
Totalis Obſcuratio, ad *Montem Sanctum*, infra Paludem *Mareotidem* accidit.
Recuperatio Luminis ferè Ibidem contigit ubi *Initium* cœpit.

2.

At Nuremburg; by M. G. C. Eimmart. n. 182. p. 146.

h.	′	″	
9	19	00	The *Penumbra* was very Obſcure, and the *Beginning* of the *Eclipſe* was at hand.
9	23	30	The *Eclipſe* was *Begun*, the Quantity almoſt half a *Digit*, and the diſtance between the Cuſps was about 42 Degrees of the Moon's Limb, and *Palus Mareotis* was juſt all *Ecliſped*; hence we may conclude the *Beginning* about 9^h. 21′ 30″.
10	23	30	As near as I can collect, was the time of the *Total Immerſion* into the Shadow, to verify which, the Azimuth of the Moon's Center was obſerved to the Eaſt, 41° 18′; 2′ 12″ of time after the ſaid *Immerſion*.
12	13	0	Or 10′ 13″ before the *Culmination* of the *Right Shoulder* of *Orion*, was the *Emerſion* or firſt Appearance of the Moon out of the Total Darkneſs.
1	14	*ferè*	Was the juſt *End* of the *Eclipſe*, being 2′ 20″ before the *Culmination* of *Syrius*.

Whence the *Middle* of this *Eclipſe* ſhould have happened at 11^h. 18′ *p. m.* at *Nuremburg*: the *Total Duration* 3^h 52′ 30″, and the *Total Darkneſs*: 1^h. 49′ 30″.

The Meridian Altitude of the Moon's upper Limb was obſerved 63° 23′ 50″, and the Moon's apparent Diameter while totally Eclipſed was found 30′ 07″.

By M. *J. Ph. Wurtzelbaur*. Ib. *p.* 147.

3. M. *Wurtzelbaur* made uſe of a Pendulum-Clock corrected by Altitudes: According to his Obſervation,

h.	′	″	
9	23	30	was the *Beginning* of the *Eclipſe*, at about 119 Degrees of the Limb of the Moon in *Hevelius's Selenography*.
9	24	50	*Palus Mareotis* was all covered.
10	25	20	The *Total Immerſion*, about the *299th* Degree of the Limb of the Moon.
12	11	30	The Moon began to *Emerge* out of the Shadow, about the *112th* Degree of her Limb.
1	14	30	The *End* of the *Eclipſe* about the *295th* Degree of the Limb.

By theſe Obſervations the *Middle* of the *Eclipſe* ought to have been about 11^h. 19′. *p. m.* at *Nuremburg*, differing but one Minute from M. *Eimmart's* Obſervation.

The *Duration* will be 3^h. 51′, and the *Total Darkneſs* 1^h. 46′.

At *Lisbon*; by Mr. *Jacobs*. n. 184. *p.* 206.

	h.	′
4. *Initium*	8	02
Immerſio	9	06
Emerſio	10	50
Finis	11	57

Phaſis

LII. An Eclipſe of the Moon, *Novemb.* 19. 1686. at *Dublin*; by Mr. *Will. Molineux*. n. 185. *p.* 236.

Phaſis.	*Temp.*			
	h.	′	″	
Penumbra Notabilis	9	15	00	
Initium ob interpoſitas Nubes præciſe determinare haud licuit, ideoque incertius pono *Eclipſin* incepiſſe ad $9^h. 25'$. vel	9	27	00	*
Umbra ad Paludem *Maræotim*	9	38	20	
Palus *Maræotis* tecta	9	40	20	*
Mons *Sinai* tegitur	9	46	30	*
Mons *Thambes* tectus	10	01	20	*
Mons *Audus* tegitur	10	08	00	*
Mons *Neptunus* tectus	10	14	00	*
Umbra ad Montem *Sipylum*	10	15	10	*
Inſula *Circinna* tegitur	16	16	40	
Ad Montem *Didymum*	10	18	20	*
Mons *Didymus* tegitur	10	20	10	
Emergit M. *Audus*	10	23	00	
Umbra ad Paludem *Maræotim*	10	48	30	
Emergit *Maræotis*	10	52	00	
Ad Montem *Sinai*	11	38	00	
Emergit *Sinai*	11	39	30	
Finis Eclipſis	11	04	00	

Notanda. Tempora ſunt Horologii Oſcillatorii ad Stellas fixas rectificati.

Quæ *Aſteriſmo* notantur Obſervationes, per hiantes Nubes captæ ſunt, adeoque accuratas haberi nolo. Quantitatem hujus Eclipſis ſex puto *Digitorum.*

An Eclipſe of the Moon, *April* 5. 1688. at *Moſcua*; by M. *Timmerman*. n. 192. *p.* 453.

LIII. Half a Quarter of an Hour after 7 in the Evening, the Moon aroſe clear, but of a deep Red Colour without any Sign of Eclipſe: At $7^h\frac{1}{2}$ the Moon went into a thick Cloud, but was again clear at $7^h. 38'$. when the Under-ſide of this Body of the Moon was *Begun* to be *Obſcured*, in a clear Sky; ſhe being then in the 25th Degree of *Libra*, and 6° ½ above the Horizon. (Suppoſe the Center.) At 9^h. the whole Under-ſide of the Moon was *Eclipſed*, and about $8'$ after 9^h, it was at the height, or rather ſeemed to decreaſe. At $9^h\frac{1}{2}$ there was ſtill a third Part of the Moon Eclipſed. (Suppoſe of her Circumference.) About 10^h. it *decreaſed* apace, and at $10^h\frac{1}{2}$ there was but little to be ſeen: At $10^h. 45'$. it was certainly Ended, the Moon being then about 22° High.

Immer-

LIV. 1.
An Eclipſe of the Moon, Octob 19.1697. at *Cheſter*; by Mr. *Ed. Halley*. n. 235. p. 784.

Immerſions.	*Times.*	*Emerſions.*	*Times.*
	h ′		h. ′ ″
The *Beginning*	6 8$\frac{1}{2}$	*Porphyrites* and the Middle of M. *Ætna*	8 7 00
Porphyrites Immerged	6 16	*Horminius*	8 17 30
North-part of *Maræotis*	6 21$\frac{1}{2}$	*Mons Herculis*	8 18 30
Lacus Niger Maj. and South End of *Mar.*	6 26	*Besbicus*	8 21 00
Besbicus	6 46$\frac{2}{3}$	*Apollonia*	8 26 15
Apollonia	6 49$\frac{1}{2}$	*Byzantium*	8 29 00
Byzantium	6 53	*Lacus Niger Major*	8 32$\frac{1}{2}$
Horminius	6 59	South-part of *Mæotis*	8 35
North-part *of Mæotis*	7 2$\frac{1}{2}$	North-part of *Mæotis*	8 43
Mons Corax	7 3$\frac{1}{2}$	The *End*	8 49$\frac{1}{2}$
Mons Herculis	7 10		
South-part of *Mæotis*	7 12$\frac{1}{2}$		

About the *Middle* there remained 9′ 26″. of the Luminous part, and conſequently the *Digits* Eclipſed 8$\frac{2}{3}$.

2.
At *Rotterdam*; by M. *Ja. Caſſini*. n. 236. p. 20. Vid. *ſup.* §. XXXIX. 2.

	Temp.Cor.	*Obſervationes.*
	h. ′ ″	
A	6 20 36	Promontorium Acutum ad Primum Obliquum.
B	6 21 27	Præcedens *Lunæ* Margo ad Perpendiculare.
C	6 22 3	Promontorium Acutum ad Perpendiculare.
C—B	0 00 36	Differentia Tranſitus per filium Perpendiculare, quæ eſt Longitudo *Promontorii Acuti* à *Margine Præcedente.*
C—A	0 1 27	Differentia Tranſitus *Promontorii Acuti* inter 1. Obliquum & Perpendiculare, quæ ejus eſt Latitudo à *Margine Boreali.*
	6 32 34	*Luna* inter Nubes conſpecta adhuc apparuit integra.

In

In Prima Phaſi.

	h.	'	"	
A	6	41	23	Initium Maris *Criſii* ad 1. Obl.
B	6	41	50	Promontorium Acutum ad 1. Obl.
C	6	42	12	*Plinius* ad 1. Obl.
D	6	42	25	*Menelaus* ad 1. Obl.
E	6	42	33	*Manilius* ad 1. Obl.
F	6	42	43	Primus Margo ad Perpendiculare.
G	6	43	00	*Proclus* ad Perpendiculare.
H	6	43	20	Promontorium Acutum ad Perpendiculare.
I	6	43	26	*Margo Sequens* ad 1. Obliquum.
K	6	43	30	*Menelaus* ad Perpendiculare.
L	6	44	00	Cornu præcedens *Lunæ* ad Perpendiculare, ipſa tangit Filum Horiz.
M	6	44	21	Cornu *Sequens* ad 1. Obl.
N	6	44	35	*Menelaus* ad 2. Obl.
O	6	44	57	*Cornu Sequens* ad Verticale.
P	6	45	07	*Margo Sequens* ad Perpendiculare.
Q	6	45	33	*Cornu Sequens* ad 2. Obl.
R	6	45	55	*Grimaldus* ad 2. Obl.
S	6	46	49	*Sequens Margo* ad 2. Obliquum.
P—F	0	02	24	Tranſitus *Lunæ* Perpendiculare.
S—I	0	03	23	*Tranſitus Lunæ* per 2. Obl.
H—F	0	00	37	*Promontorii Acuti Longitudo à Margine Præcedente.*
H—B	0	01	30	*Promontorii Acuti Latitudo à Margine Boreali.*
K—F	0	00	47	*Menelai Longitudo à Margine Præcedente.*
K—D N—K	0	01	05	*Menelai Latitudo à Margine Boreali.*
L—F	0	01	17	*Cornu Præcedentis Longitudo à Margine Præcedente.*
O—F	0	00	00	*Latitudo* nulla.
O—M	0	02	14	*Cornu Sequentis Longitudo à Margine Præced.*
Q—O	0	00	36	*Latitudo* ejuſdem Cornu *à Margine Boreali.*

In Secunda Phaſi.

	h.	′	″	
A	7	6	40	*Margo Præcedens* ad Perpendiculare.
B	7	7	8	*Cornu Præcedens* ad Verticale.
C	7	7	47	*Cornu Sequens* ad 1. Obl.
D	7	8	19	*Margo Sequens* ad 1. Obl.
E	7	9	4	*Cornu Sequens* ad Verticale.
	7	12	22	Umbra ad *Manilium.*
B—A	0	0	28	*Cornu Præced. Longitudo* à Margine præcedente.
E—A	0	2	24	*Cornu Sequentis Longitudo* à Margine Orientali.
E—C	0	1	17	*Cornu Sequentis Lat.* à Margine Auſtrali.

In Tertia Phaſi.

	h.	′	″	
A	7	19	30	*Cornu Præced.* ad 1. Obliquum.
B	7	21	9	*Margo Præced.* ad Verticale.
C	7	21	19	*Cornu Præced.* ad Verticale.
D	7	21	51	*Margo Præced.* ad 2. Obliquum.
E	7	22	24	*Cornu Sequens* ad 1. Obliquum.
F	7	22	47	*Margo Sequens* ad 1. Obl.
G	7	23	9	*Cornu Præcedens* ad 2. Obl.
H	7	23	31	*Cornu Sequens* ad Verticale.
I	7	24	40	*Cornu Sequens* ad 2. Obliquum.
	7	26	4	Umbra ad *Dionyſium.*
C—B	0	0	10	*Cornu Præcedentis* Longitudo à Margine Orientali.
C—A	0	1	49	*Cornu Præced. Latitudo* à Margine Auſtrali.
G—C	0	1	50	Eadem *Latitudo.*
H—B	0	2	22	*Cornu Sequentis Long.* à Margine præcedente.
H—E	0	1	7	*Cornu Sequentis Lat.* à Margine Auſtrali.
I—H	0	1	9	Eadem *Latitudo.*

In Quar-

In Quarta Phaſi.

	h.	′	″	
A	7	40	24	*Cornu Præcedens* ad 1. Obliquum.
B	7	41	35	*Cornu Præced.* ad Verticale.
C	7	42	18	*Margo Præced.* ad 1. Obl.
D	7	42	44	*Cornu Præced.* ad 2. Obl.
E	7	42	51	*Cornu Sequens* ad 1. Obl.
F	7	43	14	*Margo Sequens* ad 1. Obl.
G	7	43	58	*Margo Sequens* ad Verticale.
H	7	45	04	*Cornu Sequens* ad 2. Obl.
G – 2′ 24″ = I	7	41	34	*Margo Præced.* ad Verticale.
B — I	0	00	01	*Cornu Præced. Long.* à Margine Præced.
B — A	0	01	11	*Cornu Præced. Lat.* à Margine Auſtrali.
D — B	0	01	09	
H — E	0	2	13	Differentia Tranſitus *Cornu Sequentis* inter Obl.
K	0	1	6½	Dimidium, *Lat. Cornu Seq.* à Margine Auſtrali.
E + K	7	43	57½	*Cornu Sequens* ad Verticale.
E + K I	0	2	23½	Longitudo *Cornu Seq.* à Margine Præcedente.

In Quinta Phaſi.

	h.	′	″	
A	7	50	22	*Cornu Præced.* ad 1. Obl.
B	7	51	04	*Promontorium Acutum* in Umbra.
C	7	51	18	*Cornu Præcedens* ad Verticale.
D	7	51	58	*Margo Præcedens* ad 2. Obl.
E	7	52	29	*Cornu Sequens* ad 1. Obl.
F	7	52	56	*Margo Sequens* ad 1. Obl.
D – 2′ 24″ = I	7	49	34	*Margo Præcedens* ad 1. Obl.
B — I	0	01	30	*Longit. Obl. Promontorii Acuti* à 1. Obliq.
D – 0′ 42″ = L	7	51	16	*Margo Præcedens* ad Perpendiculare.
C — L	0	00	02	*Cornu Præced. Long.* à Marg. Præced.
C — A	0	00	56	*Cornu Præced. Lat.* à Margine Auſtrali.
E — I	0	02	55	*Long. Obliq.* à 1 obl. ad Max. Long. 3′. 24″.
	0	00	29	*Complementum.*
F — E	0	00	27	Idem *Complementum. Medium.* 0′ 28″.

In Sexta Phaſi.

	h.	′	″	
A	7	54	06	*Cornu Præcedens* ad 1. Obl.
B	7	54	44	*Promont. Acutum* ad 1. Obl.
C	7	54	59	*Cornu Præced.* ad Perpend.
D	7	55	38	*Margo Præced.* ad 2. Obl.
F	7	55	52	*Cornu Præced.* ad 2. Obl.
G	7	56	04	*Cornu Seq.* ad 1. Obl.
H	7	56	36	*Margo Seq.* ad 1. Obl.
I	7	57	20	*Cornu Sequens* ad Verticale.
D — 2′ 24″ = L	7	53	14	*Margo Præced.* ad 1. Obl.
D — 0 42 = K	7	54	56	*Margo Præced.* ad Perpend.
C — K	0	00	03	*Cornu Præced. Long.* à Marg. Præced.
C — A }	0	00	53	*Cornu Præced. Lat.* à Marg. Auſtrali.
F — C }				
I — K	0	02	24	*Longitudo Cornu Sequentis* à Marg. Præced.
I — G	0	01	16	*Latitudo Cornu Seq.* à Marg. Auſtrali.
B — L	0	01	30	*Longitudo obliqua Prom. Acuti* à 1. Obl.

In Septima Phaſi.

	h.	′	″	
A	8	6	58	*Cornu Præced.* ad 1. Obl.
B	8	7	39	*Cornu Præced.* ad Perpend.
C	8	8	15	*Margo Præced.* ad 2. Obl.
D	8	8	21	*Cornu Præced.* ad 2. Obl.
E	8	8	55	Medium *Umbræ* ad Perpend. ferè.
F	8	9	14	*Margo Sequens* ad 1. Obl.
G	8	9	55	*Cornu Sequens* ad Verticale.
H	8	9	58	*Margo Sequens* ad Verticale.
C — 2′ 24″ = I	8	5	51	*Margo Præced.* ad 1. Obl.
C — 0 42 = K	8	7	33	*Margo Præced.* ad Perpend.
B — K	0	0	06	*Cornu Præced. Long.* à Margine Præced.
B — A	0	0	41	} *Cornu Præced. Lat.* à Margine Auſtrali.
D — B	0	0	42	}
G — K	0	2	22	*Cornu Sequentis Long.* à Marg. Præced.

In Octava Phaſi.

	h.	′	″	
A	8	23	53	*Cornu Præcedens* ad 1. Obl.
B	8	24	24	*Cornu Præced.* ad Perpend.
C	8	24	55	*Margo Præced.* ad 2. Obl.
D	8	24	57	*Cornu Præced.* ad 2. Obl.
E	8	25	55	*Margo Seq.* ad 1. Obl.
F	8	26	26	*Cornu Sequens* ad Perpend.
G	8	26	38	*Margo Sequens* ad Vert.
H	8	28	20	*Cornu Sequens* ad 2. Obl.
C — 0′ 42″ = K	8	24	13	*Margo Præced.* ad Perpend.
B — K	0	00	11	*Longitudo Cornu Præced.* à Marg. Auſtrali.
B — A	0	00	31	} *Latitudo Cornu Præced.* à Marg. Auſtrali.
D — B	0	00	33	
F — K	0	02	13	*Longitudo Cornu Seq.* à Marg. Præcedenti.
H — F	0	01	54	*Latitudo Cornu Seq.* à Marg. Auſtrali.

In Nona Phaſi.

A	8	49	25	*Cornu Præced.* ad 1. Obl.
B	8	49	42	*Eclipſis Concavitas* ad 1. Obl.
C	8	50	08	*Cornu Præced.* ad Verticale.
D	8	50	41	*Margo Præced.* ad 2. Obl.
E	8	50	51	*Cornu Præced.* ad 2. Obl.
F	8	51	34	*Cornu Seq.* ad Verticale.
G	8	51	43	*Margo Seq.* ad 1. Obl.
H	8	52	26	*Margo Seq.* ad Vert.
H — 2′ 24″ = I	8	50	02	*Margo Præced.* ad Perpend.
C — I	0	00	06	*Longitudo Cornu Præced.* à Marg. Præced.
C — A	0	00	43	} *Latitudo Cornu Præced.* à Marg. Auſtrali.
E — C	0	00	43	
F — I	0	01	32	*Long. Cornu Seq.* à Marg. Præced.

In Decima Phaſi.

	h.	′	″	
A	9	4	29	*Cornu Præcedens* ad 1. Obl.
B	9	5	4	Umbra recedit à *Plinio.*
C	9	5	53	*Cornu Seq.* ad. 1. Obl.
D	9	6	17	*Cornu Præced.* ad 2. Obl.
E	9	7	2	*Margo Seq.* ad 1. Obl.
F	9	7	47	*Margo Seq.* ad Vert.
E – 2′ 42″ = H	9	5	20	*Margo Præced.* ad Verticale.
D — A	0	1	48	Tranſitus *Cornu Præced.* inter Obliquos.
M	0	0	54	Dimidium *Lat. Cornu Præced.* à Marg. Auſtrali.
D — M = I	9	5	23	*Cornu Præced.* ad Verticale.
I — H	0	0	3	Longitudo *Cornu Præced.* à Marg. Præced.
E – 3′ 24″ = L	9	3	38	} *Margo Præced.* ad 1. Obl.
F – 4.06 = L	9	3	41	
C — L	0	2	25	*Long. Obliqua Cornu Seq.* à 1. Obliquo.
	9	9	4	Umbra recedit à *Langreno.*
	0	9	19	Finis *Maris Tranquillitatis.*
	0	13	40	*Ariſtoteles.*
	0	14	39	Finis *Maris Criſii.*
	0	21	34	*Finis.*

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LV.
A Tranſit of the Moon above *Venus*. *Oct.* 11. (*ſt.n.*) 1670. at *Dantzick*; by M. *Hevelius*. n. 66. *p.* 2026.

Tempus juxta Hor. Amb. manè.		*Diſtantia & Altit.*	*Tempus ex Alt. Corr.*
h. ′ ″		° ′ ″	h. ′ ″
3 47 20	Altitudo *Lucidæ* ♈	41 45 0	3 54 10
3 50 35	Eadem Altitudo.	41 21 6	3 57 28
5 47 44	Diſtantia ♀ à ♃		5 54 44
5 48 35	Diſt. ☽ Limb. Orient. à ♃	35 25 30	5 55 35
5 51 59	Diſt. ♀ à ♃	35 50 0	5 59 0
5 53 40	Diſt. ☽ Limb. Orient. à ♃	35 29 10	6 0 40
6 12 30	Diſt. ♀ a ♃	35 50 20	6 6 0
6 14 3	Diſt. ☽ Limb. Orient. a ♃	35 37 35	6 8 0
6 37 0	Diſt. ♀ a ☽, quantum nudo oculo dijudicari potuit.	0 6 0	6 51 0
7 12 0	Diſt. ♀ a ☽ Limbo Infer.	0 5 0	7 7 0
7 17 0	Diſt. ♀ a ☽ Limb. Infer.	0 4 0	7 12 0
7 35 0	*Venus* clare apparuit.		7 30 0
9 11 0	*Venus* permanſit conſpicua.		9 12 0
9 15 48	Altitudo *Solis.*	19 8 0	9 17 20
9 17 39	Altitudo *Solis.*	19 58 0	9 19 4

LVI. Initium Occultationis accidit 3ʰ. 38′ 27″. mane circa Montem *Germaniciamum*. Linea Itineraria, quantum ex ſolo Ingreſſu haud obſcurè colligere licuit, tranſiit per M. *Ætnam*, *Centrum* ferè *Lunæ*, per M. *Horminium*, M. *Herculis*, & ſuperiorem partem *Maris Caſpii*. Ego quantum memini, bis tantum, ſi hujus Anni Obſervationem excipias, intra 41 Annos, *Saturnum* à *Luna* tectum vidi; *Anno* nimirum 1630. Die 29 *Junii*, veſp. h. 11. cum in *Freto Danico* circa Inſulam *Huennam* verſarer: Rurſus Anno 1661. Die 3. *Augusti* hic *Dantisci*, h. 7. 58′. 20″. Veſp.

An Occultation of *Saturn* by the Moon, *June* 1. (*ſt. n.*) 1671. at *Dantzick*; by M. *Hevelius*. n. 78. *p.* 3027.

LVII. Sudo admodum Cælo, *Lunam* protinus Orientem, nec non paulò poſt *Jovem*, Tubo 20. pedum ſummâ Aviditate excepimus, atque deprehendimus Octante noſtro permagno Orichalcico, 9 ferè pedum Radio, *Jovem* à *Lunæ* Limbo Orientali adhuc 1° 23′ 40″. eſſe remotum; Jovialeſque omnes quatuor à Dextra, à qua *Luna* accedebat, adeſſe. Ipſum quidem Conjunctionis momentum inſperatus quidam Caſus infelicior deprehendere prohibuit. Cum enim *Jupiter* ad Limbum *Lunæ* Orientalem ad 3′ jam accideret, atque ad 6 duntaxat à Linea Conjunctionis, per utrumque Cornu ducta, diſtaret, ecce

A Tranſit of the Moon above ♃ *Sept.* 30. (*ſt.n.*) 1671. at *Dantzick*; by M. *Hevelius*. Ib. *p.* 3031.

ſupervenientes Nubeculas, quæ tam *Jovem* quam ipſam *Lunam*, nobis è conſpectu eripuerunt. Tabulæ *Rudolphinæ* Occultationem, eamque multò citiùs promiſere, nulla tamen omninò fuerit, ſed, arctiſſimus ſolummodò ad duos propemedum Digitos, Tranſitus extiterit, *Horâ* ſcil. 7 26′ 0″.

An Occultat. of the *Pleiades* by the *Moon*, Feb. 23. 167½, at *Derby*; by Mr. *Flamſteed*. n. 86. *p*. 5034.

Fig. 116.

LVIII. Alta *Luna* 20° 50′. cepi ipſius Diametrum 32′ 48″; & Altâ ipsâ 19° 23′ rurſus eam cepi, 32′ 47″. Ergo *Lunæ* in Horizonte Semidiameter erat vera 16′ 19″. Plus tamen etiamnum ab Occidentali Stellâ *Pleiadum* abfuit quam commode caperet Teleſcopium. At 11h 19′½, *p. m.* Altâ ✱ª *b*, Occidentali *Pleiadum* 9° 50′; ejuſdem Stellæ diſtantiam cepi à Cornu *Lunæ* Proximo 11′ 58″. divertens deinde ſubitò ad ✱æ altitudinem (oſtenſam Quadrante, 20 digitorum Radio, ad Tubi latus affixo) notandam, & continuo reverſus, Stellam (quippe tunc à *Lunâ* Tectum) non comperi. Interea *Luna* deſcenderat Minuta 10, ſimulque tantundem *Stella*, quam ſubiiſſe *Lunam* h. 11. 20′½ ex ſequente Phaſi conjicio: Etenim 11h 30′½, Alta ✱ª *e*. 8° 43′. Stellam *c*, à *Luna* tectam conſpexi. Ejus cum cepiſſem à Cornu Proximo Diſtantiam 16′ 55″. ſpatium Temporis inter hujus & præcedentis Occultationem, editis ſupputationibus, conſtitui 9′ 37″; quæ Tempori hujus Phaſeos ſublatæ, dant utique præcedentis Occultationis Tempus ut conſtitui.

H. 11. 37′½, alta ✱ª *c*. 11° 37′½, ipſa *Lunam* ſubiit, me interea diſtantiam ejus dimetiente 22′ 36′. à Cornu *Lunæ* apparenter Inferiori, ſed Superiori verè. Erat, Stella evaneſcente, *Lunæ* Semidiameter apparens 16′ 21″. quæ propterea occultata erat 87° 25′. *Peripheriæ Lunaris* à *Cuſpide Superiori*, cujus erat Reclinatio (à Linea per Centrum ejus, *Eclipticæ* ducta perpendiculari) 1° 37′. Sic ſubingreſſus Stellæ fuit 4° 12′. ſuprà Lineam per Centrum *Lunæ Eclipticæ* ductam Parallelam, & *Lunæ* Centrum in Antecedentia ✱æ 16′ 18″. cum minori *Latitudine* 1′ 12″.

Fixæ *Locus* Authori *Carolino* ♉ 25° 1′ 24″; *Latitudo* perpetua 4° 20′ 39″, quamobrem *Lunæ* Locus Apparens hora Apparenti *Derbiæ* 11h 37′½. *p. m.* erat ♉ 24° 45′ 6″, & *Latitudo* viſa 4° 19′ 27″. Bor.

Notatu præterea digniſſimum quod etiamſi omnes ferè omnium Aſtronomorum Hypotheſes, *Lunæ Plenæ Perigeæ* in Quadraturis Majorem tribuant Diametrum, & proinde Minorem à *Terra* Diſtantiam quam in *Syzygiis* aut *Oppoſitionibus Perigæis*: contrarium tamen Cælitus fieri & evenire, *Luna* etenim *Plena Perigea* tranſiens juxta *Pleiadas*, *Nov*. 6. 1671. Majorem habebat Diametrum quam in hoc Tranſitu, quando in eodem ferè loco à *Sole* diſtitit gradus 70, *Lunæ* Semidiameter Horizontalis

Nov. 6. 1671.	*Bullialdo*	71′ 00″	*Streetio*	16′ 30″	*Obſervata*	17′ 00″
Feb. 23. 1672.		17 50		17 13		16 19.
		+ 50		+ 43		— 41.

Amplius non nunc miramur *Lunam* tam diu numerorum recuſaſſe vincula & de Tabulis ſupputata apparentiarum tempora uſque adeò expectationes noſtras fefelliſſe, à falſis quandoquidem Hypotheſibus ipſas plerumque constructas fuiſſe liquet.

The *Moon's* Place. *Mar.* 23. 167½; by M. *Caſſini*. n. 82. p. 4047.

LIX. *April* 2d. (*ſt. n.*) 6h 50′. *v.* A Line drawn through the Horns of the *Moon* paſſed through the Star that is at the Point of the *Northern Horn* of *Taurus*, and the Diſtance of this Star to the *Northern Horn* of the *Moon*, was by a Minute greater than the Semidiameter of the *Moon*.

An Occultation of a Fix'd Star by the *Moon*, Feb. 29. (*ſt. n.*) 1676. at *Paris*; by M. *Caſſini*. n. 123. p. 564.

LX. Immerſio Stellæ *Sequentis Duarum in Siniſtro Pede poſteriori Leonis* fuit 1h 19′ 54″. Immerſionis plaga fuit juxta Finem *Schicardi* verſus *Phocilidem* in Selenographia *Riccioli*.

Emerſio vero fuit 11h 16′ 40″: in æquali à recta diſtantia à *Vendelino* & *Petavio*.

Per puncta *Immerſionis* & *Emerſionis*, diligenter notata, ducta recta Linea Diametrum illi perpendicularem abſcidit in ratione 6′ 45″, ad 26′ 5″.

Fuit autem Diameter *Lunæ* ad Meridianum accedentis 32′ 50″.

H. 12. 29′. Margo *Lunæ* Superior fuit in eodem Parallelo cum Stella, quæ tunc præcedebat *Lunam* minuto horario 1′ 50″.

H. 12. 40′ 18″. Stella præcedebat Marginem Occidentalem *Lunæ* minutis horar. 2′ 11″. *Lunæ* Diameter pertranſibat 2′ 14″.

H. 12. 52′ 35″. Stella præcedebat eundem Marginem 2′ 25″.

Altitudo Meridiana Limbi Inferioris *Lunæ* capta eſt 39° 25′ 25″.

LXI.

A Tranſit of the Moon above *Jupiter*, *Feb.* 28. *m.* 167$\frac{5}{6}$, at *Greenwich*; by Mr. *Flamſteed.* Ib. *p.* 566.

Hora Horologii correcta.			*Alt. & Diſtantiæ.*		
h.	′	″		′	″
4	20	15	♃ à Limbo *Lunæ* Lucido ——	26	9
4	47	0	☽ *æ Capta Diameter* ——	31	30
4	49	30	♃ à Cuſpide Proximo ——	26	28
4	52	15	♃ Rectam per Cuſpides ductam prætererat decimâ parte Diſtantiæ vel 3′ circiter, oculari per Tubum conjecturâ. ——		
4	56	0	♃ à Cuſpide —— ——	27	33
5	1	15	— à Recta per Cuſpides ——	7	53
5	3	30	— à Cuſpide —— ——	28	22
5	7	25	— à Recta —— ——	9	58
5	10	50	— ab eadem —— ——	11	55
5	15	50	— à Cuſpide —— ——	30	27
5	21	20	— à Limbo Remotiori dub. —	62	4
5	26	0	— à Cuſpide Proximo ——	33	0
5	31	25	— à Recta per Cuſpides ——	20	9
5	37	0	— à Cuſpide —— —	36	15
5	41	10	— ☽ æ Altæ 10½ *gr.* Diameter circ. —— ——	31	53
5	48	30	Differentia Altit. Limbi ☽ æ Inferioris & ♃ ——	23	1
5	52	40	♃ à Cuſpide Proximo aberat.	41	40
6	9	40	— à Cuſpide dub.	47	29

An Occultation of *Mars* by the *Moon*, *Aug.* 21. 1676. at *Greenwich*; by Mr. *Flamſteed.* n. 129. *p.* 723.

LXII. 1. *Aug.* 21. A. 1676. ante meridiem pro correctione Horologii has Limbi Solaris Altitudines acceperam.

Hora Horologii.			*Altitudines.*			*Hor. Supp.*			*Hor. Err.*		
h.	′	″		°	′	h.	′	″		′	″
8	04	31	Alt. Limbi *Solis* Infer.	26	04	8	09	26	†	4	55
8	5	42	—— —— ——	26	14	8	10	35	†	4	53
8	7	58	—— —— ——	26	34	8	12	53	†	4	55
8	9	10	—— —— ——	26	44½	8	14	03	†	4	53
8	10	15	—— —— ——	26	54	8	15	12	†	4	57
8	17	15	—— —— ——	27	54	8	22	09	†	4	54

Deinde poſt Meridiem, cœlo ſereniſſimo.

Hor. Horol.			*Correcta.*			*Distantiæ.*			
h.	'	''	h.	'	''			'	''
10	45	03	10	49	58	*Mars* à Limbo Lucido *Lunæ* —	5125=	42	08
11	06	11	11	11	05	Eadem Diſtantia — —	3829=	31	29
11	20	00	11	24	55	Iterum — — —	3007=	24	44
11	35	57	11	40	52	Denuo — — —	1982=	16	18
11	57	31	12	02	26	♂ Z. ſive Diff. Alt. Limb. Inf. ♂	1912=	07	35
12	05	00	12	09	55	Jamq; tubo ped. 16. ♂ à Limbo Planeta nudis oculis diutius conſpici non potuit. —	1158=	5	47
12	9	44	12	14	39	♂ Lux cum lumine Lunæ confuſa. ♂ Z — — —	1185=	9	44
12	10	03	12	14	58	♂ penitus Tect. à Cuſp. Boreo.	3475=	17	20
12	18	38	12	23	33	41ª ♉ in Recta per Cuſpides ducta apparuit — —			
12	20	36	12	25	31	41ª ♉ à Limbo vel Cuſp. Tubo breviori — — —	3912=	32	10
12	24	58	12	29	53	41ª ♉ à Cuſpide iterum eodem Tubo — — —	3935=	32	21
12	46	00	12	50	55	*Lunæ* Diameter longiori tubo —	5971=	29	47
1	04	30	1	09	25	Iterum eodem tubo.	5973=	29	48
1	10	56	1	10	51	♂ Emerſ. forſan 4'' vel 5' citius			
1	13	29	1	18	24	♂ à Cuſpide Boreo — —	3675=	18	20
~~1~~	~~18~~	~~25~~	~~1~~	~~23~~	~~10~~	~~Eadem Diſtantia — —~~	~~4035=~~	~~20~~	~~08~~
1	22	00	1	26	55	*Lunæ* Altæ 23° Tubo longiori Diameter — — —	5988=	29	55
1	39	00	1	39	55	*Lunæ* Diameter breviori Tubo —	3645=	29	58

41ª ♉ ſecundum *Tychonem* Locus nunc eſt ♉ 17° 58' ½: *Latitudo* 1° 20' Auſtralis; unde cúm *Lunæ* túm *Martis* Locus accurate deduci poteſt.

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2. At Oxford; by Mr. *Halley*. Ib. p. 724.

Temp. Cor.			*Diſtantia.*			
h.	′	″			′	″
11	43	30	The Center of *Mars* from the neareſt Limb of the *Moon* ——— ———	$719\frac{1}{2}$=	12	40
11	49	2	Again ——— ———	571=	10	3
11	54	48	Again ——— ———	409=	7	12
12	3	25	The Center of *Mars* from the North Cuſp of ☽. ——— ———	1118=	19	41
12	10	28	The Gibbous part of *Mars* touched the *Moon*'s Limb.			
12	10	42	*Mars* was wholly covered, being diſtant from the Cuſp ———	963=	19	14
1	10	41	*Mars* did *Emerge*, I ſuppoſe, his Center.			
1	12	45	*Mars* was diſtant from the Northern Horn of ☽	1018=	71	55
1	31	10	*Mars* paſſed over a Point noted in the Teleſcope.			
1	33	15	The Southern Limb of *Ætna* paſſed by the ſame Point.			
1	34	00	The Lucid Limb paſſed over the ſame Point.			
1	52	35	The *Moon*'s Diam. obſerved 1698=30′ 1″. *Alt.* ☽ 31° circ.			
1	57	52	*Mars* from the Northern Horn of the *Moon*.	2042=	36	5
2	2	53	*Mars* from the Southern Horn of the *Moon*.	2266=	40	3

3. At *Dantzick*; by M. *Hevelius*. Ib. p. 721.

Temp. Sec. Horol. Oſcil.			*Fixarum Nomina.*	*Altitudines.*		*Tempus ex alt. corr.*			*Animadvertenda.*
h.	′	″		°	′	h.	′	″	
1	1	25	*Caudæ Cygni*	57	10	1	0	24	
1	9	45				1	8	45	♂ Diſtabat ferè tanto interſtitio à Limbo ☽ Lucido, quanto M. *Porphyrites* à M. *Ætna* removetur.
1	36	39				1	33	42	*Mars* à *Luna* omninò tectus.
1	45	25	*Caudæ Cygni*	51	17	1	44	7	
2	47	54				2	46	29	*Mars* emicuit; *Finis* nempe Occultationis.
3	19	50	*Scheat Peg.*	45	3	3	18	19	

Mars obtectus eſt circa Montem *Audum*, incedens quaſi per *Loca Lunæ Paludoſa*, per M. *Ætnam*, infra Inſulam *Besbicam*, ſupra Paludem *Acheruſiam*, ſupra M. *Coracem*, per Paludem *Mæotidem*, & paulò ſupra Inſulam *Alopeciam*, & ipſum *Lunæ Centrum*; ſicque rurſus ad Lacum *Majorem Occidentalem* exiens.

Si quæras, unde viam itinerariam hanc adeò accuratè mihi determinare licuerit, & quidem ad partem *Lunæ* Obſcuram, ſcias, eò eveniſſe, quod Tubis illis meis præcipuas Maculas majores in parte *Lunæ* Umbroſâ ſatis diſtinctè deprehendere potuerim; atque ita dilucidè conſpexerim, *Martem* circa medium ferè *Paludis Mæotidis* Emicuiſſe.

LXIII. Obſervavit *Bullialdus Initium*, Alto ſup. Horiz. ad occaſum *Capite Andromedæ* 18° 11′. unde datur à Meridie 7h. 20′ T. A. ſed Med. 7h. 29′ 55″. *Finem* vero vidit, Alta ad Occaſ. *Cinguli Androm. Auſtraliori* Mag. 2. 21° 17′. unde à Meridie colligitur T. A. 8h. 30′ 22″.

An Occultation of Saturn *by the* Moon, *Feb.* 27. ſt. n. 1678, *at* Paris; *by M.* Bullialdus. *n.* 139. *p.* 969.

Monere hic neceſſum eſt Tabulas *Philolaicas* ♄ Promotiorem in Longitudine oſtendere, quam in Cœlo apparet, ſcrupulis primis ut minimum 19. ita ut ♄ tunc fuerit in Cœlo in ♊ 3° 28′. & Lat. Auſt. 1° 38′.

In hac porrò Obſervatione adhibita Illuſt. Viri *Joh. Hevelii* Lunaris Diſci deſcriptione, in illa Limbi parte, quæ in recta linea à medio Montis *Beroſi* per Montes *Riphæos* ducta, paulò ſupra *Alanum* Montem, infra Terminos Auſtrales *Paludum Hyperborearum*, ſita eſt, Saturnum Emerſiſſe aſpeximus.

LXIV. 1. Etiamſi nunc per quinquaginta Annos (pro quo D. O. M. immortales & debeo & habeo gratias) Obſervationibus Rerum Cœleſtium operam dederim, non niſi tamen ſemel tantummodò *Jovem* à *Luna* vidi obtectum, Anno nempe 1646. die 24 *Decembris*, *St. n.* veſperi, *Sole* ſcilicet exiſtente ſub Horizonte. Gratulor igitur mihi magnoperè, quod hanc obſervationem, non ſolùm Cœlo perquam ſereno, ſed etiam ex voto & quidem cum gratiſſimo meo Hoſpite, Clariſ. & Doctiſſimo Domino *Edmundo Halleio* obſervare potuerim.

An Occultation of Jupiter, Jan. 5. (*ſt. n.*) 1679. *at* Dantzick; *by M.* Hevelius. *Ph. Coll.* n. 1. *p.* 29.

Lunam intraverit ad M. *Audum*, & quantum conjicere dabatur ex *Jovis* Exitu, viam carpſit per *Loca Paludoſa* Inſulæ *Cercinnæ*, ſupra Montem *Ætnam*, per Inſ. *Besbicam*, per *Byzantium*, Inſ. *Apolloniam* & ſuperiorem partem *Paludis Mæotidis*; ſic ut paulò ſupra *Centrum Lunæ* inceſſerit, *Luna* habenti aliqualem Latitudinem Auſtrinam. Deinde, quod rariſſimum, *Jovis* apparentem Diametrum in hac obſervatione accuratiſſimè (ut mihi videor) dimenſi ſumus. Memini quidem me aliquoties *Jovis* Diametrum per Maculas Lunares obſervaſſe, eandem videlicet ad 55″. plus minus accedere; ſed hac vice diameter *Jovis* longè extitit minor. Cognita enim totâ Duratione hujus Occultationis 58′ 10″, atque datâ ſimul Diametro *Lunari* 32′ 40″, protinus innoteſcit, ex illo temporis Momento, cum ſcilicet primum *Jupiter* Limbo ſuo *Lunæ* Limbum attingeret, & cum rurſus Occultaretur (id quod factum eſt ſpatio 55″) Diameter *Jovis*, 30″ 53‴.

Ord. Obſerv.	Temp. juxta Horol. Oſcil.	*Altitudines Fixarum & Solis, cum Diſtantiis Jovis à Lunæ Limbo.*		Temp. ex Alt. corr.
	h. ′ ″		° ′ ″	h. ′ ″
	1 18 55	Altit. Capitis *Andromedæ* ——	24 52 0	1 20 54
	1 29 0	Altit. *Arcturi* —— ——	31 3 00	1 31 24
1	1 52 0	*Luna* Oriebatur —— —— —— ——		1 54 25
2	3 26 0	*Sol* Oritur —— —— —— ——		3 28 25
3	3 33 0	Diſt. *Jovis* à Limbo *Lunæ* æquabat fere Diametrum *Lunarem* —— —— ——		3 35 0
4	3 41 0	*Jupiter* à *Lunæ* Limbo in tanta diſt. aberat quanta eſt diſt. M. *Porph.* à Mont. *Sinai* —		3 43 0
5	3 58 0	*Jupiter* à Limbo *Lunæ* Orient. ut M. *Ætna* à Palude *Mareotide.* —— —— ——		4 00 0
6	4 13 50	*Jupiter* diſtabat à Limbo ☽ duab. Diam. ♃ —		4 16 15
7	4 14 40	*Jupiter* ad unicam Diam. *Jovis* removebatur—		4 17 5
8	4 15 40	*Jupiter* ſtringebat *Lunæ* Limbum Orient.——		4 18 5
9	4 16 9	*Jupiter* dimidiâ parte Occultabatur —— ——		4 18 34
10	4 16 35	*Jupiter* Totus omnino Tectus —— ——		4 19 0
11	5 14 0	*Jup.* nunc notabili particula prodire videbat—		5 16 25
12	5 14 20	Dimidius *Jupiter* jam exiverat —— ——		5 16 45
13	5 14 45	Totus *Jupiter* omnino prodierat —— ——		5 17 10
	10 22 30	Altitudo ☉ —— —— ——	53 34 44	10 25 2
	10 27 16	Altitudo ☉ —— —— ——	53 59 45	10 29 42
	10 30 8	Altitudo ☉ —— —— ——	54 14 0	10 32 26
	10 38 0	Altitudo ☉ —— —— ——	54 53 40	10 40 26
	10 45 28	Altitudo ☉ —— —— ——	55 27 20	10 47 46

At Paris; *by M.* Caſſini. *ib.* p. 33.

2. At $3^h\ 0'\ 11''$. the firſt *Satellite* was hid by the Eaſt Limb of the *Moon.* At $3^h\ 2'\ 0''\frac{1}{2}$ the Eaſt-ſide of the *Moon* touched the Weſt-ſide of *Jupiter*; then I took the Height of *Jupiter*, which was 8° 01′; at $3^h\ 2'\ 51''$. At $3^h\ 2'\ 57''$. *Jupiter* was intirely hid by the *Moon.* It entered at equal Diſtance from the two Spots *Grimaldi* and *Ariſtarchus*; the laſt of which was in the Section of the *Moon*, which diſtinguiſhed the Light from the Dark part. At $3^h\ 5'\ 1''$. the Second *Satellite* was hid by the Eaſt-ſide of the *Moon.* At $3^h\ 5'\ 48''$. the third *Satellite* was hid. At $3^h\ 56'$. we perceived by the Eye that *Jupiter* was parted from the obſcure Side of the *Moon.*

M. *De la Hire* took the Height of *Jupiter* two Minutes after parting, and found it, 17° 17′.

LXV.
An Occultation of the Bull's Eye, *at* Greenwich, *Sept.* 4. 1680. *by Mr.* Flamſteed.
Ph. Coll. n. 4. *p.* 99.

Temp. Corr. mane. h.	′	″	*Obſervat.*		′	″	
2	49	52	*Lunæ* Cuſpis Auſtrina à *Palilicio* —	5300	= 26	26	
2	53	10	*Palilicium* à Limbo Proximo —	1028	= 05	08	
2	56	54	à Cuſpide dicta — —	4830	= 24	06	
3	01	16	— — rep. —	4370	= 21	48	
3	05	42	*Immerſio*, à Cuſpide — —	4049	= 20	12	vel
4	08	20	*Lunæ* Diameter — — —	6595	= 32	54	paulo
			ab Amanuenſi repetita. —	6588	= 32	52	minus
4	14	02	*Palilicium* Emerſerat.				
4	15	12	diſtitit à Cuſpide Auſt. —	3411	= 17	01	
4	18	52	rep. —	3725	= 18	35	
4	22	00	iter. —	3980	= 19	51	
4	23	52	denuo —	4098	= 20	56	

$3^{h}\,05'\,40''$. Limbo Lunæ Lucido Fixa videbatur adhærere, & poſt duo ſcrupula ſecunda Temporis, nihil in Limbo apparuit. Locus *Immerſionis* erat juxta Auſtraliſſimam revera trium Macularum parvularum medio jacentium inter Paludem *Mareotida* & Montem *Climacem*.

$4^{h}\,13'\,45''$ Fixa non *Emerſerat*; tunc, vel paulo poſtea, neſcio qua occaſione amovi oculum à Teleſcopio, at cum iterum adhibui $4^{h}\,14'\,2''$. *Emerſam* vidi & plena Luce effulgentem.

h.	′	″		°	′	″
4	53	20	*Lunæ* Limb. Prox. à Pede Lucido *Orionis* —	26	12	25
4	56	20	Pes *Orionis* Lucidus à *Palilicio* — —	26	29	25
4	58	20	*Lunæ* Limbus Prox. à Pede *Orionis* iter. —	26	12	00
5	04	02	*Lunæ* Limbus Prox. à *Polluce* — —	44	09	10
5	06	33	— rep. —	44	08	15
5	11	55	Lucidus Pes *Orionis* à *Palilicio* — —	26	29	40

LXVI.
An Occultation of the Bull's Eye *at* Greenwich, *Oct.* 28. 1630; *by Mr.* Flamſteed.
Ph. Coll. n. 5. *p.* 100.

Temp. Corr. h.	′	″	*Obſervat.*		′	″
7	11	52	*Lunæ* Diameter — —	6745	= 33	39
7	16	26	— — — rep. —	6744	= 33	39
7	19	46	*Palilicium* à Limbo Lucido —	5895	= 29	24
7	24	44	— — rep. —	5095	= 25	25
7	40	14	— — —	3203	= 15	59
7	51	22	— — —	1817	= 09	04

h. ′ ″			′ ″
7 59 34	——— ——— ——— ———	810	= 04 02
8 02 09	——— ——— ——— ———	490	= 02 26
8 06 09	*Attingebat* Limbum		
8 06 30	*Evanuit*, Longitudinem Paludis *Miridis* ad Boream ab ejus fine Boreo.		

Boreus Limbus *Mæotidis*, idem *Ætnæ*, eandem habuere Declinationem cum Loco ſubingreſſus.

Differentia Declinationum Loci *Immerſionis* & Limbi *Lunæ* Borei erat 2770 = 13′ 49″.

9^{h} 02′ 58″. *Emergebat* ab obſcuro Limbo, Longitudinem *Inſulæ Majoris* ab ejus Boreo Termino.

Itineraria per Locum *Emerſionis*, ad Boream à *Creta* ipſius Diametrum, per Limbum Boreum *Sirbonis*, & Montem *Climacem*, tranſibat.

9^{h} 10′ 26″ *Lunæ* Diameter ——— ——— 6791 = 33′ 52″.

At London; *by Mr*. Halley, *and M*. Haines. *Ph. Coll*. n. 5. *p*. 124.

2. At *London*, we noted the *Immerſion* at 8^{h} 6′ 00″, and that Star was newly *Emerged* at 9^{h} 2′ 52″.

At Ballaſore *in* India; *by Mr*. Benj. Harry. *ib*.

3. Mr. *Benj. Harry*, Maſter of the *Berkley Caſtle*, Riding at Anchor in *Ballaſore* Road, about 20 Miles E. S. E. from the Town, obſerved that the *Moon* paſſed to the Northward of the *Bull's Eye* about 24 or 25 Min. and by his Pendulum Watch, Rectify'd by Altitudes and the Riſing and Setting of the Sun, he noted that preciſely at 16^{h} 00′ the *Bull's Eye* was in equal Altitude with the *Moon*'s Center, and that at 16^{h} 30′ the Star was in equal Altitude with the lower Limb of the *Moon*, and at 17^{h} 12′ the Occidental Limb of the *Moon* was in a Right Line with the *Bull's Eye* and *Capella*.

LXVII. 1. *An Occultation of the* Bull's Eye *at* Dantzick, *Jan*. 1. ſt. n. 1681. *by M*. Hevelius. *Ph. Coll*. n. 3. *p*. 65.

Temp.	Obſervat.	Alt.
h. ′ ″		
6 47 00	*Paliliciu m* diſtabat à confinio Lucis & Umbræ tanto ſpatio quanto *Mons Chriſti* removetur à Limbo Lunæ Inferiori. ——— ——— ———	
		° ′
7 37 00	Stella occulta eſt ad *Mare Syrticum* ſub Inſula *Cercinna*, alto Capite *Andromedæ* ——— ———	50 32
7 46 00	Altitudo Capitis *Andromedæ* ——— ———	49 27
7 49 30	Eadem Altitudo ——— ——— —	48 55
8 46 00	*Palilicium* rurſus affulſit ad *Inſulam Majorem* Maris *Caſpii*.	
	Tranſiit itaque via ejus Itineraria, ratione Macularum *Lunarium*, per Mare *Syrticum*, Montem *Athos* ſub Inſula *Lemnos*, & Montem *Didymum*, ſub *Sinu Athenienſi*, per *Fretum Ponticum*, atque *Inſulam Majorem* Maris *Caſpii*.	
8 51 00	Altitudo Capitis *Andromedæ* ——— ———	40 22

2. Mr. *Benj. Harry* in *Ballaſore* Road Obſerved, that the *Moon* paſt to the Northward of the *Bull's Eye*, and that the Star and the *Moon's* under-Limb were in equal Altitude when they were both 13° 45′. high to the Weſt, which gives the time 14h 49′. and when the *South Horn* of *Taurus* was 23° 30′ high, which makes the Time 15h 13′. the Weſtern Limb of the *Moon* was in a Line with *Capella* and the *Bull's Eye*.

At Ballaſore; *by Mr.* Ben. Harry. *Ph. Col.* n. 5. p. 125.

3. The Correct Time of the *Immerſion* was 6h 18′ 22″. and the *Emerſion* at 7h 19′ 46″.

At Avignon; *by* M. *Gallet*. ib.

LXVIII. *Sept.* 27. *ſt. n.* 1682. hora 3. *m. Lunam* tum tres reliquos *Planetas* nudo quidem conſpexi Oculo; ſed *Luna* eo tempore adhuc ad 7. circiter Gradus removebatur, *ſ. ſ. ſ.* Occaſum verſus.

Quantum autem ex *Inclinatione Cornuum Lunæ* quoad Planetarum ductum colligere licuit, protinus proſpiciebam, nullas fore *Occultationes*, ſed tantum *Tranſitus*; ſic ut *Luna* infra illos *Superiores Planetas* incederet. In qua Opinione magis magiſque etiam ſum confirmatus: Cum *Die* ſubſequente, 28 *ſcil. Sept. m.* nec *Regulus* fuerit à *Luna* Tectus, quæ Stella ratione utriuſque Latitudinis potius Occultari debuiſſet. *Regulus* namque in ipſa Conjunctione, *ſcil.* 4h 6′. Diſtabat à Superiori ☽ Cornu Boream verſus adhuc 31′ 17″. id quod optimo *Micrometro* Tuboque egregio, accuratè obſervatum eſt; adeo ut nulla prorſus fuerit *Occultatio Reguli*, ſed tantummodo *Lunæ Tranſitus*. Ita pariter accedit die 25 *Octob.* Nam *Jupiter* & *Saturnus* nec non ♂ Die 26. *Octob. ſt. n.* minime fuerunt à *Luna* Obtecti; ſed *Luna* ſatis longè infra *Planetas* inceſſit.

A Tranſit of the Moon *below the* 3 Superior Planets, *and* Regulus. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 17. n. 151. *p.* 325.

n. 143. *p.* 18. *min.* 31. *ſec.* 17. n. 151. *p.* 326. *min.* 31. *ſec.* 55.

LXIX. *Feb.* 11. ſt. n. 1683. h. 9. cum primum *Luna* in Oculos incurreret, *Regulus* ſatis longè Occaſum verſus removebatur; ita ut ipſa Conjunctio (quantum ruditer colligere dabatur) Oriente circiter *Luna*, *ſcil.* 5h vel 6h contigerit: Utrum autem *Regulus* omnino Tectus ſit, an vero tantummodo Tranſitus fuerit, haud adeo accurate deprehendere licuit.

An Occultation of Regulus *by the* Moon, Feb. 11. [illegible] 1683. [illegible] Dantzick; [illegible] Hevelius. n. 151. *p.* [illegible]

Temp.

LXX.
An Occultation of Two Fixt Stars by the Moon, *and a Transit above a Third,* April 2. *ſt. n.* 1683. *at* Dantzick; *by* M. Hevelius. *ib.*

Temp. Sec. Horol. Ambul.		Diſt. & Altitud.
h. ′ ″		
9 53 30	*Initium* Occultationis Stellulæ Majoris A. 5. *Mag.*	
10 08 30	Conjunctio *Lunæ* & Stellulæ C, diſtabat à *Lunæ* Cornu Inferiori. — — —	° ′ ″ 0 04 00
10 29 36	*Initium* Occultationis Stellulæ B, 6. *Magn.*	
10 52 50	*Finis* Occultationis Stellulæ A.	
11 45 30	Altitudo *Lyræ* —— —	31 44 00
11 46 30	Altitudo eadem —— —	31 55 00
11 47 50	Denuò — — — —	32 06 00

Sectio Luminis & Umbræ hac Die per Montes *Serrorum & Carphatos*, per *Sinum Peronticum*, inter *Byzantium* & Inſ. *Cyaneam*, per M. *Amanum*, *Taurum*, *Urijque Montes* incidit.

Prior Stellula A, in Catalogo *Tychonico* non invenitur; ſed in meo *Novo* Vocatur *ſub Cornu Tauri Sequens 5ª magn.* Verſatur hoc tempore in ♊ 19° 11′ 35″. & in Lat. 4° 43′ 44″. Auſt. Altera vero B, quantum ex hac obſervatione colligere potui, exiſtit in ♊ 19° 17′ 00″ & in Latit. 4° 47′ 0″. Auſtr. At tertia C, quæ forte nudis oculis non conſpicitur, degit modo in ♊ 19° 9′ 0″. & in Latit. 5° 2′ 0″ Auſtr. Cæterum Stella A, *Lunam* ſubingreſſa eſt ad montem *Audum*, tranſiit per Inſulam *Cercinnam*, per M. *Neptunum*, Mare *Adriaticum*, inter M. *Horminium* & M. *Amanum*, per M. *Herculis*; ſic ut inter *Paludem Mæotidem* & *Inſ. Majorem Caſpii* rurſus Emerſerit: unde liquidum eſt hanc Stellulam A, fere Centralem cum *Luna* celebraſſe Conjunctionem.

Altera vero Stella B, *6a Magn. Lunam* ingreſſa eſt ad *Paludem Mareotidem*, Tranſiit per *Sinum Syrticum*, ad M. *Athos*, per M. *Latmum*, inter Montes *Sipylum* & *Macyſtum*, infra *Centrum Lunæ*, per Superiorem M. *Moſchum*, per *Fretum Ponticum*, atque ſic infra *Inſulam Majorem Caſpii.*

LXXI. *Maii* 2. ſt. n. 1683. 11^{h} 0′ 0″ veſp. *Luna* ſupra Stellulam in radice Caudæ *Cancri* tranſibat, quæ modo verſatur in ♋ 27° 53′ 37″. & in Lat. 2° 18′ 42″. Auſtr. ſic ut in ipſa Conjunctione non niſi ad 12′ à *Lunæ Cornu* Infer. abeſſet.

12^{h} 0′ 0″. etiam *alia Fixa*, ſed minutiſſima, tecta eſt, quæ in Catalogo alias non habetur. Quantum conjicere dabatur, hærebat in ♋ 28° 30′ & in Latit. 1° 54′ Auſtr.

An Occultation of a Fixt Star, and a Tranſit above another, May 2. *ſt. n.* 1683. *at* Dantzick; *by M.* Hevelius. *ib.*

LXXII. Ipſum Momentum *Immerſionis* accuratiſſime notavi, id quod incidit 11^{h} 17′ 20″. veſp. ſecundum Horolog. Ambulat. Linea itineraria inceſſit per Mare *Pamphilium*, infra Inſulam *Carpathos*, Inſ. *Cyprum*, infra Sinum Extremum *Ponti*, & Sinum Inferiorem Maris *Caſpii*. 11^{h} 24′ 42″ ſecundum Horol. Ambul. Altitudo Lucidæ *Lyræ* Obſervata eſt 44° 39′ 0″; ex qua *Initium Occultationis* corrigi poteſt. Sectio Luminis & Umbræ per *Lacum Nigrum Majorem*, ad Inſ. *Corſicam*, M. *Nyconium*, per *Lacum Strymonicum*, ad Inſ. *Rhodus*; per M. *Sinai* & M. *Techiſandam* incidit.

An Occultation of Regulus *by the* Moon, *May* 4. ſt. n. 1683. *at* Dantzick; *by M.* Hevelius. *ib.*

LXXIII. 1. At 9^{h} 26′ the under Limb of the *Moon* was juſt Riſen, and ſoon after *Jupiter* appeared near the Eaſtern Limb of the *Moon*, within a few Minutes of being *Ecliſped*.

9^{h} 33′ as near as could be gueſſed, was the Time of the *Central Immerſion*; which was very difficult to be obſerved by reaſon of the Aſperity of the *Moon*'s Limb, which Undulated and Sparkled very much, as it appeared through the Vapours near the Horizon: The *Ingreſs* happened much about the length of the Spot, called by *Hevelius*, *Palus Maræotis*, to the North of the ſaid Spot, or about the 124*th* Degree of the outer Limb of his *Selenography*, nearly in the ſame Latitude with the *Moon*'s Center.

10^{h}. 30′. The Weſtern Edge of *Jupiter* began to *Emerge* out of the dark Limb of the *Moon*.

10^{h} 31′ 20″. The whole Diſk of *Jupiter* was entire, ſo that he was about a Minute and a third in coming out from behind the *Moon*.

The *Emerſion* was exactly in a right Line with the *Moon*'s *Center* and the Northern part of *Palus Mæotis*, or about the 324*th* Degree of the Inner Limb of the *Selenographick Table* of *Hevelius*.

An Occultation of Jupiter *by the* Moon, *March* 31. 1686. *at* London; *by Dr.* Hook, *and M.* Halley. n. 181. *p.* 85.

2. *At* Greenwich; *by Mr.* Flamſteed. *n.* 184. *p.* 206.

Temp. Corr. per ho. Oſcil.					
h.	′	″		′	″
9	32	30	♃'s Limbus tangebat Limbum *Lunæ* Lucidum, *Maræotidis* Diametrum à termino ejus Boreo.		
9	33	42	♃ Totus tectus erat. Quantum per vapores Horizontis, & undulationem Limbi valde turbidam, licuit conjicere.		
10	11	12	Differentia Declinationum Limbi *Lunæ* verè Auſtrini & loci *Ingreſſus*, Tubo ped. 8 & *Micrometro* — — — 1546=	12	42
10	30	30	*Jovis* particula *Emerſerat* è regione Borealis Limbi *Mæotidis*		
10	31	36	*Jupiter* Totus liber.		
10	35	50	Differentia Declinationum Centri *Jovis* & Limbi *Lunæ* Auſtrini — — 3436=	28	15
10	41	40	Repet. — — — 3585=	29	28
10	44	00	*Lunæ* Diameter. — — 3906=	32	07
10	45	44	Repet. — — — 3915=	32	11

At Nuremburg; *by M.* Zimmerman. *n.* 183. p. 177.

3. At 10^{h} 19′ 56″. *M. I. Ja. Zimmerman* obſerved the firſt *Contact* of the Limbs of ♃ and the ☽, and at 10^{h} 20′ 47″. ♃ was all *Eclipſed*.

At 11^{h} 22′ 51″. ♃ was wholly *Clear* from the *Eclipſe*.

The *Immerſion* was about the 117th, the *Emerſion* at the 321ſt. Degree of the Limb, in the *Chart* of *Hevelius*.

At 11h 31′ 06″. The third *Satellite* of ♃ *Emerged*. Theſe times were collected from the Culminations of fixt Stars, and the Vibrations of a Pendulum.

By M. Wurtzelbauer. *ib.*

4. At 10^{h} 20 50″. ♃ applyed to the Limb of the ☽, over-againſt the *Loca Paludoſa Inſulæ Cercinnæ*.

At 10^{h} 22′ 00″. he appeared about half *Eclipſed*.

At 10^{h} 22′ 30″. he was wholly *Hid*.

At 11^{h} 19′ 40″. ♃ began to *Emerge*.

At 11^{h} 21′ 20″. he was quite *Free* from the Interpoſition of the ☽. The point of the *Emerſion* was ſomewhat to the North of the *Palus Mæotis*. No Spot in the ☽ was ſo near the apparent *Magnitude* of ♃'s Disk at the *Inſula Besbicus Hevelii*.

At 11^{h} 40′ 00″. the Altitude of *Procyon* was 8° 37′. whence the Pendulum Clock, which had been ſet by Altitudes of the ☉, the afternoon preceding, may be examined.

5. Etiamſi hucuſque per 56 Annos nullam Obſervationem alicujus Momenti neglexerim, non niſi tres *Jovis* Eclipſes rite deprehendere & annotare potuerim: Utpote primam *Anno* 1646, die 24 *Decemb.* veſperi, ſed tantummodo ejus *Finem*; Secundam *Anno* 1679, die 5 *Junii* ante meridiem de die, quo tempore res omnis felicius ſucceſſit; Tertiam hoc *Anno currente* 1686, die 10 *April.* veſperi.

At Dantzick *by M.* Hevelius. *ib.* p. 178.

Inter alia autem notandum occurrit, quod hæcce *Occultatio* non *Luna* omnino exiſtente *Plena*, ſed altera die circiter poſt ipſum *Plenilunium* veſperi acciderit; & quidem eodem Tempore (quod permirum ſanè accidit, & eſt caſus, qualis haud facile unquam continget) eademque facie, ut illa *Occultatio Anni* 1646, die 24 *Decemb.* veſperi viſa eſt, quo Tempore *Luna* jam ad biduum pariter decreverat, & ſine dubio eandem *Librationem* etiam exhibuit, quam in hac noſtra ultima *Obſervatione*. Nam Sectio Luminis atque Umbræ plane fuit eadem, & per eaſdem Maculas tranſiit (quod ſatis admirari nequeo) nimirum ad *Lacum Hyperboreum Majorem* & *Minorem*, tum ad *Montes Riphæos*, per Paludem *Mæotidem*, per *Lacum Majorem* Maris *Caſpii* & *Sinum* ejus *Inferiorem*, ad Montem *Neroſum*.

E contrario, *Jovis Occultatio Anno* 1679 à me habita, plane extitit diverſa, ſiquidem illa non circa *Plenilunium*, ſed *Novilunium* accidit, tertia circiter die ante *Conjunctionem* ipſam.

Horol. ambulat.		*Altit. Quadrant. captæ.*	*Tempus ex Alt. Cor.*
h. ′ ″		Gr. ′ ″	H. ′ ′
5 10 10	Altitudo *Solis.*	13 47 0	5 11 43
5 12 30	Altitudo *Solis.*	13 28 0	5 13 55
5 17 40	Altitudo *Solis.*	12 41 0	5 19 21
5 23 50	Altitudo *Solis.*	11 46 0	5 25 43
8 7 10	Altitudo *Arcturi.*	29 55 0	8 12 50
8 11 15	Altitudo *Arcturi.*	30 32 0	8 17 4
8 15 10	Altitudo *Arcturi.*	30 59 0	8 20 51
	Luna oritur circit.		9 24 0
9 44 50	*Jupiter* diſtabat ab Inſ. *Cercinna* 43. circit. minut.		9 52 50
10 21 30	*Jovis* diſtantia erat tanta, quanta diſtantia M. *Sinai* à *Palude Maraotide.*		10 31 30
10 40 35	*Jovis* diſtantia erat ferè æqualis diſtantiæ inter M. *Ætnam* & M. *Porphyritem.*		10 51 5

h. ′ ″			h. ′ ′
10 51 30	*Jovis* Limbus à ☽ limbo diſtabat tanto interſtitio, quanto *Pal. Mareotis* à *Limbo Lunæ.*		11 2 0
10 56 9	♃ Limbo ſuo Tangere incipiebat *Lunæ* Limbum, atque ſic *Initium Occultationis* accidit.		11 7 9
10 56 54	Dimidius *Jupiter* Occultabatur.		11 7 54
10 57 39	Totus *Jupiter* omnino à ☽ *Tectus.*		11 8 39
11 8 31	Occultatio *Comitis Jovis ultimi* ad M. *Alabaſtrinum* accidit. Duo tantummodo *Comites* à parte Orientali conſpecti ſunt.		
11 15 54	Altitudo *Lyræ*	32° 59′ 0″	11 26 0
11 19 0	Inſula *Besbica* & *Rhodus* reperiebantur ſub eodem perpendiculo; id quod ad 35 gr. circ. à Linea ☽ verticali removebatur.		
11 21 37	Altitudo *Lyræ.*	33 50 0	11 32 15
11 24 57	Altitudo *Lyræ.*	34 24 0	11 36 24
11 38 15	*Emerſionis* Initium *Jovis.*		11 49 15
11 39 0	Dimidius *Jupiter* Emergebat.		11 50 0
11 39 45	Totus *Jupiter* apparebat. Diameter *Lunæ Micrometro* obſervata erat 31′ 0″.		11 50 45
11 54 10	Diſtantia *Jovis* à confinio Lucis & Umbræ erat æqualis diſtantiæ M. *Ætnæ* à M. *Porphyritæ.*		12 5 40
11 57 20	Diſtantia *Jovis* à Confinio Lucis & Umbræ elongabatur intervallo inter Inſulam *Besbic.* & M. *Ætnam.* Et *Comes* ♃ *Remotiſſimus* à *Jove* tantum aberat, quantum ipſe *Comes* à dicto Confinio Lucis		12 9 20
12 6 9	Altitudo *Lyræ.*	40 19 0	12 18 39
12 9 18	Eadem Altitudo denuo.	40 46 0	12 21 40
12 13 20	Altitudo *Lunæ.*	16 15 0	

Primo liquidum eſt ex ipſa Obſervatione, quod *Orbita*, ſeu Linea *Jovis* Itineraria, per *Montem Alabaſtrinum*, per M. *Christi*, M. *Carpathes*, infra M. *Macrocemnios*, & per *Lacum Hyperboreum Inferiorem* inceſſerit. Secundo quod Inſula *Besbica* & Inſula *Rhodus* ſub uno eodemque perpendiculo, tempore *Occultationis*, circiter 11ʰ 30′ extiterit; ſic ut 35 gradus *Lunæ* Limbi culminaverit. Intravit itaque *Jupiter* Limbum *Lunæ* Illuminatum circa 61 gradum,

dum, à Linea *ſcil.* perpendiculari Nonageſimi atque puncto Zenith, Ortum verſus; Exivit vero circa 31 gradum à dicta Linea Perpendiculari Nonageſimi occaſum verſus, ad Limbum *Lunæ* obſcuratum. Proinde linea *Jovis* itineraria fuit ſubtenſa 104. ferè graduum, attenta videlicet parte *Lunæ* Boreali.

Præterea etiam maximè notatu dignum, quod ex hac obſervatione Diametrum *Jovis* exquiſite elicere potuerim; nimirum $51''\ 42'''$; & tantæ magnitudinis extitit etiam Diameter *Jovis* $50''$ circ. quoties illam per *Maculas Lunæ* dimenſus ſum. Quod autem *Anno* 1679. die 5 *Jun.* cum ſimilem *Jovis* Eclipſim obſervarem, longè ea extitit minor, nimirum tantum $30''\ 53'''$. id ex eo eveniſſe puto, quod obſervatio illa, tempore Diurno, Splendente *Sole* fuerit obſervata; quo Radii Stellarum & Planetarum adventitii magis à Luce *Solis* abſterguntur, quam Tempore Nocturno, nocte obſcura. Quod ſi autem quæras, quamnam Diametrum apparentem veriorem exiſtimem? Scias illam, quam *Anno* 1679. 5 *Jun.* de die, Sole ſplendente obſervavi. Non equidem ex eo quod non æquè diligenter hanc quam illam determinaverim; ſed quod tempore Nocturno Radii Adventitii magis obſtent, ſicuti diximus, quam tempore Diurno.

6. At $9^{h}\ 31'\ 6''$. *Jupiter* was in a Perpendicular falling on the Limb of the *Moon* over-againſt the Northern-part of the Spot *Grimaldi*, (*Mareotis*) near to *Riccioli* (*Stag. Miris*) and diſtant from the Limb about 4 times as much as the ſaid Spot.

At Paris; *by* Mr. Caſſini. n. 183. p. 175.

$9^{h}\ 40'\ 21''$. *Jupiter* touched the Circumference of the *Moon*, which undulated by reaſon of the Vapours near the *Horizon*.

$9^{h}\ 41'\ 20''$. He quite diſappeared in the Inequalities of the *Moon*'s Limb, the *Total Immerſion* might be ſome Seconds later. So the *Central Immerſion* was at $9^{h}\ 40'\ 51''$. *Jupiter* entered over-againſt that Part of *Grimaldi* next *Riccioli*.

$10^{h}\ 30'\ 2''$. The *Outermoſt Satellite* which preceeded *Jupiter* appeared over againſt the Middle of the *Caſpian Spot* (*Pal. Mæotis*) through which the *Section of Light and Darkneſs* paſſed, and made nearly an Equilateral Triangle, with the Extremities of that Spot.

$10^{h}\ 40'\ 24''$. The firſt Limb of *Jupiter* began to come out of the Dark ſide of the *Moon*, over-againſt the North-part of the *Caſpian* Spot, about *Cleomedes*, (*ad Montes Riphæos.*)

$10^{h}\ 40'\ 56''$. The Center of *Jupiter* did *Emerge*. It was difficult to diſtinguiſh the Moment when *Jupiter's* Disk was fully clear, but at $10^{h}\ 41'\ 36''$. the Eclipſe was certainly paſt.

At the *Emerſion* of the Center, the Altitude of *Jupiter* was 11° 31′.

At $10^{h}\ 42'\ 49''$. The *Second Satellite* being the neareſt of the three that followed the Planet, *Emerged*.

At $10^{h}\ 45'\ 1''$. The *Innermoſt Satellite*, being near its greateſt *Elongation*, *Emerged*.

At $10^{h}\ 50'\ 40''$. The *Third*, or *Penextimus Satelles*, being likewiſe near its greateſt *Elongation*, began to appear over-againſt the Northern-Edge of the *Caſpian* Spot,

At $11^h\,45'$. The Diameter of the *Moon* was $32'\,27''$. and according to the *Calculus* of M. *Caſſini*, her *Parallax* was 61 Min.

At Avignon; *by* R. P. Bonfa. *ib. p.* 176.

7. The Central *Immerſion* was at $9^h\,42'\,13''$. and the Central *Emerſion* at $10^h\,45'\,26'$. over-againſt the Southern-part of the *Caſpian* Spot.

An Occultation of Jupiter *by the* Moon, Apr. 28. 1686. *n.* 181. p. 87.

LXXIV. 1. The *Immerſion* was ſeen at *Totteridge* (which Place is about 9 Miles from *London*, and nearly 25″ of Time to the Weſtwards thereof) by Mr. *Ed. Haines:* who between a Gap of the Clouds obſerved the *Contact* of the *Moon*'s Limb and *Jupiter*'s, at $3^h.\ 3'\frac{1}{2}$.

The Clouds cloſing again permitted him to obſerve no more: however from this we may conclude the *Central Immerſion* at *London*, to have been $3^h\ 4\frac{1}{2}'$. *mane.*

The *Emerſion* was obſerved at *London*, by Mr. *Edm. Halley*, to fall out at $3^h\,49'$. for at $3^h\,50'$. *Jupiter* was all out, and the Limbs ſo little ſeparated, that he judged, that a Minute before, the *Center* of *Jupiter* had been upon the *Moon*'s Edge: The Point of the *Emerſion* was over-againſt the Southern-part of the Spot, call'd by *Hevelius Inſula Macra*, or at the 342*d* Diviſion of the Inner Limb of his *Map* of the *Moon*.

At Avignon; *by* R. P. Bonfa. *n.* 183. p. 177.

2. The *Immerſion* of the *Center* happened at $3^h\,37'\,23''$. on the Eaſt-ſide of the Spot *Xenophanes*. The *Emerſion* was at $4^h\,28'\,24''$. between *Seneca* and *Beroſus*, according to *Riccioli*, or *ad Montes Alanos*, *Hevelii*, a little to the Northward of the *Palus Meotis*.

3. *At* Dantzick; *by* M. Hevelius. *ib.* p. 184.

Secund. Horol. Amb.			*Temp. Cor.*
h. ′ ″		° ′ ″	h. ′ ″
3 23 20	Altitudo *Arcturi.*	31 16 0	3 20 12
3 24 25	Eadem Altitudo.	31 04 0	3 21 35
3 44 30	*Jupiter* à Limbo *Lunæ* diſtabat Majori adhuc Intervallo quam M. *Sinai* à M. *Ætna.*		3 41 30
3 47 00	*Jovis* diſtantia erat tanta, quanta M. *Porphyritidis* à *Byzantio.*		3 44 00
3 52 00	*Jovis* à Limbo *Lunæ* Diſtantia erat æqualis Diſtantiæ Inſulæ *Sardiniæ* & *Paludis Mareotidis.*		3 49 00
3 59 00	*Jupiter* à Limbo *Lunæ* paulo plus diſtabat quam *Pal. Mareotis* ab *Ætna.*		3 56 00
4 16 40	Diſtantia *Jovis* à Limbo *Lunæ* æquabatur ferè Diſtantiæ M. *Porphyritidis* ab Inſ. *Cercinna.*		4 13 40
	Planetarum *Occaſus* factus eſt.		4 17 00

Hora Horolog. ma.			*Temp. Corr.*
h. ′ ″		° ′	h. ′ ″
12 06 10	Altitud. *Pollucis.*	28 35	12 12 14
12 16 47	———————	26 59	12 22 47
1 18 00	*Lunæ* Limbus tangebat Anſam Occid. ♄*ni*. ———		1 24 00
1 18 30	*Immerſio* Centri *Saturni* paulo infra *Pal. Mareotin.*		1 24 30
1 19 00	Jam *Saturnus* omnino latuit.		1 25 00
4 01 25	Altitudo Centri *Solis.* ———	20 00	7 07 17
4 09 06	———————	18 55	4 14 43

LXXV. 1. *An Occultation of* Saturn *by the* Moon, *Mar.* 19. 168$\frac{6}{7}$. *at* Totteridge; *by Mr.* Ed. Haines. *n.* 186. p. 268.

2. *March* 18. At Night I obſerved here the *Occultation* of *Saturn* by the *Moon*, which happened at 12ʰ 13′ 55″. it paſſed directly under the Midſt of the *Moon's Diſcus*.

In Ireland; *by Dr.* Aſh, *Bp. of* Cloyn. *n.* 243. p. 293.

LXXVI. 1. *Oct.* 13. 1665. at ſix of the Clock, with a very good Teleſcope near 38 Foot long, and a double Eye-glaſs, *Saturn* appeared to me ſomewhat otherwiſe than I expected, thinking it would have been Decreaſing, but I found it as full as ever, and a little hollow above and below.

Phaſes of Saturn, *An.* 1665. *at* Mainhead *near* Exeter; *by Mr.* Will. Ball. *n.* 9. p. 152. *Fig.* 132.

2. *Jun.* 29. 1666. between 11 and 12 at Night, I obſerved the Body of *Saturn* through a 60 Foot Teleſcope, and found it exactly of the Shape Repreſented in the Figure. The *Ring* appeared of a ſomewhat brighter Light than the Body; and the black Lines *a a*, croſſing the *Ring*, and *b b*, croſſing the Body (whether Shadows or not I diſpute not) were plainly viſible; whence I could manifeſtly ſee, that the Southermoſt-part of the *Ring*, was on this ſide of the Body, and the Northern-part, behind or covered by the Body.

An. 1666. *at* London; *by Dr.* Hook. *n.* 14. p. 246. *Fig.* 133.

3. *Aug.* 17. 1668. at $11\frac{1}{2}^{h}$ theſe *Pariſian* Obſervers, imploying a Teleſcope of 21 Foot, ſaw the *Globe* of *Saturn* in the Middle manifeſtly appearing above and below, beyound the Ovale of his *Anſes*; which was hardly diſcernible the laſt Year. They meaſured divers ways the *Inclination* of the greater Diameter of the *Oval* to the *Æquator*, and found it of about 9 Degrees. By this Obſervation and other like ones of this and the preceeding Year, Mr. *Hugens* finds, that, inſtead of 23° 30′, the Angle of the Planes of the *Ring* and of the *Ecliptick* muſt be of 31°, or thereabouts.

An. 1668. *at* Paris; *by Mr.* Hugens *& Mr.* Picart. *n.* 45. p. 900.

4. *Aug.* 26. *ſt. n.* 1670. Teleſcopium illud 50 Pedes longum, quod non ita pridem mihi tranſmiſiſtis, Faciem *Saturni* utut *Lunæ* fuerit præſens nitidiſſimè ac clariſſimè detegebat. Quali autem mihi apparuit, adjecta Delineatio commonſtrabit; alia planè Facie, quàm *Cl. Hugenio*, tum vobis. *An.* 1666.

An. 1670. *at* Dantzick; *by* M. Hevelius. *n.* 65. p 2039. *Fig.* 134.

tum *Pariſienſibus* 1668, videbatur. Siquidem *Annulus*, qui *Saturnum* circundat, multo nunc arctior, compreſſiorque animadverſus quam illo tempore, quaſi obliquiori, reſpectu *Terræ*, nunc via incedit.

At Paris; *by* *Mr*. Hugens. *Ib*. p. 2093.

5. This Summer Mr. *Hugens* obſerved *Saturn* with his Teleſcope of 22 Feet, and ſaw his Figure to be very conformable to what it ſhould be according to his *Hypotheſis*; viz. the *Anſæ* or Arms to be very narrow, in ſo much that their opening appeared not but very obſcurely.

At London; *by Dr*. Hook. *ib*.

6. *Sept*. 16. Dr. *Hook* obſerved the Phaſe of Saturn as here repreſented. *Fig*, 135.

An. 1671. *at* Paris; *by Mr*. Caſſini. *n*. 78. p. 3024.

7. *Saturn* according to the *Hypotheſis* of Mr. *Hugens* was to have retaken his *Round Figure* in the Months of *July* and *Auguſt* 1671. But this Appearance hath been perceived ever ſince the End of *May*, at a Time when he was diſtant enough from the *Sun* and the *Horizon*, to be well Obſerved. He hath remained in this Figure unto the 11*th* of *Auguſt*, and Mr. *Caſſini* did then obſerve him thus; but three Days after he ſaw him with *Arms*, though very Narrow ones.

By Mr. Hugens. *ib*. p. 3025, 3026.

8. Our *Philoſophers* here, know very well, that as ſoon as Mr. *Caſſini* had told me that the *Arms* of *Saturn* were returned in *Auguſt*, I ſaid that aſſuredly they would diſappear before the End of this Year. I ſtill obſerved them, *Nov*. 6, *ſt*. *n*. in the Evening, but they were ſo faint and obſcure, that it was hard to diſcern them; ſo that within a few Days they will appear no more at all. This confirms altogether my *Hypotheſis* of the *Ring*, which now diſappears in Proportion that the Rays of the *Sun* do obliquely illuminate the flat Surface of it, Obverted to our Sight.

At Dantzick; *by Mr*. Hevelius. *ib*. p. 3032. *Fig*. 136.

9. Quali Facie nuper, Die *ſc*. *Sept*. 11. *ſt*, *n*, apparuerit, quàm rectiſſimè & diligentiſſimè delineavi atque hic tranſmiſi. An vero Menſe *Jun*. *Jul*. & *Aug*. eum planè *Rotundum* conſpexeritis, ut volunt *Pariſienſes*, vix mihi imaginari poſſum. Etenim utut *Brachia Saturni* ad latera apparuerint arctiſſima, etiam Tubo 60 vel 70 Pedum; haud tamen credo, ea omnino evanuiſſe, ita ut ne Veſtigium aliquod fuerit reliquum. Fortaſſe *Pariſienſes* Teleſcopiis brevioribus in ipſo Crepuſculo, Luna præſente, *Saturnum* contemplati ſunt.

At Derby *by* *Mr*. Flamſteed. *ib*. p. 3034.

10. *Oct*. 12. with my leſs Tube I thought I ſaw ſomething on each ſide of *Saturn*, amidſt the Colours of my Glaſs, and the Spurious Rays of his Body. Directing my longer Tube (of 14 Feet) to him, I could ſee his *Anſæ* ſomewhat more diſtinctly, but very ſlender, and to one, that thought not of them, ſcarce diſcernible.

Nov. 30. I obſerved him with my 14 Foot Teleſcope, the Aperture being 1½ Inch, and its Eye-Glaſs drawing two Inches. He appeared perfectly *Round*, free from Rays and Colours, and no *Anſæ* to be ſeen. Mr. *Townley* in his laſt (*Nov*. 20.) tells me, that he looked at him one Night, and could hardly diſtinguiſh his Line of the *Anſulæ*, but plainly ſaw a dark Line thro' him near his upper Part.

At Paris; *by* Mr. Caſſini. *n*. 92. p. 5180. An. 1675. *at* London; *by* *Mr*. Flamſteed. *n*. 116. p. 372. *At* Dantzick; *by Mr*. Hevelius. *n*. 127. p. 661.

11. *Dec*. 16. we found that *Saturn* had retaken his *Round Figure*.

12. *Jun*. 27. m. 1675. *Saturnum* Patulis auctum *Anſis* vidimus.

13. In *Auguſt*, The Figure of *Saturn* appear'd, as *Fig*. 137.

14. Ex Schemate *Saturni* à *Cl. Hevelio* ante Annum obſervato video, eum Teleſcopiis, noſtris longè inferioribus, uti. Tunc enim Temporis (ut & nunc (*Aug*, 1776.) cernebatur nobis in *Saturni* Globo Zona ſubobſcura, paulo Auſtralior Centro, inſtar Zonarum *Jovialium*. Deinde Latitudo *Annuli* dividebatur biſariam, Lineâ obſcurâ apparenter Ellipticâ, revera Circulari quaſi in duos *Annulos* Concentricos, quorum Interior Exteriori Lucidior erat. Hanc Phaſim ſtatim poſt Emerſionem *Saturni* è Solaribus Radiis per totum Annum uſque ad ejus Immerſionem conſpexi; primo quidem, Teleſcopio Pedum 35; deinde minori, Pedum 20.

An. 1676. *at* Paris; *by M.* Caſſini. *n.* 128. p. 690.
Fig. 137.
Fig. 138.

LXXVII. 1. Vidi *Saturnum Aug.* 26. *ſt. n.* 1670. alta *Aquila* 24° 32′ 0″. in Diſtantia ab Extrema Alæ *Pegaſi* 33° 48′ 0″. & ab *Ore* ejus 24° 51′ 40″. in 4° 11′. *Piſcium*, in 1° 53′. Latit. Auſtr. in ipſa nempe Oppoſitione *Solis* exiſtentem.

Places of Saturn *Obſerved* An. 1670. *at* Dantzick; *by Mr.* Hevelius. *n.* 63. p. 2089.

2. *Dec.* 29. *ſt. n.* 1677. 8^{h} 58′. Vidimus in eodem Azimutho inque Nonageſimo Eclipticæ Gradu ab Horizonte *Saturnum* & *Boreum Oculum* ♉, qui infra *Saturnum* erat, unde Planetam & Fixam eandem in *Zodiaco* Longitudinem obtinere deprehendimus, *viz.* juxta *Tychonem* ♊ 3° 58′ 53″.

An 1677. *at* Paris; *by Mr.* Bullialdus. *n.* 139. p. 973.

LXXVIII. 1. About the End of *October* 1671. we diſcovered, by a Teleſcope of 17 Feet, 11 ſmall Stars near *Saturn*, one of which by its particular Motion ſhew'd it ſelf to be a true *Planet*: which we found by comparing it not only to *Saturn* and his ordinary *Satellite*, diſcovered 1655. by *M. Hugens*, but alſo to the Fix'd Stars. The Motion of it was very manifeſt in reſpect of the Fix'd Stars, but leſs ſenſible in reſpect of Saturn; yet it appear'd that from *October* 25. unto *November* 1. his Diſtance from *Saturn* increaſed Weſtward, and from that time unto *November* 6. it diminiſhed: So that his greateſt *Digreſſion* from *Saturn* happened in the Beginning of *November*.

The Outermoſt Satellite *of* Saturn *diſcovered by Mr.* Caſſini. *n.* 92. p. 5178.

Dec. 16. We found that on the Eaſt of *Saturn*, there was a ſmall Star, far diſtant in a ſtreight Line to *Saturn*, and to his ordinary *Satellite*, which was Oriental alſo but little diſtant from *Saturn*. And *Dec.* 24*th.* we ſaw this *Satellite* in the Weſt, and a Star, Oriental likewiſe, leſs diſtant from *Saturn* than that we had ſeen the 16*th.*

Dec. 13. and 17. 1672. We perceived, with an Excellent Teleſcope, (of 35 Foot made by *Campani*) an Occidental Star, remote from *Saturn*, which in both thoſe Obſervations had a Southern Latitude in reſpect of the Line of his *Wings*; but in the firſt it was further diſtant from *Saturn* than in the Second: ſo that, if this was the ſame Star, as I ſuppos'd it to be, it mov'd towards *Saturn* on the Eaſt, and conſequently (ſuppoſing it to be his *Satellite*) it was in the Superior part of his Circle.

Feb. 6. 1673. We began to ſee him again, and we obſerved him almoſt all the Days following till the 20*th.* This New *Planet* did more and more remove from *Saturn* till the 19*th* of *Feb.* when we meaſured the Difference between his Paſſage and that of the Center of *Saturn* to be 30″ of an Hour, which gave at leaſt 10 Diameters of *Saturn*, and on the 20*th* the Diſtance was judged by Eſtimate to be yet greater.

Ib. p. 5179. 5184. This Digreſſion, being treble to that of the *Ordinary Satellite*, enabled us at firſt to judge the Time of this Revolution to be Quintuple, applying to the *Satellites* that Proportion, which *Kepler* hath noted in the *Principal Planets*, between the Periodical *Times* and their *Diſtances*. We were afterwards confirmed in this Opinion; For by the Apparent Swiftneſs of his Motion it was eaſie to ſee that this *Planet* had been in Conjunction with *Saturn Feb.* 3. 1673. and by his Motion on the Weſt it appears, that he was in the Inferior part of his Circle: And becauſe during this time of 17 Days he remov'd more and more from *Saturn*, 'tis certain that he remained in the ſame Quadrant of the Inferior Occidental Circle above 17 Days, and that his whole *Revolution* is more than 68 Days. He was theſe laſt Days at a Diſtance almoſt equal to that which he had about the end of *Octob.* 1671; ſo that in 480 days or thereabout he made a certain Number of Intire *Revolutions*, which can be no more than 7; ſince each of them is without Queſtion of more than 68 days. If you ſhould count 7 of them, each would be 68' Days; if you count 6, each would be 80 Days; and if you count but 5, each would be 96 Days. But this laſt Suppoſition can by no means be made to agree with the two Obſervations of *Dec.* 1672, and the firſt doth not agree ſo well with them as the Second.

n. 133. p. 831. *Mr. Caſſini* has ſince found that this *Outtermoſt Satellite*, is diſtant from the Center of *Saturn* $10\frac{1}{2}$ Diameters of his *Ring*; that the Period of his *Revolution* in 80 Days is ſo juſt, that he doth not Anticipate 9 Revolutions, which are made in two Years, but by one whole Day; and that in the Conjunctions with *Saturn*, his Latitude encreaſes according as the *Ring* of *Saturn* enlargeth it ſelf; though the Line of his Motion is not Parallel to the Circumference of the *Ring*.

Mr. Caſſini hath alſo diſcovered, after many Revolutions, that this *Satellite* hath a Period of *Apparent Augmentation* and *Diminution*, by which Period he becomes viſible in his greateſt Occidental Digreſſion, and Inviſible in his greateſt Oriental Digreſſion; he begins to *Appear* two or three Days before his Conjunction in the Inferior part, and to *Diſappear* two or three Days after his Conjunction in the Superior part: So that he remains Inviſible in every Revolution of 80 Days for a whole Month together.

This Viciſſitude of *Phaſes* makes it ſeem probable, that one part of his Surface is not ſo capable of Reflecting to us the Light of the Sun, which maketh it viſible, as the other part is. Whence we may conjecture, that the Globe of the *Satellite* hath ſome Diverſity of Parts Analogous to that of the Earth, the one part of whoſe Surface is cover'd by the Sea, which is not ſo fit to Reflect from all parts the Light of the Sun, as the Continent which maketh up the other part: So that this *Planet* by a Converſion about his *Axis*, or by an Expoſition of the ſame Hemiſphere to *Saturn* (much after the manner of the Hemiſphere of the *Moon* to the *Earth*,) ſometimes turns to us the part Analogous to the Continent, ſometimes that part which anſwers to the Sea.

LXXIX. *Dec.* 23. 1672. We found a ſmall Star Weſtward of *Saturn*, between him and his *Ordinary Satellite,* which was on the Weſt alſo, almoſt at a double Diſtance, *Dec.* 30. We ſaw a little Star, on the Eaſt of him and his *Ordinary Satellite,* which had paſſed alſo to the Eaſt of him.

The Third Satellite of Saturn *diſcovered, by M.* Caſſini. *n.* 92. p. 5181.

Jan. 10. 1673. This little Star appeared to have returned almoſt to the ſame Poſition in reſpect of *Saturn* and his *Ordinary Satellite,* where it had been *Dec.* 23. *Jan.* 15. the *Ordinary Satellite* was Oriental, and the *New* one Occidental, as it had been in the precedent, but a little nearer to *Saturn.* We had that Evening time enough attentively to Obſerve this *Planet* for a whole Hour together, during which we perceived, it approached to *Saturn* on the Weſt, and conſequently was in the Superiour part of his Circle: Which did fully confirm us in the Suppoſition we were inclined to, that it was an *Interior Satellite.*

Comparing the Obſervations together, we began to find the Rule of the Motion of the new *Interior Satellite.* For the two laſt ſhewed us, that in 5 Days he had made more than a whole Revolution. The firſt Obſervation compared with the 3d made us judge, that in 18 Days he had made a Number of Revolutions almoſt whole ones, which certainly were 4; each of them was of $4\frac{1}{2}$ Days: So that between the 10*th* and 15*th* it might be, that there had been one Revolution of $4\frac{1}{2}$ Days, or two Revolutions of $2\frac{1}{4}$ Days each. But the Combination of the firſt with the 2d, made us ſeclude the Period of $2\frac{1}{4}$ Days. We therefore Judged by theſe Obſervations; that this laſt *Planet* finiſhes his Revolution about *Saturn* in $4\frac{1}{2}$ Days; that the Semidiameter of his Circle is $1\frac{5}{8}$ of the Diameter of *Saturn's Ring*; and that he was towards his greateſt Occidental Digreſſion the 23d of *December*, and *Jan.* 1. about 7 a Clock in the Evening. We have ſince found, that his greateſt Digreſſion from the Center of *Saturn* is only $1\frac{2}{3}$ of his *Ring*, and the Period of his Revolution is 4 Days 12^h and $27'$. His Latitude Augments alſo according as the *Ring* enlargeth, and at the preſent that the largeneſs of the *Ring* is greater than the Diameter of the Globe of *Saturn,* he is to paſs in the Conjunctions without touching either *Saturn* or his *Ring*. Yet notwithſtanding we have not yet been able to diſtinguiſh him in the Conjunctions either in the upper or lower part of his Circle; but only in his Greateſt, as well Oriental, as Occidental Digreſſions.

n. 133. p. 833.

LXXX. Theſe two *Satellites* were firſt of all ſeen in *Mar. An.* 1684. by two excellent Object Glaſſes of 100 and 136 Feet, and afterwards by two others of 93 and 70 Feet, all made by *S. Campani,* after the Diſcovery of the 3d and 5th *Satellites,* which had been made by others of his Glaſſes of 47 and 34 Feet. We have ſince ſeen all theſe *Satellites* with that of 34 Feet, and continued to obſerve them with Glaſſes of *M. Borelli* of 40 and 70 Feet, and by thoſe which *M. Artouquel* hath lately made, of 80, 155, and 220 Feet. It was eaſy for us to ſee theſe two *Satellites* by theſe different Sorts of Glaſſes, after having found the Rules of their Motion, whereby we might with more particular Attention look upon the Places where they ought to be.

Two Interior Satellites *of* Saturn *diſcovered; by M.* Caſſini. *n.* 181. p. 79.

The *Firſt Satellite* was obſerved 45° Diſtant from its *Perigee*, moving toward the Weſt, *Mar.* 11. 1686. *ſt. n.* at $10^{h}\ 40'$. at Night, and returned to the ſame Poſition on the *14th* of *April* at the ſame Hour.

The *Second* was 36° Diſtant from the *Perigee* to the Weſt, the *30th* of *Mar.* 1686. *ſt. n.* at 8 a Clock in the Evening.

The *Firſt* or *Innermoſt Satellite*, is never diſtant from *Saturn's Ring* above $\frac{3}{5}$ of the apparent length of the ſame *Ring*; It makes one *Revolution* in 1^{d}. $21^{h}\ 19'$ and the Circle of its Orb is nearly in the ſame Plain with the *Ring*.

The *Second* or *Penintime Satellite* is $\frac{3}{4}$ of the Length of the *Ring* diſtant therefrom, and makes his Revolution in $2^{d}\ 17^{h}\ 43'$.

After a great Number of Choice Obſervations, it was concluded, that the Proportion of the *Digreſſions* of the *Second* to that of the *Firſt*, (counting both from the Centre of *Saturn*) is as 22 to 17; and of its *Revolution* as $24\frac{3}{4}$ to 17. This is that very ſame Proportion which *Kepler* Obſerves between the *Diſtances* and *Period* of the *Primary Planets*, and which we have found between the other *Satellites* of *Saturn*, and is verified in the *Satellites* of *Jupiter*. There is nothing that better ſhews the admirable Harmony of the Particular *Syſtems* with the Great *Syſtem* of the World.

The antient Aſtronomers having tranſlated the Names of their Heroes among the Stars, whoſe Names have continued down to us Unchanged, notwithſtanding the Endeavour of the following Ages to alter them; and *Galileo* after their Example, having honoured the Houſe of the *Medici* with the Diſcovery of the *Satellites* of *Jupiter*, made by him under the Protection of *Coſmus* II. (which Stars will be always known by the Name of the *Sidera Medicea*.) Wherefore the Diſcoverer Concludes that the *Satellites* of *Saturn*, being much more exalted and more difficult to Diſcover, are not unworthy to bear the Name of *Louis Le Grand*, under whoſe Reign, and in whoſe *Obſervatory* the ſame have been detected, which therefore he calls *Sidera Lodoicea*. not doubting but to have Perpetuated the Name of that King, by a Monument much more laſting than thoſe of Braſs and Marble, which ſhall be erected to his Memory.

M. Hugens's *Theory of the Fourth* Satellite *of* Saturn *corrected*; *by Mr.* Ed. Halley. n. 145. *p.* 82. *Mar.* An. 1683.

LXXXI. The *Fourth* or *Penextime Satellite* of *Saturn*, firſt Diſcovered by M. *Hugens* 1655, I have of late frequently Obſerved with a 24 Foot *Teleſcope*: And I perceived that M. *Hugens*'s Numbers were conſiderably run out, and about 15° in 20 Years too ſwift; this made me reſolve more nicely to enquire into it's *Period*, and accordingly I waited till I had gotten a Competent Number of Obſervations, the moſt Conſiderable whereof are theſe.

1682. *Nov.* 13. $13^{h}\ 00'$. *p. m.* the *Satellite* appeared on the North ſide of *Saturn*, and a Perpendicular let fall from it on the Tranſverſe Diameter of the *Ring*, fell upon the middle of the Dark ſpace of the following *Anſa*; and the ſame Night $19^{h}\ 00'$. it had paſt the Conjunction, and the Perpendicular fell exactly on the Weſtern Edge of the *Globe* of *Saturn*; The Northern Latitude, and Retrograde Motion, made it evident that the *Satellite* was then in *Perigaeo*.

Again, *Nov.* 21. 16^h 15′. this *Satellite* of *Saturn* was on his South-ſide, the Perpendicular on the Line of the *Anſa* fell on the Middle of the dark Space of the Weſtern *Anſæ*, and the ſame Night 19^h 00′, the Perpendicular fell preciſely on the Center of *Saturn*, and the Diſtance therefrom was ſomewhat leſs than one Diameter of the *Ring*. By this it was evident that the *Satellite* was in *Apogæo*.

I obſerved it in *Apogæo* again on the *24th* of *Jan.* 1683. at 8^h 00′ *p. m.* the Perpendicular on the Line of the *Anſa* fell exactly on the Weſtern Limb of the *Globe* of *Saturn*, and at 9^h 30′. *p. m.* the ſaid Perpendicular fell within the *Globe* more than half way to the *Center*, and the Diſtance from the Line of the *Anſa* towards the South, ſeemed much about one Diameter of the *Ring*.

Laſtly, *Feb.* 9. 1683. 8^h 10′. *p. m.* it was again in *Apogæo*, and I could by no means diſcern towards which ſide it inclined moſt, nor whether the Tranſverſe Diameter of the *Ring*, or the Diſtance of the *Satellite* therefrom, were the greater; ſo that at that time it was preciſely *Apogæon*.

To compare with theſe, I choſe two out of thoſe of *Hugens*, which ſeemed the moſt to be confided in; the firſt made 1659. *March* 14. *ſt. n.* 12^h 00′ at the *Hague*; when the *Satellite* appeared about one Diameter of the *Ring* under *Saturn*, but it was gone ſo far to the Weſtward, that he concluded, that about 4 Hours before, or 7^h 40′ at *London*, it had been in *Perigæo*.

Again, *March* 22. 1659. 10^h 45′. the *Satellite* was a whole Diameter above the Line of the *Anſæ*, and the Perpendicular thereon fell nearly upon the Extremity of the Eaſtern *Anſa*.

By the Firſt of my Obſervations it appears that the *Satellite* was in *Perigæo* 1682. *Nov.* 13. 17^h 00′. *circiter*, at which time *Saturn* was 3^s 29° 39′ from the firſt Star of *Aries* in the *Ecliptick*, but the *Earth* reduced to *Saturn's Equinoctial*, and the *Satellite* was 9^s 23° 46′ à 1^a ✱ ♈. And *March* 4. 1659, 7^h 40′. *Saturn's* Place in the *Ecliptick* was 6^s 0° 41′. but the *Earth* reduced, and conſequently the *Satellite*, in 11^s 28° 18′. *à Prima Stella Arietis*. The Interval of Time is 8655 Days, 9^h 20′; in which the *Satellite* had made a certain Number of Revolutions to the Fix'd Stars, and beſides 9^s 25° 28′. or 295 Degrees 28′, whoſe Complement to a Circle 64° 32′ is 2 Days 20^h 36′ Motion of the *Satellite*, according to *Hugens*. So that 8655 Days 5^h 56′, or 12467846 Minutes of Time, is the time of ſome Number of intire Revolutions; and dividing that Interval by 15 Days 22^h 39′, or 22959′ (the Period of *Hugens*) the Quotient 543 ſhews the Number of Revolutions; and again dividing 12467876 by 543, the Quotient 22961$\frac{1}{10}$ *min.* or 15 Days, 22^h 41′ 6″. appears to be the true Time of this *Satellite's Period*. Hence the Diurnal Motion will be 22° 34′ 38″ 18‴, and the Annual beſides 22 Revolutions 10^s 20° 43′. Having made Tables to this Period, I found that in the *Apogæon* Obſervation of *Hugens*, the *Satellite* was above 3 Degrees faſter than by my *Calculus*, and that in the three other Obſervations of my own, being likewiſe in the Superior part, it was 2½ *Deg.* ſlower than by the ſame Calculation. Now 'tis evident that the Differences muſt ariſe from ſome *Eccentricity* in the *Orbit* of this *Satellite*, and that in *Mar.* 1659.

the

the *Apocronion* (as I may call it) was ſomewhere in the Oriental Semicircle, and that in *Nov.* 1682. it was in the Weſtern Semicircle, and ſuppoſing the *Apocronion* fix'd, it muſt neceſſarily be between $9^s\ 23^\circ\ 46'$, and $11^s\ 28^\circ\ 18'$, à 1^a ✱ ♈, that being the common Part between thoſe two Semicircles; and becauſe the Difference was greater in *Hugens*'s Obſervation than in Mine, 'twill follow that the *Linea Abſidum*, or *Apocronion*, ſhould be nearer to $9^s\ 23^\circ\ 46'$, than to $11^s\ 28^\circ\ 18'$. I will ſuppoſe $10^s\ 22^\circ\ 00'$ à *Prima Stella Arietis*, (which happens to be alſo the Place of *Saturn's Equinox*) and the greateſt Equation about 2½ *Degrees*. Upon the Score of this Inequality the mean Motion of the *Satellite* will be found about $2^\circ\ 45'$. ſlower in 23½ Years, or 7 Minutes in a Year, whence I ſtate the Annual Motion $10^s\ 20^\circ\ 36'$. above 22 Revolutions, and the Correct *Epocha* for the laſt Day of *December* 1682. at Noon in the Meridian of *London* $9^s\ 10^\circ\ 15'$ à 1^a ✱ ♈, from which *Elements* I compoſe the following Table.

Tabula

Tabula Motus Medii Satellitis Saturnii *ab* Hugenio *inventi*, à Prima * ♈.

Anno Christi *Curr.*	*Epocha.* s.	°	'
1641	8	29	17
1661	10	14	10
1681	11	29	3
1682	10	19	39
1683	9	10	15
1684	8	00	51
1685	7	14	2

Menſ. Ann. Com.	*Mot. Med.* s.	°	'
Jan.	0	0	0
Feb.	11	9	53
Mar.	8	12	2
April.	7	21	56
Maii	6	9	14
Jun.	5	19	7
Jul.	4	6	26
Aug.	3	16	19
Sept.	2	26	12
Oct.	1	13	31
Nov.	0	23	24
Dec.	11	10	43

Annis.	*Mot. Med.* s.	°	'
1	10	20	36
2	9	11	12
3	8	1	48
4	7	14	59
5	6	5	35
6	4	26	11
7	3	16	47
8	2	26	57
9	1	20	23
10	0	11	9
11	11	1	45
12	10	14	56
13	9	5	32
14	7	26	8
15	6	16	44
16	5	29	54
17	4	20	30
18	3	11	6
19	2	1	42
20	1	14	53

Diebus.	*Mot. Med.* s.	°	'
1	0	22	35
2	1	15	9
3	2	7	44
4	3	0	18
5	3	22	53
6	4	15	28
7	5	8	2
8	6	0	37
9	6	23	12
10	7	15	46
11	8	8	21
12	9	0	55
13	9	23	30
14	10	16	5
15	11	8	39
16	0	1	14
17	0	23	48
18	1	16	23
19	2	8	58
20	3	1	32
21	3	24	7
22	4	16	42
23	5	9	16
24	6	1	51
25	6	24	25
26	7	17	00
27	8	9	35
28	9	2	9
29	9	24	44
30	10	17	18
31	11	9	53
32	0	2	28

Hor. Min	*Mot. Med.* ° '	' "	*Min.*	*Mot. Med* '	"
1	0	56	31	29	10
2	1	53	32	30	6
3	2	49	33	31	3
4	3	46	34	31	59
5	4	42	35	32	55
6	5	39	36	33	52
7	6	35	37	34	48
8	7	32	38	35	45
9	8	28	39	36	41
10	9	24	40	37	38
11	10	21	41	38	34
12	11	17	42	39	31
13	12	14	43	40	27
14	13	10	44	41	24
15	14	7	45	42	20
16	15	3	46	43	17
17	16	0	47	44	13
18	16	56	48	45	10
19	17	52	49	46	6
20	18	49	50	47	3
21	19	45	51	47	59
22	20	42	52	48	56
23	11	38	53	49	52
24	22	35	54	50	49
25	23	31	55	51	45
26	24	27	56	52	42
27	25	24	57	53	38
28	26	20	58	54	35
29	27	17	59	55	31
30	28	13	60	56	27

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

I here ſuppoſe the *Linea Apſidum* fix'd, as having no Argument from Obſervation to prove the contrary, tho' it be very probable that as the *Apogeum* of our *Moon* has a Motion about the *Earth* in about 9 Years, ſo that of this *Satellite* ought to have about *Saturn*, but with a much Longer Period, which future Obſervation may diſcover.

The diſtance of this *Satellite* from the Center of *Saturn* ſeems to be much about 4 Diameters of the *Ring*, or 9 of the *Globe*, and the Plane wherein it moves very little or nothing differing from that of the *Ring*, that is to ſay interſecting the Orb of *Saturn* $4^s\ 22^\circ$ and 10 22° *à* 1^a ✱ ♈, with an Angle of $23\frac{1}{2}$ Degrees, ſo as to be nearly Parallel to the *Earth's Equator*, whence the *Latitude* of the *Apogæon* Semicircle from $4^s\ 22^\circ$ to $10^s\ 22^\circ$ of *Saturn's Longitude* from the Firſt Star of ♈, will be Northern, and of the other Semicircle Southern; and the contrary in the other half of *Saturn's Longitude*, to wit, from $10^s\ 22^\circ$ to $4^s\ 22^\circ$ of his diſtance from the Firſt Star of ♈.

It follows now to ſhew how by the help of this Table to compute the place of this *Satellite*, to any time required.

Firſt we muſt have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the Firſt Star of ♈, (or rather the Place of the Earth viewed from *Saturn* together with its Latitude from the Orb of *Saturn*, but that being never fully $\frac{1}{2}$ of a degree we neglect it as a Nicety) and therefrom ſubſtract $10^s\ 22^\circ$, there remains the diſtance of *Saturn* from this *Equinoctial* Point, with which diſtance as with the *Longitude* of the *Sun*, take out the *Right Aſcenſion* and *Declination* thereto ($23\frac{1}{2}$ degrees being the Obliquity common to both) and to the *Right Aſcenſion* adding $10^s\ 22^\circ$ the Summ ſhall be the *Longitude* of the *Satellite's Apogæon*. Then ſay, as *Radius* to *Sine* of the *Declination*, ſo 8 to the greateſt *Latitude* in *Apogæo*, or *Perigæo* in the parts of the Semidiameter of the *Ring*.

Next Collect the *Middle Motion* of the *Satellite*, and from it Subſtract $10^s\ 22^\circ$ the remainder ſhall be the mean Anomaly, with which in the Table of the *Moon's* Primary *Equation*, take out the *Equation* anſwering thereto, and the half thereof added or ſubſtracted to or from the *Middle Motion*, according to the Table, gives the True Motion of the *Satellite*, from which ſubſtract the *Apogæon*, and if the remainder be more than 6 *Signs*, the *Satellite* is Occidental, if leſs Oriental; and as *Radius* to Sine of the remainder, ſo 8 to the Semidiameters of the *Ring*, or 18 to the Semidiameters of the *Globe*, that the *Satellite* is to the Eaſtward or Weſtward of the Center of *Saturn*, according to the aforegoing Precept.

Laſtly, as *Radius* to *Co-ſine* of the ſaid Remainder, ſo is the greateſt *Latitude* from the Line of the *Anſa*, to the *Latitude* ſought.

Here Note, that I purpoſely neglect the Inequality of the diſtance ariſing from the *Eccentricity*, as being too ſmall to be any way obſervable.

Laſtly to clear all difficulties that may ariſe to them that are but little verſed in this ſort of Calculation, I have added an Example of the Work, that where the precept may ſeem obſcure it may be thereby illuſtrated.

An. 1657. *Maii* 19. *ſt. n.* M. *Hugens* obſerved the *Satellite* very near *Saturn* on the Weſtern ſide, and very little above the Line of the *Anſæ*. I ſuppoſe this about 10ʰ *p. m.* Let us now *Calculate* to that time.

1657 *Maii*	9ᵈ 9ʰ	10′	*Londini*
Saturni Locus	♍	28°	57′
♄ *à* 1*a* ✱ ♈.	5ˢ	0	32
Æquinoct. Sub.	10	22	00
♄ *ab Æquinoct.*	6	8	32
Aſcen. Recta	6	7	50
Apogæon.	4	29	50
Declin. Auſt.		3	23

Mot. Med. Satel.	s.	°	′
1641	8	29	17
16	5	29	54
Maii	6	9	14
9ᵈ	6	23	12
9ʰ 40′	0	9	5
Long. Med. Satel.	4	10	42
Apocron.	10	22	00
Anomalia Med.	5	18	42
Æquatio Sub.	0	0	31
Long. Vera Satel.	4	10	11
Apogæon	4	29	50
Reſiduum	11	10	21
h. e. ante *Apogæum.*	0	19	39

Ergo $2\frac{7}{10}$ *Semid. Annuli ad occaſum & $\frac{9}{10}$ ad Boream*; Agreeing exactly with the Deſcription and Figure of M. *Hugens.*

I here call the Plane of this *Satellite's* Orb, which hitherto I ſuppoſe the ſame with that of the *Ring*, *Saturn's Equinoctial*, not that any diſcovery hath been able to prove that the *Axis* of that *Globe* is at right Angles thereto, but becauſe it hath pleaſed M. *Hugens* to call it ſo, and likewiſe becauſe it is ſo nearly Parallel to our *Globe's Equinoctial*; Nevertheleſs to ſpeak my Opinion, I believe that the *Axis* is Inclined, and that not a little, to the Plane of the *Ring*; for as the Reflection of the *Sun's* Light from the *Ring* is a great convenience to that Hemiſphere of *Saturn*, which beholds its Illuminate ſide; ſo that the other Hemiſphere is very much Incommoded by the Shadow of the *Ring*, which for many Months, and in ſome Parallels for ſeveral Years, occaſions a continual Night by the Interception of the *Sun's* Beams, which is a conſequence that Demonſtratively follows the Poſition of the *Ring*, in the Plane of *Saturn's Æquator*. Now this great inconvenience would be in ſome meaſure relieved by the Oblique Poſition of the *Axis*, for then the Parallels of Latitude interſecting the Plane of the *Ring*, many, and in moſt caſes all of them, might for ſome time in every Diurnal Revolution of the Globe, free themſelves from this *Eclipſe*, which otherwiſe were ſufficient to render this Globe of *Saturn* unfit for any ſettled Habitation; but this is but Conjecture.

The other two *Satellites* of *Saturn* diſcovered by *S. Caſſini* at *Paris*, *An.* 1672. and 1673. I muſt confeſs I could never yet ſee; I have been told that they Diſappear for about $3\frac{2}{3}$ of *Saturn*'s Revolution, and were only to be ſeen when the *Anſæ* were very ſmall, it being ſuppoſed that the Light which proceeds from the *Anſæ*, when conſiderably opened, might hide theſe *Satellites*.

The Theory of the 5 Satellites *of* Saturn *corrected; by M.* Caſſini. n. 187. p. 299.

LXXXII. 1. La *Diſtance* du *Premier Satellite* du Centre de *Saturn* m'a paru variable, & ſon *Mouvement* ſenſiblement Inegal, plus viſte, en ce temps, dans le Demicercle Occidental, que dans l'Oriental. J'ay dernierement determiné ſa Moyenne Diſtance de $\frac{19}{42}$ du Diametre de l' *Anneau* de *Saturne*, ſon *Mouvement* Journalier de $6^s\,10^{\circ}\,41'\,31''$. Ainſi ſi ſon *Mouvement* eſtoit Egal, la durée de ſa Conjonction avec *Saturne*, c' eſt à dire, tout le temps qu'il met à parcourir ſon *Anneau*, ſeroit de $7^h\,46'$. Elle m' a paru plus grande par les obſervations immediates, mais il eſt à remarquer que je n'ay juſqu'à preſent pû voir ce *Satellite* plus prés de *Saturne*, que d' un quart d' un *Anſe*.

J'ay Calculé l' *Epoque* de ſon *Mouvement*, pour le dernier *Decembre* 1685. à Midi au Meridien de *Paris* en ♑ 24° 50′.

La *Diſtance* du *Second Satellite* du Centre de *Saturne* m'a paru plus Uniforme. Je l' ay determinée d'un Diametre de l' *Anneau* & $\frac{1}{4}$: Son *Mouvement* paroit auſſi plus Egal. J' ay Calculé le Journalier de $4^s\,11^{\circ}\,31'\,30''$. Ainſi la Durée de ſa Conjonction devroit eſtre de $8^h\,36'$. Je n'ay pas non plus vû juſqu'à preſent ce *Satellite* plus proche de l' *Anneau* de *Saturne* que d'un $\frac{1}{4}$ d' un *Anſe*. Comme ce *Satellite* ſe voioit la plus part du temps dedans les Confins de la *Diſtance* du *Premier*, au quel il eſt Egal en Grandeur, & Semblable dans la Couleur, la difficulté de diſtinguer l' un de l' autre à éſté extreme, de ſort que ſans un aſſiduité particuliere aux Obſervations, & ſans une grande multitude de Combinaiſons je n'en ſerois pas venu à bout.

J' ay determiné l' *Epoque* de ce *Satellite* pour le 31 *Decembre* 1685. à Midien ♍ 9° 10′.

La *Diſtance* du *Troiſieme* du Centre de *Saturne* paroit d' un Diametre de l' *Anneau* & $\frac{3}{4}$. Son *Mouvement Journalier* $2^s\,18^{\circ}\,41'\,50''$. Ainſi ſa Conjonction doit durer 10 heures. L' *Epoque* de ſon *Mouvement* pour le Midi du Dernier de l'Année 1685 ♍ 9° 39′.

La *Diſtance* du Quatrieme *Satellite* au Centre de *Saturne* paroit de 4 Diametres de l' *Anneau*. Son *Mouvement Journalier* de $22^{\circ}\,34'\,38''$. la Durée de ſa Conjonction $15^h\,6'$. L' *Epoque* de ſon *Mouvement* au meſme temps & lieu que les autres, en ♓ 18° 1′.

La *Diſtance* du *Cinquieme Satellite* au Centre de *Saturne* de 12 Diametres de l' *Anneau*. Son *Mouvement Journalier* de $4,\,32'\,17''$. Ses Conjonctions durent 24 heures. L' *Epoque* de ſon *Mouvement* au meſme temps & lieu, en ♓ 16° 19′. Sur ces Principes on peut conſtruire les *Tables*, & les *Ephemerides*.

By .... *ib.* p. 300.

2. The following Tables are Calculated from theſe Elements, and Reduced to the *Meridian* of *London*.

Tabula Motus Medii Intimi Satellitis Saturni, à Caſſino *Detecti Anno* 1686.

Anno Chriſti *Curr.*	*Epochæ*			*Annis.*	*Mot. Med.*			*Diebus.*	*Mot. Med.*			*Hor. Mi.*	*Mot. Med.*			*Min.*	*Mot. Me.*	
	s.	°	′		s.	°	′		s.	°	′		s.	° ′	′ ″		°	′
1681	♒	19	34	1	4	2	34	1	6	10	42	1	0	7	57	31	4	6
1685	♑	10	30	2	8	5	7	2	0	21	23	2	0	15	53	32	4	14
1686	♉	13	4	3	0	7	41	3	7	2	5	3	0	23	50	33	4	22
1687	♍	15	37	4	10	20	56	4	1	12	46	4	0	31	47	34	4	30
1688	♑	18	11	5	2	23	30	5	7	23	28	5	0	39	44	35	4	38
1689	♐	1	26	6	6	26	4	6	2	4	9	6	0	47	40	36	4	46
1701	♌	4	14	7	10	28	38	7	8	14	15	7	0	55	37	37	4	54
				8	9	11	52	8	2	25	32	8	1	3	34	38	5	2
Menſ.	*Mot. Med*																	
Anni				9	1	14	26	9	9	6	14	9	1	11	31	39	5	10
Com.	s.	°	′	10	5	17	00	10	3	16	55	10	1	19	28	40	5	18
				11	9	19	34	11	9	27	36	11	1	27	24	41	5	26
Jan.	0	0	0	12	8	2	48	12	4	8	18	12	1	35	21	42	5	34
Feb.	5	1	27															
Mar.	3	0	49	13	0	5	22	13	10	19	00	13	1	43	18	43	5	42
April.	8	2	16	14	4	7	56	14	4	29	42	14	1	51	15	44	5	50
				15	8	10	29	15	11	10	23	15	1	59	11	45	5	58
Maii	6	23	2	16	6	23	43	16	5	21	4	16	2	7	8	46	6	5
Jun.	11	24	29															
Jul.	10	15	15	17	10	16	17	17	0	1	46	17	2	15	5	47	6	13
Aug.	3	16	42	18	2	18	51	18	6	12	28	18	2	23	1	48	6	21
				19	7	1	25	19	0	23	9	19	2	30	58	49	6	29
Sept.	8	18	9	20	5	14	39	20	7	3	50	20	2	38	55	50	6	37
Oct.	7	8	54															
Nov.	0	10	21					21	1	14	32	21	2	46	52	51	6	45
Dec.	11	1	7					22	7	25	13	22	2	54	49	52	6	53
								23	2	5	55	23	3	2	45	53	7	1
								24	8	16	36	24	3	10	42	54	7	9
								25	2	27	18	25	3	18	39	55	7	17
								26	9	7	59	26	3	26	35	56	7	25
								27	3	18	41	27	3	34	32	57	7	33
								28	9	29	22	28	3	42	28	58	7	41
								29	4	10	3	29	3	50	25	59	7	49
								30	10	20	45	30	3	58	22	60	7	57

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

Tabula Motus Medii Penintimi Satellitis Saturni, à Caſſino *Detecti Anno* 1686.

Anno Chriſti *Curr.*	*Epocha.* s.	°	'
1681	♓	20	41
1685	♐	28	42
1686	♉	5	20
1687	♍	11	57
1688	♑	18	35
1689	♎	6	44
1701	♒	0	48

Menſ. Anni Com.	*Mot. Med.* s.	°	'
Jan.	0	0	0
Feb.	3	27	16
Mar.	6	19	58
April.	10	17	15
Maii	10	3	0
Jun.	2	0	16
Jul.	1	16	1
Aug.	5	13	18
Sept.	9	10	34
Oct.	8	26	19
Nov.	0	23	36
Dec.	0	9	21

Annis.	*Mot. Med.* s.	°	'
1	4	6	37
2	8	13	15
3	0	19	52
4	9	8	1
5	1	14	39
6	5	21	16
7	9	27	54
8	6	16	3
9	10	22	40
10	2	29	18
11	7	5	55
12	3	24	4
13	8	00	42
14	0	7	19
15	4	13	57
16	1	2	6
17	5	8	43
18	9	15	21
19	1	21	58
20	10	10	7

Diebus.	*Mot. Med.* s.	°	'
1	4	11	31
2	8	23	3
3	1	4	34
4	5	16	6
5	9	27	37
6	2	9	9
7	6	20	40
8	11	2	12
9	3	13	43
10	7	25	15
11	0	6	46
12	4	18	18
13	8	29	49
14	1	11	21
15	5	22	52
16	10	4	24
17	2	15	55
18	6	27	27
19	11	8	58
20	3	20	30
21	8	2	1
22	0	13	33
23	4	25	4
24	9	6	36
25	1	18	07
26	5	29	39
27	10	11	10
28	2	22	42
29	7	4	13
30	11	15	45

Hor. Mi.	*Mot. Med. Sex.* °	'	"
1	0	5	29
2	0	10	58
3	0	16	26
4	0	21	55
5	0	27	24
6	0	31	53
7	0	38	22
8	0	43	51
9	0	49	19
10	0	54	48
11	1	00	17
12	1	5	46
13	1	11	15
14	1	16	44
15	1	22	12
16	1	27	42
17	1	33	11
18	1	38	39
19	1	44	8
20	1	49	37
21	1	55	6
22	2	00	34
23	2	6	3
24	2	11	31
25	2	17	00
26	2	22	29
27	2	27	58
28	2	33	26
29	2	38	55
30	2	44	24

Min.	*Mot. Me.* °	'
31	2	50
32	2	56
33	3	1
34	3	7
35	3	12
36	3	17
37	3	23
38	3	28
39	3	34
40	3	40
41	3	45
42	3	50
43	3	56
44	4	1
45	4	7
46	4	12
47	4	17
48	4	23
49	4	28
50	4	34
51	4	39
52	4	45
53	4	50
54	4	56
55	5	1
56	5	7
57	5	12
58	5	18
59	5	23
60	5	29

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

Tabula Motus Medii Satellitis Saturnii Medii, à Caſſino *detecti Anno* 1673.

Anno Chriſti *Curr.*	*Epochæ.* s.	°	'
1661	♎	22	50
1681	♍	16	3
1685	♒	2	41
1686	♏	17	10
1687	♍	1	39
1688	♊	16	9
1689	♊	19	20
1701	♌	9	15

Menſ. Ann. Com.	*Mot. Med.* s.	°	'
Jan.	0	0	0
Feb.	9	9	37
Mar.	10	23	8
April	8	2	45
Maii	2	23	40
Jun.	0	3	17
Jul.	6	24	12
Aug.	4	3	49
Sept.	1	13	25
Oct.	8	4	20
Nov.	5	13	57
Dec.	0	4	52

Annis.	*Mot. Med.* s.	°	'
1	9	14	29
2	6	28	58
3	4	13	27
4	4	16	38
5	2	1	8
6	11	15	37
7	9	00	6
8	9	3	17
9	6	17	46
10	4	02	15
11	1	16	45
12	1	19	55
13	11	4	24
14	8	18	54
15	6	3	23
16	6	6	34
17	3	21	3
18	1	5	32
19	10	20	1
20	10	23	12

Diebus.	*Mot. Med.* s.	°	'	*Hor. Min.*	*Mot. Med. Sex.* °	'	''	*Min.*	*Motus Med.* °	'
1	2	18	42	1	0	3	17	31	1	41
2	5	7	24	2	0	6	33	32	1	45
3	7	26	5	3	0	9	50	33	1	48
4	10	14	47	4	0	13	7	34	1	52
5	1	3	29	5	0	16	24	35	1	55
6	3	22	11	6	0	19	40	36	1	58
7	6	10	53	7	0	22	57	37	2	1
8	8	29	35	8	0	26	14	38	2	5
9	11	18	16	9	0	29	31	39	2	8
10	2	6	58	10	0	32	47	40	2	11
11	4	25	40	11	0	36	4	41	2	14
12	7	14	22	12	0	39	21	42	2	18
13	10	3	4	13	0	42	38	43	2	21
14	0	21	46	14	0	45	55	44	2	24
15	3	10	27	15	0	49	11	45	2	28
16	5	29	9	16	0	52	28	46	2	31
17	8	17	51	17	0	55	45	47	2	34
18	11	6	33	18	0	59	1	48	2	37
19	1	25	15	19	1	2	18	49	2	40
20	4	13	57	20	1	5	35	50	2	44
21	7	2	39	21	1	8	52	51	2	47
22	9	21	20	22	1	12	8	52	2	50
23	0	10	02	23	1	15	25	53	2	54
24	2	28	44	24	1	18	42	54	2	57
25	5	17	26	25	1	21	59	55	3	00
26	8	6	8	26	1	25	15	56	3	4
27	10	24	50	27	1	28	32	57	3	7
28	1	13	32	28	1	31	49	58	3	10
29	4	2	13	29	1	35	6	59	3	13
30	6	20	55	30	1	38	22	60	3	17

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

Tabula Motus Medii Penextimi Satellitis Saturni, *ab* Hugenio *inventi Anno* 1673.

Anno Chriſti *Curr.*	*Epocha.* s.	°	′
1641	♑	24	43
1661	♓	11	19
1681	♈	27	56
1685	♐	13	15
1686	♏	3	56
1687	♍	24	37
1688	♌	15	19
1689	♋	28	34
1701	♊	14	32

Menſ. Ann. Com.	*Mot. Med.* s.	°	′
Jan.	0	0	0
Feb.	11	9	54
Mar.	8	12	3
April.	7	21	57
Maii	6	9	16
Jun.	5	19	10
Jul.	4	6	29
Aug.	3	16	22
Sept.	2	26	16
Oct.	1	13	35
Nov.	0	23	29
Dec.	11	10	48

Annis.	*Mot. Med.* s.	°	′
1	10	20	41
2	9	11	22
3	8	2	3
4	7	15	19
5	6	6	00
6	4	26	41
7	3	17	22
8	3	00	39
9	1	21	20
10	0	12	1
11	11	2	42
12	10	15	58
13	9	6	39
14	7	27	20
15	6	18	1
16	6	1	17
17	4	21	58
18	3	12	40
19	2	3	21
20	1	16	36

Diebus.	*Mot. Med.* s.	°	′	*Hor. Min.*	*Mot. Med.* ° ′	′ ″	*Min.*	*Motus Med.* ′	″
1	0	22	35	1	0	56	31	29	10
2	1	15	9	2	1	53	32	30	6
3	2	7	44	3	2	49	33	31	3
4	3	0	18	4	3	46	34	31	59
5	3	22	53	5	4	42	35	32	55
6	4	15	28	6	5	39	36	33	52
7	5	8	2	7	5	35	37	34	48
8	6	0	37	8	7	32	38	35	45
9	6	23	12	9	8	28	39	36	41
10	7	15	46	10	9	24	40	37	38
11	8	8	21	11	10	21	41	38	34
12	9	0	55	12	11	17	42	39	31
13	9	23	30	13	12	14	43	40	27
14	10	16	5	14	13	10	44	41	24
15	21	8	39	15	14	7	45	42	20
16	0	1	14	16	15	3	46	43	17
17	0	23	48	17	16	0	47	44	13
18	1	16	23	18	16	56	48	45	10
19	2	8	58	19	17	52	49	46	6
20	3	1	32	20	18	49	50	47	3
21	3	24	7	21	19	45	51	47	59
22	4	16	42	22	20	42	52	48	56
23	5	9	16	23	21	38	53	49	52
24	6	1	51	24	22	35	54	50	49
25	6	24	25	25	23	31	55	51	45
26	7	17	00	26	24	27	56	52	42
27	8	9	35	27	25	24	57	53	38
28	9	2	9	28	26	20	58	54	35
29	9	24	44	29	27	17	59	55	31
30	10	17	18	30	28	13	60	56	27

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

Tabula Mediorum Motuum *Extimi Satellitis Saturnii,* à Caſſino *Detecti Anno* 1671.

Anno Chriſti *Curr.*	*Epochæ.* s.	°	′
1661	♓	24	45
1681	♈	25	15
1685	♍	25	21
1686	♉	1	44
1687	♐	8	7
1688	♋	14	31
1689	♒	25	27
1701	♉	25	45
Menſ. Ann. Com.	*Mot. Med* s.	°	′
Jan.	0	0	0
Feb.	4	20	41
Mar.	8	27	45
April	1	18	25
Maii	6	4	34
Jun.	10	25	15
Jul.	3	11	23
Aug.	8	2	4
Sept.	0	22	45
Oct.	5	8	53
Nov.	9	29	34
Dec.	2	15	43

Annis.	*Mot. Med.* s.	°	′	*Diebus.*	*Mot. Med.* s.	°	′	*Hor. Min.*	*Mot. Me* ° ′	′ ″	*Min.*	*Mot. Me·* ′	″
1	7	6	23	1	0	4	32	1	0	11	31	5	51
2	2	12	47	2	0	9	5	2	0	23	32	6	3
3	9	19	10	3	0	13	37	3	0	34	33	6	14
4	5	00	6	4	0	18	9	4	0	45	34	6	25
5	0	6	29	5	0	22	41	5	0	57	35	6	37
6	7	12	53	6	0	27	14	6	1	8	36	6	48
7	2	19	16	7	1	1	46	7	1	19	37	7	00
8	10	0	12	8	1	6	18	8	1	31	38	7	11
9	5	6	35	9	1	10	50	9	1	42	39	7	22
10	0	12	59	10	1	15	23	10	1	53	40	7	34
11	7	19	22	11	1	19	55	11	2	5	41	7	45
12	3	0	18	12	1	24	27	12	2	16	42	7	56
13	10	6	41	13	1	28	59	13	2	27	43	8	8
14	5	13	5	14	2	3	32	14	2	39	44	8	19
15	0	19	28	15	2	8	4	15	2	50	45	8	30
16	8	0	24	16	2	12	36	16	3	1	46	8	42
17	3	6	47	17	2	17	8	17	3	13	47	8	53
18	10	13	11	18	2	21	41	18	3	24	48	9	4
19	5	19	34	19	2	26	13	19	3	35	49	9	16
20	1	0	30	20	3	0	46	20	3	47	50	9	27
				21	3	5	18	21	3	58	51	9	38
				22	3	9	50	22	4	9	52	9	50
				23	3	14	22	23	4	21	53	10	1
				24	3	18	54	24	4	32	54	10	12
				25	3	23	27	25	4	43	55	10	24
				26	3	27	59	26	4	55	56	10	35
				27	4	2	31	27	5	6	57	10	46
				28	4	7	4	28	5	17	58	10	58
				29	4	11	36	29	5	29	59	11	9
				30	4	16	8	30	5	40	60	11	21

In *Anno Biſſextili* poſt *Februarium* adde unum diem, motumque ei competentem.

I ſhall only add that the Proportion of the Squares of the *Times* of the *Periods* to the Cubes of the *Diſtances*, (which is propoſed as probable by *Kepler*, but now demonſtratively found true by Mr. *Newton*,) gives us nicely the Proportion of the *Diſtances* of theſe *Planets* from the Center of *Saturn*; and ſuppoſing the *Satellite* of *Hugens* four Diameters of *Saturn's Ring* diſtant from him, we ſhall find by the *Periods* the Diſtances as follows.

	Periodus.			*Diſtantia.*
	d.	h.	′	*Diam.*
Intimi ——	1	21	$18\frac{1}{2}$	0, 964
Penintimi ——	2	17	$41\frac{1}{2}$	1, 235
Medii ——	4	13	$47\frac{1}{4}$	1, 740
Penextimi ——	15	22	41	4, 000
Extimi ——	79	7	54	11, 621

Theſe *Diſtances* may be uſed, as more accurate than thoſe obtained by Obſervation, which yet differ but little therefrom.

The Phaſis of Jupiter; *by* *Dr.* Hook. n. 14. *p.* 245. *Fig.* 139.

LXXXIII. *An.* 1666. *Jun.* 26. between 3 and 4 of the Clock in the Morning, I obſerved the Body of *Jupiter* through a 60 Foot Glaſs, and found the apparent Diameter of it through the Tube, to be ſomewhat more than two Degrees, that is about 4 times as big as the Diameter of the *Moon* appears to the naked Eye.

I ſaw the Limb pretty round, and very well defined without Radiation. The parts of the *Phaſis* of it had various Degrees of Light. About *a*, and *f*, the North and South Poles of it, ſomewhat darker, and by degrees it grew brighter towards *b* and *e*, two *Belts* or *Zones*; the one of which, *b*, was a ſmall dark *Belt* croſſing the Body Southward; Adjoining to which was a ſmall Line of a ſomewhat lighter Part; and below that again, Southwards, was the Great black *Belt* *c*. Between that, and *e*, the other ſmaller black *Belt*, was a pretty large and bright *Zone*, but the Middle *d*, was ſomewhat darker than the Edges.

The Revolution of Jupiter *upon his* Axis; *by* S. Campani. n. 1. *p.* 3.

LXXXIV. 1. *S. Campani* affirms that, by the Goodneſs of his Glaſſes, he hath Obſerved certain *Protuberances* and Inequalities of *Jupiter*: and he is now Obſerving whether they do not change their Situation.

By Dr. Hook. *ib.*

2. *An.* 1664. *May* 9. about 9 a Clock at Night, Mr. *Hook* with an Excellent 12 Foot Teleſcope obſerved a ſmall *Spot* in the biggeſt of the three *Obſcurer Belts* of *Jupiter*, and Obſerving it from time to time, he found that within two Hours after, the ſaid *Spot* had moved from Eaſt to Weſt, about half the Length of the Diameter of *Jupiter*.

By S. Divini. n. 12. *p.* 209.

3. *Euſtachio de Divinis* pretends, that the *Permanent Spot* in *Jupiter* hath been firſt of all diſcover'd with his Glaſſes; that *P. Gotignies* is the firſt that

hath thence deduced the *Motion* of *Jupiter* about his *Axis*; and that M. *Caſſini* at firſt oppoſed it: *But that Spot was obſerved in* England *a good while before.*

4. There are two Sorts of *Spots* at certain times to be ſeen in the Diſque of *Jupiter*. One Sort are nothing but the *Shadows* of his *Satellites*: but the other have ſome reſemblance to thoſe that are ſeen in the *Moon*: and they are perhaps of the ſame Nature with thoſe that are called *Belts*. They Move from the Eaſtern to the Weſtern Limb; their apparent Motion is unequal, and ſwifter near the Center than the Circumference; and they never are ſo well ſeen as when they approach to the Center; they being very narrow and almoſt imperceptible when they approach to the Circumference; which makes us believe, that they are flat and ſuperficial to *Jupiter*.

By M. Caſſini.
n. 8. *p.* 143.
n. 10. *p.* 172.
n. 35. *p.* 687.
n. 82. *p.* 4039.

Among theſe *Spots* there is none ſo ſenſible, as that which is ſituated in the Northern-part of the *Southern-Belt*. Its Diameter is about the tenth part of that of *Jupiter*; and at the time that its Center is neareſt to that of *Jupiter*, it is Diſtant from it about the third part of the Semidiameter of that *Planet*.

M. *Caſſini*, after he had made many Obſervations of this *Spot* during the Summer of the Year 1665, found, that the *Period* of its apparent *Revolution* is of $9^{h}\,56'$. He continued to Obſerve it till the Beginning of 1666, when *Jupiter* approach'd to the Beams of the *Sun*: But after he was got free of the *Sun-Beams* it was difficult to be diſcern'd. This gave grounds that it might be of the Nature of the *Spots of the Sun* (which after having appeared for a while, diſappear for ever) M. *Caſſini* ceaſed at length to obſerve them.

But *Jan.* 19. 1672. (*ſt. n.*) when he obſerv'd *Jupiter*, at $4\frac{3}{4}$ h in the Morning, he perceived in the ſame Place of his Diſque the Figure of the ſame *Spot*, adhering to the ſame *Southern-Belt*. It was already gone beyond the Moiety of this *Belt*, and he ſaw it advance little by little towards the Weſtern Limb, to which it ſeemed to be very near at $6\frac{1}{4}$h.

By the Celerity of its Motion near the Center, and by the Place where he had begun to ſee it, he judged that it might have been in the midſt of the *Belt* at $4^{h}\,35'$ in the Morning. And as he prepared himſelf to make Ephemerides of its Motion for that *Year* 1672, he perceived that in thoſe he made for the Year 1666, this *Spot* had been in the midſt of *Jupiter* the ſame Day, namely the 19th of *Jan.* at the ſame Hour. So that in ſix Years, of which one is a *Biſſextile*, it is found to have made, in reſpect of the Earth, at leaſt 5294 Revolutions, each of $9^{h}\,55'\,85''$, compenſating one Revolution by another; and at moſt 5294 Revolutions of $9^{h}\,55'\,51''$; foraſmuch as he was aſſured of the preciſeneſs of one *Mean Revolution* to one eighth of a Minute.

Until then he had never ſeen an immediate Return of this *Spot* after 9 hours and 56 minutes, becauſe it had not happened, that *Jupiter* after the Apparition of the *Spot* had ſtayed, in one and the ſame Night, long enough above the *Horizon*, at leaſt a ſufficient height, to obſerve him with due Diſtinctneſs. He had only concluded the time of this *Revolution* by Returns obſerved after about 20, 30, and 50 hours; and he had more preciſely limited it by Obſervations more diſtant. But the Night after *Mar.* 1, at $7\frac{1}{4}^{h}$ in the Evening, he ſaw this *Spot* in the midſt of the *Belt*; and the ſame Night at

5^h 26′. in the Morning, he ſaw it again returned preciſely to the ſame Place. *Mar*. 3. He together with M. *Buot* and M. *Mariotte*, began to ſee at 8^h 4′ the *Spot* already ſomewhat removed from the Oriental Limb, but yet obſcure and ſmall. At 8^h 47′. they ſaw it very diſtinctly advancing towards the middle of the *Belt*. At 9^h 15′ 40″. until 9^h 8′. they ſaw it in the midſt of the *Belt*. At 9^h 15′. it was paſt the middle, and was come nearer to the Occidental Limb. And a little after the Heavens being over-caſt, he could obſerve it no further.

Places of Jupiter *obſerv'd; by Mr.* Flamſteed *at* Derby. n. 82. *p*. 4036.

LXXXV. 1. *An*. 167$\frac{1}{2}$. *Feb*. 16. 7^h 44$\frac{1}{2}$′. Alto *Jove* 18° 10′. ejus Diſtantiam à Fixa Lucis 4^{ae} (cujus Lat. 1° 40′ Bor. Locus *Mihi* ♍ 14° 7′ 16″. at *Streetio*, 14° 3′ 54″) Tubo Longiori dimenſus ſum, 16′ 33″. & Differentiam Altitudinum Centrorum ♃ & ✱^{ae} 1′ 1″.

17 *Feb*. 7^h 25′. *p. m*. Alto ♃ 15° 54′. ipſe à *Fixa* diſtitit 21′ 50″; Altitudinum differentia erat 8′ 40″.

18 *Feb*. 7^h 0′. *Fixa* diſtantia à Centro ♃ erat 28′ 15″; Altitudinum differentia circ. 15′ 29″. In utraque Obſervatione *Erro* Altior erat *Fixâ*, à qua ſemper Meridianum verſus Stetit.

Inito dein Calculo ad dies ſingulos & horas obſervationum, inveſtigavi

	d.	h.	′	d.	h.	′	d.	h.	′
Februar.	16	7	44$\frac{1}{2}$	17	7	25	18	7	0
	°	′	″	°	′	″	°	′	″
Jovis à *Fixa* Longitudinem in Antecedentia	0	9	16	0	17	22	0	25	12
Latitudinem ad Auſtrum ab ea	0	13	30	0	13	14	0	12	45
Ergo ♃^{is} Latitudo Borealis	1	26	30	1	26	46	1	27	15
Locus verus ♍ …. { *Mihi*	13	58	0	13	49	54	13	42	4
{ *Streetio.*	13	54	38	13	46	32	13	38	42

2. *Mar*. 15. *v*. Obſervare cœpi *Jovis* diſtantias & Poſitiones à *Stella* ♌ 38. cujus Latitudo 1° 20′$\frac{1}{2}$ Bor. Locus *Streetio* ♍ 9° 54′ 0″; *mihi* vero ♍ 9° 57′ 20″. 7^h 25′. *p. m*. Alto ♃ 32° 52′. Diſtantia Centri Ipſius ab ipſa 33′ 50″. Altitudinum Differentia circit. 20′ 42″.

Mar. 16. 7^h. 8′. Alta *Fixa* 36°. *Jovis* ab ea diſtantia erat 27′. 7″. Altitudo minor 16′ 3″.

Mart. 19. Alto ♃ 29° 35′. *i. e.* 6^h 45′. *Fixa* Altior erat quam *Planeta* 2′ 24″; à quo 6^h 55′. diſtitit. 10′ 21″.

Hor. 7. 11. *Satelles* 4^{tus} à *Fixa*, 7′ 28″. Etiamnum *Erro* ſemper altior apparuit, ſed verè fuit depreſſior quam *Fixa*: poſtea humilior viſus eſt, ſed revera fuit Altior.

Mar. 20. melius præparato ad Altitudinum Differentias capiendas Micrometro, obſervationes habui (ſic putem) accuratiſſimas, quæ ſequuntur.

Jov.

Jov. Alt.		*Temp.*				
°	′	h.	′		′	″
30	0	6	44	*Jovis* Centrum vere altius erat *Stella*	2	13
30	47	6	51⅓	Altitudinum eadem differentia rurſus capta	2	14
32	0	6	59	*Jovis* Centrum à *Fixa* diſtitit	7	0
38	30	7	54¾	Centrum *Jovis* verè altius *Fixa*	3	14
40	50	8	18⅓	Altitudinum differentia denuò capta	3	42
				Et deinde Centrorum diſtantia erat	7	5

Ad Locum *Jovis* ex his Obſervationibus acquirendum ſupputavi, ad

	h.	′	″	h.	′	″
	6	51	30	8	18	40
	°	′	″	°	′	″
Angulos *Circuli Verticalis* cum *Ecliptica*	35	39	—	46	15	—
Jovis erat à *Fixa* diſtantia	0	7	0	0	7	0
Altitudinum differentia	0	2	14	0	3	42
Ergo ♃ erat in conſequentia *Fixa*	0	2	3	0	1	44
Cum Latitudine majori	0	6	42	0	6	47
Quare *Jovis* Latitudo vera	1	27	12	1	27	17
Locus verus ♍ { *Streetio*	9	56	3	9	55	44
{ *Mihi*	9	59	23	9	59	4

Maii.	*Temp.*		*Alt. Jov.*					
d.	h.	′		′				
24	10	00	14	10	Humilior erat *Jovis* Centrum quam	0	′	″
					Stella dicta ♌ 38′	0	7	46
					A qua ſemel ejus cepi diſtantiam	0	20	00
					deinde	0	19	54
26	8	46	33	30	Diſtantia centri ♃ à *Fixa*	0	10	4
Cœlo	9	00	31	50	*Fixa* altior erat quam ♃	0	6	30
sereno.	9	20	29	10	Altitudinum differentia erat	0	6	38 *
	9	33	27	20	Differ. Azymuthorum ♃ & *Fixa*	0	7	19
	9	36	27	00	Diſtantia ♃ à *Fixa* denuò capta	0	10	2

2.
N. 86. p. 5037.

	h.	′	°	′		°	′	″	
27. *Ventis interdum tubum moventibus.*	8	59	31	24	Centri ♃ à *Fixa* diſtantia ——	0	6	2	*
	9	7	30	30	Azymuthorum differentia ——	0	1	50	
	9	16	29	13	Eadem rurſus differentia ——	0	1	52	*
	9	23	28	10	Centri ♃ à *Fixa* diſtantia ——	0	6	1	
28. *Ventis valide flantibus & Teleſcopium concutientibus.*	8	57	31	8	A *Fixa* diſtantia centri ♃ ——	0	6	2	*
	9	19	28	10	Differentia Azimuthorum centri, &c	0	3	29	
	9	31	26	30	Iſthæc Differentia rurſus ——	0	3	32	
	9	34	26	8	Diſtantia rurſus ——	0	6	7	
30. *Cælo ſatis ſereno.*	9	36	26	15	*Jovis* diſtantia à *Fixa* ——	0	15	38	
	Hac & præcedente nocte in Conſequentia *Fixæ* erat, antea ſemper in Antecedentia.								

Harum obſervationum certiſſimas habeo, quas aſteriſco (*) notavi Azymuthorum differentias; diebus 27 & 28 obſervatas, nimis ſtrictas acceptas æſtimo, ob vacillationem Tubi, quapropter, quando non ut volui eas accuratè dimetiri licebat, ne nimis amplas caperem curabam.

Ad *Jovis* locum ab his obſervationibus obtinendum ſupputavi.

	d.	h.	′	d.	h.	′	d.	h.	′
Maii.	26	0	0	27	9	7	28	9	19
	°	′	″	°	′	″	°	′	″
Angulos Parallacticos ſive Circuli *Verticalis* cum *Ecliptica.* ——	80	47	—	79	49	—	78	36	—
Jovis à *Fixa* diſtantia obſervata ——	9	10	4	0	6	2	0	6	3
Differentia abſ. Alt. ——	0	6	30	*Azym.* 1	52		*Azym.* 3	30	
Ergo *Angulus Poſitionis* ——	31	1		62	38	0	66	3	0
Et *Planeta* in antecedentia *Fixa* ——	0	8	38	0	2	51	*conſ.* 2	27	
Cum minori Latitud. Boreali ——	0	5	11	0	5	19	0	5	32
Fixa Latitudo *Tychonica* Bor. ——	1	20	30	1	20	30	1	20	30
Fixæ Locus ♍ —— *Streetio* ——	9	54	9	9	54	9	9	54	9
Mihi. ——	9	57	30	9	57	30	9	57	30
Jovis itaque Locus *Streetio* ——	9	45	31	9	51	18	9	56	36
Mihi ——	9	48	52	9	54	39	9	59	57
Cum Latitudine Boreali ——	1	15	19	1	15	11	1	14	58

3. An-

5. *An.* 1673. *Mar.* 13. veſp, *Jupiter Aphelius* Pronus ad Phaſin *Acronitam*, & *Limitem Orbitæ* Boreum paululum tranſgreſſus, *Retrogradus* inceſſit verſus 9^{am} ♍ *Lucis quartæ*, à qua (Alto eo ſex circiter gradus) Limbi ejus Remotiſſimi diſtantiam, ſeptempedali Tubo & *Micrometro Townleiano* cœpi 4560 52′ 34″. n. 94. p. 6033.

Mar. 17. circa horæ dimidiam poſt exortum *Jovis* ejus eodem Tubo, Limbi Remotiſſimi à *Fixa* cepi iterum diſtantiam 2073=23′ 54″.

Mar. 20. Sequentes habui obſervationes, Primam Breviori Tubo, digitorum tantum 85, reliquas Longiori, viz. 164. dig.

	Fix. Alt.		*Hor. Sup.*				*Limb. ph.*		*Cent.*	
	°	′	h.	′			′	″	′	″
1	6	0	7	14	Limbi ♃ remotior. à *Fixa* diſt.	850	9	48	9	24
2	——		——		Eadem diſt. Tubo Long. capta	1650	9	52	9	28
3	14	40	8	16	Limb. ♃ Inf. depreſſ. ac *Fixa*	784	4	41	4	17
4	15	40	8	23	Alt. eadem repetita Differen.	786	4	41	4	17
5	——		——		*Jovis* Diameter ——	135	0	48		
6	16	25	8	26	Limb. iterum capta diſtantia	1665	9	57	9	33
7	——		——		Denuo ——	1658	9	54	9	30
8	19	00	8	50	Differ. Alt. Limbi *Jov.* & *Fixæ*	138	5	00	4	36

Mar. 26. *Veſp.* Alto *Jove* 15° 50′. Limbi ſui remotioris à *Fixa* diſtantiam, eodem minori Tubo dimenſus ſum 4205=48′ 30″.

Ad *Planetæ* Locum ex his annotationibus eliciendum, ſtructis ſupputationibus, invenio.

	d.	h.	′		h.	′
Mar.	20	8	16	—	8	50
	°	′	″	°	′	″
Angulum Parallacticum ——	34	44	0	37	30	0
Centrum *Jovis* à *Fixa* diſtitit ——	0	9	38	0	9	30
Altitudinum differentia erat ——	0	4	17	0	4	36
Ergo, *Angulus* erat *Poſitionis* ——	80	6	0	78	21	0
Et *Jupiter* in antecedentia *Fixæ* ——	0	1	38	0	1	55
Cum Latitudine Minori ——	0	9	19	1	9	18

Fixæ *mihi* Locus, accepto motu Annuo 50″, erit ♎ 13° 37′ 11″. quem vult *Author Carolinus* 13° 33′ 47″. Latitudo ejus Borea. 1° 45′. Locus ergo verus *Jovis* erit *Mihi*.

8h 16′ ♎ 13° 35′ 33″ } Latitudo { 1° 35′ 40″ ½
8 50 ♎ 13 35 16 } vera { 1 35 42.

At *Fixæ* conceſſo Loco *Carolino*, prodibit ♃is Locus ♎ 13° 32′ 09″.

Loco ſic *Planetæ* & Latitudine perceptis Orbitæ *Jovialis* & *Terreſtris* Orbitæ Planum Inclinationem inde eruere conabimur.

Huic equidem inveniendæ, una cum Loco *Solis*; ejuſdem, *Jovis* & *Terræ* intermutuæ diſtantiæ poſtulantur: quas à Tabulis quibuſvis probatioribus tutiſſime haurire licet: Ego Tabulis utor plerumque *Carolinis*; quippe quas, accuratiores, & faciliores cæteris omnibus comperi; ex quibus ad 8^{h} 16 *p. m.* deprompſi;

Solis locum verum ———————— ♈. 10° 40′ 18″.
diſtantiam à *Terra* ———————— 100084.
Jovis à *Sole* diſtantiam ———————— 544921.
à *Terra* ———————— 444952.

Fig. 140. Jam in appoſita Figura ſint, S *Sol*, T *Terra*, ♃ *Planeta*, S E Radius *Eclipticæ*, ad ♃n *Orbitam* protenſæ, & Angulus ♃ T S, viſa *Planetæ* à *Terra* Latitudo 1° 35′ 40″ ½.

Ex datis (in Triangulo ♃ S T) Angulo, ♃ T S, viſæ Latitudinis ad Circulum complemento, ♃ S, & ♃ T, *Planetæ* à *Sole* & *Terra* Diſtantiis, ut ſupra repertis, eruetur Angulus ♃ S E, Latitudo ſive Inclinatio *Planetæ* à *Sole* conſpecta 1° 18′ 7″.

Jovis Locus Geo-centricus erat, ♎. 13° 35′ 33″. ab iis ergo datis ♃is & *Terræ* à *Sole* diſtantiis, invenietur Locus Helio-centricus *Planetæ* ♎ 13° 03′ 33″; è quo ſubductis ſigillatim iis *Nodi* Locis, quo Autores, quorum Nomina in ſequenti Tabella exaravimus, aſſumpſerunt, annexa produnt Argumenta Latitudinis; è quibus videre eſt, nullis plus *Jovem* à Limite promotum haberi quàm 6° 29′ 59″, nec minus quàm 3° 58′ 59″. quæ quantavis videtur differentia in maximâ Orbitæ Inclinatione inveſtiganda, Errorem ſcrupulis ſecundis 23″ Majorem inferre nequit.

Authores.	☊ *Loca.*				*Argumenta Latitudinis.*			
	S.	°	′	″	S.	°	′	″
Keplerus.	3	6	33	37	3	6	29	56
Streetius.	3	6	33	47	3	6	29	46
Wingius.	3	7	11	39	3	5	21	54
Ricciolus.	3	7	18	00	3	5	45	33
Caſſinus.	3	8	45	00	3	4	18	33
Bullialdus.	3	9	4	34	3	3	58	59

Fig. 141. Iſte *Nodi* Locus, quem Cl. *Caſſinus* elegerit mihi etiamſi aliquantulum juſto promotior videtur, Magis tamen cæteris variis de cauſis, placet: ſumptis propterea, Triangulo ♃ A ☊, Argumento Latitudinis, ☊ A, 94° 18′ 33″. & Inclinatione ♃ A, 1° 18′ 07″, eruetur Angulus Inclinationis Plani *Orbitæ*

Jovialis

Plate 3. Vol. 1.

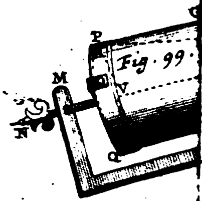


Fig. 99

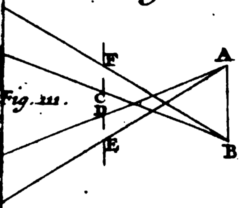


Fig. 111.

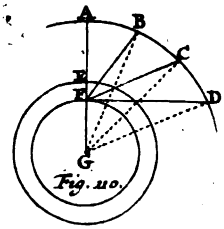


Fig. 110.

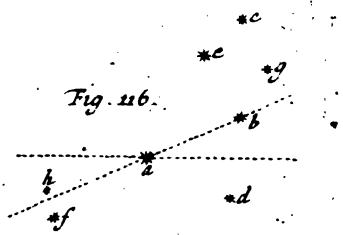
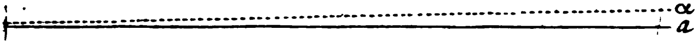


Fig. 116.



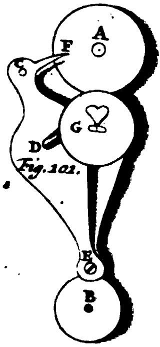
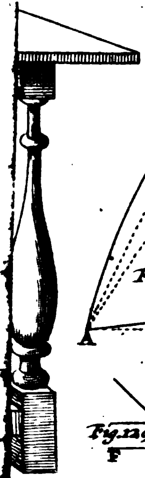


Fig. 101.



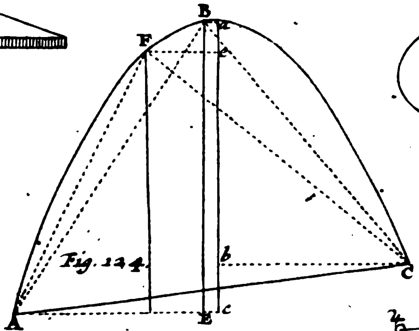


Fig. 124.

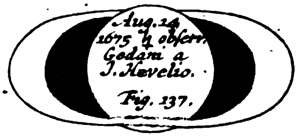

Aug. 14
1675 ♄ obſerv.
Gedani a
J. Hevelio.

Fig. 137.

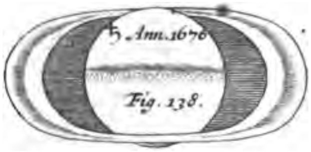
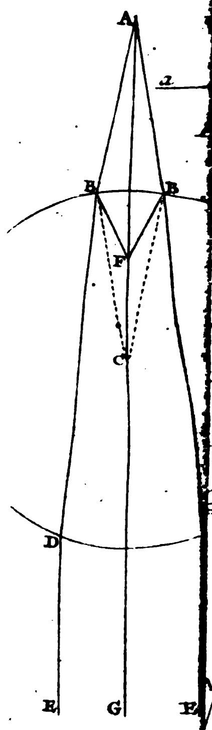


Fig. 138.



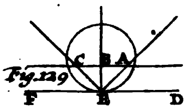


Fig. 129

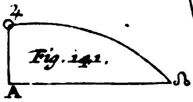
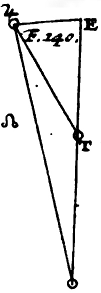


Fig. 141.



F. 140.

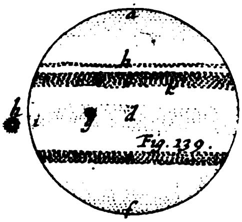


Fig. 139.

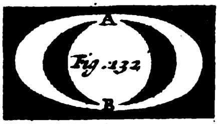
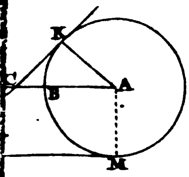


Fig. 132



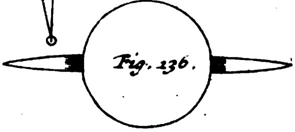


Fig. 136.

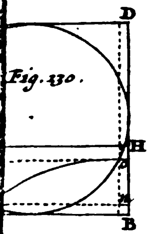


Fig. 130.

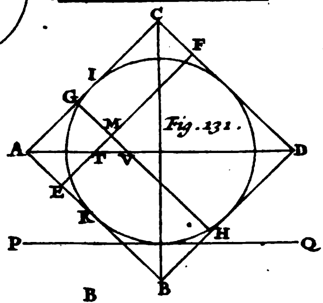
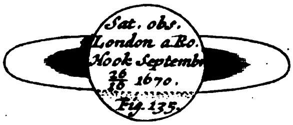


Fig. 131.


Sat. obs.
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26/16 1670.

Fig. 135.

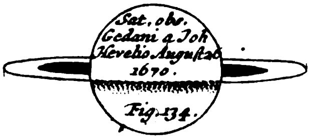

Fig. 134.

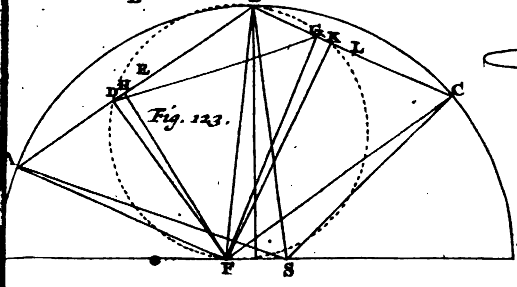

Fig. 123.

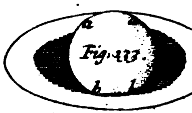

Fig. 133.

Jovialis ad *Eclipticam* 1° 18′ 20″. quem ſtatuunt *Keplerus* 1° 19′ 00″. *Streetius*, 1° 20′ 00″. *Bullialdus* & *Wingius* 1° 21′ 48″. omnes juſto nonnihil Majorem.

Tantamque eſſe Inclinationem, vel ſaltem non Majorem, cum heſternæ noctis, tum Menſium *Februarii*, *Martii*, & *Maii*, *Anni Elapſi*, obſervationes ſuadent. Interea verò non diſſimulandum, poſſe & majorem (ſcilicet 1° 20′ 20″.) à tranſitu ♃ prope 8^{am} ♍ *Anno* 1649. *Maii.* 29 & 30. St. Juliano *Bononiæ* & *Majorcæ* à *Ricciolo* & *Muto*, Viris Doctiſſimis, obſervato Demonſtrari: id quod nobis (ſiquidem Orbitarum Inclinationes ab omnibus invariabiles habentur,) videtur innuere, Errorem vel huic, vel illis *Fixarum* Latitudinibus à *Tychone* aſſignatis, ineſſe aliquem: quæ propterea donec accuratius reſtituantur, à præciſa hujus Inclinationis quantitate determinanda meritò nos arcent: Hoc tantum, quoniam *Fixarum* eæ Latitudines etiam Immutabiles reperiuntur, auſim affirmare; Angulum Maximæ Inclinationis Plani *Orbitæ Jovialis* ad *Eclipticam* Minorem eſſe ſcrupulis 26′ 40″. quam Latitudo *Stellæ* $9^{æ}$ ♍s *Lucis* $4^{tæ}$ quæ *Tychoni* dicitur, *Ultima quatuor in Siniſtra Ala Virginis*; quæ propterea ſi quando correcta dabitur, eadem certa dabitur Inclinatio.

Almag. Nov. Par. 1. *p.* 710.

LXXXVI. Examining our Ancient *Ephemerides* I do not find that three *Conjunctions* of *Saturn* and *Jupiter* have ever happened in one Year's ſpace, ſince they were firſt in uſe to this preſent. Thoſe of *Moletius* Calculated from the *Alphonſine Tables* indeed make three in the ſpace of eight Months betwixt *Auguſt* 1563. and *April* 1564. Incluſive; but the *Ephemerides* of *Stadius* Calculated from the *Prutenick*, make only one, and the 26 of *Auguſt* of which *Junctinus* gives us the following Obſervation in the Preface to his *Aſtronomical Tables*, *An.* 1563. *Aug.* 24 14^h 30′. *p. m.* Aurangæ, Jupiter à *parte Septentrionis cooperiebat quaſi* Saturnum, *qui erat à parte Meridionali, utraque autem harum Stellarum, in fine* 28 *gradus* Cancri *deprehendebatur*; *Riccioli* hence concludes, that the Planet ♃ covered ſome part of *Saturn* at this time. But without Reaſon, for the Words *quaſi cooperiebat* intimate not that the one did corporally cover the other, but rather that there was ſome ſmall Interval betwixt them. The *Caroline Tables* make the viſible Latitude of *Saturn* now, 11′ 45″. of *Jupiter* 20′ 10″. both North, the Conjunction being ſome few days paſt: but becauſe their Latitudes alter ſlowly we may hence conclude the Difference 8′ 25″. to have been nearly their Diſtance at that time, theſe Tables being grounded on the *Tychonick* Obſervations made within leſs than 40 Years after, and ſhewing the Latitudes of the *Planets* well at this time near 100 Years later we may Conclude to have anſwer'd them as well then; and if we conſider how ſmall a ſpace the Diſtance of 8½ minutes appears to the naked Eye in the Heavens, eſpecially betwixt two ſuch bright Planets as *Saturn* and *Jupiter* are, that the *Caroline* Diſtance agrees very well with the Words of *Junctinus*, and that *Riccioli* was miſtaken.

The Conjunctions of Saturn *and* Jupiter. An. 1682. *and* 1683. *at* Greenwich; *by Mr.* Flamſteed. n. 149. *p.* 244.

Their next *Conjunction* according to *Maginus*'s *Ephemerides* founded on the *Prutenick* Numbers, was *April* 29. 1583 in 21 *Deg.* of ♓, the *Sun* being then in 17 *Deg.* of ♈, ſo that that Planet's Riſing before him in *Signs* of ſhort

Aſcenſion and with South Latitude this *Congreſs* could not be obſerved by the Noble *Tycho* who was mindful of it, as appears by this Note in Page 55 of his *Hiſtoria Cœleſtis.* May 30 A. M. *quo primum poſt* Conjunctionem Saturnum *vidimus, capta ſunt diſtantiæ inter* Jovem & Saturnum *per Radium.*

1^h 47′	3° 24′.
1 50	3 24.

The ſame *Ephemerides* ſhew the next *Conjunction* of *Saturn* and *Jupiter* 1603. *December* 14 at Noon in 9° 36′ of ♐, but the Ingenious *Kepler* and our Sir *Chr. Heydon* found it by Obſervation 7 days ſooner, or the 7 Day of the ſame Month in the Morning, in near 8 *Degrees* of ♐, the Planets being but then newly Emerged from the Rays of the *Sun.*

The *Emphemerides* of the Learned *Kepler* Calculated from his own *Rudolphine Tables,* make the next *Conjunction* 1623. betwixt the 7th and 8th of *July* in 6° 46′. of ♌ the Planet of *Saturn* being then only 4 Minutes to the North of *Jupiter,* but this firſt *Conjunction* in the *Fiery Trigon* happening under the *Sun*'s Beams was not obſervable.

By the ſame *Tables,* and *Emphemerides* of *Eichſtade* Calculated from them; theſe *Planets* met again in the 25° of ♓, betwixt the 15 and 16 of *Febr.* 1643. with a degree Difference of Latitude.

By the joint Conſent of *Eichſtade's* and our *Wing*'s *Ephemerides* the ſame Planets were in *Conjunction* again 1663. on the 10 of *October* at Noon in 13° 30′. of ♐ with one degree of Difference of Latitude; this *Conjunction* was obſervable after *Sun-ſet* in our Latitude, but I hear not that any one obſerved it.

In every one of theſe Years there happened only one *Conjunction* of the two *Superiors,* nor is it poſſible that there ſhould be more, except the *Heliocentrical Conjunction* fall near the *Oppoſition* of the *Sun*; for then there may be three, two *Direct,* and one *Retrograde,* as has been within the Space of 7 Months, betwixt *October* and *May* laſt incluſive, of which the true times are determined from the following Obſervations.

An. 1682.							
	d.				°	′	″
Oct.	5	17	51	Betwixt the Centers of *Sat.* and *Jup.*	0	34	54
		17	54	——————————— rep.	0	34	48
	12	13	49	Betwixt their Centers ———	0	16	2
		13	54	——————————— rep.	0	16	4
		14	3	Betwixt their Next Limbs ———	0	15	22

Oct. 17.

	d.	h.	′		°	′	″
Oct.	17	14	10	Betwixt their Centers ———	0	20	9
		14	17	——————— rep.	0	20	12
		14	21	——————— again	0	20	14
		14	25	Betwixt their next Limbs ——	0	19	44
		14	33	Betwixt their remoter Limbs ——	0	20	37
		15	9	*Saturn* from the *Heel* of *Caſtor* ——	48	32	25
		15	14	*Jupiter* from the ſame *Star* ———	48	45	5
		15	17	——————— rep.	48	45	20
		15	20	*Saturn* from the ſame *Star* again ——	48	32	20
		15	50	Betwixt their Centers again ———	0	20	30
	19	15	41	Betwixt their Remote Limbs ———	0	26	2
		15	45	Their Centers ————	0	25	37
		15	47	Their Next Limbs ————	0	25	11
	22	18	25	Betwixt their Centers ———	0	33	19
		18	29	——————— rep.	0	33	26

The *Diſtances* betwixt the *Planets* were meaſured with the *Micrometer* and 16 Foot Glaſs, from the *Fixt Stars* with the *Sextant:* thoſe of the 12th by my Aſſiſtant, the reſt by my ſelf.

On the 22 day, the Planet *Jupiter* was in Conſequence of *Saturn* ſomething leſs Diſtant from him than he had been obſerved on the 5th day near the ſame hour. Hence the Middle Time, betwixt theſe Obſervations is pointed out for the time of their true *Conjunction*, but to determine it more accurately, I ſhall Examine the Obſervations made with the Sextant on the ſeventeenth day, which being neareſt the time are moſt proper for this purpoſe.

The Correct Longitude of the *Heel of Caſtor* now ♋ 0° 50′ 42″. its Latitude 51′ 40″. South. The Latitude of *Saturn* by the *Caroline Tables* 56′ 20″ of *Jupiter* 41′ 30″. both North.

By the aſſumed Latitude of *Saturn* 56′ 20″. and his Diſtance from the *Heel* of *Caſtor* obſerved and corrected 48° 32′ 30″. I found their Difference of Longitude 48° 30′ 37″. therefore *Saturn* in ♌ 19° 21′ 19″.

By the Latitude of *Jupiter* aſſumed 41′ 30″. and his Diſtance from the *Star* 48° 45′ 20″. their Difference of Longitude 48° 43′ 56″. and *Jupiter's* Place in ♌ 19° 34′ 39″.

Hence *Jupiter's* Place in conſequence of *Saturn's* 13′ 20″. with which and the Diſtance of their Centers obſerved, the ſame night 20′ 12″. I find the true Difference of their Latitudes 15′ 20″ but half a Minute Different from what I aſſumed it on the Authority of the *Tables.*

The Apparent Motion of *Jupiter* from the 14 to the 18 day of *October* by an *Ephemeris* exactly calculated and made agreeable to theſe obſervations is 29′ 16″. of *Saturn* 15′ 01″. both Direct, hence the Motion of *Jupiter* from *Saturn* in four *days* is 14′ 15″. I ſay therefore as 4 days Motion, or 14′ 15″. is

15″. is to 4 Days, or 96 Hours; ſo is 13′ 20″ (which *Jupiter* is paſt the *Conjunction* of *Saturn*,) to 90 Hours or 3 Days 18 Hours; the time interlapſed ſince the *Conjunction*; which taken from the 17 Day 15 Hours the time of my Obſervation, gives the true time of the *Conjunction* of the two Planets on the 13 Day, one and twenty Hours after Noon, or according to the common Account, the 14 Day, at 9 a Clock in the Morning.

At which time *Saturn* is with *Jupiter* in ♌ $19^{\circ}\,07'\frac{1}{5}$ with $15'\frac{1}{3}$. more Northern Latitude.

The *Acta Eruditorum Lipſienſia* p. 366. make this *Conjunction* to have happened the ſame Day in the ſame Longitude with the *Eleventh Star of Leo*; whoſe Place they ſtate in ♌ 19° 04′ Lat. 0′ 16″. N. with 14 Minutes difference of Latitudes betwixt the two *Planets*. But their Obſervation ſeems to have been made only by the Judgment of the bare Eye, without an Inſtrument, which conſidered, I wonder not that it differs at all, but rather that the Difference is ſo ſmall from this Determination.

On the 19 of *January* following, viewing then the Planets both *Retrograde* with the 16 Foot Glaſs, I found them approached within a meaſurable Diſtance of each other.

An. 1683.						
d.	h.	′		°	′	″
Jan. 19	6	41	Betwixt their Centers	0	33	28
	6	45	rep.	0	33	24
	6	49	Betwixt their remote Limbs	0	33	52
26	6	3	Betwixt their Centers	0	15	8
Both being in ☍ *to the* Sun.	6	7	rep.	0	15	6
	7	00	Betwixt their remote Limbs	0	15	31
	7	8	By *T. Smith* rep.	0	15	9
	7	12	Betwixt their Centers	0	15	5
	7	14	rep.	0	15	2
	7	17	Betwixt their next Limbs	0	14	29
	7	20	rep.	0	14	31
	7	21	again	0	14	26
	9	24	*Jupiter* from the *Heel* of *Caſtor*	46	18	10
	9	26	rep.	46	18	5
	9	28	*Saturn* from the ſaid *Heel*	46	8	50
	9	30	rep.	46	9	55
	9	37	*Jupiter* from the bright Star of the *Lion's Head* E.	8	42	5
	9	39	rep.	8	42	5
	9	40½	*Saturn* from the ſame Star	8	29	35
	9	42½	rep.	8	29	40

	h.	′		°	′	″
	9	48	*Jupiter* from the *Lions Heart* —	8	18	00
	9	50	— — — rep.	8	17	55
	9	52	*Saturn* from the ſame ——	8	29	35
	9	54	— — — rep.	8	29	35
	9	59	The *Lions Heart* from *E* in the *Head* — —	12	58	50
	10	3	The *Heel* of *Caſtor* from the *Lions Heart* — —	54	34	20
	10	8½	The *Heel* of *Caſtor* from *E* ♌ —	46	24	45
Jan. 30.	5	28	Betwixt their Centers —	0	11	36
	5	30	— — — rep.	0	11	33
	5	34	Betwixt their Remote Limbs —	0	11	58
	5	36	— — — rep.	0	12	1
	5	38	Betwixt their Next Limbs —	0	11	1
	5	41	— — — rep.	0	11	00
Feb. 7	7	37	Between their Centers — —	0	28	35
	7	40	— — — rep.	0	28	34

From Obſervations formerly made, I have determined the true Places and Latitudes to this preſent time of,

	s	°	′	″		°	′	″	
The Heel of *Caſtor* ——	♋	0	51	10	*Lat.*	0	51	40	*South.*
Bright * in the *Lions Head*, E —	♌	16	15	27		9	41	07	*North.*
Lions Heart —— ——	♌	25	24	45		0	26	20	*North.*

And from the above recited Meaſures, the true Diſtances of the *Planets* from theſe Stars *January* the 26th at $9^h\ 40'$ *p. m.* as follows.

	°	′	″
Saturn from the *Heel* of *Caſtor* ——	46	09	00
Jupiter from the ſame ——	46	18	10
Saturn from the *Lions Heart* ——	8	29	40
Jupiter from the ſame ——	8	18	00
Saturn from the bright * in the *Lions Head* E —	8	29	40
Jupiter from the ſame ——	8	42	10

Whence I collect the true Places at this time.

	s	°	′	″		°	′	″
Of *Saturn*	♌	16	57	10	*Lat.*	1	13	10
Of *Jupiter*	♌	17	07	10	*Lat.*	1	01	30
Difference of *Long.* —		10	00		of *Lat.*		11	40

The *Retrograde* Motion of *Jupiter* from *Saturn* in 4 Days, betwixt the Twenty ſixth and thirtieth of this Month, by my correct *Ephemeris* is 12′ 15″. I ſay therefore as 12′ 15″. is to 4 Days or 96 Hours; ſo is 10′ 00″ (the Difference of the *Planets* preſent Longitudes,) to 78 Hours or three Days ſix Hours, which therefore added to the time of that Obſervation *Jan.* 26^{d} $9^{h}\frac{1}{3}$, gives the true time of the *Conjunction* Jan. 29^{d} 16^{h}. or according to the common account *Jan.* 30 at 4^{h} in the Morning. At which time both the *Planets* are in ♌ 16° 51′$\frac{1}{3}$. with 11′$\frac{1}{2}$. Difference of Latitude or Diſtance from each other, which is further confirmed by the Meaſured Diſtances of the *Planets* on the 30*th* at Night before recited.

On the 26*th* Day at 9^{h} 40′ the *Sun*'s True Place was by my Tables in ♒ 17° 21′$\frac{1}{2}$, ſo that he was now about $\frac{1}{3}$ of a Degree paſt their Oppoſition.

Towards the Latter end of the following *April*, the Planet *Jupiter* began to approach *Saturn* again both being now Direct; the 28 at Night with the 16 foot Glaſs and *Micrometer* I meaſured the Diſtances.

An. 1683.								
	d.	h.	′			°	′	″
Apr.	28	10	21	Betwixt their Centers —		0	32	35
		10	23	— —	rep.	0	32	33
		10	24	Betwixt their Next Limbs —		0	32	4
		10	26	— —	rep.	0	32	2
				Betwixt their Remote Limbs but not accurately, Clouds interpoſing —				
		10	29	— — —		0	33	22
May	7	8	59	*Jupiter* from the *Lions Heart* —		10	59	00
		9	1	— —	rep.	10	59	00
		9	3$\frac{1}{2}$	*Saturn* from the *Lions Heart* —		10	58	50
		9	5	— —	rep.	10	58	50
		9	11	*Jupiter* from *E* in the *Lions Head* —		8	55	35
		9	15	— —	rep.	8	55	40
		9	17	*Saturn* from the ſame Star —		8	39	40
		9	18	— —	rep.	8	39	40
		9	30	Betwixt their Centers —		0	15	38
		9	33	— —	rep.	0	15	37
		9	35	Betwixt their Next Limbs —		0	15	5
		9	36	— —	rep.	0	15	00
		9	40	Betwixt their Remote Limbs —		0	16	2
		9	42	— —	rep.	0	15	58
		10	20	*Jupiter* from *B* in ♍ —		38	11	45
		10	23	— —	rep.	38	11	45
		10	26	*Saturn* from the ſame Star —		38	10	55
		10	28	— —	rep.	38	10	45
May	11	9	28	Betwixt their Centers —		00	20	2
			31	— —	rep.	00	20	2
	16	9	22	Betwixt their Centers —		00	34	4

From theſe Obſervations I ſtate the Diſtances of the *Planets* from the Fixed Stars *May* 7th at 9^h 5′. *p. m.* as follows.

		°	′	″
Saturn from the *Lyons Heart*	——	10	58	50
Jupiter from the ſame	——	10	59	00
Saturn from E in the *Lyons Head*	——	8	39	40
Jupiter from the ſame	——	8	55	35

Hence the true Longitude of *Sat.* ♌ 14° 27′ 42″. *Lat.* 1° 12′ 46″. *North.*
of *Jupiter* ♌ 14 26 37. *Lat.* 0 56 43. *North.*
Difference of Longitude —— 1 4. *Lat.* 0 16 3.

The Difference of Latitudes ſomething exceeds the Diſtance meaſured with the *Micrometer*, by reaſon that the Wind then ſhaking the *Sextant* permitted us not to be ſo exact as uſually, but the Difference, being leſs than half a Minute, I eſteem inconſiderable.

The Diurnal Motion of *Jupiter* from *Saturn* was now 3′ 15″. It holds therefore as 3′ 15″. (one days Motion,) is to one Day or 24 Hours; ſo 1′ 4″. (the Diſtance of *Jupiter* from the ☌ with *Saturn*) to 8 Hours, the Interval betwixt the Obſervation and following *Conjunction*, which was therefore 17^h after Noon, or according to the vulgar reckoning, *May* 8. at 5 a Clock in the Morning. At which time the true Place of the *Planet* is ♌ 14° 28′$\frac{3}{4}$, the Difference of their Latitudes, 15′ 40″. *Saturn* being ſo much more Northerly than *Jupiter.*

In all, beſt eſteemed, *Aſtronomical Tables* extant, the Mean Motions of the *Planet Saturn* are too Swift, of *Jupiter* too Slow conſiderably: Hence it came to paſs that they made the *Direct Conjunctions* ſome Days Later, the *Retrograde* earlier than they were found by Obſervation.

2. *Oct.* 26. ſt. n. 1h 40′. mane Situm *Jovis* & *Saturni* Tubo & *Micrometro* ex voto deprehendere mihi obtigit: quo tempore ſimul Fixa quædam ſatis conſpicua (quod notatu dignum) dictis *Planetis* ſatis prope adhærebat. *Jupiter* ſeſe cum tribus *Comitibus* tum offerebat, forte & Quartus adfuit, ſed ob Nubeculas haud fuit conſpectus. *Saturnus* diſtabat à *Jov.* 16′ 44″; *Jupiter* à Stella (ni fallor in *Armo Dextro* ♌) 27′ 55″; Rurſus *Saturnus* à dicta Stella 38′ 1″. Stella dicta verſatur modo juxta noſtrum Catalogum in 19° 2′ 9″. ♌ & Latit. 0° 20′ 45″. B. Die 30 *Octob.* 5^h mane Diſtantia ♄ & ♃, 25′ 5″ extitit; unde certo colligere licuit *Conjunctionem* jam ante complures dies celebratam eſſe quam Ephemerides Calculuſque primum die 3 *Novem.* exhibent.

At Dantzick; *by M.* Hevelius. *n.* 143. *p.* 18. & n. 151. *p.* 326.

Id quod ſubſequentes Obſervationes adhuc clarius demonſtrant. Nam Loco, quod Diſtantia ♃ & ♄ de die in diem (ſi ☌ inſtaret) paulatim Minor fieri debebat, continuo Aucta eſt. Die 1 *Nov.* hor. 2 mane, ope *Micrometri* noſtri dicta Diſtantia extitit 31′ 35″. Et Die. 2. *Nov.* eandem Diſtantiam ruiſus reperi 35′ 21″. Die 3. *Nov.* mane hor. 1. jam 39′ 9″; Die 4. *Nov.* cælo perquam ſereno adhuc paulo Major dicta Diſtantia inter ♄ & ♃ deprehenſa.

Alteram,

Alteram *Conjunctionem*, quod attinet quæ ex Retrogreſſionibus horum *Planetarum* incidere debebat ſecundum Ephemeridum Scriptores die 26 *Januarii* hujus Anni currentis 1683. Obſervationes nonnullas ac præcipuas habitas hic apponam.

An. 1683.	*Temp. ex alt. corr. veſp.*			*Obſervationes.*		*Diſtantiæ.*		
	h.	′	″			°	′	″
Feb. 1	6	40	0	Diſt. *Satur.* & *Jov.* inventa eſt —	3300	0	35	5
Feb. 2	9	30	0	Diſt. *Satur.* & *Jov.* ———	2900	0	22.	3
Feb. 3	9	00	0	Diſt. *Satur.* & *Jov.* ———	2500	0	19	0
Feb. 4	10	00	0	Diſt. *Satur.* & *Jov.* ———	2300	0	17	29
Feb. 5	8	30	0	Diſt. *Satur.* & *Jov.* ———	2100	0	15	59
Feb. 6	7	51	0	Diſt. *Satur.* & *Jov.* ———	1850	0	14	6
Feb. 7	8	17	19	Diſt. *Satur.* & *Jov.* ———	1700	0	12	55
Feb. 8	6	10	0	Diſt. *Satur.* & *Jov.* ———	1600	0	12	10

Feb. 9. veſp. 9^{h} 0′ 8″. *Planetas* Tubo conſpexi per dehiſcentes denſiſſimas nubes; Oculoque fugitivo deprehendebam *Conjunctionem* ipſam jam celebratam eſſe nocte præcedente inter 8 & 9 *Febr.* Nam dicta Diſtantia paulo amplior modo apparebat. Prout etiam 11 *Febr.* veſp. 9^{h} 0′ 0″ factum eſt: nam Diſtantia inter *Saturnum* & *Jovem* erat 2000, hoc eſt 0° 15′ 12″ *Micrometro*, quæ die 8 *Feb.* tantum inventa eſt 0° 12′ 0″.

Ad hæc *Conjunctionem* jam eſſe celebratum, exinde certò conſtabat, quod uterque *Planeta* cum *Ventre Urſæ Majoris* non amplius, ut quidem die 8 *Feb.* contingebat, in linea ſubſiſteret recta; tum etiam quod *Saturnus* non amplius ad Orbitam *Jovialium* ſub Angulo Recto commoraretur.

Quo autem hoc ipſum eo evidentius redderetur, Obſervationes nonnullas diebus ſubſequentibus continuatas hic apponam.

	h.	″			′	″
Feb. 12	9	0	Diſt. *Satur.* & *Jov.* capta eſt —	2200	17	6
13	7	15	——— ———	2550	19	24
14	9	0	——— ———	2900	22	3
17	6	0	——— ———	3750	28	30
20	9	0	——— ———	5250	30	12

Ex hiſce igitur Obſervationibus continuatis ſatis ſuperque liquet, cum de die in diem *Planeta* ab invicem magis magiſque diſceſſerint, quod *Conjunctio* ipſa inter 8 & 9 *Febr.* revera jam fuerit obſervata.

Denique adjiciendum pariter cenſeo Aſtrophilorum gratia, quidnam circa Tertiam eorum *Conjunctionem*, Menſe *Maii* obſervaverim.

An. 1683.		*Temp. Sec. Hor. ambul. veſp.*		*Obſervationes.*		*Diſtantiæ*	
		h.	'	Diſt. *Saturni* & *Jov.* inventa		'	"
Maii	8	9	6	eſt *Micrometro* ——	4300	32	41
	10	9	14	——	3750	28	30
	11	9	10	——	3450	26	13
	12	8	45	——	3050	23	10
	13	9	15	——	2800	21	17
	14	9	45	——	2550	19	23
	15	9	30	——	2400	18	15
	16	9	30	——	2250	17	6
	17	9	40	——	2150	16	10
	18	10	00	——	2100	15	58
				Hac die extitit Vera *Conjunctio.*			
	20	11	45	——	2450	18	37
	21	9	15	——	2650	20	9
	22	9	20	——	2900	22	3
	23	9	5	——	3250	24	43
	24	10	6	——	3600	27	22
	25	9	30	——	4000	30	25
	26	11	0	——	4453	33	50
	27	9	25	——	4900	37	15
	28	9	56	——	5325	40	29

Ex quibus Obſervationibus cuilibet nunc liquidum eſt, cum Diſtantia *Saturni* & *Jovis* de die in diem continuò decreverit ad. 18. *Maii*, & ab hac die rurſus creverit, *Conjunctionem* horum *Planetarum* eadèm ipſa die etiam accidiſſe, & quidem, (uti ex obſervationibus diei 15, & 20. *Maii* patet) hora antemeridiana 10: quæ ſecundum Ephemeridum Scriptores die 26. primum ingruere debuit. Sic ut hæc tertia & ultima hujus Anni *Conjunctio Magna* pariter haud mediocriter Tabulas eluſerit; ita quidem quod citius ultra 8 integros dies revera contigerit.

De cætero, hanc ultimam *Conjunctionem* jam celebratam die ſc. 18 *Maii* veſp. hor. 10. fuiſſe, ex eo liquet, quod *Saturnus* jam non amplius eo tempore ad Orbitam *Jovialium* ſub Angulo verſabatur Recto; deinde etiam (uti ex ſubſequentium dierum obſervationibus videre eſt,) quod à die 18 *Maii* ad diem 28 quouſque *Micrometro* Diſtantiam *Saturni* & *Jovis* dirimere potui, continuo aucta fuit.

Ultimo, notandum quoque occurrit, cum die 21 *Maii* veſp. inter reliquas obſervationes etiam Diſtantiam utriuſque *Planetæ* à Stella *Superiori in Pede*

Anteriore Dextro Urſæ Majoris ſextante obtinuerim, atque eadem dicta Stella cum utroque *Planeta* in eadem ſimul fere recta tum conſiſterent linea, quod promptum ſit cuilibet Rerum Cæleſtium cultori dijudicare, an obſervationes meæ, *Sextante* noſtro novo ſimul obtentæ, cum obſervationibus *Micrometro* captis omnino etiam conveniant. Sextante Diſtantia *Jovis* à dicta Stella erat 32° 38′ 40″, & *Saturni* 32° 19′ 45″; ſic ut genuina utriuſque *Planetæ* Diſtantia extiterit 18′ 55″; *Micrometro* vero ea ipſa Diſtantia inventa eſt ea ipſa die 21 ſc. *Maii* 20′ 9″; ſic ut *Sextante* obtenta 1′ 14″. Minor extiterit. Non eſt autem, mi Amice, quod exiſtimes, hoc vel illo Inſtrumento me aberraſſe, minime profecto; quippe *Saturnus* & *Jupiter* cum dicta Stella non omnino in linea recta ſubſtiterunt; prout cuilibet ex Globo & Calculo patet, hincque neceſſario illa diſtantia, *Sextante* derivata, etiam paulo minor oportuit eſſe.

The Mean Conjunctions of Saturn *and* Jupiter; *by Mr.* Flamſteed. *n.* 149. *p.* 254. *July* An: 1683.

LXXXVII. *Riccioli* in the Second Part of the firſt Tome of his *Almageſt*, has given us a Table of all the mean *Conjunctions* of the two Superiors from the Creation till the *Year* of *Chriſt* 2358, but very courſe and incorrect. I have therefore made a new one for 43 *Revolutions* which are completed in 853 *Julian Years*, and 235 Days from their mean Motions, which I have corrected by very late Obſervations. This being the *Period* of the *Greateſt Conjunctions*, after which Space of Time they return to the ſame Place of the *Zodiack* within $\frac{3}{5}$ of a Degree.

The *Ordinary Conjunctions* happen once in Twenty Years, or more preciſely in 19 *Julian* Years, and 312 Days, in which time *Saturn's* Mean Motion is 8: 2°.48′$\frac{1}{2}$, *Jupiter's* the ſame above one *Revolution*.

Theſe are commonly termed the *Leſſer* of the *Great Conjunctions*, which continue in *Signs* of the ſame *Triplicity* for 10 *Revolutions* to each other, or 198 Years: each *Conjunction* according to the Mean Motions being 8^s 2° 48′$\frac{1}{3}$ removed from the preceeding, ſo that if any *Conjunction* was made upon the firſt Point of ♈, the next following ſhall be in 2° 48′ of ♐, and all the following for 198 Years ſhall fall in ♈, ♌ and ♐, *Signs* of the ſame *Triplicity*.

But the 11 *Conjunction* after, ſhall happen in the firſt Degree of ♍, and the following ten *Conjunctions* in ♉, ♍, and ♑, *Signs* of the ſame *Triplicity*. Of theſe the firſt is called by our Aſtrologers the *Greater Conjunctions*.

But the *Greateſt* is, when after 43 *Conjunctions* compleated in 853 Years, 235 Days, the *Mean Conjunctions* having been made in all the *Signs* return to that Point of the *Ecliptick* from whence they began: tho' I muſt confeſs, had I been to name them, I would have called thoſe the *Greateſt* which happen in the *Signs*, ♋ and ♌, becauſe then the *Planets* Riſe higheſt, and are longeſt viſible in our *Horizon*, as alſo being near their North *Nodes*, they approach neareſt, and if they have any extraordinary Influence (which *Naboyd* thinks either they have not, or if they have, we underſtand not) it muſt according to their Axioms be ſtrongeſt.

Thoſe which happen in ♑ and ♒ I ſhould call the *Greater* or *Middle*, becauſe the *Planets* being then near their South *Nodes*, may approach each other

again very nearly tho' they riſe not high in our *Horizon*, being in Southern *Signs*; the reſt might be accounted the *Leſſer* or *Ordinary*.

The *Mean Conjunction* of *Saturn* and *Jupiter* this Year 1683. was on the 14*th* Day of *January* old Stile, at 12 Hours after Noon in the Meridian of *London*, at which time the Mean Motions of both the *Planets* were $4^{s}\ 11^{\circ}\ 45'$. This may be the *Radix* of the following *Table*.

A Table of the *Mean Conjunctions* of *Saturn* and *Jupiter*.

	Intervals.						*Intervals*.				
Revolutions Complete.	*Time*.		*Motion*.			*Revolutions Complete*.	*Time*.		*Motion*.		
	y.	d.	s.	°	′		y.	d.	s.	°	′
1	19	312	8	2	48	23	456	219	6	4	31
2	39	258	4	5	37	24	476	165	2	7	19
3	59	204	0	8	25	25	496	111	10	10	8
4	79	150	8	11	13	26	516	57	6	12	56
5	99	96	4	14	1	27	536	3	2	15	44
6	119	42	0	16	50	28	555	315	10	18	32
7	138	353	8	19	38	29	575	261	6	21	21
8	158	299	4	22	26	30	595	207	2	24	9
9	178	245	0	25	15	31	615	153	10	26	57
10	198	191	8	28	3	32	635	99	6	29	45
11	218	137	5	00	51	33	655	45	3	2	34
12	238	83	1	3	40	34	674	356	11	5	22
13	258	29	9	6	28	35	694	302	7	8	10
14	277	340	5	9	16	36	714	248	3	10	59
15	297	286	1	12	4	37	734	194	11	13	47
16	317	232	9	14	53	38	754	140	7	16	35
17	337	178	5	17	41	39	774	86	3	19	24
18	357	124	1	20	29	40	794	32	11	22	12
19	377	70	9	23	18	41	813	343	7	25	00
20	397	16	5	26	6	42	833	289	3	27	49
21	416	327	1	28	54	43	853	235	0	00	37
22	436	273	10	1	43						

By this *Table* to find the Time of any *Mean Conjunction*, Paſt or Future; neareſt to any Place of the *Zodiack*; for Times Paſt, ſubſtract the Longitude of the given Place from the Longitude of the *Radix* $4^{s}\ 11^{\circ}\ 45'$. the reſidue ſeek in the laſt Column of the *Table*; if you find not the preciſe Number take the next to it, againſt this you have in the ſecond Column the Years and Days, and in the firſt the Number of *Conjunctions* paſt ſince any was made in that Place, ſubſtract the Years and Days from 1683. *Jan.* 14. and the Motion from $4^{s}\ 11^{\circ}\ 15'$. ſo have you the true Time of the *Mean Conjunction*, and Longitudes of the *Planets* then.

But for Times to come ſubſtract the *Radix* from the given Place, ſeek the Reſidue as before in the laſt Column; if you find it not, take that you find neareſt it; againſt which, as before, you have in the ſecond Column, the Years and Days; in the firſt, the Revolutions future; for Example,

If it were required to know when the laſt *Conjunction* happen'd in the firſt Deg. of ♒, Subſtracting ♒ or Ten *Signs* from $4^{s}\ 11^{\circ}\ 15'$. the Reſidue is $6^{s}\ 11^{\circ}\ 15'$. which ſeeking I cannot find in the third Column of the *Table*, but I find $6^{s}\ 12^{\circ}\ 56'$. which is not two Degrees more, and againſt them 516 Years, 57 Days, and in the firſt Column 26, for the Number of *Conjunctions* interlapſed. Subſtracting 516 Years, 57 Days from 1683. *Jan.* 14. there remains 1166 Years, 322 Days, which ſhews me that the *Conjunction* was in the *Year* 1166. *Nov.* 18. and ſubſtracting the Motion $6^{s}\ 12^{\circ}\ 56'$, from $4^{s}\ 11^{\circ}\ 45'$. it points me to the Place in $9^{s}\ 28^{\circ}\ 49'$.

Or if the Time of the firſt *Conjunction* in ♎ to come were demanded. I ſubſtract the *Radix* $4^{s}\ 11^{\circ}\ 45'$. from 6 *Signs*, the Reſidue $1^{s}\ 17^{\circ}\ 15'$. I ſeek in the *Table* but find it not, I take therefore the next to it $1^{s}\ 20^{\circ}\ 29'$. againſt which ſtands 357 Years, 124 Days, theſe added to 1683. *Jan.* 14. give me the Year 2040, and 138 Days, *May* 18, for the Time of this *Conjunction*, and adding the $1^{s}\ 20^{\circ}\ 29'$. to the *Radix* $4^{s}\ 11^{\circ}\ 45'$. it makes $6^{s}\ 2^{\circ}\ 14'$. for the true mean Longitude of this *Conjunction*.

From the *Mean Conjunction* the Apparent may be found by the help of a Planetary Inſtrument, or the uſual Aſtronomical Tables.

The Shadows of Jupiter's Satellites *obſerved*; *by* S. Campani. *n.* 1. p. 3. *By* M. Caſſini *and others*. n. 8. *p.* 143. n. 10. *p.* 171. n. 82. *p.* 4039.

LXXXVIII. 1. *S. Campani* affirms, that he hath remarked in the *Belts* of *Jupiter*, the *Shadows* of his *Satellites*, and followed them, and at length ſeen them Emerge out of his *Disk*.

2. M. *Caſſini*, after he had diſcover'd (by the means of thoſe excellent *Glaſſes* of 35 Foot made by *S. Campani*) the *Shadows* caſt by the *Satellites* of *Jupiter* upon his *Disk* when they happen to be between the *Sun* and *Him*; and after He had alſo diſtinguiſh'd their Bodies upon the *Disk* of *Jupiter*; made ſome Predictions when they ſhould appear, to the End that the Curious might be convinc'd of this Matter by their own Obſervations.

Some of theſe Predictions have been verified, not only at *Rome*, and in other Places of *Italy*, but alſo at *Paris* by M. *Auzout*, and in *Holland* by M. *Hugens*; particularly *Sept.* 26. 1665. at half an Hour after ſeven a Clock, one of theſe *Shadows* was ſeen both in *France* and in *Holland*.

The *Spots*, have this Peculiar, which diſtinguiſheth them from all others, that they are found preciſely in that Place of *Jupiter*, where ſome *Satellite* is ſeen by the *Sun*; that they go from the Oriental Limb to the Occidental of the Diſque of *Jupiter*, with a Motion always equal to that of the *Satellite*; that in reſpect to us they precede the *Satellite*, before the Oppoſition of *Jupiter* to the *Sun*, and follow him after the Oppoſition; that the further *Jupiter* is diſtant from the Oppoſition, the greater is the apparent Diſtance of the ſame *Satellite*; that at divers Times of the Year, this Diſtance changeth in Proportion of the Annual *Parallax* of the *Satellite*, according as he is differently ſeen by the *Sun*, and by the Earth; and that at one and the ſame time of the Year, when divers *Satellites* happen to be between *Jupiter* and the *Sun*, the *Spots* correſpondent to them are diſtant from them in Proportion of the Semidiameters of the Circles of the ſame *Satellites*.

By Dr. Hook. *n.* 14. *p.* 246. *Fig.* 139.

3. *An.* 1666. *Jan.* 26. about $3^h\ 15'$ in the Morning, I perceived (with a 60 Foot Glaſs) near the Middle of the Zone *d*, a very round *Spot*, like that repreſented at *g*, which was not to be perceived about half an Hour before; and I obſerved it in about 10'. time to be gotten almoſt to *d*, keeping equal Diſtance from the *Satellite h*, which moved alſo Weſtwardly, and was joined to the Disk at *i*, at $3^h\ 25'$, ſo that it was ſufficiently evident that this black *Spot* was nothing elſe, ſave the Shadow of the *Satellite h*, Eclipſing a part of the Face of *Jupiter*. The other three *Satellites* in the time of the *Eclipſe* were Weſtwards of the Body of *Jupiter*.

The Elongations of Jupiter's Satellites; *by Mr.* Flamſteed. *n.* 81. *p.* 4036, 4037. *n.* 94. *p.* 6033. *n.* 96. *p.* 6094.

LXXXIX. 1. Anni duo & amplius elapſi ſunt ex quo Eruditiſſimus *Richardus Townleius* Armiger, mihi Maximas *Jovialium* Siderum a Centro *Jovis Digreſſiones*, à ſeipſo obſervatas, necnon & Motus cujuſque Medios, Motuumque illorum Radices, ab Obſervationibus ejus deductas, *Townleio* ſuo accommodatas, communicavit. Ab eodem deinceps *Ephemerides* tuas, Clariſſime *Caſſine*, *Mediceorum Siderum An.* 1668. impetravi; quibus quando cum Motus tum Motuum Radices, necnon & ſummas Elongationes à Te conſtitutas, nonnihil à D. *Townleii* inventis diſſidere comperui, & Ego, quod ipſe impenſius hortatus eſt, nonnullas primâ quoque Occaſione Obſervationes inſtituere Operæ fore pretium duxi. *Micrometro* itaque & Tubo 14. *Ped. An.* 1672. Menſe *Mar. Stil. Jul.* ſequentia qua potui Cura Experimenta prima feci, Obſervationibus, in majorem Certitudinem, identidem quaque Nocte iteratis.

An. 1672. *Mar.* $19^d\ 7^h\ 11'$. Limb. *Jovis* remotior à 4*to*. *Satellite* diſt.
1601 * = 9' — 34'' * 9' 37''.
27. 8. Limb. remotior ab eodem 4*to*. *Satellite* *n.* 28. *p.* 4037.
1591 = 9 — 30
28. 8. Eadem Diſtantia ——— 1598 = 9 — 33

Jovis Diameter pluribus obſervationibus reperta 128. Ergo Semidiameter ejus 64; quâ diviſis Diſtantiis obſervatis, apparentes fient *Satellitis* à Limbo *Jovis* Remotiori Diſtantiæ in Semidiametris ejus.

	d.	ſd.	′		ſd.	′
Mar.	19.	25	1	Sublatâ ſemidiam. à centro fient	24	1
	27.	24	51		23	51
	28.	24	58		23	58

Cujus tunc Motus à *Jove* & diſtantiæ à centro ipſius fuere, ſecundum numeros tuos, ut hic.

	d.	h.	′	s.	°	′	ſd.	′
Mart.	19	7	11	8	25	33	22	56
	27	8	..	2	19	35	22	37
	28	8	..	3	11	13	22	34

Aberat ergo *Satelles* ab extima Elongatione, in prima obſervatione, tantum 4′; in ſecunda 23′; in tertia 26′: Semidiametri ſcrupulos ſexagenarios; quos propterea ſi obſervatis *Elongationibus* modo debito adjiciamus, fient maximæ Digreſſiones, hujus *4ti Satellitis*, à centro *Jovis* per primam Obſervationem $24^{\text{ſd}}$ 5′; per ſecundam $24^{\text{ſd}}$ 14′; per tertiam, $24^{\text{ſd}}$ 24′; quam *Tu* ſtatuiſti tantum $23^{\text{ſd}}$. *R. Townleius* $24^{\text{ſd}}$ 72.

Harum Elongationum poſteriores duas accuratiores exiſtimo, quippe quibus inveſtigandis obſervationes commodas, omni qua cura poteram, peregi; priorem deinde inter plura noctis $19^{\text{æ}}$ Adverſaria inveni quam perinde exquiſite captam haud auſim affirmare. Utcunque tamen Obſervationem adjeci, quippe qui non adeò à ſequentibus diſſentit, quin eas poſſit confirmare, necnon oſtendere, perparùm (ſi quicquam) minùs, Siniſtram quam dextram verſus, hunc Satellitem à *Jove* Elongari.

Sed tamen inter obſervandum ſenſi, Aeris & Venti motum,, quatiendo vel agitando Tubum (ad erectam Abietem, ope funis, & trochleæ ſub dio penſilem) obſervationem reddere difficilem; quin & frequenter efficere, ut nimis ſtrictas acciperem Diſtantias. Quamobrem à pluribus hujuſmodi obſervationibus, quæ ſummam curam & præciſionem depoſcunt, eo uſque ſuperſedere conſtitui, donec commodiorem iis inſtituendis locum aptarem, quem tandem paravi. In Feneſtra quadam Ligneam Machinam, brevis adinſtar Scalæ, aptari curavi; cujus operi impoſitus Tubus quaquaverſum converti potuit, nec à ventis, niſi admodum turbidis, hinc inde, ut ſub dio fuit, agitari. Huic impoſito Tubo, *An.* 1683. *Apr.* 4. Veſp. Meipſum obſervationibus omni diligentia peragendis accinxi, nec fruſtra quidem: Etenim, cælo admodùm ſereno, omnes quatuor *Satellites*, per Tubum Lentium Convexarum, conſpexi, & eorum infra ſcriptas à Limbo *Jovis*, cuique remotiori, Diſtantias Dimenſus ſum; ſcil.

3 985 iterum — 988
2 628 ——— 636
1 425 ——— 427
4 272 ——— 272 Altitudo *Jovis* Quadrante ferè bipedali capta 24° — 00'. Ergo hora apparens *Derbiæ* 8^h 26' *p. m.* & tunc 4^{tus} *Satelles* infra lineam utrinque per extimos *Satellites*, apparuit; ſed vix plenam, ni fallor, Semidiametrum.

Jovis Diameter, identidem repetitis obſervationibus, reperta 133; Simidiameter ergo $66\frac{1}{2}$, quæ obſervatis ſublata Diſtantiis, fient interſtitia inter Centrum *Jovis* & *Comitis Primi*, 360; *Secundi* 569; *Tertii*, 921; *Quarti*, 205; quibus per $66\frac{1}{2}$ diviſis, prodibunt viſæ Elongationes à centro *Jovis*, in ipſius Semidiametris.

	ſd.	'			s.	°	'	ſd.	'
1	5	25	Motus *Satellitum* à *Jove* &	1	9	4	52	4	59
2	8	33	Remotiones apparentes ſe-	2	2	22	47	7	57
3	13	51	cundum *Tuas Tabulas*, *Derbiæ*	3	2	20	26	12	48
4	3	5	reductas, fuere;	4	4	23	49	2	29

Defecit ergo *Satelles Primus*, 1' tantum; *Secundus* 3; *Tertius* 12; Scrupulis Semidiametri ſexagenariis à ſumma Elongatione, quos propterea ſi obſervatis addamus, fient extremæ Digreſſiones.

	ſd.	'		ſd.		ſd.	'
Primi,	5	26	Quas *Tu* ponis,	5	Attamen D. *Towneleius*,	5	31
Secundi,	8	37		8	ut in Schedis aliquibus	8	47
Tertii,	14	2		13	reperio,	13	28

Commoda rurſus præviſa dari opportunitas *Apr.* 11. veſperi: quamobrem, cum non ab uno aut altero Experimento diſtantias has duxerim definiendas, habitis tunc etiam Obſervationibus, ulterius mecum inquirere inſtitui; quas cùm primum auſpicabar, Cælum circa *Jovem* raris adeo nubibus tectum erat, ut ſubobſcurius non niſi aliquando *Satellites* potuerim conſpicere; quorum tamen à limbo *Jovis* remotiori, ut tulit aer, cepi diſtantias; nimirum,

Hor $7\frac{1}{2}$ *p. m.* 3 947 iterum 932
2 628 ——— 614
1 405. Facto tamen Cælo protinus ad votum ſereno, accuratius notavi.

3 947
2 622
1 405
4 942. Iterum 957, *Alto* ♃ve. 24° 00' ergo hora apparens 7^h 56'. *Satelles Quartus* paulo ſupra lineam, per *Primum* & *Secundum* ductam, apparuit;

apparuit; *Tertius* infra eam, ſed & aliquando exiſtimavi in ea. *Jovis* capta diameter 132, Semidiameter ergo 66, obſervatis quæ ſubducta Diſtantiis interſtitium dabit inter centrum *Jovis* & *Primi*, 339, *Secundi*, 556; *Tertii*, 881; *Quarti*, 891; quibus ſigillatim per 66 diviſis, prodeunt Elongationes apparentes à centro *Jovis* in Semidiametris ejuſdem, *Primi* quidem $5^{ſd}\,8'$; *Secundi*, $8^{ſd}\,25'$; *Tertii*, $13^{ſd}\,21'$; *Quarti*, $13^{ſd}\,30'$.

Satellitum Motus Medii à *Pleni-Mediceis*, cum Diſtantiis eorum à centro *Jovis*, ſecundum numeros tuos fuere, ut in hac tabellula exarantur. Unde videre eſt, *Primum* à ſumma Elongatione abeſſe ſcrupulos Semidiametri 10′; *Secundum*, 26′; *Tertium*, 38′; quos propterea ſi obſervatis Elongationibus adjiciamus, fient *Maximæ* hinc deducendæ *Digreſſiones*.

	s.	°	′	ſd.	′
1	8	15	35	4	50
2	2	10	59	7	34
3	2	12	2	12	22
4	10	25	8	13	15

Primi	5	18
Secundi	8	51
Tertii	13	59

perparum ab iis, quas obſervationibus noctis Quartæ deduximus, diſſentientes.

His tamen utriſque vicibus *Intimus Satelles* ad Lævam, *Secundus* & *Tertius* ad dextram, à *Jove* apparuere; ſed *Apr.* 15. veſp. *Tertium* à ſiniſtra, in maxima Elongatione appariturum prævidi, cui propterea Phænomeno invigilare operæ fore pretium duxi, *nimirum* ut perſpicerem, num eadem eſſet ejuſdem *Satellitis* ad manum utramque à centro *Jovis*, ſumma Remotio. Cælum nocte obſervationi antedicta ſudum erat; ſic pro voto obſervavi circa *hor.* $7\frac{1}{2}$. *Tertii* Diſtantiam 955; & *Jovis* Diam. 131. *Jovis* ergo Semidiameter $56\frac{1}{2}$, obſervatæ quâ ſubductâ Diſtantiæ, fit interſtitium inter centrum *Jovis* & *Satellitem*, 889; quod per eandem Semidiametrum diviſum viſibilem dat Elongationem *Comitis* à Centro *Jovis* in Semidiametris ipſius $13^{ſd}\,35'$. Motus *Satellitis* medius erat $3^{s}\,04^{\circ}\,9'$. Locus *Jovis* verus ♎ $10^{\circ}\,27'$. Ergo *Planeta* à *Pleni-Mediceo* $9^{s}\,3^{\circ}\,42'$ aberat, à ſumma Elongatione tantum ſcrupulos 3; quos ſi obſervatæ digreſſioni $13^{ſd}\,35'$ adjiciamus, fiet maxima ad *Siniſtram* hac vice $13^{ſd}\,38'$. parte nimirum tertia Semidiametri minor quam ad *Dextram*, bis conſpirantibus notis, obſervavimus: Quod mihi videtur innuere, eſſe aliquam Centri Orbitæ hujus *Planetæ* à Centro *Jovis Excentricitatem*.

An Inſtrument for finding the diſtances of Jupiter's Satellites *from his Axis; by Mr.* Flamſteed. n. 178. *p.* 1262. Dec. An. 1685. *Fig.* 142.

2. The Little Circle in the Middle repreſents the Planet *Jupiter*, the four Concentrick Circles, the proper Orbits of his four *Satellites*, duly proportioned to the Breadth of his Body; the Diſtances betwixt the parallel Lines interſecting them, being each equal to one of his Semidiameters.

The 4 divided Circles next without theſe, are diſtinguiſhed into ſo many parts as there are days and hours in each *Satellite's* Revolution; the innermoſt of them ſerving for the *Firſt* or *Innermoſt Satellite*; that next it, for the 2^{d}; that next without this for the 3^{d}; and outermoſt for the 4^{th}; above which is a ſmall divided Arch of 15 Degrees.

By this to find the Diſtances of the *Satellites* from ♃'s *Axis* to a propoſed Time.

1. Find the *Parallax* of *Jupiter's* Orb to the Time propoſed, and note whether it be to be Added or Subſtracted.

2. Extend the Thread from the Center of the *Inſtrument* over the *Parallax* numbred in the ſmall Arch, it cuts off in the four divided Circles, ſo many hours as each *Satellite* ſpends in paſſing from the *Axis* of the Shadow to the *Axis* of ♃ viewed from our Earth; theſe I call the Simple *Parallactick Intervals*, which if the Parallax was to be added, are alſo Additional, if to be ſubſtracted, Subductive.

3. To theſe *Parallactick Intervals* add the Times of half the Duration of the *Eclipſe* of each *Satellite*, which for the *Firſt* may be aſſumed 1h 10′; for the 2*d*, 1h 30′, (greater Exactneſs being needleſs); but for the 3*d*, and 4*th* when *Eclipſed*, (their *Immerſion* into the Shadow and *Emerſion* from it being commonly given in the *Catalogues*) take half the Difference of theſe Times at the next *Eclipſe*, to the Time propoſed, for half the Duration, and Add them to the Simple *Parallactick Intervals*, ſo have you them Augmented. But as often as the 4*th Satellite* is not *Eclipſed*, (which is two Years in every ſix) its Interval needs no Augmentation.

4. Find in the *Tables* the Times of the *Eclipſes* of each *Satellite* next preceding the Time propoſed, and when the 4*th* is not *Eclipſed*, of its paſſing the *Axis* of the Shadow, to which if the *Parallactick Intervals* Augmented were Additional, Add them to, if Subductive, Subſtract them from, each the Time of its proper *Satellite's Eclipſe*, ſo have you very near the Apparent Times when each *Satellite* laſt paſt over the *Axis* of ♃ viewed from our Earth.

5. Subſtract each of the Times thus got from the Time propoſed, the Remainders are the Intervals of the Motion of each *Satellite* from ♃'s *Axis*.

6. Extend the Thread, from the Center over each of theſe Intervals of Motion numbred ſeverally in the divided Circles belonging each to its proper *Satellite*, where it cuts the proper *Orbit* of that *Satellite*, whoſe Interval was numbred in its peculiar Circle, it ſhews amongſt the Parallels, how many Semidiameters of ♃, that *Satellite* is diſtant from him, and on which ſide of him 'tis Poſited.

Note further, that the Thread as it lay extended over the *Parallax* of the *Orb* numbred in the ſmall Arch, where it cut the ſeveral proper Orbits of each *Satellite*, ſhewed amongſt the Parallels, how many Semidiameters of ♃, the Center of the Shadow was diſtant from the Center of ♃, viewed from our Earth. And that if the *Parallax* of the *Orb* were Additional, the Shadow lies on the Right-hand from ♃, if Subductive on the Left.

To explain theſe Precepts, I ſhall give two brief Examples. Let it be then propoſed to know how far each *Satellite* appears diſtant from ♃ on the 26*th* of *Dec.* this preſent Year 1685. at 6h 52′. *p. m.* when the *Third Satellite* falls into the Shadow; alſo on *Jul.* 16. 1686. at 10h 00′. *p. m.* when there is no *Eclipſe*.

Fig. 143. *An.* 1685. *December* 26^d 16^h 52′. *p. m.* the *Parallax* of the *Orb* is 9° 20′ Additional; Therefore

	1			2			3			4		
	d.	h.	′	d.	h.	′	d.	h.	′	d.	h.	′
The *Simple Parallactick* Intervals *add.* ———	0	1	5	0	2	10	0	4	25	0	10	20
The half *Durations* of the *Eclipſes* to be added ———	0	1	10	0	1	30	0	1	18	0	0	0
The *Parallactick Intervals* augmented ———	0	2	15	0	3	40	0	5	43	0	10	20
Laſt *Immerſions*, and ☌. *Dec.* ———	25	9	37	25	5	47	19	12	58	10	00	30
Times of laſt paſſing *Jupiter's Axis, Dec.*-	25	11	52	25	9	27	12	18	41	10	10	50
Subſtracted from the Time propoſed. *Dec.*	26	16	52	26	16	52	26	16	52	26	16	52
Leaves the *Intervals* of *Motion.* ———	1	5	00	1	7	25	6	22	11	16	6	2
Over which numbred in their peculiar Circles the Thread being ſeverally laid, cuts the proper *Orbit* of each at their viſible *Diſtances* from *Jupiter* ———	ſd. 5. *dext.*			ſd. 6½. *ſin.*			ſd. 3. *dext.*			ſd. 4½. *dext.*		

Again, *An.* 1686. *Jul.* 16. 10h *p. m.* the *Parallax* of the *Orb* is 10° 46′ Subductive. Hence *Fig.* 144.

	1	2	3	4
	d. h. ′	d. h. ′	d. h. ′	d. h. ′
The *Simple Parallactick Intervals Sub.* ——	0 1 12	0 2 35	0 5 10	0 12 00
Half *Duration* of the *Eclipſes* add. ——	0 1 10	0 1 30	0 1 21	— — —
The *Parallactick Intervals* augmented. ——	0 2 22	0 4 5	0 6 31	0 12 00
The next laſt *Emerſions*, and paſſing the *Axis* of the *Shadow*, *Jul.* ——	15 5 55	15 22 2	15 9 19	15 17 52
The Time of laſt paſſing the viſible *Axis* of *Jupiter* ——	15 3 33	15 17 57	15 2 48	15 5 52
The Time propoſed ——	16 10 00	16 10 00	16 10 00	16 10 00
The *Intervals* of *Motion* ——	1 6 27	0 16 3	1 7 12	1 4 8
Therefore *Diſtan.* from *Jupiter's Axis* ——	ſd. $5\frac{1}{2}$. *Dext.*	ſd. $8\frac{1}{5}$. *Sin.*	ſd. $13\frac{1}{2}$. *Sin.*	ſd. $10\frac{1}{2}$. *Sin.*

And the *Satellites* ſtand at the two propoſed Times as in the two Figures.

In drawing of which, tho' I have conſidered their *Latitudes* from the Line of their utmoſt *Elongations* paſſing through *Jupiter*'s Center; yet I give no Rules for determining it, the Contrivances and Directions neceſſary on that Account, being too many and troubleſome to be inſerted here: My Deſign is only to ſhew the Ingenious Obſerver, how to find at what Diſtance from ♃, each *Satellite* appears, that ſo he may not miſtake one for another when he is to Obſerve any of their *Eclipſes*.

XC. 1. *An.* 1688. The *French Aſtronomers* have made theſe Obſervations by a 14 Foot Teleſcope.

Eclipſes and Places of the Satellites *of* Jupiter *Obſerved, at* Paris; *by* . . . . . *n.* 44. *p.* 892.

Octob. 7. 10h 32′ *p. m.* The *Firſt Satellite* (called *Pallas*) Entred upon the Face of *Jupiter*.

Octob. 8. 8^h 11′. The *Second Satellite* (called *Juno*) Went out behind *Jupiter.*

Octob. 9. 8^h 54′. The *Second Satellite* went out from the Face of *Jupiter.*

Octob. 16. 10^h 4′. The *Second Satellite* entred upon the Face of *Jupiter.*

Octob. 22. 10^h 41′ 33″. The *Firſt Satellite* entred into the Shadow of *Jupiter.*

Octob. 23. 8^h 32′. The *Firſt Satellite* entred upon the Face of *Jupiter.*

Nou. 12. 10^h 40′. The *Second Satellite* entred into the Shadow of *Jupiter.*

Nov. 20. 2^h 38′ 30″. After Midnight, the *Third Satellite* (called *Themis*) entred into the Shadow of *Jupiter.*

At Dantzick; *by M.* Hevelius. *n.* 78. *p.* 3029. 2. Cum *An.* 1671. die 25. *Sept.* ſt. n. manè ex condicto Dⁿ *Caſſinus Pariſſiis*, & Dⁿ *Picard Uraniburgi*, ſuſcepiſſent, ad Occultationem *Primi Jovialium* attendere, volui haud minus ego huic Phænomeno diligenter invigilare. Itaq; *Hor.* 4. 27′. ubi primum *Jupiter* emicuit, deprehendi adhuc *Joviales* omnes adeſſe, tres ſc. à Læva & unum ad dextram. Duo illi propinquiores ad Siniſtram haud procul videbantur à Limbo *Jovis*, non minus ille qui ad Dextram apparebat, aliorum Comitum Minimus. Ad quintam uſque & 7′ ferè, omnes quatuor (utut Cælum jam liveſceret) diſtincte apparebant. Præter tamen omnem ſpem, *hor.* 5. 12′ videbatur mihi propinquior ille *Comes* ad Lævam (reſpectu Tubi mei, qui inverſo ordine objecta exhibet) penitus evaneſcere, remanentibus illis tribus, quanquam ille dexterior magis magiſque etiam ad *Jovem* accedebat. An planè momentum ipſum temporis id fuerit *Immerſionis* illius *Comitis*, vix auſim adeo certò affirmare; nihilominus tardius non incidit illa *Occultatio*; ſed anne unico minuto ferè, adhuc citius forte, ingruerit, facilè conceſſerim.

Sec. Horol. amb. mane.			*Obſervationes.*	*Diſtantiæ & Altitudines.*			*Temp. correct.*		
h.	′	″		°	′	″	h.	′	″
4	36	25	*Jupiter* primum conſpectus ————	—	—	—	4	32	0
5	7	25	Altitudo *Procyonis* ————	34	43	0	5	2	7
5	16	35	Primus *Jovialium* Evanuit ————	—	—	—	5	12	0
5	26	5	Altitudo *Procyonis* ————	36	39	0	5	23	27

At Derby; by *Mr.* Flamſteed. *n.* 82. *p.* 4036. 3. An. 167½. *Feb.* 17. 7h 25′. *p. m.* Alt. ♃ erat 15° 54′. At 8^h 59′. *p. m.* vel forſan 1. min. maturius, *Satelles Primus* ad dextram ♃is, in ipſius Umbram incidit, adeo tamen Evaneſcentis exigua erat à Limbo Diſtantia, ut quanta fuerit dijudicare non potuerim.

Mart. 19^d. 6^h 45'. Alt. *Jov.* 29° 35' *Satelles Primus* ad Limbum *Jovis* appropinquabat, cui 7h 51'. jungebater.

ib. 4037.

An. 1672. *Ap.* 15. 7^h 43'. veſp. *Alt.* ♃. 25° 00'. *Satelles Primus* mox *Jovem* à tergo ſubiturus $\frac{1}{6}$ circiter diametri à Limbo ejus apparuit.

n. 96. *p.* 6099.

8. 6'. *Alt.* ♃ 27° 20'. ſubivit *Jovem*. Alt. *Jove* 27° 26' *certe* non conſpiciebatur.

4. *Jul.* 6. (ſt. n.) 1675. ante mediam noctem, hora ſcil. 11. & 16. Secundis præciſè, *Secundus Jovis Satelles* Egredi incipiebat ex Planetæ hujus, qui ipſum obſcuraverat, umbra.

At Paris; *by M.* Caſſini.
n. 117. *p.* 389.

5. An. 1679. *Jun.* 5. ſt. n. 3h. *m.* I diſcovered 3 *Satellites* of *Jupiter*: The *Firſt* was diſtant Weſtward of the Limb of *Jupiter*, a little leſs than a Diameter; the *Second* was diſtant, on the Eaſt-ſide, a little more than a Diameter. The *Third* was more Eaſtward than the *Second*, by ſomewhat leſs than a Diameter of *Jupiter*.

Ph. Col. n. 1.
p. 33.
§. LXIV. 2.

XCI. 1. Let A be the *Sun*, B *Jupiter*, C the *Firſt Satellite* of *Jupiter*, which enters into the *Shadow* of *Jupiter*, to come out of it at D; and let E F G H L K be the *Earth* placed at divers Diſtances from *Jupiter*.

The Equation of Light; by M. Romer.
n. 136. *p.* 893.
Fig. 145.
June An. 1677.

Now ſuppoſe the *Earth*, being in L towards the 2*d* Quadrature of *Jupiter*, hath ſeen the *Firſt Satellite* at the Time of its *Emerſion*, or iſſuing out of the Shadow, in D; and that about 42½ Hours after (*vid.* after one Revolution of this *Satellite*) the *Earth* being in K, doth ſee it returned in D; it is manifeſt, that if the Light require time to traverſe the Internal LK, the *Satellite* will be ſeen returned later in D, than it would have been if the *Earth* had remained in L, ſo that the Revolution of this *Satellite* being thus obſerved by the *Emerſions* will be retarded by ſo much time, as the Light ſhall have taken in paſſing from L to K, and that on the contrary, in the other Quadrature FG, where the *Earth* by approaching goes to meet the Light, the Revolutions of the *Emerſions* will appear to be ſhortened by ſo much, as thoſe of the *Emerſions* had appeared to be lengthened.

This new *Equation* of the Motion of Light, which hath been eſtabliſhed by the *Royal Academy*, and in the *Obſervatory* for the Space of 8 Years, was confirmed by the *Emerſion* of the *Firſt Satellite* obſerv'd at *Paris* 1676. *Nov.* 9. 5^h 35' 45", at Night, 10' later than it was expected, by deducing it from thoſe that had been Obſerved in the Month of *Auguſt*, when the *Earth* was much nearer to *Jupiter*.

The Theory of Jupiter's Satellites; *by M.* Caſſini.
n. 128. *p.* 681.
Sept. An. 1676.

XCII. 1. *M. Caſſini*, having formed a new Hypotheſis for the *Satellites* of *Jupiter*, different from that of *Galileo*, thinks that the Plain of their *Orbs* is Inclined to the Plain of the *Ecliptick*; He ſettles their *Nodes* with the *Orbs* of *Jupiter* towards the 13° of *Leo* and *Aquarius*; and finds that the *Obliquity* of their Circles to the Orbit of *Jupiter*, is almoſt double to the *Obliquity* of this Orbit to the *Ecliptick*.

M. Caſſini's *Tables, for the Eclipſes of the* Firſt Satellite of Jupiter, *Abridged, and Reduced to the Meridian of* London; *by Mr.* Edm. Halley.
n. 21. *p.* 238.
Nov. An. 1694.

2. M. *Caſſini*, in the laſt Treatiſe of a Book, Entitul'd, *Recueil d' Obſervations faites en Pluſieurs Voyages*, &c. has employed his Skill, to make eaſy

the Calculation of the *Eclipſes* of the *Firſt Satellite* of *Jupiter*, which is otherwiſe Operoſe even to the Skilful. The Tables have for Principles, that this *Satellite* Revolves to the *Sun* in $1^{d}\ 18^{h}\ 28'\ 36''$, ſo preciſely, that in 100 Years the Difference is not ſenſible; That in the Time of the Revolution of *Jupiter* to his *Aphelion*, which he ſuppoſes in $4332^{d}\ 14^{h}\ 52'\ 48''$, this *Satellite* makes exactly 2448 Months or Revolutions to the *Sun*; and dividing the Orbite of *Jupiter* into 2448 parts, he has in a Large Table of *Equation* ſhewn what is the Inequality of the Motion of *Jupiter* in each Revolution reduced to time, aſſuming, *Thirdly*, the greateſt *Equation* of *Jupiter* $5^{\circ}\ 32'\ 40''$. whence the hourly Motion of the *Satellite* from *Jupiter* being $8^{\circ}\ 28'$, it follows that the greateſt Inequality (*Jupiter* paſſing the Signs of *Cancer* and *Capricorn*,) amounts to $39'\ 8''$ of Time, to be added in *Cancer*, ſubſtracted in *Capricorn*. *Laſtly*, As to the *Epocha*, or beginning of this Series of Revolutions, he has determined the *Aphelion* of *Jupiter* about 1½ Degree forwarder than *Aſtronomia Carolina*, and above two Degrees more than the *Rudolphine* Tables, *viz*. preciſely in 9° of *Libra*, in the beginning of this Century, which perhaps he finds the proper Motion of *Jupiter* about the *Sun* at this time to require; and the Number of Revolutions ſince *Jupiter* was laſt *in Perihelio*, is here ſtiled *Num*. I.

A Second Inequality is that which depends on the Diſtance of the *Sun* from *Jupiter*, which he ſays Mr. *Romer* did moſt ingeniouſly explain by the Hypotheſis of the Motion of Light; to which yet *Caſſini* by his manner of *Calculus* ſeems not to aſſent, though it be hard to imagine how the *Earth's* Poſition in reſpect of *Jupiter* ſhould any way affect the Motion of the *Satellites*. This Inequality he makes to amount to two Degrees in the *Satellite's* Motion, or $14'\ 10''$ of Time, wherein he ſuppoſes the *Eclipſes* to happen ſo much ſooner when *Jupiter* Oppoſes the *Sun*, than when he is in Conjunction with him. The Diſtribution of this Inequality he makes wholly to depend on the Angle at the *Sun* between the *Earth* and *Jupiter*, without any Regard to the *Excentricity* of *Jupiter*, (who is ſometimes ½ a Semidiameter of the *Earth*'s Orb farther from the *Sun* than at other times) which would occaſion a much greater Difference, than the Inequality of *Jupiter* and the *Earth*'s Motion, both of which are accounted for in theſe Tables with great Skill and Addreſs. But what is moſt ſtrange, he Affirms that the ſame Inequality of two Degrees in the Motion, is likewiſe found in the other *Satellites*, requiring a much greater Time; as above two Hours in the 4th *Satellite*: which if it appeared by Obſervation would overthrow M. *Romer*'s Hypotheſis entirely. Yet I doubt not herein to make it Demonſtratively plain, that the Hypotheſis of the Progreſſive Motion of Light is found in all the other *Satellites* of *Jupiter* to be neceſſary, and that it is the ſame in all; there being nothing near ſo great an Annual Inequality as M. *Caſſini* ſuppoſes in their Motions, by his Table, *p*. 9. and his *Præcepta Calculi*. The Method however uſed to compute this is very Curious; for having found that whilſt the *Sun* Revolves to *Jupiter* there paſs $398^{d}\ 21^{h}\ 13'$. wherein are made 225⅜ Revolutions of the *Satellite* to *Jupiter*, the Number of Revolutions

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ſince *Jupiter* was laſt in Oppoſition to the *Sun*, is what he calls *Num.* II. in which the Inequality of the *Earth*'s Motion is allowed for in the Months, and that of *Jupiter*'s Orb by a Table of the *Equation* of *Num.* II. amounting in all to $3\frac{1}{2}$ Revolutions of the *Satellite* to *Jupiter*. This in the Tables following I have thought fit to leave out, ſhewing how to find it by the help of the former Equation of *Num.* I. The Numbers are in effect the ſame with M. *Caſſini*'s, only Reduced to our Stile and Meridian, and the Form of them Abridged, and 'tis hoped Amended.

Epocha Revolutionum Primi Satellitis *ad* Jovis *Umbram* *ſub Meridiano* Londinenſi.

Anno Jul. Curr.					Num. I.	Num. II.
	d.	h.	′	″		
1660	0	11	5	48	968	200 6
1661	0	1	17	24	1174	181 2
1662	1	9	57	36	1381	162 2
1663	1	00	9	12	1587	143 5
1664	1	8	49	24	1794	125 1
1665	0	23	1	00	2000	105 7
1666	0	13	12	36	2206	86 4
1667	0	3	24	12	2412	67 0
1668	0	12	4	24	171	48 6
1669	0	2	16	00	377	29 2
1670	1	10	56	12	584	10 9
1671	1	1	7	48	790	216 9
1672	1	9	48	00	997	198 5
1673	0	23	59	36	1203	179 1
1674	0	14	11	12	1409	159 7
1675	0	4	22	48	1615	140 3
1676	0	13	3	00	1822	121 9
1677	0	3	14	36	2028	102 5
1678	1	11	54	48	2235	84 1
1679	1	2	6	24	2441	64 7
1680	1	10	46	36	200	46 4
1681	1	00	58	12	406	27 0
1682	0	15	9	48	612	7 6
1683	0	5	21	24	818	213 6
1684	0	14	1	36	1025	195 3
1685	0	4	13	12	1231	175 9
1686	1	12	53	24	1438	157 5
1687	1	3	5	00	1644	138 1
1688	1	11	45	12	1851	119 7
1689	1	1	56	48	2057	100 4

Epochæ Revolutionum Primi Satellitis *ad* Jovis *Umbram* *ſub Meridiano* Londinenſi.

Anno Jul. Curr.	d.	h.	'	''	*Num. I.*	*Num. II.*
1690	0	16	8	24	2263	81 0
1691	0	6	20	00	21	61 6
1692	0	15	00	12	228	43 3
1693	0	5	11	48	434	23 9
1694	1	13	52	00	641	5 5
1695	1	4	3	36	847	211 5
1696	1	12	43	48	1054	193 1
1697	1	2	55	24	1260	173 7
1698	0	17	7	00	1466	154 4
1699	0	7	18	36	1672	135 0
1700	0	15	58	48	1879	116 6
1701	0	6	10	24	2085	97 3
1702	1	14	50	36	2292	78 9
1703	1	5	2	12	50	59 5
1704	1	13	42	24	257	41 1
1705	1	3	54	00	463	21 8
1706	0	18	5	36	669	2 4
1707	0	8	17	12	875	208 4
1708	0	16	57	24	1082	190 0
1709	0	7	9	00	1288	170 6
1710	1	15	49	12	1495	152 3
1711	1	6	00	48	1701	132 9
1712	1	14	41	00	1908	114 5
1713	1	4	52	36	2114	95 1
1714	0	19	4	12	2320	75 8
1715	0	9	15	48	78	56 4
1716	0	17	56	00	285	38 0
1717	0	8	7	36	491	18 6
1718	1	16	47	48	698	0 3
1719	1	6	59	24	904	206 3
1720	1	15	39	36	1111	187 9

Tabula

Tabula Revolutionum Primi Satellitis *Jovis* in *Anno.*

Januarius.				N.I.	N.II.	
d.	h.	′	″			
0	0	0	0	0	0	0
1	18	28	36	1	1	0
3	12	57	12	2	2	1
5	7	25	48	3	3	1
7	1	54	24	4	4	1
8	20	23	00	5	5	2
10	14	51	36	6	6	2
12	9	20	12	7	7	2
14	3	48	48	8	8	2
15	22	17	24	9	9	3
17	16	46	00	10	10	3
19	11	14	36	11	11	3
21	5	43	12	12	12	3
23	00	11	48	13	13	4
24	18	40	24	14	14	4
26	13	9	00	15	15	4
28	7	37	36	16	16	5
30	2	6	12	17	17	5
31	20	34	48	18	18	5
Februarius.						
0	20	34	48	18	18	5
2	15	3	24	19	19	6
4	19	32	00	20	20	6
6	4	00	36	21	21	6
7	22	29	12	22	22	6
9	16	57	48	23	23	7
11	11	26	24	24	24	7
13	5	55	00	25	25	7
15	00	23	36	26	26	7
16	18	52	12	27	27	7
18	13	20	48	28	28	7
20	7	49	24	29	29	7
22	2	18	00	30	30	8
23	20	46	36	31	31	8
25	15	15	12	32	32	8
27	9	43	48	33	33	8

Martius.				N.I.	N.II.	
d.	h.	′	″			
1	4	12	24	34	34	8
2	22	41	00	35	35	8
4	17	9	36	36	36	8
6	11	38	12	37	37	9
8	6	6	48	38	38	9
10	00	35	24	39	39	9
11	19	4	00	40	40	9
13	13	32	36	41	41	9
15	8	1	12	42	42	9
17	2	29	48	43	43	9
18	20	58	24	44	44	9
20	15	27	00	45	45	9
22	9	55	36	46	46	9
24	4	24	12	47	47	9
25	22	52	48	48	48	9
27	17	21	24	49	49	9
29	11	50	00	50	50	9
31	6	18	36	51	51	9
Aprilis.						
0	6	18	36	51	51	9
2	00	47	12	52	52	9
3	19	15	48	53	53	9
5	13	44	24	54	54	9
7	8	13	00	55	55	9
9	2	41	36	56	56	9
10	21	10	12	57	57	9
12	15	38	48	58	58	9
14	10	7	24	59	59	9
16	4	36	00	60	60	8
17	23	4	36	61	61	8
19	17	33	12	62	62	8
21	12	1	48	63	63	8
23	6	30	24	64	64	8
25	00	59	00	65	65	7
26	19	27	36	66	66	7
28	13	56	12	67	67	7
30	8	24	48	68	68	6

Tabula Revolutionum Primi Satellitis *Jovis* in *Anno.*

Maius.				N. I.	N. II.	*Julius.*				N. I.	N. II.
d.	h.	′	″			d.	h.	′	″		
0	8	24	48	68	68 6	1	7	5	48	103	102 5
2	2	53	24	69	69 6	3	1	34	24	104	103 5
3	21	22	00	70	70 6	4	20	3	00	105	104 4
5	15	50	36	71	71 6	6	14	31	36	106	105 4
7	10	19	12	72	72 5	8	9	00	12	107	106 5
9	4	47	48	73	73 5	10	3	28	48	108	107 3
10	23	16	24	74	74 5	11	31	57	24	109	108 3
12	17	45	00	75	75 5	13	16	26	00	110	109 3
14	12	13	36	76	76 4	15	10	54	36	111	110 2
						17	5	23	12	112	111 2
16	6	42	12	77	77 4	18	23	51	48	113	112 2
18	1	10	48	78	78 4	20	18	20	24	114	113 1
19	19	39	24	79	79 3	22	12	49	00	115	114 1
21	14	8	00	80	80 3	24	7	17	36	116	115 1
23	8	36	36	81	81 3						
25	3	5	12	82	82 3	26	1	46	12	117	116 0
26	21	33	48	83	83 3	27	20	14	48	118	117 0
28	16	2	24	84	84 2	29	14	43	24	119	118 0
30	10	31	00	85	85 2	31	9	12	00	120	119 0
Junius.				N. I.	N. II.	*Augustus.*				N. I.	N. II.
1	4	59	36	86	86 1	0	9	12	00	120	119 0
2	23	28	12	87	87 1	2	3	40	36	121	119 9
4	17	56	48	88	88 0	3	22	9	12	122	120 9
6	12	25	24	89	89 0	5	16	37	48	123	121 9
						7	11	6	24	124	122 9
8	6	54	00	90	90 0	9	5	35	00	125	123 8
10	1	22	36	91	90 9	11	00	3	36	126	124 8
11	19	51	12	92	91 9	12	18	32	12	127	125 8
13	14	19	48	93	92 9	14	13	00	48	128	126 8
15	8	48	24	94	93 8	16	7	29	24	129	127 7
17	3	17	00	95	94 8	18	1	58	00	130	128 7
18	21	45	36	96	95 7	19	20	26	36	131	129 7
20	16	14	12	97	96 7	21	14	55	12	132	130 7
22	10	42	48	98	97 7	23	9	23	48	133	131 7
24	5	11	24	99	98 6	25	3	52	24	134	132 7
25	23	40	00	100	99 6	26	22	21	00	135	133 6
27	18	8	36	101	100 6	28	16	49	36	136	134 6
						30	11	18	12	137	135 6
29	12	37	12	102	101 5						

Tabula Revolutionum Primi Satellitis *Jovis* in *Anno.*

September.				N. I.	N. II.	*November.*				N. I.	N. II.
d.	h.	′	″			d.	h.	′	″		
1	5	46	48	138	136 6	0	9	59	12	172	170 7
3	00	15	24	139	137 6	2	4	27	48	173	171 8
4	18	44	00	140	138 6	3	22	56	24	174	172 8
6	13	12	36	141	139 6	5	17	25	00	175	173 8
8	7	41	12	142	140 6	7	11	53	36	176	174 8
10	2	9	48	143	141 5	9	6	22	12	177	175 9
11	20	38	24	144	142 5	11	00	50	48	178	176 9
13	15	7	00	145	143 5	12	19	19	24	179	177 9
15	9	35	36	146	144 5	14	13	48	00	180	178 9
						16	8	16	36	181	180 0
17	4	4	12	147	145 5	18	2	45	12	182	181 0
18	22	32	48	148	146 5	19	21	13	48	183	182 0
20	17	1	24	149	147 5	21	15	42	24	184	183 0
22	11	30	00	150	148 5	23	10	11	00	185	184 0
24	5	58	36	151	149 5	25	4	39	36	186	185 1
26	00	27	12	152	150 5	26	23	8	12	187	186 1
27	18	55	48	153	151 5	28	17	36	48	188	187 2
29	13	24	24	154	152 5	30	12	5	24	189	188 2
October.						*December.*					
1	7	53	00	155	153 5	0	12	5	24	189	188 2
3	2	21	36	156	154 5	2	6	34	00	190	189 2
4	20	50	12	157	155 5	4	1	2	36	191	190 3
6	15	18	48	158	156 5	5	19	31	12	192	191 3
8	9	47	24	159	157 5	7	13	59	48	193	192 3
10	4	16	00	160	158 5	9	8	28	24	194	193 4
11	22	44	36	161	159 5	11	2	57	00	195	194 4
13	17	13	12	162	160 5	12	21	25	36	196	195 5
15	11	41	48	163	161 6	14	15	54	12	197	196 5
17	6	10	24	164	162 6	16	10	22	48	198	197 6
19	00	39	00	165	163 6	18	4	51	24	199	198 6
20	19	7	36	166	164 6	19	23	20	00	200	199 7
22	13	36	12	167	165 6	21	17	48	36	201	200 7
24	8	4	48	168	166 6	23	12	17	12	202	201 8
26	2	33	24	169	167 7	25	6	45	48	203	202 8
27	21	2	00	170	168 7	27	1	14	24	204	203 9
29	15	30	36	171	169 7	28	19	43	00	205	204 9
31	9	59	12	172	170 7	30	14	11	36	206	206 0

Tabula Prima Æquationis Conjunctionum Primi Satellitis *cum* Jove.

N. I.	Æquat.		N. I.	Æquat.		N. I.	Æquat.	
	′	″		′	″		′	″
0	0	0	400	34	20	810	33	21
10	1	3	410	34	51	820	32	50
20	2	5	420	35	21	830	32	17
30	3	8	430	35	47	840	31	44
40	4	12	440	36	6	850	31	10
50	5	15	450	36	26	860	30	32
60	6	16	460	36	47	870	29	56
70	7	19	470	37	8	880	29	19
80	8	20	480	37	29	890	28	40
90	9	23	490	37	44	900	27	59
100	10	25	500	37	59	910	27	19
110	11	25	510	38	16	920	26	37
120	12	25	520	38	29	930	25	53
130	13	25	530	38	39	940	25	8
140	14	25	540	38	49	950	24	23
150	15	22	550	38	55	960	23	37
						970	22	50
160	16	18	560	38	59	980	22	3
170	17	17	570	39	3	990	21	15
180	18	11	580	39	6	1000	20	26
190	19	9	590	39	8	1010	19	37
200	20	5	600	39	7	1020	18	47
210	20	56	610	39	5	1030	17	56
220	21	49	620	39	03	1040	17	5
230	22	41	630	38	58	1050	16	13
240	23	32	640	38	51	1060	15	19
250	24	20	650	38	44	1070	14	25
260	25	7	660	38	34	1080	13	32
270	25	57	670	38	24	1090	12	37
280	26	43	680	38	10	1100	11	42
290	27	27	690	37	56	1110	10	47
300	28	9	700	37	40	1120	9	52
310	28	54	710	37	24	1130	8	57
320	29	35	720	37	5	1140	8	00
330	30	11	730	36	45	1150	7	3
340	30	45	740	36	25	1160	6	7
350	31	28	750	36	4	1170	5	1
360	32	10	760	35	40	1180	4	13
370	32	44	770	35	15	1190	3	15
380	33	15	780	34	49	1200	2	19
390	33	49	790	34	19	1210	1	21
400	34	20	800	34	49	1220	0	20
			810	33	21	1224	0	0

Tabula Secundæ Æquationis Conjunctionum Primi Satellitis *cum* Jove.

N. II.	*Æquat. a dd.*		N. II.	*Æquat. add.*		N. II.	*Æquat. add.*		N. II.	*Æquat. add.*	
	′	″		′	″		′	″		′	″
0	0	0	28	2	4	56	7	0	84	12	0
1	0	0	29	2	13	57	7	12	85	12	9
2	0	1	30	2	21	58	7	24	86	12	16
3	0	2	31	2	30	59	7	36	87	12	24
4	0	3	32	2	39	60	7	47	88	12	32
5	0	4	33	2	48	61	7	59	89	12	40
6	0	6	34	2	58	62	8	11	90	12	47
7	0	8	35	3	8	63	8	22	91	12	53
8	0	10	36	3	17	64	8	34	92	13	00
9	0	14	37	3	27	65	8	46	93	13	6
10	0	17	38	3	37	66	8	57	94	13	13
11	0	20	39	3	48	67	9	8	95	13	19
12	0	23	40	3	59	68	9	20	96	13	24
13	0	27	41	4	9	69	9	32	97	13	30
14	0	32	42	4	20	70	9	44	98	13	35
15	0	37	43	4	31	71	9	54	99	13	39
16	0	42	44	4	41	72	10	3	100	13	45
17	0	47	45	4	53	73	10	14	101	13	48
18	0	53	46	5	4	74	10	25	102	13	51
19	0	58	47	5	15	75	10	35	103	13	54
20	1	4	48	5	27	76	10	45	104	13	57
21	1	11	49	5	39	77	10	55	105	14	00
22	1	18	50	5	50	78	11	5	106	14	3
23	1	25	51	6	2	79	11	15	107	14	5
24	1	32	52	6	14	80	11	25	108	14	7
25	1	40	53	6	25	81	11	34	109	14	8
26	1	47	54	6	37	82	11	43	110	14	9
27	1	56	55	6	49	83	11	52	111	14	10
28	2	4	56	7	00	84	12	00	112	14	10

Tabula

Tabula Dimidiæ Moræ Primi Satellitis *in Umbra* Jovis.

N.I.	h.	′	″	*N.I.*	h.	′	″
0	1	4	56	1200	1	5	6
40	1	4	33	1240	1	4	48
80	1	4	12	1280	1	4	26
120	1	3	59	1320	1	4	7
160	1	3	48	1360	1	3	54
200	1	3	39	1400	1	3	38
240	1	3	38	1440	1	3	38
280	1	3	48	1480	1	3	44
320	1	4	1	1520	1	3	52
360	1	4	16	1560	1	4	7
400	1	4	36	1600	1	4	24
440	1	4	56	1640	1	4	42
480	1	5	18	1680	1	5	00
520	1	5	41	1720	1	5	22
560	1	6	1	1760	1	5	46
600	1	6	21	1800	1	6	10
640	1	6	39	1840	1	6	28
680	1	6	53	1880	1	6	45
720	1	7	3	1920	1	6	57
760	1	7	11	1960	1	7	7
800	1	7	15	2000	1	7	13
840	1	7	13	2040	1	7	14
880	1	7	9	2080	1	7	15
920	1	7	2	2120	1	7	15
960	1	6	54	2160	1	7	10
1000	1	6	39	2200	1	6	49
1040	1	6	22	2240	1	6	32
1080	1	6	5	2280	1	6	15
1120	1	5	45	2320	1	5	58
1160	1	5	26	2360	1	5	38
1200	1	5	6	2400	1	5	18
				2440	1	5	2

From theſe Tables, to any given Year, Month and Day, to find the next Eclipſe of the Firſt Satellite *of* Jupiter *proceed thus.*

1. In the Table of the *Epocha* find the Year of our Lord, and ſet down the Day, Hours, Minutes and Seconds, with *Num.* I. and *Num.* II. thereto annex'd; and in the Table of Revolutions, ſeek the Month, and Day of the Month, with the Hours and Minutes, and *Num.* I. and *Num.* II. affix'd, and add them together; and the reſpective Sums ſhall ſhew the Mean Time of the Middle of the *Eclipſe* ſought, with *Num.* I. and *Num.* II. required. But it muſt be Obſerved, that in *Jan.* and *Feb.* in the *Leap Year,* one Day is to be added to the Day thus found.

2. If *Num.* I. be found leſs than 1224, with *Num.* I. or if the greater than 2448, ſubſtracting 2448 therefrom, with the Reſidue; enter the Table, and you will have the Firſt Æquation to be Added to the Mean Time before found. But if *Num.* I. be leſs than 2448, but greater than 1224, ſubſtract it from 2448, and entring the ſame Table with the remainder, you ſhall have the Firſt Æquation to be Subſtracted from the Mean Time. Then divide the Minutes of the ſaid Firſt Æquation by 11, or rather $^{5}\frac{4}{5}$, and the *Quote* ſhall be the Æquation of *Num.* II. (anſwering to the Eccentrick Motion of *Jupiter*) to be Added thereto when the Firſt Æquation Subſtracts, and *è contra* Subſtracted when that Adds.

3. If *Num.* II. thus Æquated exceed 225,4, Subſtract 225,4, therefrom; and if the Remainder or *Num.* II. be leſs than 113, with the ſaid Remainder or Number; or if greater than 113, with the Complement thereof to 225,4, ſeek in the Table the ſecond Æquation, which being Added to the Time before found, gives the True Time of the Middle of the *Eclipſe.*

4. With *Num.* I. ſeek the Half Continuance of the Total *Eclipſe,* which is to be Added for the *Emerſion* when the Æquated *Num.* II. is leſs than 113, or if more than 225,4, it be leſs than 338. But if it exceed 113, or 338, then is the *Semimora* to be Subſtracted for the *Immerſion.*

Vid. Sup. § XXII.

5. Laſtly, With the *Sun*'s True *Place* take out the Æquation of Natural Days, which Added or Subſtracted according to the Title, gives the Time of the *Immerſion* or *Emerſion* ſought.

Now how few Figures ſerve for this Computation, will beſt appear by an Example.

An. 1677. *Sept.* 17 8h 9′ 40″. at *Greenwich*, Mr. *Flamſteed* obſerved the *Firſt Satellite* to begin to *Emerge*; that is 8h 9′ 26″: at *London*.

	d.	h.	′	″	N. I.	N. II.
1677.	0	3	14	36	2028	102, 5
Sept.	17	4	4	12	147	145, 5
Sept.	17	7	18	48	2175	248, 0
Æquat. 1.	—		26	11	2448	2, 3
Æquat. 2.	17	6	52	37	273	250, 3
	+	0	1	39		225, 4
Semimora	+	1	7	00		24, 9
Equal Time	17	8	1	16	11)	26, 2 (2, 3
Equation	+	0	9	25	☉ in ♎	5°, 00
Apparent T.	17	8	10	41		
Obſerv.	00	8	9	20		
Error			1	21		

An *Immerſion* of this *Satellite* being computed after the ſame manner according to theſe Tables, ought to have happened, *An.* 1683. *Nov.* 30. 16h 52′ 7″. but I obſerved it at 16h 48′ 40″. ſo that the Error was, − 3′ 27″.

Again, M. *Caſſini* obſerved an *Emerſion* at *Paris*, *An.* 1693. *Jan.* 14d 10h 40′ 28″. that is, at *London* 10h 30′ 48″. Which theſe Tables give at 10h 30′ 39″. and therefore the Error was no more than + 9″.

After this manner I have compared theſe Tables with many good and certain Obſervations, and ſcarce ever find them Err above 3 or 4 Minutes of Time; which Errors are exceeding ſmall in compariſon of the ſhort Time that the *Satellites* have been diſcovered.

In the Conſtruction of the Table, which ſhews the Half Continuance of theſe *Eclipſes*, the Semidiameter of the Shadow of *Jupiter* is made by *Caſſini* juſt 10 Deg. and that of the *Satellite* 30′; and the *Satellite*'s *Aſcending Node*, being ſuppoſed in 15° of *Aquarius*, at the End of this Century (that is 55° 20′ before the *Perihelion* of *Jupiter*) it will thence follow, that *Num.* I. being 116, or 2102, *Jupiter* paſſes the *Nodes* of the *Satellites* Orb, and conſequently theſe *Eclipſes* are Central, and of the Greateſt Duration. But *Num.* I. being 215, or 1481, the *Satellite* paſſes the Shadow with the Greateſt Obliquity, *viz.* 2° 55′ from the Center, whence the *Semimora* becomes of all the ſhorteſt.

3. The Tables of the other 3 *Satellites* not being ſo perfect or exact as thoſe of the Firſt, are here given in another Form. The Periods of their Revolutions to *Jupiter*'s Shade are as follows.

Of the other 3 Satellites *Ib. p.* 253.

	d.	h.	′	″	‴			
Period. Secundi	3	13	17	54	3	*ſive*	$2\frac{1}{123}$	*Rev.* Primi.
Period. Tertii	7	3	59	39	22	*ſive*	$4\frac{3}{61}$	*Rev.* Primi.
Period. Quarti	16	18	5	6	50	*ſive*	$9\frac{7}{15}$	*Rev.* Primi.

Whence the Table of the First Æquation of the *Firſt Satellite*, or M. *Caſſini*'s larger Table, may by an eaſy Reduction ſerve the other three; the Æquation of the *2d* being $2\frac{1}{123}$, or twice the Minutes with half ſo many Seconds as there are Minutes in the Æquation of the *Firſt*, and the greateſt Æquation thereof 1^h 18 35″. Æquation of the *3d* is $4\frac{1}{26}$ times greater than that of the *Firſt*, and when greateſt amounteth to 2^h 29″. And the Æquation of the *4th* being $9\frac{7}{15}$ times that of the *Firſt*, is had by ſubſtracting $\frac{1}{2}$ and $\frac{1}{30}$ from 10 times the Equation of the *Firſt*, whence the greateſt becomes 6^h 10′ 28″. ſo that *Num.* I. and *Num.* II. as here collected for the *Firſt*, may indifferently ſerve all the reſt.

M. Romer's *Æquation of Light defended. Ibid. p. 254. Vid. Sup.*

4. As to the Second Æquation of the other *Satellites*, M. *Caſſini* has, by his *Pracepta Calculi* (as is before mentioned) ſuppoſed the Minutes thereof to be increaſed in the ſame Proportion, as inſtead of 14′ 10″ in the *Firſt* to be 28′ 27″ in the *Second*, 57′ 22″ in the *Third*, and no leſs than 2^h 14′ 7″ in the *Fourth*; whereas if this Second Inequality did proceed from the Succeſſive Propagation of Light, this Æquation ought to be the ſame in all of them, which M. *Caſſini* ſays, was wanting to be ſhewn, to perfect M. *Romer*'s Demonſtration; wherefore he has rejected it as ill founded. But there is a good Cauſe to believe, that his Motive thereto, is what he has thought not proper to diſcover. And the following Obſervations do ſufficiently ſupply the Defect complained of in the making out of that Hypotheſis.

An. 1676, *Oct.* ſt. n. 6^h 10′ 37″. *App.* but 5^h 59′ 37″. *Equal Time.* M. *Caſſini* at *Paris* obſerved the *Emerſion* of the *3d. Satellite* from *Jupiter*'s Shadow. And again *Nov.* 14. following 6^h 20′ 54″. *App.* Time, but 6^h 5′ 55″. *Eq. Time*, he obſerved the like Emerſion of the ſame *Satellite*. The obſerved Interval of Time between theſe *Emerſions*, was 43^d 0^h 6′ 18″, which is 8′ 22″ more than 6 Mean Revolutions of this *Satellite*, of which 4′ 27″ ariſes from the Difference of the Firſt Equation, and the greater Continuance of the Latter *Eclipſe*; ſo that the other 4 Minutes is all that is left to anſwer for the Difference of the *2d* Æquations; and *Num.* II. in that Time increaſing from 48 to 72, gives 4′ 36″ for the Difference of the *2d* Æquations of the *Firſt Satellite*. So that here the *2d* Æquation of the *Third* is found rather leſs than that of the *Firſt*, but the Difference is ſo ſmall, that it may rather be attributed to the Uncertainty of Obſervation. Whereas according to M. *Caſſini*'s Method of Calculating, inſtead of four Minutes it ought to be 18′ 38″. and the Interval of theſe two *Emerſions* 43^d 0^h 21′, exceeding the Time obſerved by a whole Quarter of an Hour; which that curious Obſerver could not be deceived in.

The like appears yet more evidently in the *Fourth Satellite.* By the Obſervations of Mr. *Flamſtead* at *Greenwich*, *An.* 1682. *Sept.* 24. $17^{h}\ 45'$. *T. App.* but $17^{h}\ 32'$. *T. Eq.* the *Fourth Satellite* was ſeen newly come out of the Shadow, ſo that about $17^{h}\ 30'$. *T. Eq.* the firſt beginning of *Emerſion* was conjectured; and after 5 Revolutions, *viz. Decemb.* $17^{d}\ 11^{h}\ 16'$ or $11^{h}\ 18'$. *T. Eq.* he again obſerved the firſt Appearance of the *Satellite* beginning to *Emerge*, that is, after an Interval of $83^{d}\ 17^{h}\ 48'$; whereas this *Satellite* makes five mean Revolutions in $83^{d}\ 18^{h}\ 25'\frac{1}{2}$. Here we have $37'\frac{1}{2}$ to be accounted for by the ſeveral Inequalities. Of this 21′ is due to the firſt Æquations, which is reduced to 19 by the Greater Continuance of the latter *Eclipſe*, *Jupiter* then Approaching to his *Deſcending Node.* So that there remains only $18\frac{1}{2}$ for the Differenee of the 2*d* Æquations, whilſt the Earth approached *Jupiter*, by more than the *Radius* of its own Orb: and the Difference of the 2*d* Æquations of the *Firſt Satellite*, being according to *Caſſini* 8′ 30″. the ſaid Difference in the *Fourth* ought to be $1^{h}\ 20'\frac{1}{2}$, inſtead of $18'\frac{1}{2}$; whence the Interval of theſe two *Emerſions* would be according to his Precepts, but $83^{d}\ 16^{h}\ 46'$, inſtead of $83^{d}\ 17^{h}\ 48'$ obſerved. And whereas $18'\frac{1}{2}$, may ſeem too great a Difference; it muſt be noted, firſt, that M. *Romer* had ſtated the whole 2*d* Æquation 22′ 00″. which M. *Caſſini* has diminiſhed to 14′ 10″. ſo that inſtead of $8'\frac{1}{2}$. M. *Romer* allows 13′, and ſecondly, that in the firſt of theſe Obſervations, being about half an Hour before Sun-riſe, the Brightneſs of the Morning might well hinder the ſeeing of this ſmalleſt and ſloweſt *Satellite*, till ſuch time as a good part thereof was *Emerged.*

The Phaſes and Revolution of Mars *about his Axis; by Dr.* Hook.
n. 11. p. 198.
n. 14. p. 239.
Mar. An. 1666.

XCIII. Having a great Deſire to obſerve the Body of *Mars*, whilſt *Acronycal* and *Retrograde* (having formerly with a Glaſs about 12 Foot long, obſerv'd ſome kind of Spots in the Phaſe of it) though it was not in the *Perihelium* of its Orb, but nearer its *Aphelium*; yet I found that the Face of it, when near its Oppoſition to the Sun (with a Charge, the 36 Foot-Glaſs, I made uſe of, would well bear) appeared very near as big, as that of the Moon to the naked Eye.

But ſuch had been the ill Diſpoſition of the Air for ſeveral Nights, that from more than 20 Obſervations of it which I had made ſince its being *Retrograde*, I could find nothing of Satiſfaction, though I often imagined, I ſaw *Spots*, yet the Inflective Veins of the Air (if I may ſo call thoſe parts, which being interſpers'd up and down in it, have a greater or leſs Refractive Power, than the Air next adjoining, with which they are mix'd) did make it ſo confuſed and Glaring, that I could not conclude upon any thing.

Fig. 146.

On the 3*d* of *Mar.* $0^{h}\ 30'$. 166$\frac{5}{6}$. in the Morning though the Air was ſtill bad enough, yet I could ſee now and then the Body of *Mars*, which I deſcribed by the Scheme B; as exactly repreſenting what I ſaw through the Glaſs as I could.

Fig. 147.

An. 166$\frac{5}{6}$, *Mar.* 10. $0^{h}\ 20'$. in the Morning finding the Air very bad, I made uſe of a very ſhallow Eye-Glaſs, as finding nothing diſtinct with the greater Charge, and ſaw the Appearance of it as in C, which I imagined might be the Repreſentation of the former *Spots* by a leſſer Charge. About 3 of the

Clock the ſame Morning, the Air being very bad (though to appearance exceeding
clear, and cauſing all the Stars to twinkle; and the Minute Stars to appear
Fig. 148. pear very thick) the Body ſeemed like D; which I ſtill ſuppoſed to be the
Repreſentation of the ſame *Spots* through a more confuſed and glaring Air.

But obſerving *Mar.* 21. I was ſurprized to find the Air (though not ſo
clear, as to the Appearance of ſmall Stars) ſo exceeding tranſparent, and the
Face of *Mars* ſo very well defined, and round and diſtinct, that I could manifeſtly
Fig. 149. see it of the Shape in E, about half an Hour after 9 at Night. The
Triangular *Spot* on the Right-ſide (as it was inverted by the Teleſcope) according
to the Appearances, through which all the preceding Figures are
drawn) appeared very black and diſtinct, and the other towards the left more
dim; but both of them ſufficiently plain and defined. About a Quarter before
12 of the Clock the ſame Night, I obſerved it again with the ſame Glaſs,
Fig. 150. and found the Appearance exactly, as in F; which I imagined to ſhew me a
Motion of the former Triangular *Spot*.

Mar. 22. about half an Hour after 8 at Night, finding the ſame *Spots* in
Fig. 151. the ſame Poſture (as in G) I concluded that the preceding Obſervation was
only the Appearance of the ſame *Spots* at another Height and Thickneſs of
the Air: and thought my ſelf confirmed in this Opinion, by finding them in
Fig. 152. much the ſame Poſture, *Mar.* 23. about half an Hour after 9 (as in H)
though the Air was nothing ſo good as before.

Mar. 28. about 3 of the Clock, the Air being light (in Weight) though
Fig. 153. moiſt and a little hazy; I plainly ſaw it, to have the Form repreſented in I.
which is not reconcileable with the other Appearances, unleſs we allow a Turbinated
Motion of *Mars* upon its Center: Which, if ſuch there be, from the
Obſervations made *Mar.* 21, 22, and 23. we may gueſs it to be once or twice
in about 24 Hours, unleſs it may have ſome kind of Librating Motion; which
ſeems not ſo likely.

The Parallax of Mars; *by Mr.* Flamſteed. n. 89. *p.* 5118. n. 96. *p.* 6100.

XCIV. *An.* 1672. *Sept. Micrometro* & Tubo 14 pedum, *Martis* Diſtantias à duabus Fixis eadem Nocte dimenſus ſum; unde didici *Parallaxin* ejus *Acronici* & *Perigei* nunquam majorem eſſe Scrupulis Secundis 25′′; unde ſequitur, *Solis* eſſe ſummum 10′′, & Diſtantiam 21000 *Terræ* Semidiametros.

Places of Mars *Obſerved at* Darby; *by Mr.* Flamſteed. *n.* 86. p. 5039.

XCV. 1. *An.* 1672. *Maii* 14 mane, ibat *Martis* Sidus prope Stellam dictam, *Quæ ad Clunes Aquarii*; cujus Latitudo 2° 0′ 0′′. Locus tum *mihi*, 24° 12′ 9′′. ♒; *Streetio*, 24° 9′ 00′′. è qua notabam.

	°	′		°	′	″
Altâ Fixæ verò	9	40	2h 29′′ mane, *Martis* in eodem *Azymutho* præcisè *Diſtantiam*	0	24	17
	11	12	eandem denuo *Diſtantiam* ——	0	24	24
	12	00	2h 51′. *Planeta* diſceſſerat ab eodem *Azymutho*, eratque ad Ortum à Linea, *Azym.* Differ.	0	00	55

2 29

	°	′	″
2^h 29′. mane *Angulus Parallacticus* erat — —	82	26	00
Ergo *Mars* in Conſequentia *Fixa* — — —	00	3	11
cum majori Latitudine Auſtrali — — —	00	24	4
Martis propterea Latitudo — — —	2	24	4
Locus verus ——— *Streetio.* ♒.	24	12	17
Locus verus ——— *Mihi.*— ♒.	24	15	40

2. *An.* 1683. Die 3. *Maii* 9^h 12′. *Mars* Conjunctionem celebrabat cum *Penultima* Stella *Alæ Auſtrinæ Virginis*, quæ modo degit in 13° 49′ 14″. ♎. & *Latit.* 1° 48′ 33″. Bor. ſic ut in ipſa Conjunctione vix ultra 40″. *Mars* infra dictam Stellam incederet.

At Dantzick; *by M.* Hevelius. *n.* 151. *p.* 332.

XCVI. 1. *M. Burattini* hath ſignified from *Poland*, that he hath Obſerved Inequalities in *Venus* as in the *Moon*.

Spots in Venus; *by M.* Burattini. *n.* 10. *p.* 173.

2. I have at laſt diſcovered towards the Middle of the Body of *Venus* a part clearer than the reſt, by which one may Judge of the Motion, or the Reſt of this Planet.

The Rotation of Venus; *by M.* Caſſini. *n.* 32. *p.* 615. *n.* 35. *p.* 687.

The firſt time I ſaw it, was *Octob.* 14. 1666. 5^h 45′. *p. m.* and then this Bright part was very near the Center, on the North-ſide. And at the ſame time I obſerv'd Weſtward two obſcure *Spots*, ſomewhat oblong; but I could not then ſee that Reſplendent part long enough to conclude any thing from thence, nor was I able to ſee any thing well of thoſe parts till *April* 28, 1667. on which Day a quarter of an Hour before Sun-riſing, I ſaw again a Bright part, ſituated near the Section, and diſtant from the Southern Horn a little more than $\frac{1}{4}$ of its Diameter. And near the Eaſtern Ring I ſaw a Dark and ſomewhat Oblong *Spot*, which was nearer to the Northern than the Southern Horn. At the Riſing of the *Sun* I perceived, that this Bright part was then no more ſo near the Southern Horn, but diſtant from it $\frac{1}{5}$ of its Diameter. This gave me great Satisfaction. But I was ſurpriſed at the ſame time to find, that the ſame Motion, which was made from South to North in the Inferiour part of the Disk was on the contrary made from North to South in the Superiour part; whence the Determination of the Motion may be better taken: For we have no Example of the like Motion, except it be in that of the Libration of the *Moon*.

The next Day, at the Riſing of the *Sun*, the ſaid Bright part was not far from the Section, and diſtant from the Southern Horn $\frac{1}{4}$ of the Diameter. When the *Sun* was 4 Degrees high, the ſame was ſituated near the Section, remote from the Southern Horn $\frac{2}{5}$ of the Diameter. The *Sun* being high 6°. 10′. it ſeemed to have paſſed the Center, and that the Section of the Disk did cut the ſame. The *Sun* being 7° high, it appeared yet more advanced Northward, together with two Obſcure *Spots* ſeated between the Section and the Circumference, and equally diſtant from one another, and from each Horn on both Sides. And the Sky being very clear, I obſerved the Motion of the Bright part for $1\frac{1}{4}^h$. which then ſeemed to be exactly made from South to North without any ſenſible Inclination Eaſtward or Weſtward.

Mean time I perceived in the Motion of the dark *Spots* ſo great a Variation, that it cannot be aſcribed to any Reaſon in *Opticks*.

May 10. and 13. Before *Sun-Riſing*, I ſaw ſtill the Bright part near the Center Northward.

Laſtly, *June* 5. and 6. Before the Riſing of the *Sun*, I ſaw the ſame between the Northern Horn and the Center of this Planet, and I noted the ſame Irregular Variation in the Obſcure *Spots*. But when *Venus* began to be farther removed from the *Earth*, it was more difficult to obſerve the *Phænomena*.

It is very difficult to determine any Thing of the Motion of theſe *Spots*; Yet this I can ſay (ſuppoſing that this Bright part of *Venus*, which I have obſerv'd, eſpecially this Year 1667, hath always been the ſame) that in leſs than one Day it abſolves its Motion, whether of Revolution or Libration, ſo as in near 23 Hours it returns to the ſame Situation in this *Planet*, which yet happens not without ſome Irregularity.

A Place of Venus, *Obſerv'd at* Dantzick; *by M.* Hevelius. *n.* 154. *p.* 419.

XCVII. *An.* 1583. *Aug.* 4. horâ fere 2. *m. Venus* à Stella Fixa 3. Magnitud. à *Ventre* ſcil. *Pollucis* non niſi 16′. removebatur Auſtrum verſus, id quod ex *Micrometro* accuratè comperi.

XCVIII. Mercury *obſerved in the Sun,* 1690. *at* Nuremberg; *by M.* Jo. Phil. Wurtzelbaur. *n.* 192. *p.* 483.

Temp. Horol. Oſcil.	*Obſervationes.*		*Tempora ſupputata.*
h. ′ ″	Die 30. *Octob.* p. m. *A.* 1690.		h. ′ ″
6 32 00	Culminat *Os Pegaſi* ——— ———		6 28 45
9 00 00	Culm. *Cap. Andromedæ* ——— ——		8 52 17
9 4 30	Culm. *Jupiter* ——— ———		
	Die 31. *Octob.* m.		
8 30 00	*Sol* è nubibus emerſit, *Mercurius* in diſco ejus ſuperne in Tab. obſervatoria, à Verticali ad dextram (revera ad Lævam) diſtans plus quam Dim. *Dig.* à Limbo exiturus apparuit.		
8 36 00	*Mercurius* poſtquam Undulanti Limbo *Solis* ad Min. Temporis adhæſerat, exiit ad 14° à *Zenith* Septentrionem verſus	° ′	h. ′ ″
8 49 00	Altitudo *Solis* ——— ——	10 5	8 38 38
8 59 45	——— ———	11 10	8 47 48
9 7 10	——— ———	12 10	8 56 24
9 50 00	——— ———	16 28	9 38 7
11 1 30	——— ———	21 31	10 56 32

Ratio Diametrorum *Solis* & *Nuclei Mercurii*, dum Lucido *Solis* Diſco immorabatur, quantum per auram haud ſatis defæcatam conjici poterat, erat ut

1000 ad $8\frac{1}{2}$. Poſtea quàm ad Limbum *Solis* pervenerat, ejuſque Limbo Undulanti ad Minutum ferè adhæſitârat, & genuinam Rotunditatem ſuam (quæ antea ex luce Diſci *Solaris* formam quaſi Ellipticam mentiebatur) recuperârat, erat ut 1000 ad $12\frac{1}{3}$.

Mercury *obſerved in the Sun.* Nov. 3. *ſt. n.* 1697; *by M.* Caſſini. *n.* 245. *p.* 372.

XCIX. *An.* 1697. *Nov.* 3. ſt. n. 7h 25′ cum *Sol* è nubibus Emerſiſſet directo ad ipſum Teleſcopio Differentia Aſcenſionis Rectæ Centri *Mercurii* Occidentalis & Centri *Solis* obſervata per Horologium fuit Hor.
— — — — — — — — — — — — 0° 11′ 52″
Differentia Declinationis *Mercurii* Meridionalis fuit Grad. — 0 6 20
8 3′. Differentia Aſcenſionis Rectæ Centrorum *Mercurii* Occidentalis & *Solis* fuit Hor. — — — } 0 15 30
Differentia Declinationis Grad. — — 0 4 42
8 8′ 38″. *Margo* Præcedens *Mercurii* pervenit ad *Solis* Marginem Præcedentem.
8 10′ 24″. *Mercurius* totus Emerſit è *Solis* diſco Teleſcopio pedum 18 obſervatus.

Ex his obſervationibus invicem comparatis, quantum ex hoc brevi intervallo inferri potuit, adventum *Mercurii* ad medium ipſius ſemitæ in *Solis* Diſco Trigonometricè deduxi 6h 11′ 18″. poſt Meridiem.

Nodum vero *Aſcendentem Mercurii* in ♉ 14° 42′. adhuc promotiorem quam per obſervationes *Anni* 1677.

Inclinationem autem *Orbitæ Mercurii* ad *Eclipticam* ex poſtremarum obſervationum comparatione inveni 6° 23′, quam nihilominus ob breve harum obſervationum intervallum præferre non auſim ei quam ex *Sinenſibus* obſervationibus R. P. *Fontenay* longè majori intervallo diſtantibus deduxi. ſc. 6° 40′. propius accedentem ad Tabulas Rudolphinas.

The Viſible Conjunctions of the Inferior Planets *with the* Sun; *by Mr.* Halley. *n.* 193. *p.* 511. Mar. An. 1691.

C. *Mercurii Veneriſque* Sidera *Solis* Diſcum ſubintrare, ac inſtar Macularum Nigricantium in Lucido ejus Orbe aliquando conſpici, tam ex Verioris *Aſtronomiæ* Principiis, quam ex indubitata Obſervantium Fide, dudum compertum eſt. Qua vero Lege, quibuſve Conditionibus, quantiſque *Annorum* Intervallis, hæc Phænomena nobis ſpectanda præbentur, neſcio an aliquis ex *Aſtronomis* Hodiernis ritè definiverit: Certe nihil hac de re inter Typis mandata hucuſque mihi viſum eſt. Quapropter non Ingratum fore arbitratus, huic Inquiſitioni ſerio operam dedi, ac Diſſertatione hanc rem maximè perplexam pauciſque intellectam me plenius enucleaturum confido.

Has *Planetarum* Horum Phaſes ſemper in Retrogradientium cum *Sole* Conjunctionibus fieri, cum ſcil. *Sol Nodis* eorum adeo vicinus ſit ut *Planetæ Soli* juncti Latitudo Semidiametrum *Solis* non excedat, per ſe ſatis conſpicuum eſt; quo vero facilius Limites ac Conditiones harum Conjunctionum perveſtigem, cumque Calculi Elementa omninò diverſi ſint, uterque Planeta ſigillatim tractandus eſt: à *Mercurio* itaque exordiamur.

Hujus Planetæ *Nodum Aſcendentem*, juxta nuperas & accuratas Obſervationes, prope 15° *Tauri*, ſeu potius ad 0ˢ 15° 44′. à 1ª ✶ ♈. hoc noſtro ſeculo reperiri, pro comperto habemus. *Deſcendentem* vero Oppoſitam ad

6^{s} 15° 44′. à 1^{a} ✱ ♈. Angulum autem quo Planum Orbitæ *Mercurialis* ad *Eclipticam* Inclinatur ſatis bene ſe habet apud *Keplerum*, viz. 6° 54′. Jam ex probatiſſimis Hypotheſibus conſtat *Mercurii* in *Nodo Aſcendente* conſtituti Diſtantiam à *Sole* eſſe partium 31365, quarum media *Solis* Diſtantia à *Terra* ſit 100000; dum vero *Nodum* Alterum occupat, Diſtantia iſta, in iiſdem partibus menſurata, ſit 45308. *Sol* vero *Nodo Aſcendenti* Oppoſitus diſtat à *Terra* eidem Juncta partium iſtarum 98955, ad *Nodum* vero *Alterum* eadem Intercapedo ſit 101007. Atque idcirco *Mercurius Soli* Conjunctus ad *Nodum Aſcend.* diſtat à *Terra* partibus 67591: ad *Nodum* vero *Deſcend.* partibus 55699. Quæ, cum inter ſe valde diſcrepent, ſeparatim etiam conſiderandæ veniunt Conjunctiones illæ quæ ad diverſos *Nodos* fiunt, Calculi Elementis Compendii gratia Synopticè expoſitis.

Conjungatur Mercurius Retrogradus *cum* Sole *Centraliter ad* Nodum Aſcendentem, *Menſe* Octobri; *ac ex dictis Hypotheſibus habebitur.*

		S.	°	′	″
Longitudo *Solis* à *Prima* Stella *Arietis* —		6	15	44	00
Longitudo *Mercurii* ex *Sole* viſi —		0	15	44	00
Diſtantia *Mercurii* à *Sole* —	31365				
Diſtantia *Mercurii* à *Terra* —	67591				
Angulus *Inclinationis Orbitæ Mercurii* —		0	6	54	00
Motus 6 Horar. *Mercurii* ex *Sole* viſi —		0	1	30	58
Motus *Solis* in iiſdem 6 Horis —		0	0	15	5
Hinc Motus *Mercurii* à *Sole*, ſex Horis —		0	1	15	53
Et Motus ejus à *Sole* ex *Terra* viſo, 6 Horis —		0	0	35	12
Et Angulus Viæ *Mercurii* intra *Solem* viſæ cum *Ecliptica* — — —		0	8	15	00
Hinc Motus *Mercurii* in *Orbita* ſua viſibili 6 Horis —		0	0	35	40
Deinde Motus *Mercurii* in *Anno Siderio* ultra quatuor Revolutiones — — —		1	24	45	8
In Annis Tredecim itaque — —		11	21	46	44
Deſunt itaque ad Revolutiones 54 Integras —		0	8	13	16
			d.	h.	′
Quod ſpatium percurrit *Mercurius* in —			2	00	11
			°	′	″
Quibus promovetur *Solis* Locus; ac ☿ in *Nodo* Situs tantundem diſtat à *Conjunctione Terræ* —			2	1	00
At Arcus iſte ex *Terra* ſpectatus ſit — —			0	56	10
Unde ex dato Angulo *Viæ* viſæ 8° 15′ provenit *Baſis*, ſive Diſtantia à *Conjunctione Viſibili* — —			0	55	34
				h.	′
Qui Arcus percurritur à *Mercurio* juxta Horariam datam — — —				9	21
Excedunt vero 13 *Anni Siderii* totidem *Julianos* cum *Intercalationibus* tribus — — —				8	0

Itaque

		d.	h.	′
Itaque *Mercurius* revertitur ad *Solem* poſt *Annos Julianos* 13 atque inſuper		2	17	34
Vel cum quatuor *Intercalationibus*, ſi præcedens Annus ſit tertius à *Biſſext.*		1	17	34
			′	″
Ex arcu vero 56′ 10″. & Angulo dato, fit perpendicularis, ſive proxima Diſtantia ☿ à *Sole*		0	8	3
Itaque ☿ poſt 13 Annos intra *Solem* conſpicuus, 8′ 3″. Borealius Incedit.				
	s.	°	′	″
Pari Argumento in 46. *Annis Sideriis* movetur ☿	11	28	36	8
Deſunt itaque ad Revolutiones 191 Integras	0	1	23	52
		h.	′	
Hoc eſt in Tempore		8	12	
		°	′	″
Quo promovetur *Sol*		0	20	41
Hic arcus è *Terra* viſus fit		0	9	36
Baſis vero ei competens		0	9	30
		h.	′	″
Tempus, vero quo *Mercurius Baſin* percurrit, fit		1	30	00
Excedunt vero 46 *Anni Siderii* totidem *Julianos* cum 11 Intercalationibus		19	3	00
		d.	h.	′
Ac *Mercurius* revertitur ad *Solem* poſt 46 Annos *Julianos* atque inſuper		1	4	51
Vel cum duodecim Intercalationibus, ut fit cum Annus præcedens ſit Secundus vel Tertius à *Biſſextili*		0	4	51
		°	′	″
Perpendicularis vero, quo *Mercurius* in Boream provehitur, fit		0	1	22
		h.	′	″
Periodus vero maxime Accurata *Mercurii* ad *Solem* abſolvitur *Annis Sideriis* 263 atque inſuper		1	11	30
Hi vero *Siderii* ſuperant totidem *Julianos* cum 66 Intercalationibus		10	20	0
Unde poſt 263 Annos *Julianos*, *Mercurius* ad *Solem* revolvitur, tardius vero		11	33	30
	d.			
Quod ſi præcedens Annus *Biſſextilis* fuerit, addantur	1	11	31	30
Poſt hoc demum Intervallum Borealius Incedit	0	0	0	10

Cæteræ vero Periodi Latiores ex jam Inventis facili negotio eruuntur, ſuntque vel 6 vel 7 Annorum.

Quæ Septem Annis abſolvitur, *Mercurium* deprimit verſus Auſtrum 22′ 47″, ac Septem Dies integros, minus 9 Minutis, citius provenit, ſi duæ fuerint Intercalationes. At cum una Intercalatione, cum ſcil. Annus prior *Biſſextilis* ſit, 6 dies ſubducendi ſunt, additis tantum 9 Minutis, ut prius.

Rarius vero poſt ſex Annos in *Solis* diſco conſpicitur iterum Vagus ille Planeta qui exacta hoc Periodo 30′ 50″. Borealius tranſit; idque tardius, 9^{d} 17^{h} 25′, ſi Annus præcedens ſit Secundus vel Tertius à *Biſſextili*, aliter 8^{d} 17^{h} 25′ addendi ſunt.

Pariter ſi fiat Conjunctio *ad* Nodum Deſcendentem *Menſe* Aprili.

		s.	°	′	″
Longitudo *Solis* à *Prima* Stella Arietis		0	15	44	00
Mercurii Longitudo ex *Sole* viſi		6	15	44	00
Diſtantia Planetæ à *Sole* ut prius	45308				
Diſtantia ejus à *Terra*	55699				
Motus *Mercurii* à *Sole* viſi Sex Horis		0	0	14	29
Motus *Solis* in eodem tempore		0	0	43	21
Motus *Mercurij* à *Sole*		0	0	28	52
Hinc Angulus *Viæ* viſæ *Mercurii* intra *Solis* diſcum cum *Ecliptica* fit		0	10	18	00
Motus vero viſus à *Terra* in 6 Horis		0	0	23	52

Unde ſequendo methodum calculi præcedentis, evincitur *Mercurium* poſt 13 Annos atque inſuper 3^{d} 7^{h} 37′ ad *Solis* Conjunctionem revolvi; quod ſi præcedens Annus fuerit Tertius à *Biſſextili*, tunc addendi ſunt 2^{d} 7^{h} 37′. tantum: ac tum *Mercurius* 16′ 55″. Auſtralius incedere reperietur.

Poſt 46 vero Annos, cum 12 Intercalationibus addantur 0^{d} 7^{h} 14′. & habebitur Mercurius *Soli* Conjunctus in Tramite Auſtraliore 2′ 23″. ſi vero Annus Prior *Biſſextilis* fuerit; vel ab eo Primus, addendus eſt 1^{d} 7^{h} 14′. ut habeatur accurate Synodus. Similiter poſt 263 Annos, quibus *Mercurius* in Auſtrum deflectitur 0′ 22″. Addendus eſt vel 1^{d} 11^{h} 49′. vel 11^{h} 49′, juxta Legem in priori Caſu præſcriptam.

At Annis ſex vel ſeptem ob viciniam *Terræ* ac *Planetæ*, atque idcirco ob ampliatos Arcus ad hunc *Nodum*, non revertitur ad *Solem*, ut intra Diſcum appareat. Poſt Annos autem 33 *Solem* Tranſit Viâ magis Boreali 14′ 2″. ac habetur momentum Conjunctionis ſubducendo à Prioris tempore 3^{d} 0^{h} 23′. ſi fuerit in Anno Tertio à *Biſſextili*; aliter ſubduc 2^{d} 0^{h} 23′. tantum.

His Inventis facile erit continuare Calculum pro omnibus hiſce Conjunctionibus *Mercurii* cum *Sole*, idque cum ſumma certitudine, ac ſine ulla hæſitatione, an omnes poſſibiles habeantur necne: Sola Additione obtinentur momenta Conjunctionum ac Diſtantiæ Planetæ à Centro *Solis*; unde etiam ope Tabellæ depromuntur Durationes harum, ut ita dicam, *Eclipſium*, ut nihil ſit quod in hac re deſiderari videatur.

Epochas vero quod ſpectat, eæ tutius Obſervatorum induſtria comparantur, quam calculi cujuſvis Subtilitate: adeoque elegimus in primo Caſu, notabilem illum Tranſitum *Mercurii* quem ipſe in Inſula *Sanctæ Helenæ* perfectiſſime obſervavi, *Oct*. 28. *An*. 1677. St. Vet. & cujus Medium ex Initio & Fine determinavi in prædicta Inſula quidem 0^{h} 4′. p. m. *Londini* vero 0^{h} 28′. p. m.

Semita vero qua incedere viſus eſt Planeta 4′ 40″. Borealior erat *Solis* centro. In altero Caſu, viz. cum *Mercurius Soli* conjungitur Menſe *Aprili*, ex Cl. *Hevelii Mercurio in Sole viſo*, p. 72. 75. *Epocham* deſumere placuit; nempe quod *Apr.* 23. *An.* 1662. St. Vet. 6h 8′. p. m. *Gedani*, hoc eſt 4h 52′. *Londini*, *Mercurius Solis* centro proximus apparuit, utpote in medio Tranſitu, ſimulque diſtabat ab eodem Centro 4′ 27″ ad Boream. Hinc juxta præcepta præmiſſa, omnes ordine viſibiles Conjunctiones *Mercurii* cum *Sole* ſimul exhibere, exigui Laboris opus erat: ac in Exemplum quod cuivis in poſterum imitari licet, accipe hujus Seculi, ab invento Teleſcopio, quotquot uſquam apparuere hujuſmodi Phænomena, vel quæ etiam in ſequentis Seculi poſteris apparitura ſunt.

Series Momentorum quibus Mercurius Soli *Conjunctus intra* Diſcum *ejus conſpicitur, per præſens & futurum Seculum, cum* Diſtantiis *Planetæ* à Solis *centro.*

APRILI.

Ann.	*Temp. Conj.*				*Diſt. à Cent.* ☉		
	d.	h.	′		′	″	
1615	22	21	38	* *	7	20	B
1628	25	5	15	*	9	35	A
1661	23	4	52	*	4	27	B
1674	26	12	29		12	28	A
1707	24	12	6		1	34	B
1720	26	19	43	*	15	21	A
1740	21	11	43		15	36	B
1753	24	19	20	*	1	19	A
1786	22	18	57	*	12	43	B
1799	26	2	34	*	4	12	A

OCTO.

OCTOBRI.

Ann.	*Temp. Conj.*				*Diſt. à Cent.* ☉.		
	d	h.	′		′	″	
1605	22	8	29		12	48	A
1618	25	2	4	*	4	45	A
1631	27	19	37	*	3	18	B
1644	30	13	11		11	21	B
1651	23	13	20		11	26	A
1664	25	6	54	*	3	23	A
1677	28	0	28	* *	4	40	B
1690	30	18	2	*	12	43	B
1697	23	18	11	*	10	4	A
1710	26	11	45		2	1	A
1723	29	5	19	*	6	2	B
1730	22	5	28		16	45	A
1736	30	22	53	* *	13	5	B
1743	24	23	2	* *	8	42	A
1756	26	16	36		0	39	A
1769	29	10	10		7	24	B
1776	22	10	19		15	23	A
	Nov.						
1782	1	3	44	*	15	27	B
	Oct.						
1789	25	3	53	*	7	20	A

Tranſitus qui ſigno * notantur, *Londini* ex parte Viſibiles ſunt, qui vero ſigno * *, toti conſpici poſſunt.

Notandum vero eſt *Solis* Diametrum ad *Nodum* ☿rii *Aſcendentem* menſe *Octobri* occupare 32′ 34″. atque adeo Maximam Durationem centralis Tranſitus eſſe 5^h 29′, Menſe vero *Aprili* Diameter *Solis* ſit 31′ 54″; unde ob tardiorem Planetæ Motum oritur Duratio Maxima 8^h 1′: Quod ſi Obliquè incidat *Mercurius*, Durationes hæ Breviores redduntur pro ratione Diſtantiæ à centro *Solis*: Quoque perfectior Calculus hic reddatur, ſequentes Tabellas adjunxi, quibus exhibentur dimidiatæ Durationes harum *Eclipſium* ad ſingula minuta Diſtantiæ Viſæ à centro *Solis*; quæ Additæ ac Sublatæ à Conjunctionis momento in priori Tabula invento, *Initium* ac *Finem* totius Phænomeni deſignant.

OCTOBRI.

Min. Diſt.	*Semi-durat.*
′	h. ′
0	2 44½
1	2 44
2	2 43
3	2 41½
4	2 39½
5	2 36½
6	2 33
7	2 28½
8	2 23
9	2 17
10	2 10
11	2 1
12	1 51
13	1 39
14	1 24
15	1 4
15½	0 50
16	0 30

APRILI.

Min. Diſt.	*Semi-durat.*
′	h. ′
0	4 0½
1	4 0
2	3 58½
3	3 56
4	3 53
5	3 48½
6	3 43
7	3 36
8	3 28
9	3 18½
10	3 7
11	2 54
12	2 38
13	2 19
14	1 55
15	1 21½
15½	0 56

Obſervationes omnes hucuſque Habitas ritè repræſentant hi Numeri, nec eſt quod dubitem de Futuris, cum ex omnibus Planetis *Mercurius*, *Soli* proximus, ejus Centro adeo vicinus ſit, ut aliorum Centrorum interventu minimè cieatur, nec deviationibus illis quæ à cæterorum Syſtemate oriuntur, quibuſque ſuperiores, præſertim *Saturnus*, obnoxii ſunt, quod ſentiri poſſit inturbetur.

Parallaxes conſultò omiſi, ut perexiguas, quæque Locis diverſis diverſæ obvenientes Generaliori Calculo immiſceri non debent; quodque etiam quantæ ſint non ſatis adhuc conſtat, ſed potius ex hujuſmodi Obſervatis tutiſſime derivari poſſint: Diametri etiam *Mercurii* rationem non habui, quia ſupra fidem Parvus perpaucula ſolum Minuta Limbo adhærere videtur. Ex Obſervatione accuratiſſimâ deprehendi vix duo Minuta elapſa dum totus è *Sole* egrederetur *Octob.* 28. 1677. unde concluſi Diametrum ejus 0′ 11″, ac juxta rationem Diſtantiarum à *Terra* ad *Nodum Alterum* $0'\ 13''\frac{1}{2}$ fere; adeoque tunc $3\frac{1}{3}$ Temporis Minuta inſumi, dum Totus Planeta *Solis* Limbum Directe pervadit: Oblique vero tranſiens paulo diutius hæret, ſecundum ac Secantes Angulorum Incidentiæ augentur. Æquationes etiam Temporis haud opus eſt ut æſtimemus, quia perplures dies hinc inde in utroque Menſe Conſtantes ac quaſi Invariatæ perſiſtunt.

De Viſibili Veneris *cum* Sole *Conjunctione.*

Venus, quamvis Syderum omnium ſpecioſiſſima, more Sexus ſui, ſine mutuato Cultu ac Splendore aſcititio in Conſpectum prodire veretur: Hoc etenim Spectaculum inter *Aſtronomica* longe nobiliſſimum, inſtar Ludorum Secularium, integri Seculi Mortalibus invident Motuum arctæ Leges. Unico vero hoc obſervato, ſumma cum certitudine Diſtantiam *Solis* à *Terra* determinari poſſe, quæ ob Parallaxin alias prorſus Inſenſibilem Vagis Terminis hucuſque definita eſt, poſthac declarabitur. Periodos vero quod attinet, illæ non adeo Accurate ac *Mercuriales* deſcribi poſſunt, cum *Venus* ſemel tantum ab Orbe Condito, idque ab *Horroxio* noſtro, intra *Solis* Diſcum deprehenſa ſit: Correctis autem Motibus, quantum per Rudiores Veterum obſervationes licet, accipe jam ſummam Calculi.

		s.	°	′	″
Longitudo *Nodi Aſcendentis Veneris* à *prima* Stella *Arietis* — — —		1	15	16	00
Sol itaque ei jungitur in puncto oppoſito; hoc eſt, per hæc Secula, circa finem *Novembris* —		7	15	16	00
Diſtantia *Veneris* à *Sole* partium —	71997				
Diſtantia *Veneris* à *Terra* —	26438				
Inclinatio Orbitæ *Veneris* ad *Eclipticam* —		0	3	23	0
Motus *Veneris* in 8 *Annis Syderiis*, ſupra 13 Revolutiones — —		0	1	30	$28\frac{1}{2}$
Motus *Veneris* in 235 *Annis Syderiis* ſupra 381 Revolutiones — —		11	29	17	39
Motus *Veneris* in 243 *Annis Syderiis* ſuprá 395 Revolutiones — — —		0	00	48	8

Ex his Principiis, inito Calculo juxta Methodum in *Mercurio* expoſitam, proveniunt intervalla Temporum ac Diſtantiarum ut ſequitur.

Poſt 18 Annos *Venus* revolvitur ad *Solem* ſc. ſublatis à prioris Tranſitus momento $2^{d}\ 10^{h}\ 52'\frac{1}{2}$. Incedit vero Planeta Semitâ $24'\ 41''$. priori magis Auſtrali.

Poſt Annos 235, Additis $2^{d}\ 10^{h}\ 9'$, *Venus* iterum *Solem* ingredi poteſt, ſed viâ $11'\ 33''$. Borealiori: Quod ſi præcedens Annus *Biſſextilis* fuerit $3^{d}\ 10^{h}\ 9'$. addendi ſunt.

Poſt Annos 243 *Venus* etiam *Solem* tranſire poteſt, auferendo tantum $0^{h}\ 43'$. à prioris Tempore; Auſtralius vero incedit $13'\ 8''$: Quod ſi præcedens Annus *Biſſextilis* fuerit, adde $23^{h}\ 17'$.

Et in omnibus his Appulſibus *Veneris* ad *Solem*, Menſe *Novembri*, Angulus Viæ Viſæ *Veneris* cum *Ecliptica* ſit 9° 5′. ac Motus ejus Horarius intra *Solem* 4′ 7″; cumq; Semidiameter *Solis* ſit 16′ 21″, provenit Maxima Duratio Tranſitus centri *Veneris* 7^{h} 56′.

Deinde conjungantur *Sol* & *Venus* ad *Nodum Deſcendentem* Menſe *Maio*; ac juxta Numeros eoſdem ſupputantur intervalla eadem. Poſt 8 Annos auferendi ſunt 2^{d} 6^{h} 55″, ac *Venus* Orbitâ 19′ 58″. Borealiori pertranſibit.

Poſt Annos 235 adde 2^{d} 8^{h} 18′, vel ſi prior Annus *Biſſextilis* fuerit 3^{d} 8^{h} 18′. & habebis *Venerem* Auſtraliorem 9′ 21″.

Denique poſt 243 Annos, adde 0^{d} 1^{h} 23′. vel ſi prior Annus *Biſſextilis* fuerit 1^{d} 1^{h} 23′. & reperitur *Venus* iterum *Soli* Conjuncta, ſed in tramite 10′ 37″. magis Boreali.

In omni ad hunc *Nodum* Tranſitu intra *Solem*, Angulus Viæ Viſæ *Veneris* cum *Ecliptica* fit 8° 28′; ac Horarius ejus Motus 4′ 00″; ac *Solis* Semidiametro ſubtendente 15′ 51″; provenit Duratio Maxima centralis Tranſitus etiam 7^{h} 56′; præciſe eadem ac ad *Nodum Alterum*.

Quoad *Epochas*; Ex Ingreſſu quem ſolum vidit *Horroxius* in *Sole* jamjam occaſuro concluditur, *Venerem Soli* Junctam fuiſſe *Londini* 1639. *Nov.* 24^{d} 6^{h} 37′. ſed verſus Auſtrum inceſſiſſe 8′ 30″. Menſe *Maio* vero à nemine Mortalium hucuſque intra *Solem* viſa eſt, ſed ex Numeris meis quos non multum à Cælo abluſuros confido, conſtat *Venerem* proximâ vice *Solem* ſubituram *A.* 1761. *Maii* 25^{d} 17^{h} 55′. mediâ ſcil. *Eclipſi*, ac tunc Diſtare à Centro ejus verſus Auſtrum 4′ 15″. Hinc & ex præmiſſis Revolutionibus facili negotio omnia hujus generis Phænomena, per Millenium integrum computavi, ut in ſequenti Tabella exhibeatur.

NOVEMBRI.

Ann.	*Temp. Conj.*			*Diſt. à Cent.* ☉		
	d.	h.	′	′	″	
918	20	21	53	6	12	B
1161	20	21	10	6	55	A
1396	23	7	20	4	38	B
1631	26	17	29	16	11	B
1639	24	6	37	8	30	A
1874	26	16	46	3	3	B
2109	29	2	56	14	36	B
2117	26	16	3	10	5	A

MAIO.

Ann.	*Temp. Conj.*			*Diſt. à Cent.* ⊙		
	d.	h.	′	′	″	
1048	24	13	45	3	50	B
1283	23	8	14	5	31	A
1291	25	15	9	14	27	B
1518	25	16	32	14	52	A
1526	23	9	37	5	6	B
1761	25	17	55	4	15	A
1769	23	11	00	15	43	B
1996	28	2	13	13	36	A
2004	25	19	18	6	22	B

Durationes harum *Veneriarum Eclipſium* quod attinet, reſpectu Centri eodem modo ſupputari poſſunt ac *Mercuriales*; ſed cum Diameter *Veneris* ſatis ampla ſit, cumque Parallaxes etiam Differentiam valde Notabilem quoad Tempus ingerere poſſunt, Calculus peculiaris pro Locis ſingulis neceſſariò ſubeundus eſt.

Veneris autem Diameter tanta eſt, ut dum Limbo *Solis* adhæret, ferme 20 Temporis Minuta præterfluunt, cum ſcil. *Solem* Directe aggreditur; Oblique vero Incidens, etiam diutius Limbo immoratur: Occupat autem Diameter iſta, juxta *Horroxii* Obſervationem 1′ 18″, dum ad *Nodum Aſcend. Soli* Jungitur, ac 1′ 12″. ad *Nodum Alterum*. Præcipuus autem harum Conjunctionum uſus eſt, *Solis* à *Terra* Diſtantiam ſive Parallaxin ejus accurate determinate, quam quidem fruſtra variis Methodis tantaverunt *Aſtronomi*; dum Inſtrumenta quantumvis Subtilia Angulorum quæſitorum Minutiæ facile eludunt. At in obſervando *Veneris* in *Solem* Ingreſſu & ab eodem Egreſſu, ſpatium Temporis inter Momenta Contactuum Internorum, ad ipſum Temporis Minutum ſecundum, hoc eſt, ad $\frac{1}{15}$ *Minuti Secundi* ſive 4‴. Arcus obſervati, ope mediocris Teleſcopii & Horologii Oſcillatorii, per 6 vel 8^h accuratè ſibi conſtantis, obtineri poteſt. Ex duabus autem talibus Obſervationibus in Locis Idoneis debitè inſtitutis intra Quingenteſimam partem certo concludi *Solis Diſtantiam* proximâ Occaſione commonſtrabo.

Fig. 154, 155. Ne quid Obſcuri Lectoribus *Aſtronomicè* minus Doctis videretur, Schemata pro utriuſque *Planetæ* Tranſitu delineavi, quibus rem oculis ſubjicere conatus ſum.

The Motion of the Comet. A. 1664. *Predicted*; *by M.* Auzout. n. 1. p. 3. n. 2. p. 18.

CI. 1. M. *Auzout*, after he had ſeen the *Comet* (which was firſt obſerved in *Holland*, *Decemb.* 2. 1664.) 4 or 5 times, made the *Ephemerides* of its Motion upon an *Hypotheſis* that it moveth juſtly enough in the Plain of a great

Circle, which inclineth to the *Equinoctial* about 30°, and to the *Ecliptick* 49° or 49°½, cutting the *Æquator* at about 45°½, and the *Ecliptick* at 28°. of *Aries*, or a little more.

n. 2. *p.* 19.

He takes Notice, that more *Comets* enter into our Syſtem by the Sign of *Libra* and about *Spica Virginis* than by all the other parts of the Heavens: For, both the preſent *Comet* and many others regiſtred in *Hiſtory* have entered that way, and conſequently paſs'd out of it by the Sign of *Aries*; by which alſo many have entered.

Obſerved; by M. Auzout. *ib.*

2. Till the *6th* of *Feb.* this *Comet* always advanced: But after that Day, I found that it returned in Augmenting always its Latitude. I left it *Mar.* 8. at the 18. of the *Horn* of *Aries*, almoſt in the ſame Latitude; and I am apt to believe it will be *Eclipſed* this Evening.

I ſhall only add, that on *Feb.* 3. we were ſurprized, to ſee the *Comet* again much Brighter than ordinary, and with a conſiderable *Train*. Some did believe, that it approached again to us. But having beheld it with a Teleſcope, I ſoon ſaid, that it was joined with two ſmall Stars, whereof one was pretty Bright, and that this *Conjunction* gave the *Comet* that Brightneſs. Hence it was, that I aſſur'd my Friends here, that we ſhould no more ſee it ſo Bright.

n. 6. *p.* 107.

M. *Auzout* alſo ſtrongly conceives, that this *Comet* could not be *Feb.* 18. *ſt. n.* where M. *Hevelius*, in his *Prodromus Cometicus*, hath placed it, viz. *in prima Arietis*; unleſs it be ſaid that it viſited that Star of *Aries* on the 18. and returned thence the *19th*, into its ordinary Courſe: For, according to his, and his ſeveral Correſpondents Obſervations, the *Comet* on *Feb.* 17. was diſtant from that *Firſt Star of Aries* at leaſt 1° 17; and on *Feb.* 19. (He having miſſed, as well as his other Friends, the Obſervation on *Feb.* 18.) was Advanced in its way 12' or 13', but yet Diſtant from the ſaid *Star* ſome Minutes above a whole Degree, and conſequently far from having then paſſed it. After which time M. *Auzout* affirms to have ſeen it as well as ſeveral others, for many Days, and that until *Mar.* 17. Obſerving, that about *Feb.* 26 or 27, when the *Comet* was neareſt to the often mentioned *Firſt of Aries*, it approached not nearer than 50'.

By ſome Engliſh Aſtronomers. *ib.* p. 108. n. 9. p. 150.

3. Some Eminent *Engliſh Aſtronomers*, who have attentively obſerved the Poſition of this *Comet*, do jointly Conclude, that whatever that Appearance was, which was ſeen near the *Firſt Star of Aries* by M. *Hevelius* (the truth of whoſe Relation concerning the ſame they do in no wiſe Queſtion) the ſaid *Comet* did not come near that Star in the *left Ear of Aries*, where the ſaid M. *Hevelius* ſuppoſes it to have paſſed, but took its Courſe near the *Bright Star* in its *Left Horn*, according to *Bayer*'s Tables.

The Principle of M. Auzout*'s Hypotheſis; and the Motion of that* Comet *Obſerved; by* M. Caſſini. r. 2. p. 17.

4. I have eaſily found the Principle of M. *Auzout*'s *Ephemerides*: and 'tis this, that this *Comet* moves about a Centre, in a ſtraight Line drawn from the Earth thro' the *Great Dog*, in ſo great a Circle, that that Portion which is deſcribed, is exceeding ſmall in reſpect of the whole Circumference thereof, and hardly diſtinguiſhable by us from a Straight Line.

Concerning the *New Comet* you mention, I obſerv'd it *Feb.* 11. about the 24° of *Aries*, with a Northern Latitude of 24°.40'.

CII.

The Motion of the Comet. A. 1665. *Predicted; by M.* Auzout. n. 3. p. 36.

CII. 1. M. *Auzout,* after 3 or 4 Obſervations, hath publiſhed another *Ephemerides* concerning the Motion of the *Comet,* which he firſt began to Obſerve *Apr.* 2. 1665. He affirms that the Line deſcribed by this *Star* reſembles hitherto a Great Circle, as it is found in all other *Comets* in the midſt of their Courſe. He finds the ſaid Circle inclined to the *Ecliptick* about 26°. 30′. and the *Nodes* where he cuts it, towards the Beginning of *Gemini* and *Sagittary*; that it declines from the *Æquator* about 26°. and cuts it towards the 11°. and conſequently that its greateſt Latitude hath been towards *Piſces,* and its greateſt Declination towards the 25°. of the *Æquator.* He puts it in its *Perigee* about 15°. of *Piſces,* a little more Weſterly than *Marchab,* or the *Wing of Pegaſus.*

Obſerved; by M. Auzout. *ib.* p. 37.

2. He obſerves in General, that this *Second Comet* is contrary to the precedent, almoſt in all Particulars: Seeing that the former moved very ſwift, this pretty ſlow; that againſt the Order of the *Signs* from Eaſt to Weſt, this, following them, from Weſt to Eaſt; that, from South to North, this, from North to South, as far as it hath been hitherto, that we hear of, Obſerved; that, on the ſide oppoſite to the *Sun,* this, on the ſame ſide; that, having been in its *Perigee* at the time of its Oppoſition, this having been there, out of the time of its Conjunction. He taketh alſo notice, that this *Comet* differs in Brightneſs from the other, as well in its *Body,* which is far more Vivid and Diſtinct, as in its *Train,* whoſe Splendor is much greater, ſince it may be ſeen even with great Teleſcopes, which were uſeleſs in the former by reaſon of its Dimneſs.

A Comet. An. 1668. *at* Bononia; *by M.* Caſſini. n. 35. p. 683.

CIII. 1. *An.* 1668. *Mar.* 10d. 1h. of the following Night (after the *Italian* way of counting) I obſerv'd a Path of Light extended from the *Whale* thro' *Eridanus*; which I judged to be the *Train* of a *Comet* both by the Figure and Colour, as alſo becauſe that the Direction of it was to the part oppoſite to the *Sun,* like other *Comets.* By its extream Point it reached to that Star in *Eridanus,* which is called the 14 by *Bayerus*: But it iſſued out of the Horizontal Clouds, ſo that I thought the *Head* of the *Comet* was either vailed by them, or hid under the *Horizon.* *Mar.* 11. there was ſeen a Brightneſs in the *Whale,* amongſt the thin Clouds, at leaſt for half an Hour, which was very like the Splendor of *Venus,* likewiſe vailed with thin Clouds.

Mar. 12. When the *Great Dog* was in the *Mid-heaven,* the ſame *Tail* appeared again. It paſſed thro' the Star in *Eridanus,* which *Bayerus* calls the 15, and left to the Southward the 14, where it did terminate *Mar.* 10. Being by the Imagination drawn out to about 3°. and further, it tended to that Southern Star which precedes the *Ear* of *Lepus.* It was therefore more Northerly than the Day before Yeſterday, and more Eaſterly. We were doubtful whether its head was hid by the Clouds or under the Horizon. But the Line from *Jupiter* to the Extremity of the ſaid *Tail* in the Clouds was Perpendicular to that *Tail*; ſo that it was in the *Whale,* and the Apparent part of the *Train* reached out in Length about 32°.

2. *Mar.* 5. *ſt. n.* The *Comet* was firſt diſcovered: but for as much as it ſet few hours after the *Sun*, there could hitherto be taken no conſiderable Obſervations of it. The *Body* thereof is not ſeen, becauſe it remains hid in the Horizon. Its *Train* is of a Stupendious Length, extended in Appearance over almoſt the 4th part of the Viſible Heaven, from Weſt to Eaſt; its apparent Breadth is of a good *Palm*, and its Splendour very great, but it Laſts but a few hours.

At Lisbon; *by* … p. 684.

At St. *Salvador Mar.* 5. (*ſt. n.*) at 7 a Clock at Night, *F. Eſtançel* began to ſee this *Comet* a little above the Horizon from Weſt to E. S. E. The Beginning of its *Tail*, was a little under the two Lucid Stars, the 15 and 16th of the *Whale's Back*, over which it then paſſed, its Point being as 'twere at the 8 and 9th which are in the Bottom of the *Whale's Belly*; and thus the whole Length thereof was about 23°. The *Globe* or *Head* of it was ſo ſmall and thin, that very few could diſcern it with the Naked Eye.

In Braſil; *by* P. Valentine Eſtançel. n. 150. *p.* 91.

Mar. 7. The former Brightneſs was ſomewhat leſs, and become ſo Thin, that the Eye could eaſily ſee the Stars that were behind it, which by Conjecture were the 14 and 20th.

The *Tail* was always directly oppoſite to the *Sun*; and when it appear'd the firſt time almoſt Horizontal, it was ſeen in the form of a Pillar, the *Head* ſtanding a little under, and on the ſide of the Star of the *Whale*, which is in the Lat. of 15°. 46′. and the Long. of 12°. 42′. of *Aries*. And the *Point* did ſhave the 14th, North of the three that are in the *Belly*, in the Lat. of 20°. 30′. and Long. of 15° 57′. of *Aries*.

This *Comet* was at firſt very Splendid, and caſt it ſelf with that Vividneſs upon the Sea, that the Rays thereof were reverberated unto the Shoar, where the Obſervers ſtood. But this Brightneſs laſted only for 3 Days, after which it did conſiderably Decay. But that which ſeemed ſomewhat ſtrange was, that having loſt ſo much of its Light, yet its Bulk was not diminiſhed, but continued rather increaſing until the *Comet* diſappear'd. It paſs'd more ſwiftly than *Venus*, whence he infers that it was under *Venus*; yet the Anticipation was not ſo great, that it could be believ'd to be under the *Moon*, as he would have it.

4. *P. Pietro Suſarte*, Rector of *Macao*, in the *Eaſt-Indies*, well verſed in Matters *Aſtronomical*, writes to have ſeen the ſame all along the Coaſt of *Bona Speranza*.

In Africa; *by* P. Pietro Suarte. *ib.*

CIV. There hath been ſeen here a New *Comet* from the 2. of *Mar. ſt. n.* 1671. It is but little, having a *Train* not above a Degree or a Degree and an half long. It is now (*Mar.* 9.) about the Stars in the *Right Arm* of *Andromeda* on her Shoulder Blade. As far as I can collect from one or two Obſervations, it tends towards the *Lucida* of *Andromeda's Girdle*, and that with a Direct Diurnal Motion of about 2 Degrees in its Courſe.

A Comet. An. 167½. *at* Dantzick; *by M.* Hevelius. *n.* 81. p. 4017.

The 6 of *March* in the Evening 7h. 40′ it was in 7°. ♈. and in 35° of Northern Latitude; as I gueſs'd by the haſty Inſpection of a Globe.

Mar. 7. in the Morning 3^{h} 30′ its Longitude was about 8° ♈. with a ſomewhat leſſer Latitude than before: in the Evening of the ſame day it's Longitude was 10° ♈. and Lat. 34°. *fere*.

Mar. 8. in the Morning 4ʰ the Long. was 12°. ♈. and the Lat. 33°. Which yet I would not have taken preciſely, becauſe I cannot yet reduce my Obſervations to a *Calculus*.

At ..... by Mr. Newton. *ib*. p. 4018.

2. Mr. *Iſaac Newton* about the 16 of *March ſt. V.* ſaw a dull Star South-Weſt of *Perſeus*, which he now takes to have been that *Comet*. It was very ſmall, and had not any Viſible *Tail*, which made him regard it no further.

At Paris; *by M.* Caſſini. n. 81. *p*. 4018. n. 82. p. 4042.

3. The *Mathematicians* of *La Fleſche* perceived him from the 18 of *March*, *ſt. n.* and gave us here at *Paris* the firſt notice of it. Thoſe of the *College* of *Clermont* being advertiſed of it, ſaw him the 25 of the ſame Month.

Mar. 26. 7ʰ 30′ in the Evening, M. *Caſſini* ſaw him between the *Head* of *Meduſa* and the *Pleiades*; without a Teleſcope he appeared no otherwiſe than a Star of the 3ᵈ Magnitude; His *Head*, ſeen with a Teleſcope of 17 Foot, appear'd almoſt Round; but it was well defined, and diſtinguiſh'd from the Miſtineſs, which formed a kind of *Chevelure*, wherewith it was encompaſſed; and even the middle was a little confuſed, and ſeemed to have Inequalities, as are ſeen in Clouds.

The *Tail* was almoſt imperceptible; yet by the Teleſcope it was ſeen turned Oppoſite to the *Sun*, and it appeared of the Length of two Diameters of the *Head* or thereabout: For it was not eaſie to meaſure it preciſely, becauſe being Thinner according as it was farther from the *Head*, its Extremity was inſenſibly loſt. And ſo the whole *Comet*, *Head*, *Tail*, and *Chevelure*, taken altogether, took up no more than 3 or 4 Minutes of a Degree. At 7ʰ. 48′. he was in a ſtraight Line with the *Lucida* in the *Head* of *Meduſa*, and with the moſt *Occidental* one of the *Pleiades*; and above the two Cleareſt Stars of the *Southern Foot* of *Perſeus*; ſo that a ſtraight Line drawn through theſe two Stars, did almoſt touch the Southern Extremity of his *Chevelure*. This Place of the *Comet*, transferred upon the Map of the fix'd Stars, fell preciſely enough upon 23° 25′. of *Taurus*, in 15° Northern Latitude.

With a Teleſcope 3 Foot, we ſaw near the *Comet* two ſmall Stars, diſtant one Diameter of the *Sun* from one another, which Stars are not in the Catalogues. The *Comet* was in a Straight Line, drawn from one of thoſe two Stars to the other preciſely at 9ʰ 15′. but a little nigher to that which was Weſtward; But 9ʰ 33′. he was equally Diſtant from them both. It was taken notice of, that from 8ʰ 5′. till 10ʰ 26′. *He* made, in reſpect of theſe two Stars, an oblique Motion ſenſible enough, going from North to South in the ſame time that he advanced from Weſt to Eaſt.

Mar. 28. 7ʰ 42′. in the Evening the *Comet* was diſtant from the leſs bright Star of the *Southern Foot* of *Perſeus*, no more than about 24′ Weſtward. He had almoſt the ſame Latitude with this Star; ſo that he was preciſely enough at 26° 8′. ♉, and in the Lat. 12°. 8′. At 8ʰ. 14′. we took, as well as we could, the Diſtance of the *Comet* to the Star in the Eye of *Taurus*, called *Aldebaran*, 19° 38′. And 8ʰ 29′. the Diſtance of the *Comet* to the Star, called *Capella*, was found to be of 22° 32′.

Mar. 30. $9^h\,35'$. at Night the *Comet*, ſeen without a Teleſcope, appeared no otherwiſe than a Star of the 4th Magnitude: thro' the Teleſcope he exceeded even thoſe of the Firſt; but he was very Dark, and in what manner ſoever we look'd upon him, we could Obſerve almoſt no *Tail* at all of him. He had paſſed one Degree and an half beneath the *Lucida* of the *Southern Foot* of *Perſeus*; ſo that this Star was exactly in the midſt of the *Comet* and the little Star of the *Leg* of *Perſeus*, marked η by *Bayerus*, which then we ſaw not but by a Teleſcope. A Straight Line drawn from one of theſe Stars to the other, did almoſt touch the *Southern Limb* of the *Comet*, which being transferred upon the Map of the *Fix'd Stars*, fell upon 28° 45′ of *Taurus* in the Northern Latit. of 9° 56′. At $9^h\,45'$. the Weſtern Limb of the *Comet* touched a Straight Line, drawn thro' this leſs bright Star of *Perſeus's Southern Foot*, and thro' the moſt Northern of the Head of *Taurus*; but that he was already got ſomewhat nearer to the Latter.

Mar. 31. 8 in the Evening, the *Comet* was in a direct Line with the *Lucida* in the *Foot* of *Perſeus*, and with the moſt Northern in the *Head* of *Taurus*; but he was more than twice as much remoter from the firſt than the other, and being transferred upon the Map of the *Fix'd Stars*, he was found at 15′. from *Gemini*, in the Latit. of 8° 49′. During the whole time that we could obſerve him this Night (which was till 10 a Clock) he quitted not this Straight Line, which was almoſt parallel to the Horizon: notwithſtanding that his own particular motion ſhould raiſe him a little above it; as the Parallax, on the contrary, ſhould ſink him beneath it in approaching to the Horizon. It may be, there was a compenſation made of theſe two contrary Motions: poſſibly alſo the effect of both was not ſenſible.

April 1. The *Comet* could not be ſeen without a Teleſcope, becauſe the *Moon*, being very near it, hid him from our ſight. But with a Teleſcope only of one Foot we diſcerned him eaſily enough, and found that he had paſſed 45′. beyond the moſt Northern Star of the *Head* of *Taurus*, and that he muſt have touch'd it by his Southern Limb; as alſo that he was Diſtant 1° 43′, from the Star that was neareſt to that toward the South; which is equally Bright, yet not marked by *Bayerus*. This place being transferred upon the Map of the *Fix'd Stars*, we found that he was at 1° 30′ of *Gemini*, in the Northern Latit. of 7° 44′.

April 2. 8 in even, M. *Caſſini*, having obſerved the *Comet* with a Teleſcope of one Foot, which diſcovered 5° found that he was two Deg. and half Diſtant from the moſt Northern Star of *Taurus*; and one Deg. from the Star of the *Ear* marked φ by *Bayerus*, and by *Tycho* called *Sequentis Lateris Borei*.

Two Lines drawn from the moſt Northern Star of *Taurus*, one to the *Comet*, the other to the Star that is wanting in *Bayerus*, made a Right Angle; and the Diſtance of the *Comet* to this Angle, was double to that which is between theſe two Stars. This Place transferred upon the Map of the *Fix'd Stars*, fell on 2° 48′ of *Gemini*, in the Northern Latit. of 6° 40′.

April 3. 9^h we ſaw him with the one Foot Teleſcope. He had paſſed over the Upper Star of the *Ear* of *Taurus*, and he made with this Star the *Baſis* of an *Iſoſceles* Triangle, on the Top whereof was the Inferior Star of the

Ear. The two Sides of this Triangle were two times and an half bigger than the *Baſis*; ſo that the *Comet* was 4° of *Gemini*, in the Northern Latit. of 5° 38'.

Apr. 5. 8h at even, the *Comet* had paſſed the Northern *Ear* of *Taurus*, and was equally Diſtant from the Upper Star of the Northern *Ear* and from that which was on the *Front* of *Taurus*. He was alſo as Diſtant from the Inferior Star of the *Ear* of *Taurus*, as this Star is from the next Weſtward, by *Tycho* called *Inferior præcedentis Lateris Quadrilateri*; and a Straight Line, drawn thro' the *Comet* and the Upper Star of the *Ear*, made an almoſt Right Angle with another Line, drawn from the *Comet* to the Inferior of the two Stars, that are above the *Eye* of *Taurus*. This Place being carried over to the Map of the *Fix'd Stars*, the *Comet* was found at 6° 18'. of *Gemini*, in the Northern Latit. of 3° 41'. He was ſo confuſed this Night, that even with the 17 Foot Teleſcope we could not exactly diſtinguiſh the *Head* from the *Chevelure* which environed him. The whole appeared a little bigger than the Diſque of *Jupiter*, ſeen by the ſame Teleſcope.

Apr. 6. 8h at even, a Straight Line drawn from the *Comet* to the Star that is in the *Front* of *Taurus*, made a Right Angle with another Straight Line drawn from this ſame Star to the Inferior of the two that are above the *Eye*: And the Diſtance of this Latter Star to that of the *Front* of *Taurus* was twice the Diſtance of the ſame Star of the *Front* of *Taurus* to the *Comet*. This Place being transferred upon the Map of the *Fixt Stars*, the *Comet* was found at 7° 25'. of *Gemini*, in the Northern Latit. of 2° 45'. At 9h 6'. we ſaw on the ſide of the *Comet* a Star ſufficiently clear, which was not farther Diſtant from him than a little more than the Diameter of the *Comet*, and that was at the ſame height of the Horizon.

Apr. 7. 9h in the Evening, the *Comet* was equally Diſtant from the Inferior Star of the *Northern Ear* of *Taurus*, and from the Superior of the root of the *Northern Horn*. He was alſo as far Diſtant from this latter Star, as this Star is from that of the *Front*. This Place, being carried over to the Map of the *Fix'd Stars*, fell on 8° 30'. of *Gemini*, in the Northern Latit. of 1° 56'.

All the Places of the *Comet*, that we have Obſerved till now, fall into a Line little differing from an Arch of a great Circle, which cuts the *Eclipick* in 10° 45'. of *Gemini*, and which conſequently hath its greateſt Latitude in 10° 45'. of *Piſces*; which Latitude is between 39° and 40° Northward. The ſame Circle cuts the *Æquator* at 101 degrees of the *Vernal Section* Eaſtward, and its greateſt *Declination* from the *Æquator* Northward is of 38°$\frac{1}{2}$.

Having choſen two of our Firſt Obſervations (becauſe the latter are not ſo proper for this purpoſe) and having taken a Mean between the firſt Obſervations of the *Mathematicians* of *La Fleſche*, we found, by our *Method* explained in the Theory of the *Comet* of 1665, that this *Comet* had been in his *Perigee* the 12 of *March* at 8 a Clock in the Morning: that in that time, which is that of his greateſt apparent Celerity, he made about 2° 32' a day in the Great Circle of his apparent Motion, and $\frac{444}{\text{[illegible]}}$ of his *Perigee* Diſtance in the Line of his Equal Motion: that he was in his greateſt

Declination the 11*th*, and 12*th* of *March*; and that at that time, he paſſed thro' the Inferior Meridian at about two a Clock after Midnight.

If we have rightly determined his *Perigee*, and that the *Hypotheſis* of the Equality of his Motion be Juſt for that time, he hath been viſible ſince the Middle of *February*, at which time he was as far Diſtant from his *Perigee* by Approaching to the *Earth*, as he is at preſent by Receding from it. He muſt then have been at the extremity of the *Southern Wing* of the *Swan*, and arrived at the *Southern Foot* of *Pegaſus* on the 23 of *Febr.* of the ſame bigneſs that he was ſeen to be of *Mar.* 28. He muſt have arrived at the Stars of the *Northern Arm* of *Andromeda Mar.* 9. at thoſe of her *Girdle* 12. when he was in his *Perigee*, and in his greateſt Declination; to her *Southern Leg*, *Mar.* 15; between her *Southern Leg* and the *Triangle Mar.* 18. very near as he was obſerved at *La Fleſche*; and under the *Head* of *Meduſa*, *Mar.* 25. The Days enſuing he muſt have arrived at the Places marked in our Firſt Obſervations: But in the laſt he hath been ſwifter than this *Hypotheſis* will bear. To repreſent theſe latter Obſervations, the Line of the Motion ought to have been made Curve, as we did for the end of the Apparent Motion of the *Comet* 1665. with this difference, that inſtead of that Lines being Convex in regard of the *Earth*, becauſe the Motion was *Retrograde*, this was to be Concave towards the *Earth*, becauſe that the Motion of this *Comet* is *Direct*.

It's a Thing worth Obſerving, that this *Comet* keeps his Courſe almoſt like that of the 2. *Comet* of 1665, and another of 1577, obſerved by *Tycho*. For they have paſſed thro' almoſt the ſame *Conſtellations*; tho' this be more inclined Northward, and cut the *Eclipt**ick* 5 or 6 Degrees more forward than that of 1665. So that it ſeems that in this Place of the Heavens there is, as 'twere, a *Zodiaque* for *Comets*.

CV. 1. *Cometam* novum D. *Romer* primùm advertit 28 *Apr.* ſt. n. 1677. & me ſtatim de rei novitate admonito 4^h 6′ 31″. p. m. n. ejus Altitudinem accepimus 12° 22′ 10″. Judicavi eum fuiſſe in Verticali declinante ab Ortu ad Septentrionem 33° circiter. Die 29 manè, momento per nubes à D. *Picardo* viſus eſt, 3^h 9′ 31″. p. m. n. in Altitudine 4° 39′.

A Comet. An. 1677. *at* Paris; *by M.* Caſſini. n. 135. p. 868.

Die 2 *Maii* manè, Aſcenſione Rectâ Medii Cœli ex Fixis exiſtente 267°. Altitudo Cometæ erat 4° 5′. Diſtantia Verticalis à Septentrione ad Ortum 42° 8′. circiter. Die 4 manè 3^h 30′. p. m. n. Altitudo *Cometæ* fuit 5° 33′. Diſtantia *Azimuthalis* à Sept. ad Ortum 42° 32′. circiter.

Die 5. 3^h 32′. Altitudo *Cometæ* fuit 5 10′. Diſtantia *Azimuthalis* à Septent. ad Ortum 44° 10′. circiter.

Obſervationes quæ habitæ ſunt, Initio *Cometam* reponunt in *Triangulo*, poſtremò prope *Caput Meduſæ*, oſtenduntque *Cometam* procedere ſecundum Signorum Seriem per Lineam proximam, & fere Parallelam Illi quam deſcripſit *Cometa An.* 1590. Menſe *Feb.* Magnitudo *Capitis*, viſi Teleſcopio, videbatur ferè æqualis *Jovis* Diſco, aut paulo minùs; nec perfectè Rotundum apparebat, ſed Figuræ Ovalis, longiore Diametro Horizonti parallelo; quod Refractioni Horizontali videtur tribuendum.

Coma ejus, Teleſcopio viſa, Latior, & ferme Parabolica; Nudo autem Oculo Anguſta, & parum Inflexa ad Occaſum, videbatur.

At Dantzick; *by* M. Hevelius *ib.* p. 869.

2. Prodiit hiſce diebus *Sidus Crinitum*, quod primâ vice hic *Gedani* die 27 *April* manè animadverſum fuit. Die 29 *Apr.* Oriebatur vel potius in Oculos incurrebat, 1^{h} 52′, Meſaquilonem verſus (*h. e.* N. E. b. N.) *Capite* quidem haud eò Amplo, ſed tamen ſatis Splendido, ex unico Nucleo clariſſimo compoſito, ad inſtar illius *An.* 1665. conſpecti. *Caudam* Lumine notabilem radiis divaricatis ſurſum verſus, duorum ferè graduum, exponebat. Linea Directionis continuata *Caudæ* inter *Alamac*, Lucidum ſc. Pedem *Andromedæ*, ejuſque *Cingulum* incedebat, & quaſi Diſtantiam harum Stellarum in duas æquales partes ſecabat. Verſabatur eo tempore ſupra *Caput Arietis* in *Triangulo*, inter *Apicem* & *Borealiorem in ejus Baſi*, nempe in 5° *Tauri*, & in Latit. 12° Bor. Diſtabat hoc tempore à *Sole* ſecundum Longitudinem tantummodò 5°. Suo Circulo verò Maximo 20°. Hincque cum adeò vicinus hic *Cometa* extiterit *Soli*, haud potuit Longiorem *Caudam*, utut meâ opinione reverà longè prolixiorem habuerit, oſtendere, imò ut puto proximis diebus aliquanto adhuc Breviorem oſtendet.

Apr. 30. Deprehenſus eſt in 9°. ♉. & Lat. 18°. Bor. totidemque ferè à *Sole* exiſtente in 12 ♉; *Caudam* rurſus duorum grad. & aliquanto Longiorem, ad Borealiorem in *Baſi Trianguli* extenſam (quæ Stella planè in Cuſpide *Caudæ* per Tubos optimè conſpecta) exhibebat.

Die 1. *Maii* 2h 32′. m. in 11°. ♉. repertus eſt, ſub Latitudine Bor. 18°, in ipſa propemodum Conjunctione *Solis*, totidem quoque gradibus à *Sole* diſtans. *Caudam* adhuc ſatis Lucidam referebat, ſed paulo Breviorem, utut Latiorem, quam ad *Lucidum Pedem Andromedæ* exporrigebat.

A Die 29. *Apr.* quâ primùm à me Obſervatus, ad hunc uſque Diem 1. *Maii*, Motu Proprio propemodum 5° 30′ abſolvit.

Quantum ex hiſce Obſervationibus conjicere poſſum, fertur Motu Directo ad *Siniſtrum Pedem Perſei*, ſupra *Taurum*, ad *Pedes Geminorum*, ſi eo uſque perdurabit. *Nodus* Deſcendens verſatur circa 20°. *Geminorum* (ſed ruditer id tantummodo refero) atque ſic ibidem *Eclipticam* pertranſibit, fietque tum Meridionalis ſub Inclinatione Orbitæ 27° ferè.

Die 2. *Maii* veſp. 8^{h} 45′. etiamſi ea in parte Cœli nullæ adhuc Stellæ emicarent, intenſumque Crepuſculum exiſteret, nihilominus *Cometam* Tubo Optico protinus inveni. Paulo poſt, illum in Altitudine 3° 30′. deprehendi: *Caudam* referebat, ratione Crepuſculi, valde Tenuem, quam inter utrumque *Genu Caſſiopeæ*, proprius tamen *Siniſtro*, exporrigebat: occidebat eâ Veſp. 10^{h} Circium verſus (*h. e.* N. N. W.)

Die 3. *Maii* mane, *Cometa* oriebatur Boream verſus (*h. e.* N. N. E.) 1^{h} 23′, quanquam *Cauda* paulò citius à nobis detecta, nempe 1^{h} 18′. Verſabatur in 14° ♉, cum *Sole* ferè in ipſa *Conjunctione*, Latitudinem habens 17° & tantam etiam Diſtantiam fere ab ipſo *Sole*. *Caudam* hâc die longè Prolixiorem & Accuratiorem ſatiſque ſplendidam, 2° vel 3° fere, oſtendebat. Hincque à me aliiſque Spectatoribus viſu pollentibus nudo oculo ad 3^{h} 34′ deprehenſus eſt, & Teleſcopio ad 3^{h} 40′. in Altitudine 11° 30′. adeo ut *Sol* eo tempore tantummodo 6°. infra Horizontem lateret, imo diutius illum vidiſſemus, niſi nubeculæ illum nobis eripuiſſent. Motus Diurnus decreſcere videbatur, quantum conjectura abſque omni calculo aſſequi potui: Nam inter 29 & 30 *April* 2° 45′ fere extitit; inter 30 *April* & 1 *Maii* 2° 15′; inter 1 & 2 *Maii* 1° 55′; inter 2 & 3 *Maii* 1° 40′; ſed ipſæ Obſervationes calculuſque id clarius oſtendent. Die 4 *Maii* veſp. Aëre admodum ſudo, 8h 53′, iterum *Cometa* detectus, ſed Obſcurior paulo extitit quam diebus præcedentibus, tum *Cauda* Brevior. Die 5 *Maii* mane 1h 41′. *Caudam Dextrum Genu Caſſiop.* verſus exponens, verſabatur in 17° ♈, in 16° Latit. Bor. pariter in tanta Diſtantia à *Sole*. Motus Proprius à Die 3 ad 5 *Maii* fuit fere 2° 40′, decreſcente Latitudine, ab ipſo Initio ſcil. fere ad 3°. ſic ut à 29 *April.* Motus Proprius *Cometæ* ad 5 *Maii* propemodum fuerit 12°. Die 6 *Maii* mane commorabatur in 18° ♈, & Lat. Bor. 15° 30′. *Sole* exiſtente in 17° ♉; Motus Diurnus erat 50′ circit. Quoad *Caput*, quam *Caudam* multò Tenuior ac Debilior videbatur, ob *Solem* non niſi 16° à *Cometa* Remotum. Die 6 *Maii* veſp. viſus quidem Tubo optico 8h 35′ *Cauda* adhuc Breviori & Dilutiori; ſed cum in Decliviori Situ, atque in Crepuſculo intenſo exiſteret, nullo modo diſtinctè in Nudos incurrebat oculos.

Die 7 *Maii* deprehenſus primùm 2h 22′. in Alt. 3°, utut valde Tenuis videretur. Occupabat eo tempore 19° ♉; Motus ejus Proprius magis magiſque decreſcebat, quantum colligere abſque calculo debatur. Die 8 *Maii* mane ab Hor. 1. ſedulo nudis quæſitus eſt oculis, ſed nuſquam apparuit: Teleſcopio tamen 12. ped. inventus, *Caudam* quidem adhuc præ ſe ferens, ſed Breviſſimam, paulo à circulo Verticali ſiniſtram verſus extenſam. Quantum conjectura aſſequi potui, verſabatur in 20° ♉, in Diſtantia a *Sole* 15°, qui tum 19° ♉ poſſidebat; ſtabat fere hoc tempore in linea recta cum *Humero Dextro Perſei* & *Algol Meduſæ*. Diameter *Cometæ*, ad *Jovis* Diametrum comparata, vix ad dimidiam partem accedebat. De reliquo, Tubi beneficio ſatis erat adhuc conſpicuus, adeo ut eum ad 3h 45′ diſtinctè conſpicere potuerimus, in Altitudine ſcil. 9° ferè: unde colligere datur, Arcum Viſionis vix 5° tum fuiſſe; *Sol* enim vix 5 ſub Horizonte hærebat, quo tempore omnes jam Stellæ, excepto unico *Jove*, evanuerunt. Die 8 *Maii* veſp. *Cometam* nec nudis oculis, nec ullo Teleſcopio detegere amplius potuimus.

3. The firſt certain notice I had of this *Comet* was on *April* 21. The 22 of *Apr.* at about 2 a Clock after the Midnight following, I ſaw the *Tail* raiſed almoſt perpendicular to the Horizon; ſoon after the *Head* appeared thro' a thin Vapour, from which the *Tail* pointed, as near as I could gueſs, upon the * in the *Knee* of *Caſſiopeia*, its Length being about 6 *Deg.* and Breadth at the Top about 7 or 8 *Min.* Viewing the *Head* with a Teleſcope of 16 Foot, I found it was not perfectly Round, but Indented, and not near one *Min.* Diameter. Afterwards I haſtened to Meaſure its Diſtances from ſeveral Fixed Stars, which were as follow.

At Greenwich, *by Mr.* Flamſteed, *ib. p.* 873.

h.	′	″		°	′	″
2	44	00	It's *Head* and the *Foot* of *Androm. Alamech* —	11	26	0
2	47	15	That Diſtance repeated —— ——	11	26	50
2	55	3	Its *Head* from *Capella* — — —	31	1	15
2	59	10	—— — — repeated	31	1	24
3	12	2	Its *Head* from *Algol* in *Meduſa's* — —	18	16	54
3	21	22	—— from *Mirach* —— ——	19	35	0
3	27	54	—— from *Alamech* again ——	11	33	30
3	36	20	—— from *Capella* again ——	30	59	45

At $4^{h}\ 21'\frac{1}{2}$ *p. m.* the height of the *Comet* was about $5^{\circ}\frac{1}{2}$, therefore the Diſtance of the *Head* of the *Comet* from *Algol* corrected by refraction, 8° 19′.
from *Mirach* — 19 37

And admitting with *Hevelius* the Place of *Mirach* now in ♈. 21° 40′ 34″ with North Latitude 25° 57′. its Diſtance from *Algol* will be 23° 42′ 40″. and the Place of the *Head* of the *Comet* in ♉. $14^{\circ}\ 48'\frac{1}{2}$, with North Latitude 17° 8′.

At $3^{h}\ 28'$. I State the correct Diſtance of the *Comet's Head* from *Capella* 31° 00′; from *Alamech* 11° 40′; and therefore its true Place ♉. $14^{\circ}\ 50'\frac{1}{2}$, with North Latitude 17° 6′ 25″: agreeing very well with the Place derived from the former Diſtances from two other and different Stars.

The *Tail* was not, it ſeems, directly Oppoſite to the *Sun*: for the *Sun's* Place was now ♉. 13° 7; but the *Comet* being in 14° 47′. of the ſame *Sign*, that is 1° 40′, in the Conſequence of the *Sun*, the *Tail* ought, if it had been exactly oppoſite to the *Sun*, to have lain in Conſequence of the *Head*; but the *Knee* of *Caſſiopeia* is now in ♉. 13° 24′. in Antecedence of the *Comet*, whoſe *Tail* lay not therefore in Conſequence, but in Antecedence of the Line paſſing thro' its *Head* and the *Sun*, at about an Angle of 10°.

Next Night, being that following the 23 of *April*, about $\frac{1}{4}$ of an Hour after two, its *Tail* appeared much ſhorter than laſt Morning: At $2^{h}\ 51'$. its *Head* was from *Mirach* 21° 9′. Hence and from a Courſe of Obſervation of it ſent me by an ingenious Friend, I found its Motion was Direct, and its Latitude Decreaſing.

A Comet. An. 1680. *at* Dantzick; *by M.* Hevelius. *Ph. Col. n.* 3. *p.* 65.

CVI. Nuperum *Cometam* Obſervavi primum m. ante *Solis* Ortum, à Die 2. ad 4 *Dec.* An. 1680. deinde veſp. à 24 *Decemb.* ad *Æquinoctium Vernum.* Mane verſabatur in ♎, & ♏ ſub Latit. Auſtrali. Veſperi vero in ♑, ♒, ♓, ♈, & ♉. ſub Latit. Bor.

A Comet. An. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 16.

CVII. Plurimas diſtantias à Fixis, tum Altitudines Nuperi *Cometæ* Meridianas, Impetravi: quas autem omnes hic recenſere nimis longum ſoret, nec vacat eas rigidiori Calculo ſubjicere. Sufficiat hac vice dixiſſe *Cometam* hunc hic *Gedani* Die 25 *Aug.* ſt. n. 1682. primum detectum, atque à Die 26 *Aug.* ad 17.

Septemb. debitè à me obſervatum eſſe. Qua via autem, qua velocitate, ſub quo Angulo Orbitæ & *Eclipticæ* progreſſus fuerit, ex adjecta Tabella patet; quam tamen (quod ſcias velim) non ex accurato Calculo, ſed ex Globo tantummodo Laxiori ratione concinnavi.

Menſ. dies.			*Long. Cometæ*		*Lat. Cometæ*	*Motus in Pro. Orbit.*	*Motus Diurnus aliquanto Accuratius.*
	h. ′		° ′	s.	° ′	° ′	° ′
Aug. 26	3 0	*Mat.*	23 30	♋	21 0 Bor.	———	———
Aug. 27	11 0	*Veſp.*	5 0	♌	23 30 Bor.	10 0 fere	5 28
Aug. 28	———	*Veſp.*	———		———	———	5 35
Aug. 29	———	*Veſp.*	———		———	———	5 41
Aug. 30	3 30	*Man.*	18 0	♌	25 20 Bor.	13 20 —	
Aug. 30	9 0	*Veſp.*	22 0	♌	25 40 Bor.	3 30 —	5 46
Aug. 31	3 30	*Man.*	24 30	♌	26 0 Bor.	2 20 —	5 50
Sept. 1	3 30	*Man.*	1 0	♍	26 0 fere	5 45	
Sept. 1	9 0	*Veſp.*	6 0 fere	♍	25 40 Bor.	4 45 —	5 46
Sept. 2	———		———		———	———	5 43
Sept. 3	8 30	*Veſp.*	20 0 fere	♍	24 30 Bor.	11 30 —	5 40
Sept. 4	———	*Veſp.*	———		———	———	5 34
Sept. 5	———	*Veſp.*	———		———	———	5 24
Sept. 6	9 0	*Veſp.*	5 0	♎	20 30 Bor.	15 0 —	5 00
Sept. 7	———	*Veſp.*	———		———	———	4 30
Sept. 8	8 0	*Veſp.*	12 0	♎	18 15 Bor.	8 0 fere	4 00
Sept. 9	8 30	*Veſp.*	15 30	♎	17 15 Bor.	3 30 —	3 30
Sept. 10	8 0	*Veſp.*	18 30	♎	15 45 Bor.	3 0 fere	3 00
Sept. 11	———	*Veſp.*	———		———	———	2 40
Sept. 12	8 0	*Veſp.*	23 0	♎	14 0 Bor.	5 0 —	2 20
Sept. 13	7 30	*Veſp.*	25 0	♎	13 30 Bor.	2 0 —	2 00

Sic ut *Motu Proprio* in ſuo *Orbita* confecerit à Die 26 *Aug.* ad 13 *Sept.* 83° 27′; & in *Ecliptica* 91° 30′. Latitudo vero Borealis creverit ad 26°. rurſus decreverit ad 12° 30′.

Not. *Nodus Boreus* in 24°. ♉, & *Nodus Auſt.* in 24°. ♍ : *Limites* vero in 24°. ♌ & ♒. extiterunt. *Angulus Orbitæ* & *Eclipticæ* fuit 26°. fere. Utrum autem toto Durationis Tempore omnino conſtans cum *Nodis* extiterit? An vero & quouſque ſeſe variaverit? ut ſæpius fieri ſolet, ex Calculo patebit.

Toto Durationis Tempore, Lucidius ac etiam aliquanto Majus *Caput*, quam iſte An. 1681. è contrario multo Breviorem *Caudam* exhibuit. In ipſo *Capite*, beneficio longioris Teleſcopii, non niſi unicum *Nucleum Figuræ Ovalis* & *Gibboſæ* conſtanter notavimus; niſi quod Die præſertim 8 *Sept.* ex dicto *Nucleo* clariſſimus ſimul Radius, ex parte etiam incurvatus, in *Caudam* exiret; quod notari meretur, cum ejus generis faciem in nullo adhuc *Cometa* (quantum memini) obſervaverim. Præterea ſciendum, quod nonnunquam, ut Die 30 *Aug.* mane, *Caudam* ſatis præciſe in *Oppoſitum Solis* direxerit; ſed ſæpius etiam notabilem Deviationem (prout in plurimis *Cometis* ſæpius fieri ſolet) exhibuerit. Longitudinem quoque *Comæ* non ſemper eandem conſervavit. Initio *Cauda* ferè 12° videbatur; deinde nonnunquam brevior, interdum etiam longior ad 15° & 16° extitit; circa finem vero quotidie diminuta eſt. *Fig.* 156.

A Comet. An. 1683. *at* Dantzick; *by M.* Hevelius. n. 154. *p.* 416.

CVIII. Die 30. *Jul.* An. 1683. $11^h\,30'$ in Novo Noſtro Sydere, *Tigride* vel *Lynce*, *Sydus Crinitum* hic *Gedani* deprehenderem, *Caudam* haud adeo Longam inter *Stellam Polarem*, & *Caſſiopeiam* ſurſum cum aliqua Inclinatione exporrigens, conſtituebat lineam Rectam cum Suprema *Capitis Aurigæ* & *Dextro Humero Perſei*; non minus cum *Ventre Urſæ Majoris* & *Dextro Humero Aurigæ*; item cum Media *Cauda* & *Latere Urſæ Majoris*. Deinde Tubo 10 pedum arrepto, iſtud Phænomenum contemplatus ſum, *Caput* erat quidem ſatis Amplum, ſed Materia non admodum Condenſata; ſic ut nullus Lucidus *Nucleus* neque diſtincta Corpuſcula, ut quidem alias in plurimis aliis deprehenſum eſt, in eo apparerent. 12^h ferè, Altitudo ejus erat 19° 57′.

Die 31 *Jul.* veſp. ſcil. $12^h\,30'$. Altus cum eſſet 21° 28′. Rectam cum *Pede Aurigæ* & *Capella* conſtituebat. *Cauda* erat dilutiſſima, ac Rarior quam die heſterna, ſed paulo Longior.

Die 4 *Aug.* mane; Removebatur eo tempore tanto ſpatio à *Dextro Humero Aurigæ*, quanto alias diſtat dictus *Humerus* à *Capite Hœdi*. Cæterum *Siniſtra Tibia Perſei*, *Capella*, & *Cometa*, Rectam referebant.

Die 16 *Aug.* veſp. hora ferè 11. *Cometa* inter quatuor Stellulas verſabatur, quarum una à parte *Cometæ* ſuperiori, in ipſa Conjunctione, non niſi 1′. diſtabat, adeo arctè limbo adhærebat: quo tempore ſimul Diametrum *Cometæ* Micrometro meo dimenſus ſum, nimirum 6′ 5″. exiſtere.

Die 20 *Aug.* veſp. Breviſſimam ac Rariſſimam *Comam* inter *Capellam* & *Caput Hœdi* exporrigebat.

Die 20 *Aug.* veſp. Utroque *Hœdo* erat viciniſſimus, ita ut cum his Triangulum ferè æquilaterum conſtitueret, cujus Latera fere Diſtantiam *Hœdorum* (quæ eſt 47′. circ.) æquabant. Ad hæc *Cometa* cum *Capella* & illa in Planta *Dextri Pedis Perſei*, Triangulum Æquilaterum, cujus Baſis erat Diſtantia dictarum Fixarum, exhibebat.

Aug. 24. veſp. verſabatur inter *Capellam* & *Pleiadas*, ſic ut à *Capella* & *Pleiadibus* in eadem fere Remotione videretur. Deinde *Capella*, *Cometa*, & *Pleiades*; item *Almanac*, *Caput Meduſæ*, & *Cometa*; nec non *Dexter Humerus Aurigæ*, *Cometa*, & *Sequens Siniſtri Pedis Perſei*, Lineam fere Rectam conſtituebant, in hac tamen ultima conſtitutione, *Cometa* fere infra paulo Rectam jam incedebat.

Aug. 25. veſp. cum *Capella* & *Cap. Hœdi* Rectam fere conſtituebat.

Aug. 29. 1h 5′. mane, plurimis minutiſſimis, & clariſſimis Fixis ſtipatus erat, nempe *Stellis Subieſcianis*, atque à Cuſpide Occidentali *Pleiadum* ſurſum verſus, non niſi 42′ 35″ removebatur.

Eadem die veſp. longe jam promotior contra S. S. ſpatio ſcil. 24 horarum, ad 4°. fere reperiebatur.

Aug. 30. veſp. *Cometa* à Stellula quadam bene conſpicua non niſi 32′ 41″. aberat; & cum *Muſca* & in *Baſi Trianguli*, deinde etiam cum præcedente in *Pede* & illa in *Genu Perſei*, conſtituebat Rectam.

Sept. 2. mane, *Cometa* inter *Pleiadas* & *Nodum Lini* verſabatur, conſtituens Lineam Rectam cum *Muſca* & *Lucida Mandib. Ceti*. & cum infima in *Arme* ♉. & *Mandibula*, Triangulum fere æquicrurum, cujus vertex dicta erat *Mandibula*. Præterea quoque Lineam referebat Rectam cum duabus in *Fronte Ceti*, tum tanto fere ſpatio ab Occidentaliori diſtabat, quam allas utraque ab invicem removentur.

Hac die iterum Diametrum *Cometæ Capitis* Micrometro diligenter dimenſus ſum, 9′ 7″. die 16. *Aug*. eodem Micrometro obtenta tantummodo 6′ 5″. ſic ut notabiliter ſpatio 17. Dierum creverit. Non nemo diceret id factum eſſe, quod in ultima Obſervatione vicinior multo fuerit Terræ: Atque ideo Clarius & Lucidius *Caput* exhibere debebat, præſertim ſi Corpus eſſet æternum (ut quidam ſtatuunt) quod rurſus certo tempore abſoluto ſuo circulo, nobis in conſpectum redit. Sed è contrario *Caput* longe Obtuſius, Rariuſque ultimo extitit, ſic ut diſtinctiſſimè notari potuerimus materiam *Capitis* ſenſim ſe diſſolvere; id quod autem multo melius cum noſtra convenit Hypotheſi.

Sept. 4. mane, videbatur *Cometa* in Linea exiſtere Recta cum illa in *Fronte Occid. Ceti* & *Lucida* ♈; item cum illa in *Ore* & *Mandibula Ceti*; ad hæc fere Triangulum æquilaterum cum illa in *Ore* & ad *Genam Ceti* conſtituebat. De cætero autem ex optato mihi extitit, quod Altitudinem Meridianam in Auſtro exactiſſimo Quadrante, Hora, *viz*. Matutina 3h 40′. ferè, nimirum 38°. 15′. eſſe impetraverim.

⁂

Menſ.	Dies.	Longitudo.	Latitudo.	Motus Diurnus.	Declinatio.	Aſcenſ. Rect.
		° ′	° ′	° ′	° ′	° ′
Jul.	30	7 0 ♋	29 15 B		51 30 B	100 0
	31	6 25 ♋	29 0 B	0 42		
Aug.	1	5 45 ♋	28 45 B	0 44		
	2	5 0 ♋	28 30 B	0 46		
	3	4 10 ♋	28 15 B	0 48		
	4	3 20 ♋	28 0 B	0 50	51 40 B	96 0
Aug.	5	2 20 ♋	27 45 B	0 52		
	6	1 20 ♋	27 30 B	0 54		
	7	0 20 ♋	27 15 B	0 56		
	8	29 20 ♊	27 0 B	0 58		
Aug.	9	28 20 ♊	26 40 B	1 0		
	10	27 20 ♊	26 20 B	1 2		
	11	26 20 ♊	25 55 B	1 4		
	12	25 20 ♊	25 30 B	1 6		
Aug.	13	24 20 ♊	25 0 B	1 8		
	14	23 20 ♊	24 30 B	1 10		
	15	22 20 ♊	24 0 B	1 12		
	16	21 10 ♊	23 20 B	1 14	46 0 B	77 0
Aug.	17	19 20 ♊	22 30 B	1 19		
	18	17 40 ♊	21 30 B	1 25	44 0 B	73 30
	19	16 0 ♊	20 30 B	1 35		
	20	14 20 ♊	19 15 B	1 45	41 0 B	69 30
Aug.	21	12 20 ♊	18 0 B	1 55		
	22	10 20 ♊	16 45 B	2 5		
	23	8 20 ♊	15 30 B	2 15		
	24	6 20 ♊	14 15 B	2 25	25 0	60 40
Aug.	25	3 50 ♊	12 45 B	2 40		
	26	1 5 ♊	11 0 B	2 55		
	27	28 15 ♉	9 0 B	3 10		
	28	25 15 ♉	6 30 B	3 25	24 30 B	51 0
Aug.	29	22 15 ♉	4 0 B	3 40	21 30 B	48 30
	30	18 55 ♉	1 30 B	3 55	18 00 B	45 40
	31	16 25 ♉	1 0 A	4 5		
	1	12 15 ♉	1 30 A	4 20		
Sept.	2	9 55 ♉	6 0 A	4 40	10 30 B	40 0
	3	6 25 ♉	9 40 A	5 5		
	4	2 35 ♉	11 20 A	5 30	1 30 B	34 0

Ex quibus nunc luculenter videre eſt, *Cometam* hunc continuo contra S. S. inceſſiſſe; ſic ut in *Ecliptica* 63° 55′. in ſua vero *Orbita* 74° 35′. peragraverit, ſub Angulo, viz. *Orbitæ* & *Eclipticæ* 39° fere, ſub Angulo vero *Orbitæ* & *Æquatoris* 56°. *Latitudo* Initio 29° 15′. *Bor.* & ultimo 11° 20′. *Auſt.* extitit; adeo ut ad 41°. ferè eam variaverit.

De *Capite* hæc notandum habeo, quod Initio, quoad Diametrum, longe minus quam ultimo; è contrario Initio longe Lucidius, quam circa Finem extiterit; nullos tamen diſtinctos & fulgentes *Nucleos*, prout in plurimis videre nobis obtingit, exhibuerit, ſed confuſam materiam, & circa Finem multo Tenuiorem. Jure hic *Cometa* (cum plerumque abſque omni *Cauda* viſus) inter *Sidera Comata*, vel *Crinita*, ſive inter *Barbata* & *Hircos* refertur. Nam non niſi ad 18 *Aug.* Breviſſimam & Dilutiſſimam *Comam* ſurſum verſus exporrigebat; quæ poſtmodum vero omnino Evanuit.

CIX. Novus *Cometes* nuper Cælo viſus eſt à Lynceo oculo Abb. *Blanchini*, Diſcipuli Cl. *Geminiani Montanarii*. *Cometes* parvus quidem, ſed in ſua *Orbita* regularis apparuit, Lumine tenui, & tanquam Stella Subobſcura; at Tubo optico exceptus, Luminoſior.

A Comet. An. 1684. *at* Rome; *by* S. Ciampini. *n.* 169. p. 920.

Jun. 30. ſt. n. *An.* 1684. *Cometa* primum mihi viſus eſt, in grad. 9. cum aliquibus minutis *Libræ*, Latitudo ejuſdem Borealis fuit graduum 8. & aliquot minutorum.

Jul. Die.	*Long. Cometæ.*			*Lat. Bor.*		*Jul. Die.*	*Long. Cometæ.*			*Lat. Bor.*	
	°	′		°	′		°	′		°	′
1	11	18	♎	13	12	10	24	42	♎	39	40
2	13	16	♎	17	57	11	25	44	♎	40	49
3	14	58	♎	22	12	12	26	38	♎	41	58
4	16	45	♎	25	40	13	27	23	♎	43	5
5	18	30	♎	28	50	14	28	32	♎	44	9
6	19	50	♎	31	34	15	29	49	♎	45	12
7	21	7	♎	33	54	16	00	50	♏	46	20
8	22	20	♎	36	6	17	2	8	♏	47	40
9	23	32	♎	38	00						

Quatuor ultimæ meliori indigent Calculo, nam aliquis fortaſſe Minutorum Error irrepſit. Obſervatio Diei Primi *Jul.* accuratiſſima eſt, *Cometa* enim Teleſcopio apparuit una cum Stella *Virginis*, quæ *Bayero* inſcribitur ω, & accidit ſub Cingulo Septentrionalis partis primæ. Omnium certiſſima eſt Obſervatio Diei Sextæ, qua Die *Arcturus* in Tubo Optico ſimul cum *Cometa* conſpiciebatur. Die pariter 14. *Cometa* & *Stella* χ. in *Colorrobo Bootis*, ſeu *Venabulo*, ſubter *Humerum*, uno Intuitu detegebantur.

A Comet. An. 1686. *at* Leipſick; *by M.* Kirk. n. 186. *p.* 256. CX. *Sept*. 8. *ſt. v. A*. 1686. 4^h. *mane*, about Day-break, M. *Kirk* found this *Comet* in the Conſtellation of *Leo*, to the Right-hand of the *Lucida in Lumbis* ♌, (as it is conceived, for the *Latin* Copy is Defective in this place) and reſembling that Star in Colour and Magnitude, with a Thin and ſhort Tail extended upright. Over the *Comet* in the ſame Vertical was the Star of θ ♌ of *Bayer*, or 21 *Tychoni*, diſtant therefrom, by the Micrometer, exactly a Degree; and a Line drawn from the *Lucida in Lumbis* ♌ to the *Comet*, paſſed much about half a Degree to the Right-hand of the θ *Leonis*. The Diſtance of the *Comet* from *Regulus* taken by a Radius was about 17°. The next Morning *Sept*. 9. at 3^h 58′. the Diſtance thereof from θ ♌ was found by the Micrometer 2° $23\frac{1}{2}$, and at 4^h 40′, again 2° $25'\frac{1}{4}$. To verify the Times, the Altitude of the *Lucida in Lumbis* ♌ was obſerved 11° 10′, at 4^h 8′. *mane*. A Right-line drawn by the *Comet*, and the ſaid θ *Leonis* towards β *Leonis*, or the *Lucida Colli*, left that Star a little to the Right-hand.

This *Comet* was ſeen by a Countryman, who firſt gave Notice thereof, from the *6th* to the *12th* of *Sept*.

The Reſult of theſe Obſervations is, that the *Comet* was Direct in Motion, that it mov'd about $1\frac{1}{2}$ Degree *per Diem*, and that it ſeemed rather to Decreaſe in Latitude. On the *7th* of *Sept*. it was about 24′ diſtant from θ *Leonis*, but its bearing therefrom is not ſet down.

This Star, θ ♌, was then in 9° 2′. of ♍, with North Lat. 9° $41'\frac{1}{2}$. Whence at the time of the firſt Obſervation it may be Concluded, that the *Comet* was in 9° 55′. of ♍, with North Lat. 9° 15′. And at the 2*d* Obſervation, the Longitude of the *Comet* will be found about 11° 20′. of ♍, with much the ſame North Latitude as before.

A Comet. An. 1699. *at* Paris. *by M.* Caſſini. *n.* 250. p. 79. CXI. *Feb*. 19. ſt. n. *An*. 1699. in *Obſervatorio Regio Pariſienſi*, videri cœpit exiguus *Cometa*, inſtar Stellæ Nebuloſæ tertiæ Magnitudinis; illi perſimilis quæ Menſe *Sept*. 1698. fuit Obſervatus.

Situs erat inter Stellas Informes 6 Magnit. prope Circulum *Polarem Arcticum* ſupra *Caput Aurigæ*, æquali ferè Intervallo inter *Cubitum Occidentalem Perſei* & *Caput Majoris Urſæ*; illas adſcribit *Tycho* Informibus circa *Urſam Minorem*. Continuatis Obſervationibus, viſus eſt, Proprio Motu, Iter ſuum dirigere *Capellam* verſus, cum exigua Deviatione ab ejus Circulo Declinationis. Ea erat ejus Velocitas ut unius Diei Spatio, Septem circiter Gradus Magni Circuli perficeret, quo Motu potuit ante Dies 4. ipſi *Polo* fermè adhærere, & *Stellæ Polari* ſociari.

Hora 6. p. m. n. Comparavimus *Cometam* cum Stella 6. Mag. quam *Tycho* appellat *Secundam earum quæ ſunt in Linea Recta cum Polo*; *Cometa* in Tranſitu per Circulum Horarium præcedebat hanc Stellam Min. Hor. 15′ 53″. quibus dabitur Differentia *Aſcenſionis Rectæ* 4° 43′. erat autem Septentrionalior eadem Stella 8′. Unde ſuppoſita hujus Stellæ Longitudine & Latitudine *Tychonica* ad hoc Tempus, *Cometa* refertur ad 15° 51′. *Gem*. cum Latitudine Sept. 37° 25′.

Movetur *Cometa* hic ad Cœli partes Oppoſitas illis ad quas tendebat *Cometa An*. præteriti, cum eſſet ferme in eadem Diſtantia à *Polo* in qua noſter hic cum primum viſus eſt, nec valde ab eodem loco remotus.

Cometa autem Menſis *Sept.* eandem proſecutus eſt viam quam inter Sidera tenuerat *Cometa An.* 1652. à Nobis *Bononiæ* Obſervatus, cujus Occaſione editis Literis ad Sereniſſimum *Fran. Eſtenſem Mutinæ* Ducem, eam viam per eadem Sidera quæ noſter tenuit *An.* 1698. diſtinctè deſcripſimus. Ille Menſe *Dec.* ab Auſtralibus Cæli partibus per Aſtra *Leporis, Orionis* & *Tauri,* ubi *Eclipticam* ſecuit cum Inclinatione 76°. & per *Perſeum* & *Caſſiopeiam* pervenit, ubi videri deſiit Menſe *Jan. An.* 1653. Hic videri cœpit Initio Menſis *Sept.* in eadem *Caſſiopeæ* parte ubi ille videri deſierat, indeque pergens per *Humeros* & *Brachia Cephei,* ubi Latitudinem Maximam ab *Ecliptica* habuit 76°. tranſit inter *Draconem* & *Cygnum,* per *Pellem Leonis* in *Hercule,* per *Ophiucum,* uſque ad Conſtellationem *Scorpii,* quam tenebat in ultimis Obſervationibus à Die 24. ad 28. *Sept.* habitis. Ex his autem Obſervationibus collegimus *Cometam* hunc *Perigeum* obtinuiſſe Die 7. *Sept.* Veſperi, cum maxima Velocitate Apparenti fere 10°. unius Diei Spatio.

CXII. Papers *of leſs Uſe General* Omitted.

1. The Conſtellation of *Cygnus,* with the *New* Star *in Pectore* in it, by *Hevelius*; together with the Names of the Stars in that *Conſtellation* by *Tycho,* and of thoſe Added by *himſelf.* Cygnus. *n.* 21. p. 372. *n.* 65. p. 2088, 2090.

2. Mr. *Flamſteed* having peruſed Mr. *Street*'s Diſcourſe, and conſidered the Contrivance of his *Moon-Wiſer,* aſſures, that for the Motion of Longitude 'tis the very ſame, and for the Motion of Latitude not much better than Mr. *Horrox*'s. *Mr.* Horrox's *Lunar Syſtem.* *n.* 110. p. 219. *n.* 116. p. 368.

But Mr. *Flamſteed* hath thought of another Contrivance that will ſhew the *Moon*'s true Place to a Minute.

3. 1. The more Notable *Celeſtial Appearances* Calculated, by Mr. *Flamſteed,* for the *Year* 1670. *Celeſtial Phenomena Calculated.*
2. The ſame for the *Year* 1671. n. 55. *p.* 1099.
3. The ſame for the *Year* 1672. n. 66. *p.* 2029.
4. The ſame for the *Year* 1673. n. 77. *p.* 2297.
5. The ſame for the *Year* 1674. n. 79. *p.* 3061. n. 86. *p.* 5040.

4. 1. The *Eclipſes* of the *Satellites* of *Jupiter* Viſible at *Uraniburg* the laſt four Months of the *Year* 1671. Calculated by M. *Caſſini.* n. 89. *p.* 5118. n. 99. *p.* 6162.
2. The *Eclipſes* of the *Satellites* of *Jupiter* Viſible at *the Obſervatory* at *Greenwich* in the three laſt Months of the *Year* 1683. Calculated by Mr. *Flamſteed.* *Satellite Eclipſes* n. 77. *p.* 2238. n. 151. *p.* 322.
3. The *Satellite Eclipſes* Calculated by Mr. *Flamſteed* for the *Year* 1684. n. 154. *p.* 404.
4. The ſame for the *Year* 1685. n. 165. *p.* 760.
5. The ſame, together with the *Parallaxes* of *Jupiter's Orb* and his *Geocentrick Places,* for the *Year* 1686. n. 177. p. 1215.
6. The ſame for the *Year* 1687. n. 184. *p.* 196.
7. The *Satellites Eclipſes* Calculated by Mr. *Halley,* for the *Year* 1688. *n.* 191. p. 435.

5. 1. An Account of the *Ephemerides* of the *Comet, A.* 1665. Calculated by M. *Auzout*; and the *Principle* of his *Hypotheſis* diſcovered by M. *Caſſini.* *Comets, n.* 1. p. 3. *n.* 2. p. 17, 18.
2. An Account of the *Ephemerides* of the *Comet, A.* 1665. Calculated by M. *Auzout.* n. 3. p. 36.

CXIII.

CXIII. Accounts *of* Books, *and* Emendations, Omitted.

n. 102. *p.* 40. 1. A new Size of *Globes* about 15 Inches Diameter Rectified by *R. Morden*
and *Will. Berry*.

Phil. Col. 2. A Repreſentation of the Heavens in two large *Hemiſpheres* of 30 Inches
n. 1. p. 44. Diameter, Stereographically Projected upon the Plain of the *Æquinox*; by
Mr. *Fr. Lamb*.

n. 90. *p.* 5150. 3. Deux Machines propres à ſaire les *Quadrans*, avec tres grande facilité,
par le P. *Ignace Gaſton Pardies*. S. J. à *Paris*. 1673. in 12°.

n. 184. p. 213. 4. *Sciotericum Teleſcopicum*, or a new Contrivance of adapting a Teleſcope
to an *Horizontal Dial*, for Obſerving the Moment of Time by Day or Night;
by *Will. Molineux R. S. S. Dublin*. 1686. *in* 4*to*.

n. 241. *p.* 240. 5. The *Meridian Line* of the *Church* of St. *Petronio*, Drawn and Fitted
for Aſtronomical Obſervations, in the *Year* 1655. Reviſed and Reſtored in
the *Year* 1695; by *Jo. Dom. Caſſini*. At *Bononia* 1695. *Fol.*

n. 66. p. 2028. 6. *Joa. Hevelii* Machinæ Cæleſtis Pars prior. *Organographiam Aſtronomicam*
n. 99. p. 6171. plurimis Iconibus illuſtratam & exornatam exhibens, *&c. Gedani* 1673. in *Fol.*

n. 109. *p.* 215. 7. *Animadverſions* on the Firſt Part of the *Machina Cæleſtis* of *Joa. Hevelius*,
together with an Explication of ſome *Inſtruments* made by *R. Hook*. *P.* of
n. 111. p. 243. *Geometry* in *Greſh. Coll.* and *R. S. S. London* 1674. *in* 4*to*. Dr. *Wallis*'s Letter
to M. *Hevelius*, concerning *Diviſions* by *Diagonals* there inſerted, but faultily,
is here *Reprinted* more correctly.

n. 175. p. 1162. 8. *Joannis Hevelii* Conſulis *Dantiſcani* Annus Climactericus. *Gedani* 1685.
Fol. Wherein (among other Things) M. *Hevelius* vindicates the Juſtneſs of
his *Celeſtial Obſervations* againſt the Exceptions by ſome made to the *Accuracy*
Ib. p. 1164. of them. The *Controverſy* between *Him* and Dr. *Hook*, about the Uſe of *Teleſcopick*
n. 111. p. 244. and *Plain Sights*, and Dr. *Wallis*'s Calculation, for *Dividing* the Limb
n. 175. p. 1176. of Inſtruments by *Diagonals*, are alſo here Abridged.

9. Excerpta ex Literis Ill. & Clariſſ. Virorum ad Nob. Ampliſſ. & Conſultiſſ.
n. 150. p. 308. D. *Jo. Hevelium* Conſ. *Gedanenſem* perſcriptis, Judicia de Rebus *Aſtronomicis*,
ejuſdemque Scriptis, exhibentia; Studio ac Operâ *Joa. Erici Olhoffii*
Secretarii *Gedani* 1683. in 4*to*.

10. A Deſcription of *Helioſcop*, and ſome other *Inſtruments*, made by *R.*
n. 118. p. 440. *Hook*. R. S. S. *Lond.* 1675. in 4*to*.

11. The Sphere of M. *Manilius* made an *Engliſh* Poem, with Annotations,
and an *Aſtronomical Appendix*; by *Ed. Sherburn*, Eſq; *Lond.* 1675. in *Fol.*

n. 101. p. 233. 12. *Albatenii* Obſervationes *Aſtronomicæ*, Quas ex *Arabico* in *Latinum* Tranſtulit
Plato Tiburtinus. *Noribergæ* 1537; & *Bononiæ* 1645. The *Arabick Copy*
n. 204. p. 913. of thoſe Obſervations does not Appear, whereby that *Tranſlation* might be *Examined*:
But Mr. *Halley*, by Calculating Tables from the *Principles* there Delivered,
hath here *Diſcovered* and *Corrected* above 30 conſiderable *Faults* in a
few Pages.

n. 43. p. 868. 13. *Hiſtoria Cæleſtis*; ex Libris & Commentariis MS. *Obſervationum* Vicennalium
Tychonis Brahe, Dani. Auguſtæ Vindelic. An. 1666. in *Fol.*

n. 102. p. 27, 14. All the *Manuſcripts* of the Famous *Kepler*, (both *Publiſhed* and *Unpubliſhed*)
& 29 which are Purchaſed, and carefully preſerved by M. *Hevelius*.

15. *Jeremiæ Horroccii Angli* Opera Poſthuma: una cum *Guil. Crabtrai* Ob- *n.* 87. *p.* 5078.
ſervationibus Cæleſtibus; nec non *Jo. Flamſtedii* de *Temporis Æquatione* Diatri-
ba, Numeriſq; *Lunaribus* ad Novum *Lunæ* Syſtema *Horroccii*. *Lond.* 1672. in 4*to*.

16. *Aſtronomia Reformata.* Auctore *Joan. Bapt. Riccioli S. J. Stephano de An-* *n.* 22. *p.* 394.
gelis, Conceiving the Arguments of this *Author*, againſt the *Motion* of the *n.* 36. *p.* 693.
Earth, to be none of the ſtrongeſt, taketh Occaſion to let the World ſee, that
they are not more eſteem'd in *Italy*, than in other places: *Manfredi*, in behalf
of *Riccioli*, endeavours to Anſwer the Objections of *Angeli*, and this latter re-
plies to *Manfredi*'s Anſwer. The Subſtance of which *Controverſy* is here given
by Mr. *Ja. Gregory*; with ſome Remarks and Explications of his own upon it.

17. An Attempt to prove the *Motion* of the *Earth* from Obſervations, made *n.* 101. *p.* 12.
by *R. Hook*. *F. R. S. Lond.* 1674. *in* 4*to*. The Method of this Undertaking *n.* 105. *p.* 901.
is Approved and Commended by M. *Chr. Hugens*, and M. *Caſſini*.

18. *Nicolai Mercatoris* Holſati, è Soc. Regia, *Inſtitutionum Aſtronomicarum* *n.* 125. *p.* 611.
Libri duo. *Lond.* 1676. in *Octavo*.

19. Annales Cæli & Temporum perpetui, ſive Myſteria *Aſtronomo-Chrono-* *n.* 104. *p.* 74.
logica à Seculo Abſcondita, nunc per Dei Gratiam Detecta & evidenter Aſſer-
ta, Libris tribus. *Kiloni*. This Book is Preparing by Dr. *Waſmuth*.

20. A *Catalogue* of *Fixed Stars* with their Longitudes, Latitudes, and Mag- *n.* 8. *p.* 145.
nitudes, according to the *Obſervations* of *Uleg Beig*. *Oxford* 1666.

21. *Catalogus Stellarum Auſtralium*, ſive Supplementum Catalogi *Tychonici*; *n.* 141. *p.* 1032.
exhibens Longitudines & Latitudines *Stellarum Fixarum* quæ prope *Polum An-*
tarcticum ſitæ, in Horizonte *Uraniburgico*, *Tychoni* inconſpicuæ fuere. Authore
Edm. Halleio, è *Col. Reg. Oxon.* in 4*to*.

22. Congietture *Phyſico Aſtronomiche* della Natura del *Univerſo*; da *Pietro* *n.* 65. *p.* 2012.
M. *Cavina*. in *Faenza* 1669. in 4*to*.

23. Proſe de *Signori Academici* di *Bologna*, in *Bologna* 1672. in 4*to*. *S.* *n.* 89. *p.* 5125.
Montanari's Diſcourſe concerning the admirable *Changes* and other *Novelties*
obſerved in the *Heavens*.

24. *Iſmaelis Bullialdi* ad Aſtronomos Monita duo. Primum de *Stella Nova*, *n.* 21. *p.* 381.
quæ in *Collo* Ceti ante An. aliquot viſa eſt. Alterum de *Nebuloſa* in *Andromedæ*
Cinguli parte Borea, ante Biennium iterum orta. Approv'd by M. *Hevelius*. *n.* 25. *p.* 460.

25. Three Letters of *Jo. Dominicus Caſſinus*, concerning his *Hypotheſis* of *n.* 84. *p.* 5001.
the *Sun's Motion*, and his Doctrine of *Refractions*. At *Bononia*. in 4*to*.

26. Refractio *Solis Inoccidui*, in Septentrionalibus Oris circa Solſtitium *n.* 233. *p.* 731.
Æſtivum, *An.* 1695. aliquot Obſervationibus *Aſtronomicis* detecta. *Holmiæ*.
in 4*to*. *Tranſlated into Engliſh. Lond.* in 8*vo*.

27. *Tabularum Aſtronomicarum* Pars Prior; de Motibus *Solis* & *Lunæ*, nec- *n.* 191. *p.* 443.
non de Poſitione *Fixarum*, ex ipſis Obſervationibus deductis: Authore *Ph. de*
la Hire. *Paris* 1687. in 4*to*. *Some Animadverſions on it are here inſerted.*

28. 1. The *Royal Almanack* for the *Year* 1675; by *N. Stevenſon*. in 12°. *n.* 108. *p.* 192.
2. ——— For the *Year* 1676. *n.* 120. *p.* 490.
3. ——— For the *Year* 1677. *n.* 130. *p.* 774.

29. Ephemeris, ad *Annum* 1686, exactiſſime ſupputata. *Lond.* in 8*vo*. *n.* 179. *p.* 35.

30. The *Celeſtial World diſcovered*, or Conjectures concerning the Inhabi- *n.* 256. *p.* 337.
tants, Plants, and Productions, of the *Worlds in the Planets*. Written in *Latin*
by M. *Chr. Huygens*. in 8*vo*.

31. Rag-

n. 1. *p.* 2. *n.* 45. 31. Ragguaglio de Nuove Obſervationi da *S. Gioſeppe Campani.* This Book
p. 900. *n.* 4. was Anſwer'd by M. *Auzout*, who gives his Opinion of *Campani*'s Glaſſes, and
p. 69. his new *Obſervations* of *Saturn* and *Jupiter* made with them; To this *S. Campani*
Ib. p. 74. publiſhes a *Reply*, and M. *Auzout* his *Animadverſions* thereon.

n. 35. *p.* 683. 32. 1. Ephemerides *Mediceorum Siderum*, ex Hypotheſibus & Tabulis *Jo.*
n. 44. *p.* 892. *Dom. Caſſini. Bononiæ* 1668. in *Fol.* The like *Tables* have been formerly publiſhed by the Learned *J. Bapt. Hodierna* at *Rome*, about 1656.

n. 214. *p.* 256. 2. The Table of the *Eclipſes* of the *Firſt Satellite* of *Jupiter* by M. *Caſſini*,
Publiſh'd at *Paris* in the *Recueil d' Obſervations faites en pluſieurs Voyages pour parfectioner l'* Aſtronomie *&* *la* Geographie, being not printed with the uſual Care of the *Imprimerie Royale*, Mr. *Halley* here amends ſome of the *Errata.*

n. 14. *p.* 242. 33. *Martis*, circa Axem proprium Revolubilis, Obſervationes *Bononiæ* à
n. 35. *p.* 687. *Joa. Dominico Caſſino* habitæ 1666. Here M. *Caſſini* judges it Evident, that the Period of this *Planets Revolution* is not performed in the Space of 12^{h} 20′, but in about 24^{h} 40′; and that thoſe, who affirm the former, muſt have been deceived by not well Diſtinguiſhing the two *Faces.*

n. 134. *p.* 853. 34. *Mercurius* in *Sole* viſus; à *Jo. Hevelio. Ged.* 1662.

n. 6. *p.* 104. 35. *Prodromus Cometicus*; by *Hevelius.*

n. 17. *p.* 301. 36. *Joannis Hevelii* Deſcriptio *Cometæ*, *An. Æræ Chriſtianæ* 1665. exortiæ una cum Mantiſſa *Prodromi Cometici*, Obſervationes omnes Prioris *Cometæ* 1654, ex iiſque Genuinum Motum accuratè deductum, cum Notis & Animadverſionibus, exhibens.

n. 40. *p.* 805. 37. *Jo. Hevelii* Cometagraphia. *Dantzick.* in *Fol.*

n. 35. *p.* 691. 38. *Staniſlai* de *Lubienietz* Theatrum *Cometicum. Amſtelod.* 1668. in *Fol.*

n. 53. *p.* 1069. 39. Del Movimento della *Cometa*, apparſa il meſe di *Decembri* 1664. da *Pietro Maria Mutoli.* in *Piſa.* in *4to.*

n. 53. *p.* 1071. 40. *Eraſmi Bartholini* de *Cometis*, *An.* 1664, *&* 1665. Opuſculum; ex Obſervationibus *Hafniæ* habitis adornatum. *Hafniæ.* in *4to.*

n. 139. *p.* 980. 41. *Joh. Walliſii*, De *Cometarum* Diſtantiis inveſtigandis. *Lond.* 1678.

Ib. 986. 42. Lectures and Collections made by *R. Hook*, Sec. of the *R. S. Lond.* 1678. in *4to.*

Ph. Col. n. 4. 43. Obſervat. of the *Comet* of 1680. and 1681. made at the *Col.* of *Cler-*
p. 106. *mont*; by *P. J. de Fontenay* è *S. J. Profeſſ.* of *Mathematicks*, Paris. 1681.

Ib. p. 114. 44. A Treatiſe concerning the late *Comet*, Publiſhed at *Turin.* 1681. by *Donato Roſſetti* S. T. D. Canon of *Leghorn*, and *Tutor* in *Mathematicks* to the *Duke of* Savoy.

Ib. p. 116. 45. An Explication of the *Comet* which appeared at the End of 1680. and in the Beginning of 1681. upon the Obſervations of D. *Anthelme*, *Carthuſian* of *Dijon.* at *Dijon.* 1681. in one ſingle Sheet.

Ph. Col. n. 7. 46. A ſmall Diſcourſe about *Comets*, publiſhed in the *High Dutch* at *Nu-*
p. 196. *renburg* 1681; by a *Lover of Aſtronomy.*

Ib. p. 199. 47. A new Introduction, ſhewing how the *Motions* of the *Comets* may be reduced to ſome certain and Geometrical Rules, ſo that their *Appearance* may be Predicted, in *High Dutch*; by *Ja. Bernouly*, at *Bazil. An.* 1681.

n. 145. *p.* 272. 48. *Joannis Jacobi Zimmermanni Cometo-ſcopia.* Or, Three *Aſtronomical Relations* concerning the *Comets* that have been ſeen in the *Years* 1680, 1681, 1682. *Stutgard*, 1682. in *4to.*

CHAP.

CHAP. V.

MECHANICKS. ACOUSTICKS.

I. 1. SI Agens ut A Efficit ut E; Agens ut 2 A Efficiet ut 2 E, 3 A ut 3 E, &c. cæteris paribus: Et univerſaliter, m A ut m E; cujuſcunque rationis Exponens ſit m.

The General Laws of Motion, by Dr. Wallis. *n.* 43. p. 864. Nov. An. 1668.

2. Ergo ſi Vis ut V moveat Pondus P; Vis m V movebit m P, cæteris paribus, puta per eandem Longitudinem eodem Tempore, *h. e.* eadem Celeritate.

3. Item ſi Tempore T moveat illud per Longitudinem L; Tempore n T movebit per Longitudinem n L.

4. Adeoque ſi Vis V, Tempore T, moveat Pondus P, per Longitudinem L; Vis m V, Tempore n T, movebit m P, per Longitud. n L. Et Propterea, ut V T (Factum ex Viribus & Tempore) ad P L (Factum ex Pondere & Longitudine) ſic mn V T, ad mn P L.

5. Quoniam Celeritatis gradus ſunt Longitudinibus eodem Tempore tranſactis Proportionales, ſeu (quod eodem recidit) Reciprocè Proportionales Temporibus eidem Longitudini tranſigendæ impenſis: erit $\frac{L}{T} : C :: \frac{mL}{nT} : \frac{m}{n} C$. *h. e.* Gradus Celeritatum, in ratione compoſita ex directa Longitudinum & Reciproca Temporum.

6. Ergo propter $VT : PL :: mnVT : mnPL$: erit $V : \frac{PL}{T} :: mV : \frac{mnPL}{nT}$; *h. e.* $V : PC :: mV : mPC = mP \times C = P \times mC$.

7. Hoc eſt, ſi Vis V movere potis ſit Pondus P, Celeritate C: Vis m V movebit vel idem Pondus P, Celeritate m C; vel eadem Celeritate, Pondus m P; vel denique quodvis Pondus ea Celeritate, ut Factum ex Pondere & Celeritate ſit m P C.

8. Atque hinc dependet omnium Machinarum (pro facilitandis Motibus) conſtruendarum ratio: nempe ut qua ratione augetur Pondus, eadem minuatur Celeritas, quo fiat, ut Factum ex Celeritate & Pondere, eadem Vi movendo, idem ſit: puta $V : PC :: V : mP \times \frac{1}{m} C = PC$.

9. Si Pondus P, Vi V, Celeritate C, latum in Pondus Quieſcens (non impeditum) m P directe impingat; ferentur utraque Celeritate $\frac{1}{1+m}$ C. Nam propter eandem Vim, majori Ponderi movendo adhibitam, eadem ratione minuetur aucti Celeritas; nempe V : P C :: V : $\frac{1+m}{1}$ P × $\frac{1}{1+m}$ C, = P C. Adeoque alterius Impetus (intellige Factum ex Pondere & Celeritate) fiet $\frac{1}{1+m}$ P C; reliqui $\frac{1}{1+m}$ m P C.

10. Si in Pondus P (Vi V) Celeritate C latum, directe impingat aliud, eadem via, majori Celeritate inſequens; puta Pondus m P, Celeritate n C, (adeoque Vi mn V latum; ferentur ambo Celeritate $\frac{1+mn}{1+m}$ C. Nam V : P C :: mn V : mn P C :: V + mn V = $\frac{1+mn}{1}$ V : $\frac{1+mn}{1}$ P C = $\frac{1+m}{1}$ P × $\frac{1+mn}{1+m}$ C. Adeoque Præcedentis impetus fiet $\frac{1+mn}{1+m}$ P C, Subſequentis, $\frac{1+mn}{1+m}$ m P C.

11. Si Pondera contrariis viis lata, ſibi directe occurrant ſive impingant mutuo, puta, Pondus P, (Vi V) Celeritate C, dextrorſum; & Pondus m P, Celeritate n C (adeoque Vi mn V) ſiniſtrorſum: utriuſque Celeritas, impetus & directio, ſic colliguntur, Pondus dextrorſum latum reliquo ſi quieſceret, inferret Celeritatem $\frac{1}{1+m}$ C, adeoque Impetum $\frac{1}{1+m}$ m P C, dextrorſum, ſibique retineret hanc eandem Celeritatem, adeoque Impetum $\frac{1}{1+m}$ P C dextrorſum (per Sect. 9.) Ponduſque ſiniſtrorſum latum (ſimili ratione) reliquo ſi quieſceret, inferret Celeritatem $\frac{mn}{1+m}$ C, adeoque Impetum $\frac{mn}{1+m}$ m P C ſiniſtrorſum; ſibique retineret hanc eandem Celeritatem, adeoque Impetum $\frac{mn}{1+m}$ P C ſiniſtrorſum. Cum itaque Motus utrinque fiat; Impetus dextrorſum prius lati, jam aggregatus erit ex $\frac{1}{1+m}$ P C dex-

trorſum, & $\frac{m\,n}{1+m}$ P C ſiniſtrorſum ; adeoque reapſe vel dextrorſum vel ſiniſtrorſum, prout hic vel ille major fuerit; eó Impetu quæ eſt duorum differentia : *h. e.* (poſito $+$ ſigno dextrorſum, & $-$ ſiniſtrorſum ſignificante) Impetus erit $+\frac{1}{1+m}$ P C $-\frac{m\,n}{1+m}$ P.C $=\frac{1-m\,n}{1+m}$ P C ; Celeritas $\frac{1-m\,n}{1+m}$ C ; adeoque dextrorſum vel ſiniſtrorſum, prout 1 vel $m\,m$ major fuerit.) Et ſimiliter Impetus prius lati, erit $+\frac{1}{1+m}\,m$ P C $-\frac{m\,n}{1+m}\,m$ P.C. $=\frac{1-m\,n}{1+m}\,m$ P C ; Celeritas $\frac{1-m\,n}{1+m}$ C : adeoque dextrorſum vel ſiniſtrorſum, prout 1 vel $m\,n$, major fuerit.

12. Si vero Pondera nec eadem directe via procedant, nec directe contrario, ſed oblique ſibi mutuo impingant ; moderandus erit præcedens Calculus pro Obliquitatis menſura. Impetus autem Oblique impingentis, ad ejuſdem Impetum qui eſſet ſi Directe impingeret (cæteris paribus) eſt in ea ratione qua Radius ad Secantem Anguli Obliquitatis ; (quod etiam intelligendum eſt, ubi perpendiculariter, ſed Oblique cadit in percuſſi Superficiem, non minus quam ubi viæ Motuum ſe mutuo Oblique decuſſant :) quæ quidem conſideratio, cum calculo priori debite adhibita, determinabit, quænam futura ſint ſic Oblique Impingentium Celeritas, Impetus & Directio, *h. e.* quó Impetu, qua Celeritate, & in quas Partes ab invicem reſilient, quæ ſic Impingunt. Eademque eſt ratio Gravitationis gravium Oblique deſcendentium, ad eorundem Perpendiculariter deſcendentium Gravitationem.

13. Si quæ ſic Impingunt Corpora, intelligantur non abſolute dura (prout hactenus ſuppoſuimus) ſed ita Ictui cedentia, ut Elaſtica tamen Vi ſe valeant reſtituere, hinc fieri potuit ut à ſe mutuo reſiliant ea corpora, quæ ſecus eſſent ſimul proceſſura ; (& quidem plus minuſve, prout hæc Vis Reſtitutiva major minorve fuerit,) nempe ſi Impetus ex Vi Reſtitutiva ſit Progreſſivo major.

In Motibus Acceleratis & Retardatis, Impetus pro ſingulis Momentis is reputandus eſt, qui gradui Celeritatis tum acquiſito convenit. Ubi autem per Curvam fit Motus, ea reputanda eſt in ſingulis punctis Motus Directio, quæ eſt rectæ ibidem Tangentis. Et ſi quando Motus tum Acceleratus vel Retardatus ſit, tum & per Curvam fiat (ut in Vibrationibus Penduli) Impetus æſtimandus erit, pro ſingulis punctis, ſecundum tum gradum Accelerationis, tum Obliquitatem ibidem Tangentis.

2. *Lex Naturæ de Colliſione Corporum.*

By Sir Chriſt. Wren.
Ibid. p. 867.

Velocitates Corporum propriæ & maximè naturales ſunt ad Corpora reciprocè proportionales.

Itaque Corpora R, S; habentia proprias Velocitates, etiam poſt Impulſum retinent proprias.

Et Corpora R, S, improprias Velocitates habentia ex Impulſu reſtituuntur ad Æquilibrium; hoc eſt, Quantum R ſuperat, & S deficit à propria Velocitate ante Impulſum, tantum ex Impulſu abſtrahitur ab R, & additur ipſi S, & è contra.

Quare Colliſio Corporum proprias Velocitates habentium æquipollet Libræ Oſcillanti ſuper bina Centra æqualiter hinc inde à Centro Gravitatis diſtantia: Libræ verò Jugum, ubi opus eſt producitur.

Itaque Corporum æqualium improprie Moventium tres ſunt Caſus. Corporum verò inæqualium improprie Moventium (ſive ad contrarias ſive ad eaſdem partes) decem ſunt omnino Caſus, quorum quinque oriuntur ex converſione.

Fig. 157. R, S, Corpora æqualia; vel R, Corpus majus, S, Corpus minus.

a Centrum Gravitatis ſive Anſa Libræ. Z, Summa Velocitatum utriuſque Corporis.

R *e*	Veloc.	R	ante Impul-	vel	S *o*	Veloc.	S	ante Impul-
S *e*	Corp.	S	ſum data.		R *o*	Corp.	R	ſum data.
o R	Veloc.	R	poſt Impul.		*e* S	Veloc.	S	poſt Impul.
o S	Corp.	S	quæſita.		*e* R	Corp.	R	quæſita.

Regula. R *e*, S *e*, faciunt *o* R, *o* S : R *o*, S *o*, faciunt *e* S, *e* R.

[Lege Syllabas (quamvis disjunctas) R *e*, S *e*, *o* R, *o* S, vel R *o*, S *o*, *e* S, *e* R, in Linea cujuſlibet Caſus, & harum quæ ſcribitur in Schemate more *Hebraico*, ea indicat Motum contrarium Motui quem notat cujuſvis Syllabæ ſcriptio *Latina*. Syllaba conjuncta quietem Corporis denotat.]

$$\text{Calcul.}\quad \begin{array}{l|l|l} R+S:S::Z:Ra & Re-2Ra=oR & So-2Sa=eS. \\ R+S:R::Z:Sa & 2Sa+Se=aS & 2Ra+Ro=eR. \end{array}$$

Natura obſervat regulas Additionis & Subductionis *Speciosæ*.

By *M.* Hugens. n. 46. *p.* 927. Apr. An. 1669.

3. *Regulæ de Motu Corporum ex mutuo Impulſu.*

1. Si Corpori Quieſcenti, duro aliud æquale Corpus durum occurrat, poſt contactum hoc quidem Quieſcet, Quieſcenti verò acquiretur eadem quæ fuit in Impellente Celeritas.

2. At ſi alterum illud Corpus æquale etiam moveatur, feraturque in eadem Linea recta, poſt contactum permutatis invicem Celeritatibus ferentur.

3. Corpus quamlibet magnum à Corpore quamlibet exiguo & qualicunque Celeritate impacto movetur.

4. *Regula Generalis* determinandi Motum, quem corpora dura per occurſum ſuum directum acquirunt, hæc eſt:

Fig. 158. *Sint corpora* A *&* B, *quorum* A *moveatur Celeritate* A D, B *vero ipſi occurrat, vel in eandem partem moveatur Celeritate* B D, *vel denique Quieſcat, hoc eſt, cadit in hoc caſu punctum* D *in* B. *Diviſa Linea* A B *in* C, *(Centro Gravitatis corporum* A B,) *ſumatur* C E *æqualis* C D. *Dico* E A *habebit Celeritatem corporis* A *poſt Occurſum*; E B *verò, corporis* B, *& utrumque in eam partem, quam demonſtrat Ordo punctorum* E A, E B. *Quod ſi* E *incidat in punctum* A *vel* B, *ad Quietem redigentur corpora* A *vel* B.

5. Quantitas Motus duorum Corporum augeri minuive poteſt per eorum occurſum; at ſemper ibi remanet eadem quantitas, verſus eandem partem, ablatâ inde quantitate Motus contrarii.

6. Summa Productorum factorum à Molè cujuſlibet corporis duri ducta in quadratum ſuæ Celeritatis, eadem ſemper eſt ante & poſt occurſum eorum.

7. Corpus durum Quieſcens, accipiet plus Motus ab alio corpore duro, ſe majori minorive, per alicujus Tertii, quod mediæ fuerit quantitatis, interpoſitionem, quam ſi percuſſum ab eo fuiſſet immediatè. Et ſi corpus illud interpoſitum, fuerit medium proportionale inter duo reliqua, fortiſſime omnium aget in Quieſcens.

Conſiderat *Author* in his omnibus (ut ipſe ait) Corpora ejuſdem materiæ, ſive id vult, ut eorum moles æſtimetur ex pondere.

Cæterum ſubjungit, notâſſe ſe miram quandam *Naturæ Legem*, quam Demonſtrare ſe poſſe affirmat in corporibus Sphæricis, quæque Generalis ipſi videtur in reliquis omnibus ſive Duris ſive Mollibus, ſive Directe ſive Oblique ſibi occurrentibus, *viz.* *Centrum Commune Gravitatis* duorum, trium, vel quotlibet Corporum, æqualiter ſemper promoveri verſus eandem partem in linea recta, ante & poſt occurſum.

Some Hiſtorical Paſſages relating to theſe Papers; by Mr. Oldenburg. *Ibid.* p. 925.

4. Cum noviſſimis Menſibus nonnulli è *Societate Regia* in publico ejuſdem conceſſu enixius urgerent, ut graviſſimum illud de *Regulis Motus* Argumentum, non ſemel inter Ipſos ante hac agitatum, ſed pluribus aliis intercurrentibus rebus, nunquam, uti par erat, diſcuſſum expenſumve, tandem aliquando Examini Rigido ſubjectum conficeretur; viſum equidem fuit Illuſtriſſimo iſti *Cœtui* decernere, ut quotquot è *Sociis* ſuis indagandæ Motus Indoli præ cæteris incubuiſſent, rogarentur, ut ſua in rem illam Meditata & Inventa depromere, ſimul & ea, quæ ab aliis Viris Præcellentibus, *Gallilæo* puta, *Carteſio*, *Honorato Fabri*, *Joachimo Jungio*, *Petro Borrelli*, aliiſque, de Argumento iſto fuerant excogitata, congerere & procurare vellent; eo ſcil. fine, ut conſultis hoc pacto collatiſque omnium ſententiis, illa dehinc Theoria, quæ cum Obſervationibus & Experimentis, debitâ curâ & fide crebro peractis, quammaximè congrueret, Civitate Philoſophica ſuo jure donaretur.

Edito hoc Celeuſmate, incitati protinus è dicta *Societate* fuerunt, imprimis *Chriſtianus Hugenius*, *Johannes Walliſius*, *Chriſtopherus Wrennus*, ut ſuas de *Motu Hypotheſes* & *Regulas*, quibus condendis aliquamdiu inſudaſſent, maturare atque expedire ſatagerent. Factum hinc, ut ſelectus ille Virorum præſtantiſſimorum Trias, poſt paucarum ſeptimanarum ſpatium, *Theorias* ſuas, eleganter compendifactas, tantum non certatim tranſmitterent, *Regiæque Societatis* ſuper iis ſententiam exquirerent. Primus omnium D. *Walliſius*, ſua de *Motibus* æſtimandis *Principia*, Literis, d. 15. *Novemb.* 1688. datis, ejuſdemque Menſis die 26 traditis & prælectis, communicavit. Mox eum excepit D. *Chriſtopherus Wren*, qui *Naturæ Legem* de *Colliſione Corporum*, proximo Menſe *Decembri*, ejuſque die 17. eidem *Societati* publice exhiberi curavit: quæ in mandatis mox dedit (præ-habito tamen utriuſque hujus Authoris conſenſu) ut ad commodiorem horum ſcriptorum communicationem, diſcuſſionemque diffuſiorem, res tota Typis mandaretur.

Hæc dum apud Nos geruntur, Ecce adfert Nobis Tabellarius d. 4. *Januarii* inſequentis (*St. Ang.*) D. *Hugenii* literas ejuſdem Menſis d. 5. (at *St. N.*) exaratas, ejuſque ſcripti, de *Motu Corporum* ex mutuo *Impulſu*, priores Regulas quatuor, una cum Demonſtrationibus, continentes. Habebam ego in promptu *Theoriæ Wrennianæ* Apographum, idque actutum eodem planè die, ſic favente Tabellione Publico, D. *Hugenio*, redhoſtimenti vicæ, remittebam, dilatâ interim Literarum *Hugeniarum* (quibus tale quid includi, ob Molem, & antegreſſum Authoris promiſſum ſuſpicabar) reſignatione, donec ferret occaſio Nobiliſſimum & Sapientiſſimum *Regiæ Societatis Præſidem*, D. *Vice-Comitem Brounckér*, compellandi. Quo facto, amborumque Regulis in modo dicta *Societate* collatis, mirus confeſtim in utroque conſenſus effulſit; id quod inſignem in nobis lubentiam pariebat, utrumque hoc ſcriptum prælo noſtro committendi. Nihil hic Nobis deerat à parte *Hugenii*, quam ejus conſenſus; abſque quo fas nequaquam judicabamus, ipſius Inventum, maximè cum illud haud integrum eo tempore nobis dediſſet, in lucem emittere. Curæ interim nobis erat, ſcriptum Ipſius publicis *Regiæ Societatis* Monumentis inferendi; ſimul & Authori d. 11. *Januarii* ſolennes pro Cordata illa Communicatione gratias reponendi, additâ dehinc (die *Scil.* 4. *Febr.*) ſollicitâ commonefactione, ut ſuam hanc *Theoriam* vel *Pariſiis* (quod proclive erat factu in *Eruditorum*, ut vocant, *Diario*) vel hic *Londini* in *Adverſariis Philoſophicis*, imprimendam curaret, vel ſaltem permitteret. Quibus expeditis Literis paulo poſt ſecundas accepimus ab *Hugenio*, ſcripti *Wrenniani* de hoc argumento rectè traditi mentionem facientes, nihil tamen quicquam de ſuimet ſcripti Editione, vel *Pariſiis* vel *Londini* paranda, commemorantes.

Unde liquere omnino autumem, ipſum ſibi defuiſſe *Hugenium* in illa publicatione maturanda; quin imo occaſionem dediſſe procraſtinando, ut Laudatus Dn. *Wren*, pro ingenii ſui ſagacitate Geminam omnino *Theoriam* eruens in Gloriæ, huic ſpeculationi debitæ, partem jure veniret; cum extra omne ſit dubium neutrum horum *Theoriæ* illius quicquam, priuſquam ſcripta eorum ſimul compararent, reſciviſſe ab altero, ſed utrumque propriâ Ingenii fæcunditate, pulchellam hanc ſobolem enixum fuiſſe.

Solvit equidem *Hugenius*, ante aliquot jam Annos, *Londini* cum ageret, illos de Motu Caſus qui ipſi tunc proponebantur; luculento ſane Argumento, eum jam tum exploratas habuiſſe *Regulas*, quarum id evidentia præſtaret. At non affirmabit ipſe, cuiquam ſe *Anglorum* ſuæ *Theoriæ* quicquam aperuiſſe; quin fateri tenentur, ſe ab eorum nonnullis ad communicationem ejus ſolicitatum, nec tamen unquam, niſi nuperrime, ad id faciendum pertractum fuiſſe.

The Synchroniſm of the Vibrations made in a Cycloide; *Demonſtrated by a Perſon of* Quality. n. 94. *p.* 6032. *Fig.* 159. May An. 1673.

II. Sint *a b*, *b c*, *c d*, *e f*, &c. omnes invicem æquales; & *b* 1, *c* 2, *d* 3, *e* 4, *f* 5, *&c.* æqualiter creſcant ut, 1, 3, 5, 7, 9, &c.

Dico in hac Linea, Grave quodlibet, Cadens ex quovis ejus Puncto, attingere fundum in eodem Temporis ſpatio, quo eum attingeret ſi caderet ex quovis ejuſdem Puncto alio.

Nam ſi ponas, $a = ab = bc = cd$, &c. & $b = b1$, & x pro quolibet Numero alterutrorum; tunc ſi xa ponatur pro af, xxb repræſentet oportet $f\delta$, proindeque Tempus Deſcenſus neceſſario erit $\frac{xxb}{xxaa}$ ſeu $\frac{b}{aa}$; atque idem in omnibus obtinet Caſibus. Ergo, &c.

Dico inſuper, Curvam hanc eſſe *Cycloidem*, quod Demonſtratu eſt facile ex Conſtructione, atque ex eo quod jam innuo; nempe, Curvam hanc $abcdefz$ æquare duplum Ultimæ Rectarum, *h. e.* $2z\omega$, & $a\omega$ æqualem eſſe Semicircumferentiæ Circuli cujus $z\omega$ eſt Diameter; ac univerſim Triangulum ♈ ♉ ♊ repræſentare Rectam $z\omega$; & Quadratum ♈ ♉ ♊ ♋, Curvam $abcdefz$, & Quadrantem ♈ ♉ ♋ repræſentare Rectam $a\omega$: ac partes unius, partes alterius reſpectivè. Uti ſi ♈ ♎ ♍ repræſentat $f\delta$, tunc ♈ ♎ ♌ ♋ repræſentat $a\delta$, & ♈ ♎ ♏ ♋ repræſentat af. At non vacat fuſius hæc proſequi.

Dico denique; Globulum ſuſpenſum è Funiculo (Juſtæ Longitudinis) & inter duas *Cycloides* vibrantem, moveri in *Cycloide*. Quare Vibrationes ejuſmodi ſunt *Synchronæ*. *Q. E. D.*

III. 1. Probl.] *Determinare Lineam Curvam data duo Puncta, in diverſis ab Horizonte Diſtantiis & non in eadem Recta Verticali poſita, connectentem, ſuper qua Mobile, Propriâ Gravitate decurrens, & à Superiori Puncto Moveri incipiens, citiſſimè Deſcendat ad Punctum Inferius.*

Senſus *Problematis* hic eſt, ex Infinitis Lineis quæ duo illa data Puncta conjungunt, vel ab uno ad alterum duci poſſunt, eligatur illa, juxta quam ſi incurvetur Lamina, Tubi Canaliſve Formam habens, ut ipſi impoſitus Globulus & liberè dimiſſus iter ſuum ab uno Puncto ad alterum emetiatur Tempore Breviſſimo.

A Problem concerning the Line of quickeſt Deſcent between two Points given; propoſed by M. Jo. Bernoulli. n. 224. p. 384. Jan. An. 1697.

2. Accepi heſterno die duo Problematum à *Joanne Bernoullo* Mathematicorum acutiſſimo propoſitorum. Exemplaria, *Groningæ* edita, *Cal. Jan.* 1697. Quorum prioris Solutio ſit hujuſmodi.

Solv'd; by . . . . . . *Ibid.*

A Dato Puncto A, ducatur Recta Infinita A P C Z Horizonti parallela, & ſuper eadem Recta deſcribatur tum *Cyclois* quæcunque A Q P, Rectæ per alterum Datum Punctum B ductæ (&, ſi opus eſt, productæ) Occurrens in Puncto Q, tum *Cyclois* alia A B C cujus Baſis & Altitudo ſit ad prioris Baſem & Altitudinem reſpective ut A B ad A Q; Et hæc *Cyclois* Noviſſima tranſibit per Punctum B, & erit Curva illa, linea in qua Grave à Puncto A ad Punctum B, Vi Gravitatis ſuæ, Citiſſimè perveniet. Q. E. I.

Fig. 160.

3. Sit A P, Linea Horizontalis; P, Punctum à quo corpus Grave deſcendit, per Curvam Lineam quæſitam A D E, C & D, Puncta duo infinitè propinqua, per quæ Corpus deciſurum ſit, C D Recta, duo Puncta connectens, D C & s C, D F & S G, F S & G C vel s H, Momenta Curvæ Abſciſſæ, & Ordinatim applicatæ reſpective. Capiatur, D r = D s, & r C = B C.

The Demonſtration; by Mr. R. Sault. *n.* 246. p. 425. Nov. An. 1698.

Fig. 161.

Quoniam in Lineolis Naſcentibus, Tempus eſt ut Via percurſa directè & Velocitas (*i. e.* in hoc caſu, ut Radix Quadrata Altitudinis corporis deſcenſi) inversè, per Hypoth. $\frac{D s}{\sqrt{QD}} + \frac{SC}{\sqrt{QF}} =$ Tempori Minimo. Et quia Velocitas in Punctis æquialtis S & B per Curvam D s C & Rectam D B C eadem eſt, Tempus per D C, quod evidenter Minimum eſt, erit ut $\frac{BD}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$; æquentur ergo hæc Tempora, & $\frac{D s}{\sqrt{QD}} + \frac{s C}{\sqrt{QF}} = \frac{DB}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$, hoc eſt, $\frac{DB - D s}{\sqrt{QD}} = \frac{s C - BC}{\sqrt{QF}}$, vel $\frac{B r}{\sqrt{QD}} = \frac{t s}{\sqrt{QF}}$.

Sed Triangula Evaneſcentia B r s, B t s, æquiangula ſunt Triangulis D s F, H s C; Ergo $\frac{B s}{D s} = \frac{B r}{s F}$, & $\frac{t s}{H s} = \frac{B s}{s t}$. Componantur hæ duæ rationes æqualitatis, & $\frac{B r}{D s \times H s} = \frac{t s}{s F \times s t}$. Ex æquo $\frac{\sqrt{QD}}{s F \times s t} = \frac{\sqrt{QF}}{D s \times H s}$. Quandoquidem autem quidvis ex Elementis æquabiliter Fluere ſupponatur, ponamus $D S = S C$, & evadet ſimpliciſſima Curvæ expreſſio $\frac{\sqrt{QD}}{s F} = \frac{\sqrt{QF}}{D s}$ ubique, *i. e.* in Puncto Flexuræ Curva ſemper erit in Ratione compoſita Velocitatis directè, & Momenti applicatim Ordinatæ inversé. Sint $\dot{x}$ $\dot{y}$ & $\dot{z}$ Fluxiones Abſciſſæ, Ordinatim applicatæ, & Curvæ reſpectivè, $\frac{x^{\frac{1}{2}}}{\dot{y}}$ conſtans eſt, ut ſupra. Ergo $\frac{x^{\frac{1}{2}}}{\dot{y}} = \frac{1}{\dot{z}}$; ſed poſuimus $\dot{z}$ $(= \sqrt{\dot{x}\dot{x} + \dot{y}\dot{y}})$ conſtans. Ergo ut hæc unitas conſtans ſit & dimenſiones debitas retineat $\frac{x^{\frac{1}{2}}}{\dot{y}} = \frac{a^{\frac{1}{2}}}{\sqrt{\dot{x}\dot{x} + \dot{y}\dot{y}}}$; & poſt reductionem $\dot{y} = \frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a - x}}$. Expreſſio Notiſſima *Cycloidis* P E L. Q. E. I.

IV. Theorema

How much the Deſcent is Quicker in the Cycloid than in a ſtraight Line, by .... n. 225. p. 424. Feb. An. 1697.

IV. Theorema.] *Si in* Cycloide A V D, *cujus Baſis* A D *eſt Horizonti Parallela, Vertice* V *deorſum ſpectante, ex* A *ducatur utcunque Recta* A B Cycloidi *occurrens in* B, *ex quo ducatur Recta* B C. *Curva* Cycloidis B D *in* B *Normalis, ad quam ex* A *demittatur Perpendicularis Recta* A C. *Dico Tempus quo Grave è Quiete cadens ex* A, *Vi ſua Gravitatis decurrit Rectam* A B, *eſſe ad Tempus quo percurrit Curvam* A V B, *ſicut Recta* A B *ad Rectam* A C.

Fig. 162.

Per B ducatur B L, Parallela *Cycloidis* Axi V E; & B K, Baſi A D Parallela, occurrens Axi in G, & Circulo ſuper Diametrum E V deſcripto in F & H, *Cycloidi* denique in K. Ducatur Recta E F, quæ ex *Cycloidis* natura parallela eſt Rectæ B C. Unde B M eſt æqualis E F, & E M æqualis B F; quæ, propter *Cycloidem,* æquatur Arcui V F; & proinde A M eſt æqualis Arcui E H V F.

Per *Prop.* 25. *Part* II. *Horologii Oſcillatorii Hugenii,* Tempus quo grave è Quiete cadens percurrit A V, eſt ad Tempus Caſus per E V, ut Semicircumferentia ad Diametrum; & per dictæ Partis *Prop. Ultimam,* Tempus quo Grave percurrit V B, poſt decurſam A V (nempe æquale Tempori quo Grave percurrit K V, poſt decurſam A K) eſt ad Tempus Lapſus per A V, ſicut Arcus V F, ad Semicircumferentiam; adeoque ad Tempus Caſus per E V, ſicut F V ad Diametrum. Quare Tempus quo Grave percurrit Curvam A V B, eſt ad Tempus Caſus per E V, ſicut Arcus E H V F, ad Diametrum E V. Sed Tempus Caſus per E V, eſt ad Tempus Caſus per L B, ſive E G, ſicut E V ad E F: Igitur ex æquo, Tempus quo Grave percurrit A V B, eſt ad Tempus Caſus per L B, ſicut Arcus E H V F ad ſubtenſum E F; hoc eſt, ut Recta A M, ad Rectam M B. Rurſus, Tempus Caſus per L B, eſt ad Tempus Lapſus per A B, ut L B ad AB: Ergo Ratio Temporis quo Grave percurrit A V B, ad Tempus quo percurrit A B, componitur ex Ratione A M ad M B, & Ratione L B ad B A; adeoque æqualis eſt Rationi A M × L B ad M B × B A. Sed A M × L B, eſt æquale M B × AC, quia utrumque æquatur duplo Trianguli A B M: Et igitur Tempus quo Grave è Quiete cadens percurrit Curvam *Cycloidis* A V B, eſt ad Tempus quo percurrit Rectam A B, ſicut M B × A C ad M B × B A; id eſt ſicut A C ad A B. *Q. E. D.* Similiterque procedet Demonſtratio, ſi Punctum B, ſit inter A & V.

Exact Portable Watches; by M. Hugens. n. 112. p. 272. Fig. 163. Mar. An. 1675.

V. 1. The upper Plate of the *Watch* is A B: The Circular *Ballance-Wheel* C D, of which the *Arbor* is E F: The *Spring* turned *Spirally,* G H M, faſtned to the *Arbor* of the *Ballance-Wheel* in M, and to the piece that is faſt to the Watch-Plate, in G, all the *Spires* or Windings of the *Spring* being free without touching any thing. N O P Q, is the *Cock*, in which one of the *Pivots* of the *Ballance-Wheel* turns; R S, is one of the *Indented-Wheels* of the *Watch* having a *Ballancing* Motion, which the *Ballance-Wheel* gives to it. And this *Wheel* R S, catches in the *Pinion* T, which holds on the *Arbor* of the *Ballance,* of which by this means the Motion is entertained as much as is neceſſary. Theſe *Watches* are exact for the Pocket,

and when made greater, will be uſeful to find the *Longitudes* both by Sea and Land.

By Dr. Goth. Guil. Leibnitz. n. 113. p. 285. Apr. An. 1675.

2. The Principle I thought upon ſome Years ago for making Exact *Portable Watches*, is altogether different from that of M. *Hugens*; His depending upon a Phyſical Obſervation, but mine upon a meer Mechanical Reflection; which hath not been taken notice of for want of the *Art* of *Combination*, the uſe of which is far more general than that of *Algebra*. For, having conſidered with my ſelf, that a *Spring* being Bent to the ſame Degree, will always Unbend it ſelf in the ſame Time, provided it find the ſame freedom of Unbending it ſelf ſuddenly; I inferred from thence, that there might be imployed two ſuch, one of which ſhould play, whilſt the *Firſt Mover* of the *Watch* did Bend the other again.

Fig. 164.

Theſe Thoughts I have executed in the following manner: Let A B, be one of the *Watch-Plates*, C and M, two indented Barrels wherein the ſmall *Springs* are incloſed. The Teeth of the Barrels catch thoſe of the Pinions *d d*, which carry the Ballances *e e*, and other Teeth of the ſaid Barrels are catched by thoſe of the Interrupted Wheel F G. Now let us imagine, that this Wheel F G, being moved towards H F, by the force of the *Firſt Mover* of the *Watch*, and turning the Barrel C, Bends the *Spring* incloſed in it, and ſtops with the Barrel as ſoon as it hath Bent this *Spring*. This piece which ſerves to ſtop, is eaſie, and hath not been thought neceſſary to be marked here, to avoid embaraſſing the Figure. But whilſt one Indented part of the Interrupted Wheel F G, *viz.* F, turns the Barrel C, the empty part, oppoſed thereunto, which is G, anſwers to the other Barrel M, and gives Liberty to the *Spring*, it incloſeth, to Unbend it ſelf. Thus whilſt the *Movement* of the *Watch* Bends the ſmall *Spring* of the Barrel C, in the ſame Time the ſmall *Spring* of the other Barrel M, Unbends of it ſelf: I ſay, in the ſame Time, except the *Spring* C, ſhall have done Bending a little ſooner than the *Spring* M ſhall have Unbent it ſelf: So that the *Spring* C, being Bent, and the Wheel F G ſtopped; both of them ſtay in this Poſture, till the *Spring* M, when it ſhall be quite Unbent, doth, at the end of its Motion, touch a piece which delivers it. And then the *Spring* C Unbends of it ſelf in its turn; the Teeth of the Interrupted Wheel, which continues its Motion the ſame way as before, ſince 'tis delivered, not being any more able to hinder it therefrom, becauſe the Barrel C, doth now meet with the Empty part H, of the ſaid Wheel. But before it hath done with Unbending it ſelf, the Indented part L, being oppoſite to the Empty part H, that turns the Barrel M, Bends its *Spring* again, and having done ſo, ſtops with it; whilſt the *Spring* C, making an end of Unbending it ſelf, delivers them by a Reciprocal good Office, and renders to the *Spring* M, the ſame Services which it had received from it, with an Expectation of receiving the like again.

Which being well conſidered, 'tis manifeſt, That the ſame *Alternative Motions* will continue always: That the Periods, taken from the very Moment that one *Spring* begins to Unbend, until the Moment it once Unbends it ſelf again, will always be of *Equal Duration*, though the two ſmall

Springs be not equally ſtrong: That the Ballance of ſuch a *Watch* will be double, and may be Charged more or leſs, and receive Delay, by Advancing or Recoiling along the two Arms two equal Weights, Counter-ballancing one another, that ſo the Change of the Situation may not at all prejudice the *Equality* of the *Watch*. For the reſt, we may in this kind of *Watches* ſpare the *Fuſee*, and conſequently the String or Chain. 'Tis alſo eaſie to judge, that ſuch *Watches* as theſe may be of a Size ſufficiently ſmall; that they will make no more Noiſe than ordinary *Watches*; that they will be as exact as *Pendulums*, and ceaſe not to go whilſt they are Winding up. And though the Motion of the Watch Wheels may be altered by many Accidents, yet the *Periods* of the *ſmall Springs* will not be concerned in all or any of them, provided the Motion of the Watch Wheels have always more Strength than it needs to Bend them again; which is in our Power.

The Objections that have been made againſt this Contrivance, if employed for *Finding Longitudes*, are theſe; that Toſſing of Ships would ſhake the *Springs* as well other pieces; that Ruſt would ſpoil them, ſince the Saltiſh Humidity of the Sea in remote Voyages, ſpares not the very Needles of Compaſſes though incloſed in Boxes; that the Changes of Seaſons and Climates will ſenſibly alter the *Springs*, eſpecially the great Heats or Rains within the Tropicks, which at length will ſomewhat Untemper the Steel; as is confirmed by the Experiments of the *Illuſtrious Academy of Florence*, ſhewing how eaſily that Heat and Cold do change *Slender Springs*: Beſides that, the Air more or leſs condenſed will alſo more or leſs reſiſt the Motion of the Ballance. To which may be added, that *Springs* by working are weakened; And Laſtly, That there will be always ſome little *Friction*, that will make the ſeveral pieces go more or leſs eaſily, and that even in length of time they will wear out.

But I anſwer, That all theſe Defects, that proceed from the Imperfection of the Matter, may be ſurmounted by a General Remedy, without Examining them here in particular: And that is, That for executing it in great, we make uſe of maſſy Springs, as are thoſe of Croſs-Bows, we being Maſters of them, not wanting Force or Place in a Ship, to govern a great Weight that may ſerve to Bend them continually again. Now theſe *Maſſy Springs* may be ſo great, and their Reſtitution ſo ſpeedy, by Augmenting their Number, that all the above-named Defects will have no conſiderable Proportion to this Strength, and the Aggregate of their Repetitions will not be ſenſible till after a very long time. And 'tis eaſie to *Demonſtrate*, That by Augmenting the Bigneſs of the Engine, and the Force of the *Maſſy Springs*, we may make the Error as ſmall as we will, provided we paſs not the Bounds of Conveniency, and content our ſelves with Exactneſs ſufficient for their Chief End, *viz*. For finding the *Longitudes*.

VI. The Circle, FGH, being placed upon a *Plain Inclined* AB, is divided into two Unequal Parts by the Line GI. To reſtore to the Leſs Secture its *Æquilibrium*, there is faſtened to the Extremity of the *Radius* DF, a Weight F, which is ſufficiently heavy to Recover, what the leſſer Secture

A Clock Aſcendent upon an Inclined Plain; by M. de Gennes.

n. 140. *p.* 1006 *Fig.* 165. July An. 1678.

loſes by its Situation. That a Wheel or Clock may thus ſtand not only in *Æquilibrium*, but alſo Aſcend upward, there is placed in the middle of the *Clock* a *Drum*, which encloſes the *Spring* of the *Pendulum*; upon which *Drum* is faſtned the *Radius* DF. For thus the *Spring* being Mounted, enforces the *Drum* to turn, and ſo to Raiſe the Weight, which it cannot Raiſe without its becoming more Heavy, in regard that coming to the Point E, it is farther from the Center, than when it was in F, and thus all the *Wheel* turns on that Side as the *Spring* gives way.

A Clock Deſcendent *on a Plain Inclined*; *by Mr.* Maurice Wheeler. n. 161. *p.* 647. May An. 1684.

VII. Although the *Marquis* of *Worceſter* is ſaid to have contrived a *Watch* that ſhould *Move* upon a *Declivity*, and M. *de Gennes* has given ſome Account of a *Clock Aſcendent on a Plain Inclined*; yet neither of them, nor any like them, was ever ſeen by me, and for ought I could ever learn, the *Reaſon* of their *Motions* remains to this Hour as great a Secret, as if they had never been. I ſhall therefore give an Account of a *Movement*, which I have deſign'd to meaſure Time after a peculiar Manner.

Fig. 166.

1. The Exteriour Structure of it is a Circular Body of 3½ Inches Diameter, conſiſting of two Plates meaſured by the ſame *Radius*, and fixed in a Parallel Poſition to each other by the Hoop *h*, the Breadth of which is about an Inch. This Hoop and the two Plates Form the Caſe of the *Movement*; of which, that which appears in the Front, is towards the Verge thereof Inſcribed with a *Horary Circle*, the Diviſions whereof anſwer the Hours of a Natural Day. The Deep Shades within this Circle are intended to repreſent a Concave, of near half an Inch deep; and the Prominence *g*, in the Middle of this Concave, is a Hemiſphere of Braſs or Silver, riding looſly on a Pin, which lies hid, and is the *Axis* of the *Movement*. The Upper half of this Hemiſphere is hollow, but the Nether filled with Lead; and the ſmall Gentleman that ſits thereon, does with an Erected Finger perform the Office of an Index. But this being only for Ornament, you may Subſtitute in the room thereof any other Index, provided the *Axis*, whereon it is ſupported, move freely in the Hole H, and the lower part thereof H L, ſo far preponderate to H P, as always to keep it Pendulous, with its Point to the Vertical Hour. *Fig.* 169.

Fig. 166.

2. For the manner of its Motion, as far forth as it appears outwardly; it is thus: S E repreſents a Board or Shelf, of a Straight and Even Surface, about 6 Foot long, and ſo Thick as not to be apt to Caſt with change of Weather; nor to grow Camber under a ſmall Weight; on this is the *Movement* placed, and here to perform its Courſe; and therefore I call it the *Stage* of the *Movement*. This *Stage* is Raiſed at the end S, about 10 *Deg.* above the Horizon or Line of Level H E; but this Angle of its Declivity D E H, is Variable. The two Plates which Form the *Caſe* of the *Movement*, are to be extant all round without the Hoop *h*, ¼ of an Inch, and the Edges of 'em lightly Indented; that while the *Movement* deſcends upon the *Stage*, it may Turn only, and not Slide. The *Movement* being placed as high as it may, near the point S, ſhall Move downward towards E, with that ſlowneſs, as to Finiſh one entire Revolution in 24 hours; and while it does ſo, the Diviſions on the

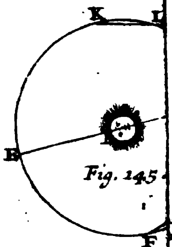


Fig. 145.

Fig. 15

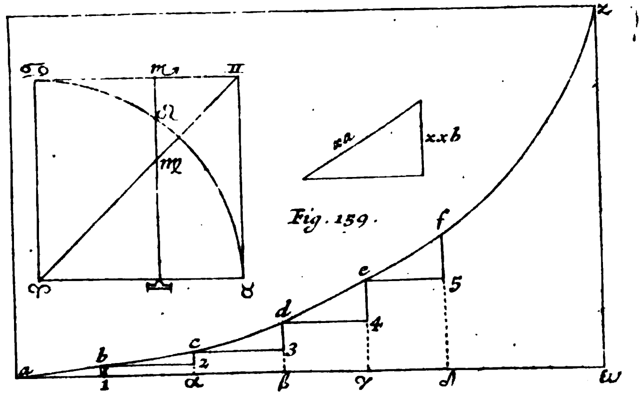
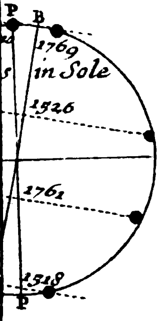
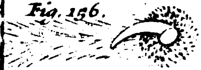


Fig. 159.





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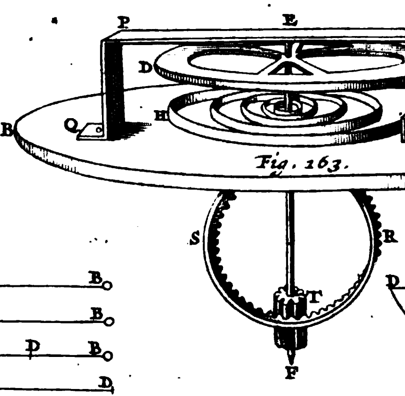

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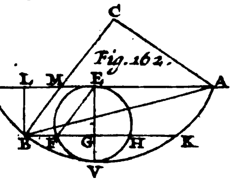

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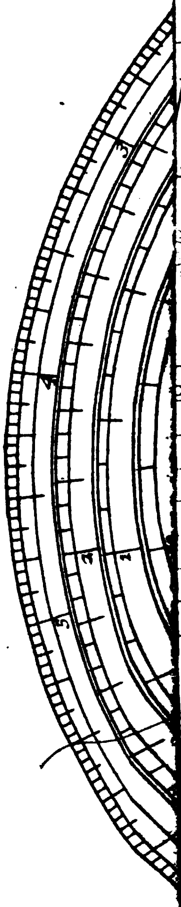

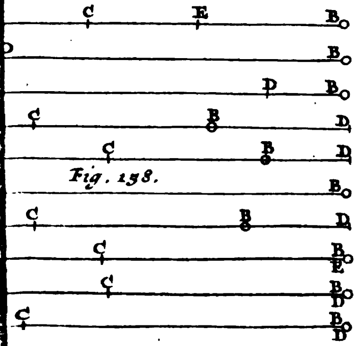

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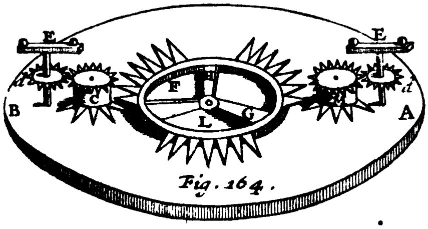
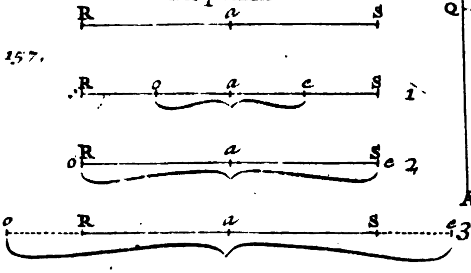


Fig. 164.

Æqualia



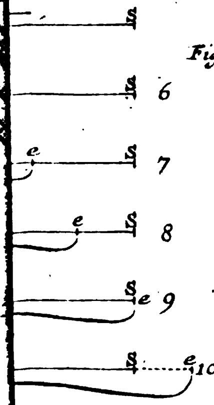
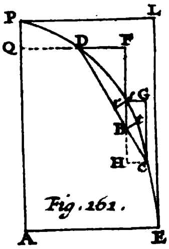


Fig. 157.



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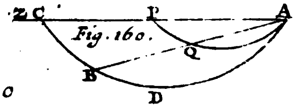
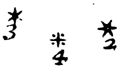


Fig. 160.



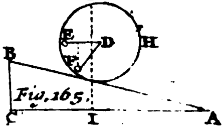


Fig. 165.

the *Horary Circle* (or Dial-Plate) ſucceſſively Culminating over the Point of the Index (which is always to keep the ſame Poſition) will ſhew the Hours of the Day and Night. And when by ſeveral repeated Revolutions, it has meaſured the length of its *Stage*, it is to be replaced at S, as before, which may be done in leſs than half the time you are Winding up a Watch; and if the *Stage* be 6 Foot long, no oftner than once in a whole Week.

3. The way of Adjuſting the Motion to the exact Meaſure of an Hour, and Rectifying its Errours, is thus: *viz.* By the turning a Skrew inſerted at S, the *Stage* may be Elevated or Depreſs'd, and accordingly the *Movement* will go Faſter or Slower: Faſter, if Raiſed up, and Slower, if let Down; and by making the *Horary Circle* Moveable, and Inſerting ſeveral ſmall Boſſes or Buttons, here and there upon the Verge thereof, it may with an eaſy touch of the Finger be moved to the right and left, as there ſhall be Occaſion, till the juſt Time be brought to the Point of the *Suſpended Index*.

The Reaſon of this *Movement* may be thus Explained: 1. Let the Circle LODN, repreſent any Circular Body, whoſe Centers both of Gravity and Magnitude are Coincident at M. Let this Circular Body be placed upon ſome Level Plain G G, and then 'tis Evident that the Angle of its Contact with that Plain at *a*, will alſo be the Point of its Libration, and conſequently it muſt Reſt there: *Quia Momentum & Impedimentum ſunt æqualia.* Fig. 167.

2. Let DE, repreſent a Deſcending Plain, making an Angle of Contact with this Circular Body at *b*; and here 'tis manifeſt it cannot Reſt; becauſe the Line of Direction *r a*, which (while it Inſiſted upon a Level) divided the Circular Body by the Centers of Magnitude and Gravity into Parts Æquiponderate, is now removed to L D; which Line L D, falling without, or beſide, the Center M, evidently deſtroys the Æquipoiſe of its Parts, and therefore muſt leave it to tumble down towards E. For here *Momentum Impedimento majus.* The Reaſon therefore of its Deſcent now, being the Over-ballance of the Parts L N D, to the remaining Section L D O, it muſt neceſſarily follow.

3. That if ſome Weight equal to the Exceſs of LND, above L O D, were affix'd to the Limb of the Quadrant O *a*, as at P; then the Circular Body would Reſt as Quietly at *b*, as it did before at *a*. The Suppoſition cannot be denied, and the Conſequence is unavoidable, becauſe L D O + P = L N D, *i. e. Impedimentum æquatur Momento.*

Let then the Numbers, 1, 2, 3, 4. repreſent a Train of Wheel-work, wherein there is no material difference from what is found in a common Watch; only the numbers of the Teeth on the Wheels and Pinions are to be ſo Calculated, that the Motion of the whole Train may correſpond to the aſſigned Revolution of the Body of the *Movement*, which is to be once in 24 Hours: It would be Expedient alſo, That a *Spiral Spring* were applied to its *Ballance*, as in later *Movements*, is uſual; but of a *Fuſee* here's no need, for the Turns of the Body of the *Movement*, as it deſcends upon the *Stage*, anſwer all the Intentions of a String or Chain; and the *Contranitence* of the Weight P, to the exceſs of LED, above LQD, ſerves inſtead of a Perpetual Spring; and the *Movement* wants only a *Perpetual Deſcent*, to make its *Mo-* Fig. 168.

tion ſo. And whereas the great Wheel in ordinary *Movements*, is placed as near the edge of the Framing Plate *f r*, as it may be; here it muſt (with its *Axis* or Arbor M) poſſeſs the Center of the *Movement*: Becauſe this Wheel is to carry the Weight of *Power* P, by the *Vectis* M P, and that Weight P, muſt always keep an Equidiſtance from the Center of the *Movement*; that while the Body thereof (*i. e.* of the *Movement*) performs its Revolutions; the ſaid Weight P, and the great Wheel (to which it is affixed) may, without any conſiderable Variation, continue in, or near the ſame Poſition, wherein they now are. Now ſuppoſe this Weight P, with its *Vectis* M P, to be taken quite out of the *Movement*, and the Body of the *Movement* to be placed on a Horizontal Plain H H, its Point of Contact in that Plain is T; where it ſhould, but cannot, Reſt; becauſe the Weight of that part of the Train, marked with the Numbers, 2, 3, 4, Removes the Center of Gravity from M, and therefore on the oppoſite part of the *Movement* as about C Q, the Inſide of the Hoop, which forms the Caſe, is to be loaded with a thin Lining of Lead, which may be a Counterpoiſe to that part of the Train; that ſo, the whole Body of the *Movement*, together with all its Furniture, within and without (excepting only P, with its *Vectis*) may on that Horizontal Plain, or while it Rides upon its own *Axis*, Reſt indifferently in any Point. This reducing of the *Movement* to an Equilibration of all its Parts in the Center M, muſt be performed *Tentando, i. e.* by Raſping the Lead at C Q, as much and in ſuch Places as is needful; which, to an Artificer of Ordinary Sagacity, will not be at all difficult.

The Center of Gravity being thus reduced to M, replace the Weight P,
Fig. 170. by the Hole H, on the Arbor of the Central-Wheel M. Then let the Body
of the *Movement* be placed on the Declivity D E, and ſuppoſing $P + LQD = LDE$, then the Body muſt needs Reſt there: But becauſe
Fig. 168. the Weight P, is not now fix'd to any part of the Quadrant Q D, but hangs
upon the Train of Wheel-work 1, 2, 3, 4, it evidently follows, That if the Power thereof be Superiour to the Reſiſtance of the Train, then the whole Body of the *Movement* muſt needs Deſcend, towards E. By this you ſee there are two Offices aſſigned to the Weight or Power P. The Firſt is, to be a Counterpoiſe to the exceſs of the Weight of L E D, above L Q D. The Second is, that it be of Force ſufficient to put the Train into a Motion ſo Adjuſted, as may exactly comport with the Time aſſigned for the Revolution of the whole Body. So that if there be any Difficulty remaining, it conſiſts in ſuch an exact Stating of the Weight and Power of P, that it may Adequately ſerve both theſe Intentions. Now how eaſie this is, will be manifeſt from theſe Propoſitions following.

1. That whatever the Intrinſick Weight of P ſhall be (as ſuppoſe it 4 Ounces *Troy*;) yet the Power of that Weight will be Augmented or Diminiſhed according to the different Degrees of its Elevation in the Quadrant T Q. Thus conſidering P M, as a *Vectis*, its *Hypomochlium* is M, the Point where it exerts its Power on the Train, is at V; I ſay then, whatever Power it has upon the Point V, in its preſent Elevation of 45 *Deg.* it will acquire a greater by being raiſed to 50, 55, *&c.* and the greateſt of all in 90 *Deg.*

at Q: And on the contrary, let it Sink to 40°. 35°. &c. its Power upon the Point V, will ſtill be Diminiſhed, inſomuch that in T, it will be utterly extinguiſh'd. And therefore if P be of a competent Weight (*i. e.* not utterly too light) to *Move* the Train at all, it will certainly Move it in ſome Degree of Elevation or other in the Quadrant Q T.

2. If the Weight P, be conſidered as to its Office of being a Counterpoiſe to the Body of the *Movement*; as I need not to prove, that it will perform this no leſs, while it hangs by upon the *Vectis* M P, than if it were faſt Rivetted in the ſame place to the Caſe of the *Movement*: ſo, in what Point of the Quadrant ſoever it will move the Train, it may be alſo a Counterpoiſe to the Body of the *Movement*. For,

1. At what Point ſoever of the Circle L E T Q, the Line of Declivity D E, makes an Angle of Contact; on the ſame Point will the Diameter S D, fall at Right Angles with D E.

2. The Line of Direction L D, will ever fall upon the Point of Contact D, making an Angle with the Diameter, as S D L.

3. The Angle S D L, will be always equal to D E H, *i. e.* As great as is the Elevation of the Line of Declivity D E, above the Horizontal E H, ſo great will the Angle of Diſtance be between the Diameter S D, and the Line of Direction L D. *Fig.* 166.

4. The Greater the Angle of Declivity is, the Leſs will be the Section L Q D; and ſo on the contrary, the Leſs that Angle is, the Greater the Section. And therefore,

5. The Exceſs of the Weight of L E D, above L Q D, muſt be alſo Greater, by Raiſing up the *Stage* with the Skrew at S: and that Exceſs Leſs by Skrewing it down.

6. The Lighter that part of the Body is, which is repreſented by the Section L Q D, the more Heavy ought the Counterpoiſe P, to be; and that either in its own Intrinſick Weight, (in Ounces and Parts of Ounces) or elſe in its Potential Weight, by being raiſed higher in the Quadrant Q T.

7. The Skrewing up the *Stage* of the *Movement* at S, will raiſe the Counterpoiſe Higher in the Quadrant Q T, by *Prop.* 3. and therefore Potentially Heavier. And from hence appears (I take it moſt clearly) both the Reaſon of the due Adjuſtment of the Motion of the Train to the exact Meaſure of an Hour, and what Weight is to be aſſign'd to P, that Moves it; and that we are not confined to Scruples and Grains, but are allowed ſuch a conſiderable Latitude, as it is not eaſie to err therein.

Having therefore ſet the *Stage* (by the help of the Arched Skrew) at the Elevation of about 10 *Deg.* place the *Movement* thereon, and try what Weight, hanging at the end of the *Vectis* M P, will ſtir the Train, mean while holding the *Movement* with the Hand in ſuch a Poſition, as the *Vectis* may make an Angle of about 30 *Deg.* with the Perpendicular M T: then let the *Movement* looſe, to Undulate upon the *Stage*; and when the Vibration ceaſes, obſerve to what Degree of the Quadrant the *Vectis* Points, and at the ſame time mind the Pulſes of the Ballance. If at this Obſervation, the Weight lies low, (as for inſtance, between 25 and 35 *Deg.* of the Quadrant) and the Beats of the Ballance are gueſſed to be not much different from their due time, the Weight P, is well enough proportioned. But if it chance to be much Heavier than is abſolutely needful, that Exceſs will be moderated by Skrewing down the *Stage*; and if it be not abſolutely too Light, its Defect will be compenſated, by Skrewing the *Stage* Higher. Therefore of theſe two Extreams, chooſe the former; for the fewer Degrees that P ariſes in the Quadrant, beyond what is abſolutely neceſſary, it will (for Reaſon very obvious) be ſo much the better.

The Effects of Gravity in the Deſcent of Heavy Bodies, and the Motion of Projects; by Mr. Halley.
n. 179. *p.* 3.
Jan. An. 1686.

VIII. *Des Cartes* his Notion, I muſt needs confeſs to be to me Incomprehenſible, while he will have the Particles of his *Celeſtial Matter*, by being reflected on the Surface of the Earth, and ſo aſcending therefrom, to drive Down into their Places thoſe Terreſtrial Bodies they find above them: This is as near as I can gather, the Scope of the 20, 21, 22, and 23 *Sections* of the laſt Book of his *Principia Philoſophiæ*; yet neither he nor any of his Followers can ſhew, how a Body ſuſpended *in Libero Æthere*, ſhall be carried downwards by a continual Impulſe tending upwards, and acting upon all its Parts equally: And beſides, the Obſcurity wherewith he expreſſes himſelf, particularly, *Sect.* 23. does ſufficiently argue, according to his own Rules, the *Confuſed Idea* he had of the thing he wrote.

Others, and among them, Dr. *Voſſius*, aſſerts the Cauſe of the Deſcent of Heavy Bodies, to be the *Diurnal Rotation* of the *Earth* upon its *Axis*, without conſidering, that according to the Doctrine of Motion fortified with Demonſtration, all Bodies moved in *Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all looſe Bodies would be caſt into the Air in a Tangent to the Parallel of Latitude. without the Intervention of ſome other Principle to keep them faſt, ſuch as is that of *Gravity*. Beſides the Effect of this Principle is throughout the whole Surface of the Globe found nearly equal, and certain Experiment ſeems to argue it rather leſs near the Equinoctial, than towards the Poles, which could not be by any means, if the *Diurnal Rotation* of the *Earth* upon its *Axis* were the Cauſe of *Gravity*; for where the Motion was ſwifteſt, the Effect would be moſt conſiderable.

Others aſſign the *Preſſure* of the *Atmoſphere* to be the Cauſe of this *Tendency* towards the *Center* of the *Earth*; but unhappily they have miſtaken the Cauſe for the Effect, it being from undoubted Principles plain, that the Atmoſphere has no other Preſſure, but what it derives from it *Gravity*; and that the Weight of the upper parts of the Air, preſſing on the lower parts thereof, do ſo far bend the Springs of that Elaſtick Body, as to give it a Force equal to the Weight that compreſſed it, having of it ſelf no Force at all: And ſuppoſing it had, it will be very hard to Explain the *Modus*, how that Preſſure ſhould occaſion the *Deſcent* of a Body circumſcribed by it, and preſſed equally above and below, without ſome other Force to *Draw* or *Thruſt* it Downwards. But to Demonſtrate the contrary of this Opinion, an Experiment was long ſince ſhewn before the *Royal Society*; whereby it appeared, that the Atmoſphere was ſo far from being the Cauſe of *Gravity*, that the

Effects thereof were much more vigorous where the Preſſure of the Atmoſphere was taken off; for a long Glaſs-Receiver having a light Down-Feather included, being *Evacuated* of Air, the Feather which in the Air would hardly ſink, did *in Vacuo* deſcend with nearly the ſame Velocity as if it had been a Stone.

Some think to Illuſtrate this Deſcent of *Heavy* Bodies, by comparing it with the Virtue of the *Loadſtone*; but ſetting aſide the Difference there is in the manner of their Attractions, the *Loadſtone* drawing only in and about its Poles, and the Earth near equally in all Parts of its Surface, this Compariſon avails no more than to explain *Ignotum per æquè Ignotum*.

Others aſſign a certain *Sympathetical Attraction* between the Earth and its Parts, whereby they have, as it were, a Deſire to be united, to be the Cauſe we enquire after: But this is ſo far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *Heavy* Bodies Deſcend, becauſe they Deſcend.

But though the *Efficient Cauſe* of *Gravity* be ſo obſcure, yet the *Final Cauſe* thereof is clear enough; for it is by this Single Principle that the Earth and all the Celeſtial Bodies are kept from Diſſolution: the leaſt of their Particles not being ſuffered to recede far from their Surfaces, without being immediately brought down again by Virtue of this *Natural Tendency*, which, for their Preſervation, the Infinite Wiſdom of their *Creator* has ordained to be towards each of their Centers; nor can the Globes of the *Sun* and *Planets* otherwiſe be deſtroyed, but by taking from them this Power of keeping their Parts united.

The Properties of Gravity. Ibid. p. 6.

The Affections or *Properties* of *Gravity*, and its manner of Acting upon Bodies Falling, have been in a great meaſure diſcovered, and moſt of them made out by Mathematical Demonſtration in this our Century, by the accurate Diligence of *Galilæus, Torricellius, Hugenius*, and others; and now lately, by our worthy Countryman Mr. *Iſ. Newton*. Which *Properties* I ſhall here enumerate.

1. The Firſt *Property* is, That by this *Principle* of *Gravitation*, all Bodies do *Deſcend* towards a *Point*, which either is, or elſe is very near to, the *Center* of *Magnitude* of the *Earth* and *Sea*, about which the Sea forms it ſelf exactly into a Spherical Surface, and the Prominences of the Land, conſidering the Bulk of the whole, differ but inſenſibly therefrom.

2. That this *Point*, or *Center* of *Gravitation*, is *Fix'd* within the Earth, or at leaſt has been ſo, ever ſince we have any Authentick Hiſtory: For a Conſequence of its Change, tho' never ſo little, would be the over-flowing of the Low-lands on that ſide of the Globe towards which it approached, and the leaving new Iſlands bare on the oppoſite ſide, from which it receded; but for this Two Thouſand years it appears, that the Low Iſlands of the *Mediterranean* Sea (near to which the Ancienteſt Writers lived) have continued much at the ſame Height above the Water, as they now are found; and no Inundations or Receſſes of the Sea arguing any ſuch Change, are Recorded in Hiſtory, excepting the *Univerſal Deluge*, which can no better way be accounted for, than by ſuppoſing this *Center* of *Gravitation* Removed for a time, towards the Middle of the then Inhabited Parts of the World; and a Change

of its place, but the two thouſandth Part of the Radius of this Globe, were ſufficient to bury the Tops of the Higheſt Hills under Water.

3. That in all Parts of the Surface of the Earth, or rather in all Points equidiſtant from its Center, the *Force* of *Gravity* is nearly *Equal*; ſo that the length of the Pendulum Vibrating Seconds of Time, is found in all Parts of the World to be very near the ſame. 'Tis true, at St. *Helena*, in the Latitude of 16 *Deg.* South, I found that the Pendulum of my Clock, which Vibrated Seconds, needed to be made ſhorter than it had been in *England*, by a very Senſible Space (but which at that time I neglected to obſerve accurately) before it would keep Time; and ſince the like Obſervations have been made by the *French Obſervers* near the Equinoctial: Yet I dare not affirm, that in mine it proceeded from any other Cauſe, than the great Height of my Place of Obſervation above the Surface of the Sea, whereby the *Gravity* being diminiſhed, the Length of the Pendulum, Vibrating Seconds, is proportionably ſhortned.

4. That *Gravity* does *Equally Affect* all *Bodies*, without regard either to their Matter, Bulk, or Figure; ſo that the Impediment of the Medium being removed, the moſt Compact and moſt Looſe, the Greateſt and Smalleſt Bodies would Deſcend the ſame Spaces in Equal Times; the truth whereof will appear from the Experiment I before cited. In theſe two laſt Particulars, is ſhewn, the great Difference between *Gravity* and *Magnetiſm*, the one affecting only Iron, and that towards its Poles, the other all Bodies alike in every part. As a *Corollary*, from hence it will follow, That there is no ſuch thing as *Poſitive Levity*, thoſe things that appear *Light*, being only comparatively ſo; and whereas ſeveral things Riſe and Swim in Fluids, 'tis becauſe Bulk for Bulk, they are not ſo Heavy as thoſe Fluids; nor is there any Reaſon, why *Cork*, for inſtance, ſhould be ſaid to be *Light* becauſe it *Swims* on *Water*, any more than *Iron*, becauſe it *Swims* on *Mercury*.

5. That this *Power Increaſes* as you *Deſcend* to, and *Decreaſes* as you *Aſcend* from the *Center*, and that in the Proportion of the *Squares* of the *Diſtances* therefrom *Reciprocally*, ſo as at a double Diſtance to have but a quarter of the Force: This Property is the Principle on which Mr. *Newton* has made out all the Phænomena of the Celeſtial Motions, ſo eaſily and naturally, that its Truth is paſt diſpute. Beſides that, it is highly rational, that the *Attractive* or *Gravitating Power* ſhould exert it ſelf more vigorouſly in a Small Sphere, and weaker in a Greater, in proportion as it is Contracted or Expanded; and if ſo, ſeeing that the Surfaces of Spheres are as the Squares of their *Radii*, this Power at ſeveral Diſtances will be as the Squares of thoſe Diſtances *Reciprocally*, and then its whole Action upon each Spherical Surface, be it great or ſmall, will be always Equal. And this is evidently the Rule of *Gravitation* towards the Centers of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, and thence reaſonably inferred, to be the General Principle obſerved by Nature in all the reſt of the *Celeſtial Bodies*.

Theſe are the Principal Affections of *Gravity*, from which the Rules of the *Fall* of *Bodies*, and the *Motion* of *Projects*, are Mathematically deducible. Mr. *If. Newton* hath ſhewed how to define the Spaces of the Deſcent of a

Body, let fall from any given Height, down to the Center, ſuppoſing the *Gravitation* to Increaſe, as in the Fifth Property ; but conſidering the ſmallneſs of Height to which any *Project* can be made Aſcend and over how little an Arch of the Globe it can be Caſt by any of our Engines, we may well enough ſuppoſe the *Gravity Equal* throughout, and the *Deſcents* of *Projects* in *Parallel Lines*, which, in Truth, are towards the *Center*, the Difference being ſo ſmall as by no means to be diſcovered in Practice.

Propoſitions concerning the Deſcent of Heavy Bodies, and the Motion of Projects.

Prop. I.] *The Velocities of Falling Bodies, are Proportionate to the Times from the Beginning of their Falls.*

Ibid. p. 9.

This follows, for that the Action of *Gravity* being continual, in every ſpace of Time, the Falling Body receives a new Impulſe, Equal to what it had before in the ſame Space of Time, received from the ſame *Power :* For inſtance, in the Firſt Second of Time, the Falling Body has acquired a *Velocity*, which in that Time would carry it to a certain Diſtance, ſuppoſe 32 Foot, and were there no new Force, would Deſcend at that Rate with an Equable Motion; but in the next Second of Time, the ſame *Power* of *Gravity* continually Acting thereon, ſuperadds a New *Velocity* equal to the former; ſo that at the end of two Seconds, the *Velocity* is double to what it was at the end of the Firſt; and after the ſame manner may it be proved to be Triple, at the end of the Third Second, and ſo on. Wherefore the *Velocities of Falling Bodies, are proportionate to the Times of their Falls.* Q. E. D.

Prop. II.] *The Spaces deſcribed by the Fall of a Body, are as the Squares of the Times from the Beginning of the Fall.*

Fig. 171.

Demonſtration.] Let A B repreſent the Time of the Fall of a Body, B C, perpendicular to A B, the Velocity acquired at the End of the Fall, and draw the Line A C; then Divide the Line A B, repreſenting the Time, into as many equal Parts as you pleaſe, as *b*, *b*, *b*, *b*, *&c.* and through theſe Points draw the Lines, *b c*, *b c*, *b c*, *b c*, *&c.* parallel to B C; 'tis manifeſt that the ſeveral Lines, *b c*, repreſent the ſeveral *Velocities* of the Falling Body, in ſuch parts of the Time, as A *b*, is of A B, by the *Former Propoſition*. It is evident likewiſe, that the Area, A B C, is the Sum of all the Lines *b c*, being taken according to the *Method* of *Indiviſibles* infinitely many; ſo that the Area A B C, repreſents the Sum of all the *Velocities* between none and B C, ſuppoſed infinitely many; which Sum is the Space Deſcended in the Time repreſented by A B. And by the ſame Reaſon, the Area's A *b c*, will repreſent the Spaces Deſcended in the Times A *b*; ſo then the Spaces Deſcended in the Times A B, A *b*, are as the Area's of the Triangle, A B C, A *b c*, which by the 20*th* of the 16*th* of *Euclid* are as the *Squares* of their *Homologous Sides* A B, A *b*, that is to ſay, of the *Times :* Wherefore the *Deſcents* of *Falling Bodies*, are as the *Squares* of the *Times* of their *Fall.* Q. E. D.

Prop. III.] *The Velocity which a Falling Body acquires in any Space of Time, is double to that, wherewith it would have moved the Space Deſcended by an Equable Motion, in the ſame Time.*

Demonſtration.] Draw the Line E C Parallel to A B, and A E Parallel to B C, and compleat the *Parallelogram* A B C E, it is evident that the *Area* thereof may repreſent the Space, a Body Moved *Equably* with the *Velocity* B C, would Deſcribe in the Time A B; and the *Triangle* A B C, repreſents the Space Deſcribed by the Fall of a Body, in the ſame Time A B, by the *Second Propoſition*. Now the *Triangle* A B C, is Half of the *Parallelogram* A B C E, and conſequently the Space deſcribed by the *Fall*, is Half what would have been deſcribed by an *Equable Motion* with the *Velocity* B C, in the ſame Time; wherefore the *Velocity* B C, at the End of the Fall, is *Double to that Velocity*, which in the Time A B, would have deſcribed the Space Fallen, repreſented by the *Triangle* A B C, with an *Equable Motion*. Q.E.D.

Prop. IV.] *All Bodies on or near the Surface of the Earth, in their Fall, Deſcend ſo, as at the End of the Firſt Second of Time, they have deſcribed* 16 *Feet, one Inch,* London *Meaſure, and acquired the Velocity of* 32 *Feet, two Inches, in a Second.*

This is made out from the 25*th*, *Prop. Par.* 2. *Horol. Oſcill. Hugen.* wherein he Demonſtrates the Time of the leaſt Vibrations of a Pundulum, to be to the Time of the Fall of a Body, from the Height of Half the Length of the Pendulum, as the Circumference of a Circle to its Diameter: Whence, as a *Corollary*, it follows, That as the Square of the Diameter to the Square of the Circumference, ſo half the Length of the Pendulum Vibrating Seconds, to the Space deſcribed by the Fall of a Body in a Second of Time: and the Length of the Pendulum Vibrating Seconds, being found 39, 125, or ⅛ Inches, the Deſcent in a Second will be found, by the aforeſaid Analogy, 16 Foot and one Inch: And by the *Third Propoſition*, the Velocity will be double thereto; and near to this it hath been found by ſeveral Experiments which by reaſon of the Swiftneſs of the Fall, cannot ſo exactly determine its Quantity.

From theſe Four Propoſitions, all Queſtions concerning the *Perpendicular Fall* of Bodies are eaſily ſolved, and either *Time, Height*, or *Velocity* being aſſigned, one may readily find the other two. From them likewiſe is the *Doctrine of Projects* deducible, aſſuming the two following *Axioms*; *viz.*

1. *That a Body ſet a Moving, will Move on continually in a Right Line with an Equable Motion, unleſs ſome other Force or Impediment intervene, whereby it is Accelerated, or Retarded, or Deflected.*

2. *That a Body being agitated by two Motions at a Time, does by their Compounded Forces paſs through the ſame Points, as it would do, were the two Motions divided and acted ſucceſſively.* As for inſtance.

Fig. 172. Suppoſe a Body moved in the Line G F, from G to R, and there ſtopping, by another Impulſe ſuppoſe it Moved in a Space of Time equal to the former from R towards K to V; I ſay the Body ſhall paſs through the point V, though theſe two ſeveral Forces Acted both in the ſame Time.

Prop. V.] *The Motion of all Projects is in the Curve of a Parabola.*

Demonſtration.] Let the Line G R F be the Line in which the *Project* is directed, and in which by the *Firſt Axiom* it would Move Equal Spaces in Equal Times, were it not Deflected downwards by the Force of *Gravity*. Let G B be the Horizontal Line, and G C a Perpendicular thereto. Then the Line G R F, being divided into Equal Parts, anſwering to Equal Spaces of Time, let the Deſcents of the Project be laid down in Lines Parallel to G C, Proportioned as the *Squares* of the *Lines*, G S, G R, G L, G F, or as the *Squares* of the *Times*, from S to T, from R to V, from L to X, and from F to B, and draw the Lines T H, V D, X Y, B C, Parallell to G F: I ſay, the Points T, V, X, B, are Points in the *Curve* deſcribed by the *Project*, and that that *Curve* is a *Parabola*. By the *Second Axiom* they are Points in the *Curve*; and the Parts of the Deſcent G H, G D, G Y, G C, = to S T, R V, L X, F B, being as the Spuares of the Times (by the *Second Prop.*) that is, as the *Squares* of the *Ordinates* H T, D V, Y X, C B, Equal to G S, G R, G L, G F, the Spaces meaſured in thoſe Times; and there being no other *Curve* but the *Parabola*, whoſe Parts of the *Diameter* are as the *Squares* of the *Ordinates*, it follows that the *Curve* deſcribed by a *Project* can be no other than a *Parabola*: And ſaying, as R V, the Deſcent in Time, to G R, or V D, the direct Motion in the ſame Time, ſo is V D, to a Third Proportional; that Third will be the Line called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* G C; which is always the ſame in *Projects* Caſt with the ſame *Velocity*: and the *Velocity* being defined by the number of Feet, moved in a Second of Time, the *Parameter* will be found by dividing the Square of the *Velocity* by 16 Feet, 1 Inch, the Fall of a Body in the ſame Time.

Lemma.] *The Sine of the Double of any Arch, is equal to twice the Sine of that Arch into its Co-Sine, divided by Radius; and the Verſed Sine of the Double of any Arch, is equal to the Square of the Sine thereof divided by Radius.*

Fig. 173. Let the Arch B C be Double the Arch B F, and A the Center; Draw the Radii A B, A F, A C, and the Chord B D C, and let Fall B E, Perpendicular to A C, and the Angle E B C, will be equal to the Angle B A D, and the Triangle B C E, will be like the Triangle A B D; wherefore it will be as A B to A D; ſo B C, or twice B D, to B E; that is, as *Radius* to *Co-Sine*, ſo *twice Sine* to *Sine of the Double Arch*; and as A B to B D, ſo twice B D or B C, to E C; that is, as *Radius* to *Sine*, ſo *twice that Sine* to the *Verſed Sine of the Double Arch*; which two *Analogies* reſolved into Equations, are the *Propoſitions* contained in the *Lemma* to be proved.

Prop. VI.] *The Horizontal Diſtance of Projections made with the ſame Velocity, at ſeveral Elevations of the Line of Direction, are as the Sines of the doubled Angles of Elevation.*

Fig. 172. Let G B, the Horizontal Diſtance be $= z$, the Sine of the Angle of Elevation, F G B, be $= s$, its Co-Sine $= c$, Radius $= r$, and the Parameter $= p$. It will be as c to s, ſo z to $\frac{sz}{c}$ = F B = G C, and by reaſon of the *Parabola* $\frac{psz}{c}$ = to the Square of C B, or G F. Now as c to r, ſo is z to $\frac{zr}{c}$ = G F, and its Square $\frac{zzrr}{cc}$ will be therefore = to $\frac{psz}{c}$: which *Equation* Reduced, will be $\frac{psc}{rr} = z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the Sine of the Double Angle, whereof s is the Sine: Wherefore 'twill be as *Radius* to *Sine* of Double the Angle F G B, ſo is Half the *Parameter*, to the *Horizontal Range* or *Diſtance* ſought; and at the ſeveral *Elevations*, the *Ranges are as the Sines of the Double Angles of Elevation.* Q. E. D.

Coroll.] Hence it follows, That *Half the Parameter is the greateſt Random, and that that happens at the Elevation of* 45°. *The Sine of whoſe Double is Radius.* Likewiſe, That *the Ranges equally Diſtant above and below* 45°. *are equal, as are the Sines of all doubled Arches, to the Sines of their doubled Complements.*

Prop. VII.] *The Altitudes of Projections made with the ſame Velocity, at ſeveral Elevations, are as the Verſed Sines of the doubled Angles of Elevation.*

As c is to s, ſo is $\frac{psc}{rr}$ = G B, to $\frac{pss}{rr}$ = B F, and V K = R V = $\frac{1}{4}$ B F, the *Altitude* of the *Projection* $= \frac{pss}{4rr}$. Now by the foregoing *Lemma* $\frac{2ss}{r}$ = to the *Verſed Sine* of the *Double Angle*, and therefore it will be as *Radius* to *Verſed Sine* of Double the Angle F G B, ſo $\frac{1}{4}$ of *Parameter* to the *Height* of the *Projection* V K; and ſo thoſe *Heights* at ſeveral *Elevations* are as the ſaid *Verſed Sines.* Q. E. D.

Coroll.] From hence it is plain, That *the greateſt Altitude of the Perpendicular Projection is a* 4*th of the Parameter, or half the greateſt Horizontal Range:* The *Verſed Sine of* 180 Degrees being $= 2r$.

Prop. VIII.] *The Lines* G F, *or Times of the Flight of a Project caſt with the ſame Degree of Velocity at Different Elevations, are as the Sines of the Elevations.*

As c is to r, ſo is $\frac{psc}{rr}$ = G B (by the 6 *Prop.*) to $\frac{ps}{r}$ = G F, that is as *Radius* to the *Sine* of *Elevation*, ſo the *Parameter* to the *Line* G F; ſo the *Lines* G F are as the *Sines* of *Elevation*, and the *Times* are proportional to thoſe *Lines*; wherefore the *Times* are as the *Sines of Elevation*: *Ergo conſtat Propoſitio.*

Prop. IX. Prob. 1.] *A Projection being made, as you pleaſe, having the Diſtance and Altitude, or Deſcent of an Object, through which the Project paſſes, together with the Angle of Elevation of the Line of Direction, to find the Parameter and Velocity*; that is, (having the Angle F G B,) G M, and M X.

Solution.] As Radius to Secant of F G B, ſo G M the Diſtance given, to G L; and as Radius to Tangent of F G B, ſo G M to L M. Then L M − M X in *Heights*, or + M X in *Deſcents*; or elſe M X − M L, if the *Direction* be *Below* the *Horizontal-Line*, is the *Fall* in the *Time* that the Direct Impulſe given in G, would have carried the Project from G to L = L X = G Y; then by reaſon of the *Parabola*; as L X, or G Y, is to G L or Y X; ſo is G L, to the *Parameter* ſought. To find the *Velocity* of the *Impulſe*, by *Prop.* 2. and 4. find the *Time* in Seconds that a Body would *Fall* the Space L X, and by that dividing the Line G L, the Quote will be the *Velocity*, or *Space Moved* in a Second Sought, which is always a mean *Proportional* between the *Parameter* and 16 Feet, 1 Inch.

Fig 172.

Prop. X. Prob. 2.] *Having the Parameter, Horizontal Diſtance, and Height or Deſcent of an Object, to find the Elevations of the Lines of Direction neceſſary to Hit the given Object*; that is, having G M, M X, and the greateſt Random equal to half the Parameter; to find the Angles F G B.

Let the Tangent of the Angle ſought be = t, the Horizontal Diſtance G M = b, the Altitude of the Object M X = h, the Parameter = p, and Radius = r, and it will be, as r to t, ſo b to $\frac{tb}{r}$ = M L; and $\frac{tb}{r} \mp h$ $\begin{Bmatrix}\text{in Aſcents}\\ \text{in Deſcents}\end{Bmatrix}$ = L X, and $\frac{ptb}{r} \mp ph$ = G L $q.$ = X Y $q.$ *ratione Parabolæ*; but $bb + \frac{ttbb}{rr}$ = G L $q.$ (47. 1. *Euclid.*) Wherefore $\frac{ptb}{r} \mp ph = bb + \frac{ttbb}{rr}$; which Equation tranſpoſed, is $\frac{ttbb}{rr} = \frac{ptb}{r} \mp ph - bb$; divided by bb, is $\frac{tt}{rr} = \frac{pt}{br} \mp \frac{p}{b}\frac{h}{b} - 1$. This Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r}$

$=\frac{p}{2b}\mp\sqrt{\frac{pp\mp4ph}{4bb}-1}$; from which I derive the following Rule. Divide half the *Parameter* by the *Horizontal Diſtance*, and keep the Quote, *viz.* $\frac{p}{2b}$; then ſay, as Square of the *Diſtance* given to half the *Parameter*, ſo half *Parameter* $\left\{\begin{matrix}-\\+\end{matrix}\right\}$ double $\left\{\begin{matrix}\text{Height}\\\text{Deſcent}\end{matrix}\right\}$ to the Square of a *Secant* $=\frac{pp+4ph}{4bb}$, the *Tangent* anſwering to that *Secant* will be $\sqrt{\frac{pp\mp4ph}{4bb}-1}$, or rr: ſo then the Sum and Difference of the afore-found *Quote* and this *Tangent*, will be the Root of the *Equation*, and the *Tangents* of the *Elevations* ſought.

Note here, That in *Deſcents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb, the *Direction* of the Lower *Elevation* will be below the *Horizon*, and if $ph=bb$, it muſt be *Directed Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: *Note* likewiſe, That if $4bb+4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, be equal to pp, there is but one *Elevation* that can *Hit* the *Object*, and its *Tangent* is $\frac{pr}{2b}$; and if $4bb+4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, do exceed pp, the *Object* is without the *Reach* of a *Project* caſt with that *Velocity*, and ſo the thing impoſſible.

From this *Equation* $4bb\mp4ph=pp$, are determined the utmoſt *Limits* of the *Reach* of any *Project*, and the Figure aſſigned, wherein are all the Heights upon each Horizontal Diſtance beyond which it cannot paſs; for by Reduction of that *Equation*, h will be found $=\frac{1}{4}p-\frac{bb}{p}$ in *Heights*, and $\frac{bb}{p}-\frac{1}{4}p$ in *Deſcents*; from whence it follows, that all the Points h are in the *Curve* of the *Parabola*, whoſe *Focus* is the Point from whence the *Project* is caſt, and whoſe *Latus Rectum*, or *Parameter ad Axem* is $=p$. Likewiſe from the ſame *Equation* may the leaſt *Parameter* or *Velocity* be found capable to Reach the Object propoſed; for $bb=\frac{1}{4}pp\mp pb$ being reduced, $\frac{1}{2}p$ will be $=\sqrt{bb+hh}\left\{\begin{matrix}+h\text{ in } \textit{Aſcents}\\-h\text{ in } \textit{Deſcents}\end{matrix}\right\}$ which is the *Horizontal* Range at 45°. that would juſt Reach the *Object*, and the *Elevation* requiſite will be eaſily had; for dividing the ſo found *Semi-Parameter* by the *Horizontal Diſtance* given *b*, the Quote into *Radius* will be the *Tangent* of the *Elevation* ſought.

But if a *Geometrical Conſtruction* of this *Problem* be required; I think I have one, that is as eaſie as any can be expected, which I Deduce from the foregoing *Analytical Solution, viz.* $\frac{t}{r} = \frac{p}{2b} \pm \sqrt{\frac{\frac{1}{4}pp \pm ph - bb}{bb}}$, and 'tis this; Having made the Right Angle L D A, make D A, DF = *p*, or greateſt *Range*, D G = *b*, the *Horizontal Diſtance*, and D B, D C = *h*, the Perpendicular *Height* of the Object; and draw G B, and make D E = thereto. Fig. 174. Then with the Radius A C, and Center E, ſweep an Arch, which if the thing be poſſible, will interſect the Line A D, in H; and the Line D H, being laid both ways from F, will give the Points K, and L, to which draw the Lines G L, G K; I ſay, the Angles L G D, K G D, are the *Elevations* required for *Hitting* the *Object*, B. But *Note*, That if B, be below the Horizon, its *Deſcent* D C = D B, muſt be laid upon A, ſo as to have A C = to A D + D C. *Note* likewiſe, That if in *Deſcents*, D H, be greater than F D, and ſo K, fall below D, the Angle K G D, ſhall be the *Depreſſion* below the Horizon.

When I gave the preceding Solution of this Problem, *viz. To Hit an Object above or below the Horizontal Line, with the greateſt Certainty and leaſt Force*, I was not aware, that the *Elevation* there ſought did conſtantly *Biſect* the Angle between the Perpendicular and the Object, as is Demonſtrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch, being always equal to the *Tangent* and *Co-Tangent* of the half Complement thereof to a Quadrant. But having diſcover'd this, I think nothing can be more compendious, or bid fairer to compleat the *Art* of *Gunnery*, it being as eaſie to Shoot with a *Mortar* at any Object on demand, as if it were on the Level; neither is there need of any Computation, but only ſimply laying the Gun, to paſs in the middle Line between the Zenith and the Object, and giving it its due *Charge*. Nor is there any great need of Inſtruments for this purpoſe: For, if the *Muzzle* of the *Mortar* be turn'd truly Square to the Bore of the Piece, as it uſually is, or ought to be, a piece of Looking-Glaſs Plate applied Parallel to the *Muzzle*, will, by its Reflection, give the true Poſition of the Piece; the *Bombardier* having no more to do, but to look Perpendicularly down on the Looking-Glaſs, along a ſmall Thread with a Plumbet, and to Raiſe or Depreſs the *Elevation* of the *Piece*, 'till the Object appear Reflected on the ſame Point of the *Speculum* on which the Plumbet falls; for the Angle of *Incidence* and *Reflection* being *Equal*, in this caſe a Line at Right Angles to the *Speculum*, as is the *Axis* of the *Chaſe* of the Piece, will *Biſect* the Angle between the Perpendicular and the Object, according as our *Propoſition* requires.

n. 216. p. 69. Mar. An. 1695.

Prop. XI. Prob. 3.] *A Shot being made on an Inclined Plain, having the Horizontal Diſtance of the Object it ſtrikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular, to find the greateſt Horizontal Range of that Piece, laden with the ſame Charge; that is, half the* Latus Rectum *of all the* Parabolæ *made with the ſame* Impetus.

Take half the Diſtance of the Object from the *Nadir*, and take the Difference of the given Elevation from that Half; the *Verſed Sine* of twice that Difference Subſtract from the *Verſed Sine* of the Diſtance of the Object from the *Zenith*: Then ſhall the Difference of thoſe *Verſed Sines* be to the Sine of the Diſtance of the Object from the *Zenith*, as the Horizontal Diſtance of the Object ſtruck to the greateſt Horizontal Range at 45°.

Prop. XII. Prob. 4.] *Having the greateſt Horizontal Range of a Gun, the Horizontal Diſtance and Angle of Inclination of an Object to the Perpendicular, to find the two Elevations neceſſary to ſtrike that Object.*

Halve the Diſtance of the Object from the *Nadir*, this Half is always equal to the Half Sum of the two *Elevations* we ſeek. Then ſay, As the Greateſt Horizontal Range, is to the Horizontal Diſtance of the Object; ſo is the Sine of the Angle of Inclination, or Diſtance of the Object from the Perpendicular, to a *4th* Proportional; which *4th* being Subſtracted from the *Verſed Sine* of the Diſtance of the Object from the *Zenith*, leaves the *Verſed Sine* of the Difference of the *Elevations* ſought; which *Elevations* are therefore had, by Adding and Subſtracting that Half Difference to, and from, the aforeſaid Half Sum.

n. 172. *p.* 18. Prop. XIII.] *To determine the Force or Velocity of a Project, in every Point of the Curve it deſcribes.*

To do this, we need no other *Præcognita*, but only the *Third Propoſition*, viz. *That the Velocity of Falling Bodies, is double to that which in the ſame Time would have deſcribed the Space fallen by an Equable Motion*: For the *Velocity* of a *Project* is compounded of the conſtant equal *Velocity* of the impreſſed Motion, and the *Velocity* of the Fall, under a given Angle, *viz*, The Complement of the *Elevation*: For inſtance, In the Time wherein a *Project* would Move from G to L, it Deſcends from L to X, and by the *Third Propoſition* has Acquired a *Velocity*, which in that Time would have carried it by an Equable Motion from L to Z, or twice the Deſcent L X; and drawing the Line G Z, I ſay the *Velocity* in the Point X, compounded of the *Velocities* G L, and L Z, under the Angle G L Z, is to the *Velocity* impreſſed in the Point G, as G Z is to G L; this follows from our *Second Axiom*; and by the 20*th* and 21*ſt Prop. Lib.* 1. *Conic. Midorgii*, X O, Parallel and Equal to G Z, ſhall touch the *Parabola* in the Point X. So that the *Velocities* in the ſeveral Points, are as the Lengths of the *Tangents* to the *Parabola* in thoſe Points, intercepted between any two Diameters: And theſe again are as the *Secants* of the Angles, which thoſe *Tangents* continued make with the Horizontal Line G B. From what is here laid down, may the comparative Force

Fig. 172.

of a Shot in any two Points of the Curve, be either Geometrically or Arithmetically diſcovered.

Coroll.) From hence it follows, That the force of a Shot is always leaſt at V, or the *Vertex* of the *Parabola*, and that at equal Diſtances therefrom, as at T and X, G, and B, its force is always Equal, and that the leaſt Force in V, is to that in G and B, as *Radius* to the *Secant* of the Angle of *Elevation*, FGB.

The *Tenth Propoſition* contains a *Problem*, untouched by *Torricellius*, which is of the Greateſt uſe in Gunnery, and for the ſake of which this Diſccurſe was principally intended. It was firſt Solved by Mr. *Anderſon*, in his Book of the *Genuine Uſe and Effects of the Gun*, Printed in the Year 1674. but his *Solution* required ſo much Calculation, that it put me upon Search, whether it might not be done more eaſily; and thereupon in the Year 1678. I found out the *Rule* I now publiſh, and from it the Geometrical Conſtruction: Since which time, there has a large Treatiſe of this Subject, Intitled, *L' Art de Jetter les Bombes*, been publiſhed in *France* by M. *Blondel*, wherein he gives the *Solutions* of this *Problem*, by Meſſieurs *Bout*, *Romer*, and *de la Hire*; but none of them are the ſame with mine, or in my Opinion more eaſie.

It was formerly the Opinion of thoſe concerned in *Artillery*, That there s. 216. p. 68. was a certain Requiſite of *Powder* for each *Gun*, and that in *Mortars*, where their Diſtance was to be varied, it muſt be done by giving a greater or leſſer *Elevation* to the *Piece*. But now our Latter Experience has taught us, That the ſame thing may be more Certainly and Readily performed, by *Increaſing and Diminiſhing the Quantity of Powder*, whether regard be had to the Execution to be done, or to the Charge of doing it. For when *Bombs* are Diſcharged with great *Elevations* of the *Mortar*, they fall too Perpendicular, and Bury themſelves too deep in the Ground, to do all that Damage they might, if they came more Oblique, and Broke upon or near the Surface of the Earth; which is a thing acknowledged by the *Beſieged* in all *Towns*, who Unpave their Streets to let the *Bombs* Bury themſelves, and thereby ſtifle the force of their Splinters. A *Second Convenience* is, That at the Extream *Elevation*, the *Gunner* is not obliged to be ſo curious in the Direction of his *Piece*, but it will ſuffice to be within a Degree or two of the Truth; whereas in the other Method of *Shooting*, he ought to be very curious. But a *Third* and no leſs conſiderable *Advantage* is, in the ſaving of the *King's Powder*, which in ſo great and ſo numerous *Diſcharges*, as we have lately ſeen, muſt needs amount to a conſiderable Value. And for *Sea Mortars* it is ſcarce Practicable otherwiſe to uſe them, where the *Agitation* of the Sea continually Changes the *Direction* of the *Mortar*, and would render the Shot very uncertain, where it not that they are placed about 45°. *Elevation*, where ſeveral Degrees above or under makes very little Difference in the *Effect*.

It only remains by Good and Valid *Experiments* to be aſſured of the *Force* Ibid. p. 70. of *Gun-Powder*; How to make and conſerve it equal; And to know the Effect thereof in each *Piece*; that is, How far differing *Charges* will caſt the ſame Shot out of it; which may moſt conveniently be Engraven on the outſide thereof, as a ſtanding Direction to all *Gunners*, who ſhall from thence forward have occaſion to uſe that *Piece*: And where this Matter well aſcertained, it might be worth the while to make all *Mortars* of the like *Diameter*, as near as may be alike in *Length* of *Chaſe*, *Weight*, *Chamber*, and all other *Circumſtances*.

n. 179. p. 19. Now the foregoing Rules would be Rigidly true, were it not for the *Oppoſition* of the *Medium*, whereby not only the Direct Impreſſed Motion is continually Retarded, but likewiſe the Increaſe of the Velocity of the Fall, ſo that the Spaces deſcribed thereby, are not exactly as the *Squares* of the *Times*: But what this *Oppoſition* of the *Air* is, againſt ſeveral *Velocities*, *Bulks*, and *Weights*, is not ſo eaſy to determine. 'Tis certain, That the *Weight* of the *Air* to that of *Water*, is nearly as 1 to 800, whence the *Weight* thereof, to that of any *Project* is given; 'tis very likely, that to the ſame *Velocity* and *Magnitude*, but of Different Matter, the *Oppoſition* ſhould be *reciprocally* as the *Weights* of the *Shot*; as likewiſe that to *Shot* of the ſame *Velocity* and *Matter*, but of Different *Sizes*, it ſhould be as the *Diameters reciprocally*: Whence generally the *Oppoſition* to *Shot* with the ſame *Velocity*, but of differing Diameters, and Materials, ſhould be as their *Specifick Gravities* into their *Diameters reciprocally*; but whether the Oppoſition, to differing *Velocities* of the ſame *Shot*, be as the *Squares* of thoſe *Velocities*, or as the *Velocities* themſelves, or otherwiſe, is yet a harder Queſtion. However it be, 'tis certain, That in large Shot of Metal, whoſe Weight many thouſand times ſurpaſſes that of the *Air*, and whoſe Force is very great in Proportion to the Surface wherewith they preſs thereon, this *Oppoſition* is ſcarce *Diſcernible*: For by ſeveral *Experiments*, made with all Care and Circumſpection with a *Mortar-Piece*, extraordinary well fixed to the Earth on purpoſe, which carried a Solid Braſs Shot of $4\frac{1}{2}$ Inches Diameter, and of about 14 Pound weight, the Ranges above and below 45°. were found nearly equal; if there were any Difference, the under Ranges went rather the fartheſt, but thoſe Differences were uſually leſs than the Errors committed in Ordinary Practice, by the unequal goodneſs and dryneſs of the ſame Sort of *Powder*, by the Unfitneſs of the Shot to the *Bore*, and by the Looſeneſs of the *Carriage*. In a ſmaller Braſs-Shot of about an Inch and half Diameter, caſt by a *Croſs-Bow*, which Ranged it at moſt about 400 Foot, the Force being much more equal than the *Mortar-Piece*, this Difference was found more curiouſly, and conſtantly, and moſt evidently, the under *Ranges* outwent the upper. From which Tryals I conclude, That altho' in Small and Light Shot, the *Oppoſition* of the *Air*, ought and muſt be accounted for; yet in Shooting of Great and Weighty *Bombs*, there need be very little or no Allowance made; and ſo theſe Rules may be put in Practice to all Intents and Purpoſes, as if this *Impediment* were abſolutely removed.

The Meaſure of the Air's Reſiſtance to Bodies, moved in it; by Dr. Wallis. n. 186. p. 269. Jan. An. 1687.

IX. 1. In order to compute the *Reſiſtance* of the *Air* to all *Projects*, I firſt Premiſe this *Lemma* (as the moſt Rational that doth occur, for my firſt footing,) *That* (ſuppoſing other things equal) *the Reſiſtance is proportional to the Celerity*. For in a double *Celerity*, there is to be removed (in the ſame time) twice as much *Air* (which is a double *Impediment*;) in a treble, thrice as much; and ſo in other Proportions.

2. Suppoſe we then the Force Impreſſed (and conſequently the *Celerity*, if there were no *Reſiſtance*) as 1; the *Reſiſtance* as r (which muſt be leſs than the Force, or elſe the Force would not prevail over the Impediment, to create a *Motion.*) And therefore the Effective Force at a firſt *Moment*, is to be reputed as $1 - r$: That is, ſo much as the Force Impreſſed, is more than the *Impediment* or *Reſiſtance*.

3. Be it as $1 - r$ to 1, ſo 1 to m (which m is therefore greater than 1.)

4. And therefore the effective Force (and conſequently the *Celerity*) as to a firſt *Moment*, is to be $\frac{1}{m}$ of what it would be, had there been no *Reſiſtance*.

5. This $\frac{1}{m}$ is alſo the remaining Force after ſuch firſt Moment; and this remaining Force is (for the ſame Reaſon) to be proportionably abated as to a Second Moment: That is, we are to take $\frac{1}{m}$ thereof, that is, $\frac{1}{mm}$ of the Impreſſed Force. And for a third Moment (at equal Diſtance of Time) $\frac{1}{mmm}$; for a fourth $\frac{1}{m^4}$; and ſo onward infinitely.

6. Becauſe the Length diſpatched (in Equal Times) is proportional to the *Celerities*; the Lines of Motion (anſwering to thoſe Equal Times) are to be as $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, $\frac{1}{m^4}$, &c. of what they would have been in the ſame Times, had there been no *Reſiſtance*.

7. This therefore is a *Geometrical Progreſſion*; and (becauſe of m greater than 1) continually Decreaſing.

8. This Decreaſing Progreſſion Infinitely continued (determining in the ſame Point of Reſt, where the Motion is ſuppoſed to expire) is yet of a finite Magnitude, and equal to $\frac{1}{m-1}$ of what it would have been in ſo much time, if there had been no *Reſiſtance*: As is demonſtrated in my *Algebra, Chap.* 95. *Prop.* 8. For (as I have elſewhere Demonſtrated) the *Sum* or Aggregate of a *Geometrical Progreſſion* is $\frac{VR - A}{R - 1}$, (ſuppoſing V the greateſt Term, A the leaſt, and R the Common Multiplier.) That is $\frac{VR}{R-1} - \frac{A}{R-1}$. Now in the preſent Caſe (ſuppoſing the Progreſſion

Infinitely continued) the leaſt Term A, becomes Infinitely Small, or $= 0$. And conſequently $\frac{A}{R-1}$ doth alſo Vaniſh, and thereby the Aggregate becomes $= \frac{VR}{R-1}$. That is (as will appear by Dividing VR by $R - 1$;) $V + \frac{V}{R} + \frac{V}{RR} + \frac{V}{R^3} +$, &c. $= \frac{VR}{R-1}$; (ſuppoſing the Progreſſion to begin at $V = 1$.) That is, dividing all by R, that ſo the Progreſſion may begin at $\frac{V}{R} = \frac{1}{m}$:) $\frac{V}{R-1} = \frac{V}{R} + \frac{V}{RR} + \frac{V}{R^3}$ + &c. That is in our preſent Caſe (becauſe of $V = 1$, and $R = m$) $\frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3}$, &c. $= \frac{1}{m-1}$. That is (putting $n = m - 1$) $\frac{1}{n}$, of what it would have been, if there had been no *Reſiſtance.*

9. This Infinite Progreſſion is fitly expreſſed by an *Ordinate* in the *Exterior Hyperbola,* Parallel to one of the *Aſymptotes*; and the ſeveral Members of that, by the ſeveral Members of this, cut in *Continual Proportion.* As is there Demonſtrated at *Prop.* 15. For let SH, be an *Hyperbola* between the Fig. 175. *Aſymptotes,* AB, AF: And let the *Ordinate* DH (in the *Exterior Hyperbola,* Parallel to AF) repreſent the Impreſſed Force undiminiſhed; or the Line to be deſcribed in ſuch Time, by a *Celerity* anſwerable to ſuch Undiminiſh'd Force. And let BS (a like *Ordinate*) be $\frac{1}{m}$ thereof, which therefore, being leſs than DH (as being equal to a part of it) will be further than it from AF. In AB (which I put $= 1$;) Let B*d*, be ſuch a part thereof, as is BS of DH. Now becauſe (as is well known) all the Inſcribed Parallelograms, in the *Exterior Hyperbola,* AS, AH, &c. are Equal; and therefore their Sides Reciprocal: Therefore as $Ad = 1 - \frac{1}{m}$, (ſuppoſing B*d*, to be taken from B toward A,) to $AB = 1$, (or as $m - 1$ to m:) ſo is $BS = \frac{1}{m}$ DH to *dh*, which is therefore Equal to $\frac{1}{m-1}$ of DH; that is (as will appear by Dividing 1, by $m - 1$,) to $\frac{1}{m} + \frac{1}{mm} + \frac{1}{m^3}$ &c. of DH.

Or if B *d* be taken beyond B; then as A *d* $= 1 + \frac{1}{m}$, to A B $= 1$, or as $m + 1$ to m, ſo is $\frac{1}{m}$ D H, to *d b*, which is therefore Equal to $\frac{1}{m+1}$ D H; that is, (as will appear by like Dividing of 1 by $m + 1$;) $= \text{to } \frac{1}{m} - \frac{1}{mm} + \frac{1}{m^3} -$, &c. of D H.

10. Let ſuch *Ordinate d h*, or (Equal to it in the *Aſymptote*) A F, be ſo Divided in L, M, N, &c. (by Perpendiculars cutting the *Hyperbola* in *l*, *m*, *n*, &c.) as that F L, L M, M N, be as $\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. That is, ſo continually Decreaſing, as that each Antecedent be to its Conſequent, as 1 to $\frac{1}{m}$, or as m to 1. Fig. 176.

11. This is done by taking A F, A L, A N, &c. in ſuch Proportion. For, of Continual Proportionals the Differences are alſo Continually Proportional, and in the ſame Proportion. For let A, B, C, D, &c. be ſuch Proportionals; and their Differences, *a*, *b*, *c*, &c. That is, $A - B = a$, $B - C = b$, $C - D = c$, &c.

Then becauſe, A, B, C, D, &c. are in Continual Proportion;
That is, A : B :: B : C :: C : D ::, &c.
And Dividing A − B : B :: B − C : C :: C − D : D ::, &c.
That is, *a* : B :: *b* : C :: *d* : D ::, &c.
And Alternately, *a*. *b*. *c*. &c. :: B. C. D. &c. :: A. B. C. &c.
That is, In Continual Proportion, as A to B, or as *m* to 1.

12. This being done; the *Hyperbolick Spaces* F *l*, L *m*, M *n*, &c. are Equal, as is Demonſtrated by *Gregory San-Vincent*; and as ſuch is commonly admitted.

13. So that F *l*, L *m*, M *n*, &c. may fitly repreſent Equal Times in which are diſpatched Unequal Lengths, repreſented by F L, L M, M N, &c.

14. And becauſe they are in Number Infinite, (though Equal to a Finite Magnitude) the Duration is Infinite: and conſequently the Impreſſed Force, and Motion thence Ariſing, never to be wholly Extinguiſhed (without ſome further Impediment) but perpetually Approaching to A, in the Nature of *Aſymptotes*.

15. The Spaces F *l*, F *m*, F *n*, &c. are therefore as *Logarithms* (in *Arithmetical Progreſſion* Increaſing) anſwering to the Lines, A F, A L, A M, &c. or to F L, L M, M N, &c. in *Geometrical Progreſſion* Decreaſing.

26. Be-

16. Becauſe FL, LM, MN &c. are as ($\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. Infinitely) Terminated at A; therefore (by *Prop.* 8.) their Aggregate FA, or *d b*, is to DH, (ſo much Length as would have been diſpatched in the ſame Time, by ſuch impreſſed Force Undiminiſhed) as 1 to $m - 1 = n$.

17. If therefore we take, as 1 to *n*, ſo AF to DH; this will repreſent the Length to be Diſpatched, in the ſame Time, by ſuch Undiminiſhed Force.

18. And if ſuch DH, be ſuppoſed to be Divided into Equal Parts Innumerable (and therefore Infinitely ſmall;) theſe anſwer to thoſe (as many) Parts Unequal in FA, or *b d*.

19. But, what is the Proportion of *r* to 1, or (which depends on it) of $1 - r$ to 1, or 1 to *m*; remains to be enquired by *Experiment*.

20. If the Progreſſion be not infinitely Continued; but End (ſuppoſe) at N, and its leaſt Term be $A = MN$: Then out of $\frac{V}{R-1} + \frac{1}{m} + \frac{1}{mm} = \frac{1}{m^3} +$ &c. is to be Subducted $\frac{A}{R-1}$, (as at *Prop.* 8.) that is, (as by Diviſion will appear) $\frac{A}{R} + \frac{A}{R^2} + \frac{A}{R^3} +$ &c. That is, (in our preſent Caſe) $\frac{a}{m} + \frac{a}{mm} + \frac{a}{m^3} +$ &c. And ſo the Aggregate will be

$$\frac{1-a}{m} + \frac{1-a}{mm} + \frac{1-a}{m^3} + \text{\&c.} = \frac{1-a}{n}.$$

And thus as to the Line of *Projection*, in which (Secluding the *Reſiſting*) the Motion is reputed Uniform; diſpatching Equal Lengths at Equal Times. Conſider we next the Line of Deſcent.

21. In the Deſcent of Heavy Bodies, it is ſuppoſed, that to each Moment of Time, there is ſuperadded a new Impulſe of *Gravity* to what was before: And each of theſe, Secluding the Conſideration of the *Air's Reſiſtance*, to proceed equally (from their ſeveral beginnings) through the ſucceeding Moments As, (in the Erect Lines) 1111 &c. 111 &c. 11 &c. 1 &c. and ſo continually, as in the Line of *Projection*.

$$\begin{array}{llll} 1 & & & \\ 1 & 1 & & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 \\ \text{\&} & \text{\&c.} & \text{\&c.} & \text{\&c.} \end{array}$$

22. Hence ariſeth (in the Tranſverſe Lines) for the firſt Moment 1, for the ſecond 1 + 1, for the third 1 + 1 + 1, and ſo forth in *Arithmetical Progreſſion*. As are the *Ordinates* in a Triangle at Equal Diſtance.

23. And ſuch are the continual Increments of the Diameter, or of the *Ordinates* in the *Exterior Parabola*, anſwering to the interior *Ordinates*, or Segments of the *Tangent*, equally Increaſing; As is known, and commonly admitted.

24. If we take in the Conſideration of the *Air's Reſiſtance*; we are then for each of theſe Equal Progreſſions, to Subſtitute a Decreaſing *Progreſſion Geometrical*; in like manner (and for the ſame Reaſons) as in the Line of Projection.

25. Hence ariſeth for the firſt Moment $\frac{1}{m}$; for the ſecond $\frac{1}{m} + \frac{1}{m^2}$; for the third $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$; &c. And ſuch is therefore the Deſcent of a Heavy Body falling by its own Weight. The ſeveral Impulſes of Gravity being ſuppoſed Equal.

26. That is, as F L, F M, F N, &c. in the Line of Deſcent, anſwering to F L, L M, M N, &c. in the Line of Projection.

$$\begin{array}{ccc} \frac{1}{m} & & \\ \frac{1}{m^2} & \frac{1}{m} & \\ \frac{1}{m^3} & \frac{1}{m^2} & \frac{1}{m} \\ \frac{1}{m^4} & \frac{1}{m^3} & \frac{1}{m^2} \\ \&c. & \&c. & \&c. \end{array}$$

Fig. 176.

27. But though the Progreſſions for the Line of Projection, are like to each of thoſe many in the Line of Deſcent: It is not to be thence inferred, that, therefore $\frac{1}{m}$ in the one is equal to $\frac{1}{m}$ in the other: But in the Line of Projection (ſuppoſe) $\frac{1}{m}f$, (ſuch a part of the Forced Impreſſed, and a Celerity anſwerable:) in the Line of Deſcent $\frac{1}{m}g$, (ſuch a part of the Impulſe of Gravity.)

28. Thoſe for the Line of Deſcent (of the ſame Body) are all equal each to other: Becauſe g, (the new Impulſe of Gravity) in each Moment is ſuppoſed to be the ſame.

29. But what is the Proportion of f to g (that is, of the Force Impreſſed, to the Impulſe of Gravity, in each Body) remains to be enquired by *Experiment*.

30. This Proportion being found as to one Known Force; the ſame is thence known as to any other Force (whoſe Proportion to this is given) in the ſame *Uniform Medium*.

31. And this being Known as to one *Medium*, the ſame is thence Known as to any other *Medium*, the Proportion of whoſe Reſiſtance to that of this is Known.

32. If a Heavy Body be Projected Downward in a Perpendicular Line, it Deſcends therefore at the rate $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, &c. of f (the Impreſſed Force) Increaſed by $\frac{1}{m}$, $\frac{1}{m} + \frac{1}{m^2}$, $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$, &c. of g, the Impulſe of Gravity: (by *Prop.* 5. and *P.* 25.) becauſe both Forces are here United.

33. If in a Perpendicular Projection Upwards; it Aſcends in the Rate of the former, Abated by that of the latter. Becauſe here the Impulſe of Gravity is contrary to the Force Impreſſed.

34. When therefore this latter (continually Increaſing) becomes Equal to that former (continually Decreaſing) it then ceaſeth to Aſcend; and doth thenceforth Deſcend at the rate wherein the latter continually Exceeds the former.

35. In an Horizon or Oblique Projection: If to a Tangent whoſe Increments are as F L, L M, M N, &c. that is, as $\frac{1}{m} f$, &c. be fitted Ordinates (at a given Angle) whoſe Increments are as F L, F M, F N, &c. that is, as $\frac{1}{m} g$, &c. The Curve anſwering to the Compound of theſe Motions, is that wherein the Project is to move.

36. This Curve (being hitherto without a Name) may be called *Linea Projectorum*, the Line of Projects, or things Projected; which reſembles a *Parabola Deformed.*

37. The *Celerity* and *Tendency*, as to each Point of the Line, is determined by a Tangent at that Point,

38. And that againſt which it makes the greateſt Stroak or Percuſſion, is that which (at that point) is at Right Angles to that Tangent.

39. If the Projection (at *pag.* 25) be not infinitely Continued, but Terminate (ſuppoſe) at N, ſo that the laſt Term in the firſt Column or Series Erect; be a; and conſequently in the ſecond, ma; in the third mma, &c. (each Series having one Term fewer than that before it:) then (for the ſame Reaſons as at *P.* 20.) the *Aggregates* of the ſeveral Columns (or Erect Series) will be $\frac{1-a}{n}$, $\frac{1-ma}{n}$, $\frac{1-mma}{n}$, and ſo forth, till (the Multiple of a becoming $= 1$,) the Progreſſion expire.

40. Now all the Abatements here, a, ma, mma, &c. are the ſame with the Terms of the firſt Column taken backward. For a is the laſt, ma the next before it; and ſo of the reſt.

41. And the Aggregate of all the Numerators is ſo many times 1, as is the Number of Terms (ſuppoſe t.) wanting the firſt Column; that is, $t - \frac{1-a}{n}$, or $\frac{nt-1+a}{n}$; and this again divided by the common Denominator n, becomes $\frac{nt-1+a}{nn}$. And therefore, $\frac{nt-1+a}{nn} g$, is the Line of Deſcent by its own Gravity.

42. If therefore this be Added to a Projecting Force downward in a Perpendicular, or Subducted from ſuch Projecting Force upward; that is, to or from $\frac{1-a}{n}f$: The Deſcent in the firſt Caſe, will be $\frac{1-a}{n}f+\frac{nt-1+a}{nn}g$; and the Aſcent in the other caſe $\frac{1-a}{n}f-\frac{nt-1+a}{nn}g$. And in this latter Caſe, when the Ablative part becomes equal to the Poſitive part, the Aſcent is at the higheſt: And thenceforth (the Ablative part exceeding the Poſitive) it will Deſcend.

43. In an Horizontal or Oblique Projection having taken $\frac{1-a}{n}f$, in the Line of Projection, and thence (at the Angle given) $\frac{nt-1+a}{nn}g$, in the Line of Deſcent; the Point in the Curve anſwering to theſe, is the place of the Project anſwering to that Moment.

44. I am aware of ſome Objections to be made, whether to ſome Points of the Proceſs, or to ſome of the Suppoſitions. But I ſaw not well how to wave it, without making the Computation much more perplexed. And in a matter ſo Nice, and which muſt depend upon Phyſical Obſervations, 'twill be hard to attain ſuch Accuracy, as not to ſtand in need of ſome Allowances.

45. Somewhat might have been further Added, to direct the *Experiments* ſuggeſted at *P.* 19. and 29. But that may be done at Leiſure, after Deliberation had, which way to attempt the *Experiment*.

46. The like is to be ſaid of the different Reſiſtance which different Bodies may meet with in the Same *Medium*, according to their different Gravities, (extenſively or intenſively conſidered) and their different Figures and Poſitions in Motion. Whereof hitherto we have taken no account; but ſuppoſed them, as to all theſe, to be Alike and Equal.

47. The Computation (in *P.* 39, 40, 41) may, if that be alſo deſired, be thus repreſented by Lines and Spaces. Fig. 176, 177. The Ablatives *a*, *m a*, *m m a*, &c. (being the ſame with the firſt Column taken backward) are fitly repreſented by the Segments of N F, (beginning at N.) and therefore by Parallelograms on theſe Baſes, aſſuming the common Height of F *h*, or N Q; The Aggregate of which is N *h*, or F Q, and ſo many times 1, by ſo many equal Spaces, on the ſame Baſes, between the ſame Parallels terminated at the *Hyperbola*; The Aggregate of which is *h* F N Q *n*. From whence if we Subduct the Aggregate of Ablatives F Q; the remaining Trilinear *h* Q *n*, repreſents the Deſcent.

48. If to this of Gravity, be joined a Projecting Force; which is to the Impulſe of Gravity, as *h* K to *h* F, (be it greater, leſs, or equal) taken in the ſame Line; the ſame Parallels determine Proportional Parallelograms, whoſe Aggregate is K Q.

49 And therefore, if this be a perpendicular Projecting downwards; then h K *k m* (the Sum of this with the former) repreſents the Deſcent.

50. If it be a Perpendicular Upwards; then the difference of theſe two repreſents the Motion: which ſo long as K Q is the greater, is Aſcendent: but Deſcendent, when *h* Q *n* becomes greater; and it is then at the Higheſt when they be equal.

51. If the Projection be not in the ſame Perpendicular, (but Horizontal or Oblique) then K Q repreſents the Tangent of the Curve; and *h* Q *n*, the Ordinates to that Tangent, at the given Angle

52. But the Computation before given, I take to be of better uſe than this Repreſentation in Figure. Becauſe in ſuch Mathematical Enquires, I chuſe to ſeperate (as much as may be) what purely concerns Proportions; and conſider it abſtractly from Lines or other matter wherewith it is incumbred.

As to the Queſtion propoſed; Whether the Reſiſtance of the Medium does not always take off ſuch a Proportional part of the Force, Moving through it, as is the Specifick Gravity of the Medium to that of the Body Moved in it: (for if ſo, it will ſave us the trouble of Obſervation.) I think this can by no means be admitted; For there be many other things of Conſideration herein, beſide the Intenſive Gravity (or, as ſome call it, the Specifick Gravity) of the Medium.

A Vicious *Medium* ſhall more reſiſt than one more Fluid, though of like Intenſive Gravity.

And a ſharp Arrow ſhall bore its way more eaſily through the Medium than a blunt headed Bolt, though of equal Weight and like Intenſive Gravity.

And the ſame Pyramid with the Point, than with the Baſe forward.

And many other like Varieties, intended in my *P.* 46.

But this I think may be admitted, namely, That Different Mediums, equally Liquid, (and other Circumſtances alike) do in ſuch Proportion Reſiſt, as is their Intenſive Gravity. Becauſe there is, in ſuch Proportion, a Heavier Object to be Removed, by the ſame Force which is one of the things to which *P.* 31. refers.

And again: The Heavier Project once in Motion, (being equally ſwift, and all other Circumſtances alike) Moves through the ſame Medium in ſuch Proportion more ſtrongly, as is its Intenſive Gravity: For now the Force is in ſuch Proportion greater, for the Removal of the ſame Reſiſtance. And this is part of what my *P.* 30. Inſinuates.

But where there is a Complication of theſe Conſiderations one with another, and with many other Circumſtances, whereof each is ſeverally to be conſidered; there muſt be reſpect had to all of them.

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X. 1. *To know how far a Gun ſhoots Point Blank, (as they call it) that is, ſo near the Level of the Cylinder of the Piece, that the Difference is either not diſcernible, or not conſiderable.*

On

Charge of Powder; and beſt Size of Guns. Propoſed; by Sir Rob. Moray. *n.* 26. p. 473. June An. 1667.

On a fit *Platform* Place and Point the *Gun* at a Mark as large as the Bullet, ſome 50, 60, or more Yards diſtant, ſo as the underſide of the Mark may be in the ſame Level or Line with the underſide of the *Cylinder* of the *Piece*. Then between the *Gun* and the Mark at convenient Diſtances, place pieces of Canvas, Sheets of Paper paſted together, or the like, upon ſtakes fix'd in the ground, ſo as the underſide, being Level with the Horizon, may juſt touch the Viſual Line, that paſſeth from the Eye to the upperſide of the Mark, when the Eye is placed in the Line that paſſeth from it to the upperſide of the *Cylinder* of the *Gun*; the Canvas being ſo broad and long, that if the *Bullet* paſs through it 2 or 3 Foot higher than the Level of the Mark, or of either hand, the Hole it makes may make it known, how much it Flieth higher than the Level of that place. If the *Bullet* falls Lower than the Mark, and touch not the Canvas, the *Gun* may be the next time raiſed a little, and ſo on till the *Bullet* Hit the Mark, or as High as it. If it fall as high as the Mark and Cut the Canvas, the Mark and Canvas may be brought nearer the *Gun*: But if it fall as high as the Mark and do not Cut the Canvas, the Mark may be removed to greater and greater Diſtances.

If this way of *Experiment* be made for further Diſtances and Raiſings of the *Piece*, as High as conveniently may be above the Level, and the Diſtances meaſured, and then all *Randoms* above theſe likewiſe tried and meaſured, the Diſtance of an Object, to be Shot at, being known, and other neceſſary Cautions, beneath to be mentioned, carefully obſerved, good *Gunners* may with great Confidence undertake to Hit the Mark, be the Diſtance what it will, ſo it exceed not the reach of the *Gun*.

2. *To know what Quantity of Powder is the juſt Charge of any Piece, ſo as it maketh the fartheſt Shot, and Fires totally.*

1. Raiſe the *Gun* to a mean *Random*, as of 20°, or 25°, and Shoot with the ordinary *Charge* of *Powder*, in ſome convenient Ground where the Fall of the Bullet may be eaſily ſeen, and having made a Shot, meaſure the Diſtances with a Chain between the Hole made by the Bullet and the *Muzzle* of the *Gun*.

2. Then inſtead of a Full *Charge* of *Powder* uſed in the firſt Shot, take $\frac{1}{16}$ part leſs, or ſome ſuch Proportion, for the next Trial, doing all things elſe as before.

3. For a third, fourth, or more Trials, diminiſh ſtill the *Quantity* of *Powder*, by $\frac{1}{16}$ at a Time, till the Shot be conſiderably ſhorter than at firſt.

4. Then take $\frac{1}{16}$ more than the firſt *Charge*, and do all things elſe as before, and ſo continued more Trials, increaſing ſtill the *Quantity* of *Powder* in the ſame Proportion every new Trial, till you find the Increaſe of the *Charge* does not make the *Piece* Shoot further: Only Over-charge not ſo far as to endanger the *Gun*.

5. Three or more Shot are to be made with every different *Charge*, and at every ſeveral Trial, that the Certainty may the better appear.

6. The firſt Shot being Meaſured and Marked, the reſt may all be Meaſured from it, or from one another to ſave Labour.

7. The *Gun* is to be Pointed, Placed and Ordered, every time in one and the ſame Place and Poſition, aiming ſtill at the ſame Mark, or Pointing ſtill at the ſame Line or Azimuth; that ſo all the Shot may fall in the ſame Line as near as is Poſſible.

8. The *Powder* (which ought to be all of the ſame *Goodneſs*) muſt be exactly Weighed every time the *Piece* is *Charged*, leſt it having been Weighed long before, the Weight may be altered; though *Experiment* may be made with *Cartridges* and without.

9. The *Powder* and *Bullet* is to be *Ramed* home equally at every Shot; though the Looſer the *Powder* lie, it *Fire* the better.

10. When the right *Charge* of a *Piece* is found, that makes the *fartheſt Shot* in the Ordinary and Plain way of *Charging*, M. *de Sons*'s Contrivance of a *Wedge* may be tried, to make it Shoot farther; which is a piece of Board, ſo long, as being thruſt home to the *Breech* of the *Piece* at one End, the other may reach farther out than the outſide of the Bullet being *Rammed* up to its place; broad about an Inch, and thin ſo far as the *Wadd* before the *Bullet* reaches on the outſide; there it is to have a Shoulder, from which forward to the end, it is to be cut a-ſlope, like a *Wedge*, being of ſuch thickneſs, as that at the place, where the Center of the *Bullet* is to be, it may make it ſtick ſo faſt, that the *Powder* finding more Reſiſtance, may at length Drive it out with the greater Violence.

11. Another of this Nature is a *Wooden Tampion*, like a piece of a Cylinder, big enough to fill the hollow *Cylinder* of the *Gun*, the length ſomewhat more than the Diameter of it, and Hollow'd towards the *Bullet*, ſo as to fit it; and either Flat or (which is better) Hollow likewiſe towards the *Powder*, and ſerving inſtead of *Wadd*. Theſe, and ſuch others, will probably render the Effect of the *Powder* greater, than otherwiſe it would be: But Care muſt be had that they do not endanger the *Piece*.

12. The *Strength* of the *Powder* muſt be examin'd by a *Powder Trier*, that raiſeth a Weight, ſuch an one as hath been contrived by Mr. *Hook*.

13. The ſame *Bullet* is to be made uſe of, if it can be had, till the Figure of it be Marred; otherwiſe another as near of the ſame Size, Shape and Weight, as is poſſible.

14. Obſerve the *Strength* and *Poſition* of the *Wind*, and at what *Azimuth* the Mark ſtands from the *Gun* at every time of Shooting: And take preciſe notice what effect it hath upon the *Bullet* in carrying it further, in hindring, or turning it aſide.

15. Note the Figure, Dimenſions, and Weight of the *Gun*, *Carriage*, and *Wheels*; and Record every thing exactly in a Book, as alſo every Accident and Obſervation.

16. After all other *Experiments* are made, every *Piece* may be Tried with the Right *Charge* of *Powder*, laying every time more and more Weight upon the *Carriage*; and at laſt Fixing the *Gun* ſo, as it may not *Recoil* at all, obſerving every time how far the *Bullet* goes, and how much leſs *Powder* than the full *Charge* will ſerve to Shoot the *Bullet*, when the *Piece* is Fixt, as far as the whole *Charge* does, when it *Recoils* freely.

17. The *Right Charge* found, the beſt *Random* is to be ſought, by Trying all *Randoms*, by Degrees at a time.

3. *To know what Gun ſhoots fartheſt.*

1. A *Gun* to be prepared of *Culverine-Bore* (as being held the beſt for Shooting Far) but much Longer (double the ordinary Length may do well) and without any *Ring* about the *Muzzle*, is to be placed as in the former *Experiments*, and Charged with the Ordinary Charge of a *Culverine*, or rather with that Quantity, which by the former *Experiments* ſhall be found the beſt; and being Shot, the fall of the *Bullet* is to be Mark'd, and Diſtance Meaſured.

2. Then Try Leſs, and More *Powder* in her, as before

3. Then Cut off two Inches of the *Muzzle* with a Saw, and Place the Pieces ſo cut off in the *Carriage*, or their *Weight* of Lead in a convenient Figure, that the *Recoil* may ſtill be the ſame; and try as before, doing every thing in the ſame manner: And ſo cut off ſtill for new Trials, till the Shot begin to fall ſhorter than before.

4. The ſame may be done with *Guns* of Different *Bores*.

Experiments for Trying the Force of great Guns; by Mr. Greaves. n. 173. p. 1090.

2. *Mar.* 18. 1651. At 200 Yards Diſtance from the Platform for great Ordinance at *Woolwich*, there were raiſed three *Butts*, one behind another: The Space between the firſt and ſecond *Butt* was 14 Yards, between the ſecond and the third, eight. The Thickneſs of each *Butt* was 19 Inches, whereof 13 was of Beams of Maſſey Oak faſtened into the Ground, and ſet ſo cloſe that they touched each other: On each ſide where Planks of Oak, 3 Inches a piece in Thickneſs and theſe were joyned cloſe, and faſtened on both Sides with Iron Bolts, and ſtrong Pins of Wood; and on the Back, at the ends, and on the middle, there were 3 Braces of Elm, a Foot in Breadth, and 5 Inches in Thickneſs.

The firſt *Experiment* was with an *Iron Demy-Cannon*, having a *Cylinder Bore*, of 3500 *lib. Weight*, the *Bullet* 32 *lib.* of Iron, the *Powder* 10 *lib.* which Pierced through the two Firſt *Butts*, and ſtuck in the third, ſo as the Ball was almoſt quite within, but the Timber not ſhivered (ſmall) nor ſcarce Split. The *Butts* being touched by me, felt not warm; the like *Execution* was done, when it was Charged with 9 *lib.* as alſo when with 8 *lib.* of *Powder*.

The ſecond *Experiment* was with an *Iron Demy-Cannon*, having a *Taper Bore*, and being 3600 *lib.* in *Weight*, and 4 Inches longer than the former, the *Iron Bullet* 32 *lib.* and the *Powder* 7 *lib.* which in three Trials ſeemed to have the ſame Force with the Firſt. One of the *Shots* piercing through the ſecond *Butt*, and Lighting near the Edge of the middle * *Butt* of Elm, Tore it, but by the yielding of it, the *Ballet* glanced aſide off the third *Butt*, and entered into the Earth.

* *Brace.*

The third *Experiment* was with a whole *Culverine* in *Braſs*, of 5300 *lib.* in *Weight*, 11 Foot one Inch in *Length*, with a *Taper Bore*; the *Iron Bullet* was 18 *lib.* in *Weight*; the *Powder* in the firſt Trial 10 *lib.* in the ſecond 9 *lib.* in the third 8 *lib.* which laſt Proportion did the beſt *Execution*, and paſſed through the two Firſt *Butts*, Entering gently into the Third, which the former two did touch, but not Enter.

The Fourth *Experiment* was with a whole *Culverine* in *Braſs*, made at *Amſterdam* for the *French*, with this Mark, 3580, being 10 Foot Long, and not very Thick in the *Breech*; the firſt Shot with 9 *lib.* of *Powder*, 18 *lib.* of *Bullet-Iron*, paſt through the three *Butts* and entered one Foot into the Ground; it paſſed by the Joints of the Timber, two Planks having been beat down before. The ſecond Shot with 8 *lib. Powder*, paſſed through two *Butts* and *Grazed* between them. The third with 8 *lib.* paſt Two *Butts*, and 7 Inches into the Third, but the Firſt *Butt* was much Battered before, where it Entered. The Fourth Shot paſſed, with 8 *lib.* of *Powder*, two *Butts*, and in both *Butts* through the midſt of a Maſſey-ſtrong Beam (below) that had not been Battered.

The Fifth *Experiment* was with an *Iron Demy-Culverine*, having 9 *lib. Bullet* in *Iron*, and 4 *lib. Powder*; this paſt one *Butt* (which was Torn before) and Entered the ſecond.

This $\frac{1}{2}$ *Culverine* was Shot 8 Times, as faſt as they could *Charge* it with *Powder*, and the *Iron-Bullet*, and yet was but ſcarce Lukewarm at the *Breech*, a little more in the Midſt, moſt at the *Muzzle*, and this laſt ſcarce ſo Hot as my Hand, and yet the *Gunners* in Charging her, wet not at all the *Scoop*, or *Spunge*.

The Sixth *Experiment* was with a *Braſs Demy-Culverine*, the *Breech* of her was 13 Inches $\frac{5}{8}$, the *Mouth* 9$\frac{5}{8}$. The Firſt *Shot*, with 4 *lib.* of *Powder*, 9 *lib. Iron-Bullet*, paſt two *Butts*: The Second *Shot* with 3 *lib.* of *Powder*, paſt almoſt two *Butts*: This proved to be the beſt Shot, becauſe the Timbers were the Strongeſt.

Shooting by the Rarefaction of the Air; by Dr. Papin. n. 179. p. 21.

Fig. 178. Jan. An. 1686.

XI. Whereas ordinary *Wind Guns* do their Effect by the *Compreſſion* of the *Air*: *Ottho Ghericke* hath found a new ſort that Shoots by *Rarefaction*; and he hath publiſhed that Device at large in his Book about *Pneumatick Experiments*. I have Contrived another which I take to be Better.

A A, is a *Pipe*, very Equal from one end to the other.

B B, a ſmall *Pipe* Soder'd in a Hole near the end of the *Pipe* A A, and apply'd to the *Plate* of the *Pneumatick Engine*.

C C C C, Some kind of *Stool*, to bear up the Hinder part of the *Pipe* A A.

D, A *Piece* of *Lead* fitted to the Bore of the *Pipe* A A.

The *Pipe* A A, is to be Shut at both Ends by *Valves* outwardly applied, and ſo the ſaid *Pipe* A A, though never ſo Big, may be *Exhauſted* of *Air* by means of the *Pneumatick Engine*: Which done, the *Valve* towards D muſt be ſuddenly opened, ſo that the whole Preſſure of the Atmoſphere Acting upon the *Lead* D, may drive it along the *Pipe* A A, with ſuch a Swiftneſs, that it will be able to carry it to a great Diſtance: And becauſe ſuch a *Valve* Shutting a great Hole, would prove very difficult to be Opened, when the *Pipe* A A, is of a great Bore, the Aperture towards D, may be left much ſmaller than the *Pipe*; the Swiftneſs of the Air being ſo great, that even through a pretty ſmall Aperture, it preſſed the *Lead* D, as freely almoſt as if the whole Bore was quite Open.

Having prepared a *Barrel* carrying a *Lead* of two Ounces, the *Experiment* was ſhewn before the *Royal Society*, and the Effect was found very Conſiderable, the Force being little leſs than that of the *Wind-Gun* by *Compreſſion*; the ſame *Experiment* being afterwards repeated with a Longer *Barrel*, 'twas found that the Length in this way of *Shooting* was very little, if any Advantage.

The Velocity wherewith the Air Ruſhes into an exhauſted Receiver; by Dr. Papin. n. 184. *p.* 193. Oct. An. 1686.

XII. My Way of Computing the *Velocity* of the *Air* (which I think is better than the Trial made by the *Royal Academy* at *Paris*) is Grounded upon this *Hydroſtatical Principle*, That *Liquors have a Strength to Aſcend as High as their Source is*; and although the *Reſiſtance* of the *Medium* does always hinder *Jets d'Eau* in the Open Air from reaching quite ſo High, nevertheleſs the *Liquor* at its firſt Spouting out, hath the neceſſary *Swiftneſs* to come to that *Height*.

Prop. I.] From this Principle may be eaſily deduced this Propoſition, That, *of two different Liquors driven by the ſame Preſſure, that which is in Specie Lighter muſt aſcend Higher than that which is heavier, and their Heights will be Reciprocally in the ſame Reaſon as their Specifick Gravities are.*

Prop. II.] From the foregoing *Propoſition* another may eaſily be Deduced, *viz.* That, *of Differing Liquors bearing the ſame Preſſure, thoſe that are Lighter in Specie muſt acquire a greater Swiftneſs, and their Differing Velocities are to one another * as the Roots of the Specifick Gravities of the ſaid Liquors.*

* Reciprocally.

For we have ſeen, *Prop.* I. That the *Heights* to be Attained are * in the ſame Reaſon as the *Specifick Gravities*; Now *Gallilæus*, *Hugenius*, and others, have Demonſtrated, That the *Velocities* of Bodies are to one another, as the *Square Roots* of the *Heights* to which they may Aſcend: And ſo in this Occaſion they are alſo * as the *Roots* of the *Specifick Gravities*.

* Reciprocally.

* Reciprocally.

If therefore we would know what is the *Velocity* of the *Air* being driven by any Degree of *Preſſure* whatſoever, we ought but to find what would be the *Velocity* of *Water* under the ſame *Preſſure*: And then take the *Square Roots* of the *Specifick Gravities* of theſe two Liquors; becauſe as much as the *Square Root* of the *Specifick Gravity* of *Water* doth exceed the *Square Root* of the *Specifick Gravity* of the *Air*; ſo much in Proportion will the *Velocity* of *Air* exceed the *Velocity* of *Water*. For Example; when I would Compute what ſhould be the *Swiftneſs* of a *Bullet* ſhot by my *Pneumatick Engine*, I ſhould at firſt Compute what was the *Velocity* of the Air itſelf that drove the Bullet: I did therefore take notice, That in this Occaſion the *Air* bears a Preſſure much about the ſame as that of *Water* when its *Spring* is 32 Foot High. Now ſuch *Water* would ſpout out with a ſufficient *Velocity* to aſcend 32 Foot Perpendicular, and therefore according to the Rules and Obſervations of *Gallilæus*, *Halley*, and others, ſuch Water hath the *Velocity* of 45 Foot in a Second. It remains therefore but to know the Proportion of the *Gravity* of the *Air* to that of *Water*: And we have found it not to be always the ſame; becauſe the Height, the Heat, and the Moiſture, of the *Atmoſphere*, are Variable: Nevertheleſs, we may ſay in general, That the Reaſon between the *Specifick Gravities* of *Water* and *Air* is much about 840 to 1. Taking then their *Square Roots* as I have ſaid above, which *Roots* are 29 and 1, we may conclude that the *Velocity* of *Air* muſt Exceed that of *Water*, by 29 Times: And ſo multiplying 45, the *Velocity* of *Water*, by 29, we ſhall find, that the *Velocity* of *Air* driven by the whole Preſſure of the *Atmoſphere*, is about 1305 Foot in a Second.

Wind produced by the Fall of Water; by Dr. Wal. Pope. n. 2. p. 25. *Fig.* 179. Apr. An. 1665.

XIII. In the *Braſs-Works* at *Tivoli*, the *Waters Blows* the *Fire*, not by Moving the *Bellows*, but by affording the *Wind*. Thus: A, is the *River*. B, the *Fall* of it. C, the *Tube* into which it Falls. L G, a *Pipe*. G, the *Orifice* of the *Pipe*, or *Noſe* of the *Bellows*. G K, the *Hearth*. E, a Hole in the Pipe. F, a Stopper to that Hole. D, a place Under-ground, by which the Water runs away. Stopping the Hole E, there is a perpetual ſtrong *Wind*, iſſuing forth at G: and G being ſtopt, the *Wind* comes out ſo Vehemently at E, that it will, I believe, make a *Ball* play, like that at *Freſcati*.

The beſt Form of Horizontal Sails for a Mill; by Dr. Rob. Hook, *Phil. Coll.* n. 3. p. 61. Dec An. 1681.

XIV. Whatever Men may imagine concerning *Horizontal Sails*, I doubt there never will be found a better, and more Advantageous way, for receiving the Strength of the *Wind*, or *Motion* of the *Air*, than *Perpendicular Vanes* made of a True Form, ſo as every part thereof may draw alike. But becauſe I find divers have of late attempted *Horizontal Vanes* for *Mills*, I ſhall explain a Way of making *Horizontal Vanes* capable of Performing the *Moſt* that is *Poſſible* with *Vanes* of equal *Extenſion*.

The *Invention* is founded upon the ſame Principle with that of the *Sailing* of *Ships*, and other *Veſſels* upon the Sea; namely, Upon Diſpoſing and Ordering of the Vane or Sail ſo, as *to ſtand in the beſt Poſture 'tis poſſible to Move the Arms of the Mill, or the Body of the Ship, in that Way it is to be Moved, by the Force of the Wind Blowing thus or thus againſt them.*

The *Firſt Principle* then common to both, is, that the *Vane* or *Sail* be as near as 'tis poſſible, a perfect *Plain* and *Smooth Superficies*, without any Bellying, Bunting, or Curvity in the Superficies thereof, upon which the Motion or Force of the *Wind* is Impreſſed.

Secondly, That the *Air* may have as many *Paſſages* between the parts of the *Vane* or *Sail* as may be, that the Moved *Air* may come to it as *freely* as may be, without being Intercepted by a *Stagnant Air* before it, to Impede or Divert its Force.

Thirdly, That the *Plain* of the *Vane* or *Sail* be put in the *Middle Inclination*, between the *Way* of the *Wind* and the *Way* of the *Arm*, or that of the Body of the *Ship*.

The *Contrivance* itſelf is *This*:

Fig. 180. Let A B, ſignifie the *Stream* or Current of the Air or *Wind*, Moving from A to B, and let C repreſent the Center of the *Axis* or *Spindle*, ſtanding Perpendicular to the *Horizon*, upon which, at the Top, is fixed at Right Angles, the piece D H, making the two *Arms* C D, and C H, upon the Ends of which the *Vanes* M N, are Moved on *Spindles*; ſo as that the Plain of the *Vane* doth always paſs through the Point D: I ſay, theſe *Vanes* ſo ordered, ſhall be always placed in the moſt *Advantageous Poſture* for Moving the *Arms* round upon the ſaid *Spindle*, whoſe Center is C, in the Order of DEFGHIKLD.

Firſt, For the *Vanes* placed at D and H, I ſay, They are ſet in the moſt *Advantageous Poſture* poſſible, in thoſe two Points: For *Firſt*, the *Vane* M N at D being to move Directly againſt the *Wind*, the moſt *Advantageous Poſture* is to turn its edge directly againſt the *Wind*, and thereby to give the leaſt *Reſiſtance* poſſible that being the only Point in which the *Vane* ſuppoſed only a Superficies, Draws not. And *Secondly*, For the *Vane* M N placed at H, it ſtandeth the moſt *Advantageouſly*, becauſe its Motion being directly from or before the *Wind*, it ſtandeth full *Croſs*, or oppoſed to the Motion thereof

Secondly, The *Vanes* at E, F, G, and I, K, L, ſtand the moſt *Advantageouſly*, becauſe they *Divide* the *Angle*, between the *Way* of the *Wind*, and that of the *Arms* in thoſe Points into two equal Parts and conſequently the Wind Impreſſeth the Greateſt Force in the moſt Direct Way: For it is eaſy to be Demonſtrated, That the Force Impreſſed on the *Vane* by the *Wind*, is Perpendicular to the Surface, and conſequently that the Obliquity of the Force to the Way of the *Arms*, Increaſed by the *Vanes* ſtanding more full againſt the *Wind*, will have a leſs Proportion of Power to promote the Motion thereof, than in the Poſture here Set. And ſuppoſing the *Vanes* ſet Sharper to the *Wind*, the Diminution of the Force Impreſſed by the *Wind* on its Surface, will be greater than the Augmentation of its Power, by being moved more Directly to the *Way* of the *Arms*. This is eaſy enough to be *Geometrically Demonſtrated.*

The *Vane* may be ſo ordered, as always to Stand in this *Poſture* by a great many ways: I ſhall only Inſtance in One, not the beſt for *Practice*, but the moſt Eaſy to be underſtood and *Demonſtrated*.

Let the *Vane* be equally Expanded on each ſide of its *Axis*, by which the Preſſure on the extreams of it are always Counterpoiſed, then Faſten upon the Lower end thereof a Wheel, which may be in Diameter about $\frac{1}{2}$ of the Length of the *Arms* from Hole to Hole; then Fix a Wheel upon the Frame in which the *Spindle* of the *Arms* do Move, that ſhall be of half the Diameter with the former, and to contain half the number of Teeth, Then by a third ſmall Wheel, Fixed under the *Arms*, of a convenient Bigneſs. Communicate the Motion of the One to the Other; for by this means each *Vane* being ſo provided, they will, being once ſet Right, always continue to be moved and Diſpoſed in the true Poſture deſired.

Fig. 181.

This *Contrivance* will not only be Uſeful for all manner of common *Wind-Mills*, but alſo for *Water-Mills* in *Rivers*, where there can no *Dam* be made, as may alſo the *Perpendicular Vanes* of other *Mills*, neither of which has been ſo much, as *Hinted* by any Perſon whatſoever that I have hitherto heard of.

An Account of Flying; by Dr. Hook. *Phil. Coll.* n. 1. p. 14.

XV. 1. The Art of *Flying* hath been in all Ages *Attempted* by many, particularly in the Times of our famous Fryar *Roger Bacon*, who lived about 500 Years ſince. He was Believed a *Magician* or *Conjurer*, and to have performed what was related of him, by the help of *Diabolical Magick*, but from the peruſal of ſeveral of his excellent *Works* yet Extant, I eſteem him no ſuch Perſon, I rather find him to have been a good *Mathematician*, a knowing *Mechanick*, a rare *Chymiſt*, and a moſt Accompliſhed *Experimental Philoſopher*, which was a Miracle for that Dark Age. This Man affirms the Art of *Flying* Poſſible, and that He himſelf knew how to make an *Engine*, in which a Man Sitting, might be able to carry himſelf through the *Air* like a *Bird* : And affirms, that there was then another Perſon who had actually Tryed it with good Succeſs. We have not wanted Later Inſtances in *England*, of ſeveral Ingenious Men, who have employed their Wits and Time about this *Deſign*. Particularly, I have been credibly informed, That one Mr. *Gaſcoigne* did about 40 Years ſince Try it with good Effect ; tho' he ſince Dying, the Thing alſo Dyed with him. And even now, there are nor Wanting ſome in *England*, who affirm themſelves able to do it, and that they have proved as much by *Experiment*. We have little or no account of the Ways they have taken to Effect their Deſigns; But we may conclude them Defective in ſomewhat or other, ſince we do not find them brought into *Common Uſe*.

The Art of Flying; by S. Beſnier. Ibid. p. 15. Fig. 182.

2. The Sieur *Beſnier*, a Smith of *Sable* in the County of *Maine*, hath Invented an *Engine* for *Flying*. It conſiſts of two *Poles* or *Rods*, which have at Each End of them an oblong *Chaſſie* of Taffety, which *Chaſſie* Folds from above Downwards, as the frame of a folding *Window Chaſſie*. He fits theſe *Poles* upon his Shoulders, ſo that two of the *Chaſſies* may be before him, and the other two behind him. The *Order* of *Moving* them is thus : When the *Right Hand* Strikes down the *Right Wing* before, A, the *Left Leg* by means of the String E, Pulls downwards the *Left Wing* behind, B; then immediately after, the *Left Hand* Moves or Strikes downwards the *Left Wing* before, C, and at the ſame time the *Right Foot*, by the String F, Moves or Pulls down the *Right Wing* behind D ; and ſo ſucceſſively, or Alternately, the Diagonally Oppoſite *Wings* always Moving downwards, or Striking the Air together.

A Flying Chariot; by Fr. Lana. Ibid. p. 18.

3. 1. *P. Franceſco Lana* in his *Prodromo*, finding by an *Experiment*, That the *Weight* of the *Air* is $\frac{1}{640}$ part of the *Weight* of a like quantity of *Water*, he concludes certainly, That if we could make a Veſſel of Glaſs or other Matter that might *Weigh* leſs than the *Air* that is in it, and ſhould draw out all it's *Air*, this Veſſel would be *Lighter in Specie* than *Air* itſelf, and therefore would Swim in it and Aſcend on High. This He *ſuppoſes* may be done, by making a Round Veſſel of Thin *Plate Braſs*, (*Weighing* 3 Ounces in a *Square Foot*) of the *Diameter* of 14 Foot. For the *Surface* of the Veſſel will be 616 *Square Feet*, and the *Braſs* will *Weigh* no more than 1848 Ounces; whereas the *Content* will be $1437\frac{1}{3}$ *Cub. Feet*, and that Quantity of *Air* will Weigh $2155\frac{2}{3}$ Ounces : So that that Air being evacuated, the Veſſel will be $307\frac{2}{3}$ Ounces *Lighter* than *Air*, and therefore will not only Aſcend into the Air, but alſo carry up with it a Weight of $307\frac{2}{3}$ Ounces. And thus by Encreaſing the Bulk of the Veſſel, without encreaſing the Thickneſs of the Plates of Braſs, he ſuppoſes a kind of Ship may be made, to Swim in the Air, and to carry two or three Men in it.

2. The

Shewn Impracticable; by Dr. Hook. Ibid. p. 27.

2. The Fallacy of the *Author*'s Reaſoning lies in this; He ſuppoſes *Copper* of 3 Ounces in a Foot Square to be of ſufficient Thickneſs to Reſiſt the Preſſure of the *Air* in a *Globe* of 14 Foot *Diameter*, nay of any *Dimenſion*. But in this we can nowiſe Aſſent to him: For the Preſſure from without Inwards, tho' it be always the ſame upon Equal Surfaces, yet upon Unequal Surfaces the Caſe is quite otherways, for there the Preſſure will be found, not the ſame, but to Encreaſe always in the ſame Proportion with the Surface, and thence conſequently the *Thickneſs* of his *Copper*, or any Metal or Material, which he ſhall make uſe of, muſt Increaſe in the ſame Proportion, with the *Diameter* of the *Sphere*, and conſequently the *Weight* of his *Copper* muſt always *Increaſe* in the ſame Proportion at leaſt to the *Solidity* of his *Sphere*; ſo that by his Augmenting the *Quantity* of his *Sphere*, he has no manner of Advantage of making it proportionably *Lighter* than the *Air*, and proportionably *Strong*, but the contrary: For it is manifeſt, That a Bigger *Sphere* ſo made of any Matter we yet know, has leſs Power of Reſiſting the ſame *Preſſure* of the *Air* than a Leſs, becauſe of Finite Reſiſtance of Matter to Preſſure, there being ſome degree of *Preſſure* that will *Cruſh* every Body.

An Engine to make Linnen Cloth; by M. de Gennes. n. 140. p. 1007. Fig. 183. July An. 1678.

XVI. This *Engine* is compoſed of four Principal Parts; the *Serpent* A A, two Foot-Steps or *Treddles* B B, one *Clapper* C, and two *Arms* D D, D D.

The *Serpent* or Iron Bar A A, has two Elbows E E, where to the Ends of the Ropes are fix'd that Raiſe and put Down the *Foot-ſteps* B B, F F are two Fourths of a Circle, that ſucceſſively Reſt upon two Arches or Bows of Iron, G G, which are above the *Clapper* C, to Raiſe it. H H are two Teeth of Iron, added to the *Serpent* making an Angle of 25 *Deg.* with F F, and K K: which ſerve to put Down a *Baſcule*, or *Sweep*, which is in the Arm that carries the *Shuttle*. The Foot-ſteps or *Treddles* differ in nothing from thoſe which are uſually made uſe of, only the Cords that hold them Pendent from the Ground are fix'd in the Elbows of the *Serpent*, which in turning Raiſes and puts them Down by the help of two little Pullies, upon which the Ropes turn.

The *Clapper* is ſupported between two Pillars, with a Rope double twiſted, which occaſions it to make a kind of Spring, and cauſes it naturally to give forwards to Beat the Cloth.

L M, is one of the *Arms* which Paſſes freely into the Canal or Pipe N N, ſupported by four Pillars of Wood O O O O. The Motion of it proceeds from the following Parts. P Q, is a *Baſcule*, which, though unequally divided by its Supporter R, is yet *in Æquilibrio*, the End P R being made to weigh exactly as much as R Q.

At the Extremity of this *Baſcule* is ty'd a Cord which paſſes through the Pully S, and terminates at the Extremity of the *Arm*, where it is faſtened, to a little Bow M. At the other Extremity of the ſame *Arm*, that is to ſay, towards L, is alſo faſtened underneath, a Cord which paſſes through the Pulley T, and which carries the Weight V.

At the ſame end of the *Arm* is added a little *Niche* Z, about the bigneſs of half the *Shuttle*: Then over a little Bar X Y, which paſſes a-thwart the *Arm* there are two other little **Pieces of Wood**, having at the end of them two Teeth, which enter into the *Niche* Z, through two Holes which are there, of the one ſide and t'other.

To the Ends of theſe little pieces of Wood, there is a little *Bow* of Whalebone or Steel, which keeps the two Ends aſunder, and forces the Teeth, which are at the other end, to enter into the *Niche*, before the ſaid pieces can themſelves. At the points 1 1, are two Ropes that paſs through the Pullies 2 2, faſtened to the Pillars, *o* 3 *o* 4, and have each of them a little Weight at the end big enough to keep it from paſſing through a little Bowl which is under each Pulley.

This *Arm* thus diſpoſed, goes and comes in the Hole NN, in the following manner. One Tooth of the *Serpent*, already deſcribed, ſtrikes upon the Extremity of the *Baſcule* P Q, and ſo cauſes the End Q to Riſe up, which drawing the Cord faſtened to the Point Q, makes the *Arm* LM, to Advance forward. But when afterwards the Tooth of the *Serpent* is come forth again, then the Weight V, tied to the other End of the ſame *Arm*, by a Cord that paſſes through the Pulley T, forces the ſaid *Arm* by its own Weight to Return again.

When the *Arm* LM is in its ordinary place, the two little pieces of Wood into which enters the Bar X Y, encloſe the *Shuttle* by means of the Walebone *Spring*. But when the ſaid *Arm* approaches the other oppoſite Arm, then the Cords tied to the Points 1 1, being a little to ſhort, and the Weight which is at the end of them not being able to paſs through, the *Spring* gives way a little, and ſo the *Shuttle* is no longer encloſed by the *Arm* which carries it, but is wholly received and graſp'd by the other; which likewiſe in its Turn, delivers it back again in the ſame manner.

The Motion of the whole *Machine* is made at the rate as you Move the Handle of the *Serpent*, for then the *Arms* cauſe the Threads to open, and immediately one of the *Arms* begins to ſlide in towards the oppoſite Arm, to which it carries the *Shuttle* and Retires, immediately: At the ſame time, one of the Quarters of a Circle, which held the *Clapper*, Elevated, forſakes it, and leaves it for to flap, and then the oppoſite Quarter of a Circle Elevating itſelf, the other Elbow changes the Threads, and the other *Arm* Retires, and ſo ſucceſſively.

The *Advantages* of this *Engine* are theſe. 1. One *Mill* will ſet 10 or 12 of theſe *Looms* at Work. 2. You may make the *Cloth* of what *Breadth* you pleaſe. 3. There will be fewer Knots in the *Cloth*, ſince the Threads will not break ſo faſt as in other *Looms*, becauſe the *Shuttle* that breaks the greateſt Part, can never touch them. In ſhort, The *Work* will be carried on *Quicker*, and at leſs *Charge*, in regard that inſtead of ſeveral Work-Folks which are required in making very large *Cloths*, One Boy will ſerve to tie the Threads at the ſeveral *Looms* as faſt as they break and to order the *Quils* about the *Shuttle*.

XVII. I Order'd a *Model* of a part of a *Waggon* to be made conſiſting of four *Wheels*, two *Axes*, and a *Board* nailed upon the *Axes*. The *Leſſer Wheels* were $4\frac{1}{8}$ Inches high, and the *Bigger Wheels* $5\frac{2}{3}$ Inches high, *viz.* $\frac{1}{12}$ of the Ordinary height of the *Wheels* of a *Waggon*: The Weight of the *Model* was almoſt $1\frac{1}{5}$ *lib.* I had alſo two other *Wheels* made $5\frac{2}{3}$ Inches high to be put on inſtead of the Leſſer. The Middle of the two *Axes* where $6\frac{1}{4}$ Inches aſunder. All the *Wheels* Turned very eaſily upon the *Axes*.

Advantages of High Wheels Experimented; by a Member of the Oxford Society. n. 167. p. 856. Jan. An. 1685.

A piece of Lead $50\frac{3}{4}$ *lib. Averdupoiſe*, was laid upon the *Model*, ſo forward, that the *Leſſer Wheels* ſeemed to bear above $\frac{2}{3}$ parts of the *Weight*. Then the *Model* was drawn with a String laid over a Pulley, the Top whereof was $\frac{1}{4}$ of an Inch higher than the Top of the *Hinder Axes*, and the Middle of this Pulley was $7\frac{1}{3}$ Inches from the Middle of the *Fore Axis*.

The *Leſſer Wheels* being put on, and the String being tied to the Top of their *Axis*.

1. *Three* Pound drew the *Model* on the ſmooth Level Table.
2. *Twenty* Pound drew the *Leſſer Wheels* over a Squared Rod $\frac{1}{4}$ of an Inch thick.
3. *Thirty* Pound drew *them* over a Round Rod a little more than $\frac{1}{2}$ an Inch thick.
4. *Thirty One* Pound drew *them* over a Square Rod half an Inch thick.
5. *Twelve* Pound drew the *Hinder Wheels* over the bigger Square Rod.

The String being laid under the *Axis*, *viz.* $\frac{1}{4}$ of an Inch lower than before.

6. *Twenty nine* Pound drew the *Leſſer Wheels* over the Bigger Square Rod.

Then the two *Bigger Wheels* being put on inſtead of the *Leſſer*, and the String lying over the *Axis*.

7. *Three* Pound drew the *Model* on the Table.
8. *Twenty five* Pound drew the *Fore Wheels* over the Round Rod.
9. *Twenty five* Pound drew them over the Bigger Square Rod.
10. The String lying under the *Axis*, 16 Pound drew them over the leaſt Rod.
11. *Twenty three* Pound drew them over the Round Rod.
12. *Twenty three* Pound drew them over the Bigger Square Rod.
13. *Thirteen* Pound drew the *Hinder Wheels* over the Bigger Square Rod.

In all theſe *Experiments*, the *Lead* was laid exactly upon the ſame part of the *Board*, but yet when the *Leſſer Wheels* were taken off, the *Lead* did not lean ſo much forward, ſo that the *Hinder Wheels* were ſomewhat more preſſed than they were before.

By comparing the ſecond, third, and fourth *Experiments*, with the tenth, eleventh, and twelfth, it appears how much more eaſily a *Waggon*, &c. might be drawn in Rough Ways, if the *Fore Wheels* were as high as the *Hinder Wheels*, and if the *Thills* were fix'd under the *Axis*. Such a *Waggon* as this, would likewiſe be Drawn more eaſily, where the *Wheels* cut in *Clay*, or *Sand*, or any Soft Ground. And moreover, *High Wheels* would not cut ſo deep as *Low Wheels*.

Low Wheels indeed are better for Turning in a narrow Compaſs than high Ones: But it ſeems probable that *Waggons* with four *High Wheels*, might be ſo contrived, that there ſhould be no great Inconvenience in that Reſpect; at leaſt, ſuch *Waggons* as ſeldom have occaſion to turn ſhort, as *Carriers Waggons*, and ſuch like.

The Difference which you may obſerve in the eighth and eleventh *Experiments*, is agreeable to what is ſaid by *S. Stevinus*, and Dr. *Wallis*, *viz.* That if a *Coach*, *&c.* muſt be Drawn over Rough, Uneven Places, it is beſt to fix the *Traces* to the Coach Lower than the Height of the Horſes Shouldiers.

14. A Table $2\frac{1}{2}$ Foot long, was ſet with one End $8\frac{1}{2}$ Inches Higher than the other End, and the *Model* being loaded as before, leſs Weight by 6 Ounces drew it up the Table, where the four *Bigger Wheels* were on, that when two *Bigger* and two *leſs* were on. Becauſe in the firſt Caſe there was almoſt the ſame *Direction* of the Motion of the *Model* and of the String that drew it; but not in the ſecond Caſe, when the *Fore Axis* was ſo much Lower than the Top of the Pulley.

A new ſort of Caleſh deſcribed by Sir R. B. n. 172. p. 1028. June An. 1685.

XVIII. This *Caleſh* goes on two Wheels; Carries one Perſon, is Light enough; Tho' it hangs not on *Braces*, yet it is eaſier than the Common Coach; A Common Coach will Overturn, if one Wheel go on a Superficies a Foot and a half Higher than that of the other, but this will admit of the Difference of $3\frac{1}{2}$ Foot in Height of the Superficies, without danger of Over-turning: We Choſe all the irregular Banks, and ſides of Ditches to run over; and I have this Day ſeen it at five ſeveral Times turn over and over, and the Horſe not at all diſordered. If the Horſe ſhould be in the leaſt unruly, with the help of one Pin, you Diſingage him from the *Caleſh* without any Inconvenience. I my ſelf have been once Overturned, and knew it not till I look'd up, and ſaw the Wheel flat over my Head; and if a Man went with his Eyes ſhut, he ſhould imagine himſelf in the moſt ſmooth way, tho' at the ſame time there be three Foot Difference in the Height of the Ground of each Wheel.

The Contrivance of a Perpetual Motion; by M. . . . . *Explained; by Dr.* Papin. n. 177. p. 1240. *Fig.* 184. Dec. An. 1685.

XIX. Let D E F, be a pair of *Bellows* 40 Inches long, that may be opened by removing the part F, from E: Let them be exactly ſhut every where, but at the Aperture E; and let a Pipe E G, 20 or 22 Inches long, be Sodered to the ſaid Aperture E, having its other end in a Veſſe G, full of *Mercury*, and placed near the Middle of the *Bellows*.

A, is an *Axis* for the *Bellows* to Turn upon.

B, A Counterpoiſe faſtened to the lower end of the *Bellows*.

C, A Weight with a Claſp to keep the *Bellows* upright.

Now if we ſuppoſe the *Bellows* opened only to $\frac{1}{3}$, or $\frac{1}{4}$, ſtanding Upright, and full of *Mercury*, it is plain that the ſaid *Mercury* being 40 Inches High, muſt Fall, as in the *Torricellian Experiment*, to the Height of about 27 Inches, and conſequently the *Bellows* muſt open before F, and leave a *Vacuity* there. This *Vacuity* muſt be filled with *Mercury* Aſcending from G through the

Pipe GE, the ſaid Pipe being but 22 Inches long: By this means the *Bellows* muſt be opened more and more till the *Mercury* continuing to Aſcend, makes the Upper part of the *Bellows* ſo heavy, that the Lower part muſt get looſe from the Claſp C, and the *Bellows* ſhould turn quite upſide down; but the Veſſel G, being ſet in a convenient place, keeps them Horizontal, and the part F, Engageth there in another Claſp C; then the *Mercury* by its Weight runs out from the *Bellows* into the Veſſel C, through the Pipe EG, and the *Bellows* muſt ſhut cloſer and cloſer until the part EF comes to be ſo Light, that the Counterpoiſe B is able to make the part F, get looſe from the Claſp C; then the *Bellows* comes to be Upright again as before; the *Mercury* left in them falls again to the Height of 27 Inches, and conſequently all the other Effects will follow, as we have already ſeen, and the *Motion* will *Continue for ever*. *Fig.* 185.

Upon this, it is to be obſerv'd, That the *Bellows* can never be opened by the Internal Preſſure, unleſs the ſaid Preſſure be ſtronger than the External. Now in the Caſe before us, it is plain, That altho' the Lowermoſt part of the *Bellows* be Preſſed Outward by 40 Inches of *Mercury*, yet the Upper part having no *Mercury* above it, bears none at all; the parts that lie in the Middle near the Axis of the *Bellows* bear but 20 Inches, and ſo all the reſt muſt bear more or leſs, according as they lie Higher or Lower: It is evident therefore, That there are as many parts that bear leſs than 20 Inches, as there are that bear more, and the Increaſe of Preſſure following an *Arithmetical Progreſſion*, it is undeniable, that all theſe Preſſures added together, will do more than one Uniform Preſſure, that would be equal to 20 Inches every where. It is alſo plain, that the Weight of the *Atmoſphere*, cannot come at the Inward Part of the *Bellows*, but through the Pipe GE, which containing 22 Perpendicular Inches, of *Mercury*, doth Counterpoiſe ſo much of the Weight of the *Atmoſphere*; ſo that this being ſuppoſed to be 27 Inches of *Mercury*, it cannot preſs the Inward part of the *Bellows*, but with a Weight equivalent to 5 Perpendicular Inches of *Mercury*. So that we find, the Inward Preſſure both of the *Mercury* and the *Atmoſphere*; is equivalent but to 25 Inches of *Mercury* in all. Whereas the Preſſure of the *Atmoſphere* upon the Outſide is every where equal to 27 Inches; from whence it appears, That the Preſſure without, is ſtronger than the Preſſure within. From this we may conclude, that the *Bellows* ſtanding Upright will rather Shut that Open. *And ſhewn Inſufficient by him, Ib. p.* 1241. *n.* 182. *p.* 138.

I ſhall ſay nothing to the Alterations this *Author* may make in his *Engine*, reſolving to leave it to others to ſhew him, that upon the Principle all he can do ſignifies nothing. And I doubt not, but if he pleaſes to conſult *M. Perault*, *de la Hire*, or any other at *Paris*, he will find them of the ſame Opinion with Mr. *Boyle*, and Mr. *Hook*, and others *Here*. *n.* 186. *p.* 2671

XX. This *Reflecting Trumpet*, conſiſts of two Parts. The Utmoſt B *b*, is a large Concave Pyramid, about a Yard long, (or may be of any manageable Length;) Open at the Baſe B, and Cloſed not with a Flat, but a Concave Head, at the Cone B. Within this is faſtened a Bended Tube A*ba*. *The Speaking Trumpet Improved; by Mr. J. Conyers. n.* 141. *p.* 1027. *Sept. An.* 1678.

Fig. 186. This *Trumpet* did at a Meeting of the *Royal Society* at *Arundel-House*, diſtinctly deliver ſome Words, croſs the Garden and the River *Thames*, and that againſt the Wind which was then ſtrong; and the Words were written down by one, that was ſent over for that purpoſe: Whereby it appeared, That a *Reflecting Trumpet*, after this, or ſome other like manner, of Wood, Tin, Pewter, Stone, or Earth, or which may be beſt of Bell-Metal, will carry the Voice as far, if not farther, than the Long one Invented by Sir *Samuel Moreland*. Fig. 187. Beſides that, it ſeems to take off from the Aſtoniſhing Noiſe near at hand, which happens in uſe of the ſaid Long *Trumpet*: By Sir *Sam. Moreland*'s *Trumpet* Fig. 188. Angularly Arched in the Middle, the delivery of Sound to any Diſtant Place was much ſhortened; and by another with three large Angular Arches, reaching almoſt from one End to the Other, the Sound was almoſt wholly Obſtructed.

The Swiftneſs of Sounds and their Reflections or Ecchoes; by Mr. Walker. n. 247. p. 433. Dec. An. 1698.

XXI. I provided a *Pendulum*, of ſmall Virginal-Wire, with a Piſtol Bullet at the end of it, which had two Vibrations in one Second of Time. I took this *Pendulum*, and ſtanding over forgainſt a High Wall, I clapt two pieces of ſmall Boards together, and obſerved how long it was ere the *Eccho* returned; and I Removed my Station till I found the place whither the *Eccho* Returned in about half a Second. But that I might Diſtinguiſh the Time more nicely, I clapt every Second of Time, 10 or 15 times together; ſo that by this means, I could the better Diſcover whether the Diſtances betwixt the Claps and the *Ecchoes*, and the following Claps, were Equal. And though it be very difficult to be Exact, yet I could come within ſome few Yards of the place I ſought for, thus: I obſerved the two Places, where I could but juſt diſcover that I was too near, and where I was too far off; and from the midway betwixt them I meaſured to the Wall, which Meaſure doubled, was the Space that the *Sound Moved* in half a Second.

Here follow the Number of *Engliſh* Feet which a *Sound Moved* in one Second of Time at ſeveral Trials.

Trials.	*Feet.*	*Trials*	*Feet.*	*Trials*	*Feet.*
1	1256	5	1292	9	1278
2	1507	6	1378	10	1290
3	1526	7	1292	11	1200
4	1150	8	1185		

Merſennus mentions an *Experiment* wherein he found the *Motion* of the *Sound* to be 1474 *Feet* in a *Second*. The *Academy del Cimento* cauſed 6 Harquebuſſes, and 6 Chambers to be fired one after another at the Diſtance of 5739 *Engliſh Feet*, and from the Flaſh to the Arrival of the Report each was 5″: And repeating the *Experiment* at the Midway, the *Motion* was exactly in half the time; and Mr. *Boyle* obſerved. That the *Motion* of *Sound* paſſes above 400 *Yards* in a *Second*.

When the *Firſt Trial* was made, there was ſome Wind ſtirring, tho' not much; the 2d, 3d and 6th were made in a Calm Morning. In the 8th, the *Eccho* was returned from a Wall at 395 *Yards* Diſtance in two *Seconds*. and in the 9th and 10th, at 213 and 215 *Yards* Diſtance, in one *Second*. The 4th was made at one end of St. *John's Cloiſter*, in *Oxford*, which is 194 *Feet* 7 *Inches* long, where the *Sound* was *Reflected* 11 *times* in two *Seconds*: And the 5th, on the North ſide of *New College Cloiſter* (which is 160 *Feet* 8 *Inches* long) where there are about 7¼ *Ecchoes* in two *Seconds*.

By ſome of thoſe *Experiments* that I tried, I am inclined to think, That the *Sound* Moved Quicker when it was Calm, than in a Wind, even when the *Sound* Moved half way with the Wind, and that it Moves ſwifter at firſt, than afterwards.

There is ſeldom any *Eccho*, where there is not ſome Wall, Wood, Bank, or ſuch like, directly Oppoſite, that may Reflect the *Sound* to the Perſon that makes it; but in St. *John's Grove*, if you ſtand near the Gate leading from the *College* to the *Grove*, and Clap, the *Eccho* will return to you from the *Ball Court*, though a Line drawn from you to the *Ball Court* be not Perpendicular to the Wall there, but as much Oblique as the Line A B, is to the Line B C; where A repreſents the Gate, B C the *Ball Court* Wall, and B D another Wall. Or, if you ſtand at E, the Corner of the *Grove* next to *Trinity*, and Clap, the *Eccho* will Return to you, from the *Ball-Court*. Fig. 180.

In the ſame *Grove*, I ſtood about 20 Yards from the ſame Gate, and the Gate being ſhut, Clapt, and at other times Stamped, and the *Eccho* Returned from the Gate as loud, if not louder than the Clap or Stamp.

An *Eccho* Reflected from a Gate or Door, has uſually a baſer and duller Sound than that which is returned from a Wall, this being much brisker.

As I have been walking towards a Wall, I have Clapped my Hands together ſeveral times, and I could diſtinguiſh the *Eccho* from the Clap, 'till I came within 7 or 8 Yards of the Wall.

In the *Cloiſters*, where, as we ſaid before, the *Eccho* was Repeated ſeveral times, the firſt Repetition ſeemed to be ſlower than the ſecond or third; but of all the Repetitions, beſides the firſt, the ſubſequent ſeemed ſlower than the precedent.

I have obſerved the Toſſing of a *Sound* forward and back again, in very many Places where there are Parallel Walls; and where the Diſtance of the Walls is leſs, there the *Ecchoes* follow one another quicker.

Whereſoever a *Sound* was thus toſſed betwixt two Walls, if I ſtood about the Middle, I could hear the *Sound* twice as quick, that is twice as often Repeated in one Second, as if I ſtood near one Wall: The *Sound* being Reflected to me from both ends, when I ſtood in the middle.

In *Trinity Ball Court*, when I ſtood and Clapt at B, three or four Yards from the End of the Wall C, or at A, which is oppoſite to B, the *Sound* was toſſed betwixt the Oppoſite Walls, but not half ſo long as when I ſtood betwixt the Walls. In Places where there are Parallel Walls, not above ſix or Fig. 190.

eight Yards aſunder, as in *Trinity Ball Court*, and at the Entrance into St. *John's Grove*, &c. I have heard the Ecchoes of a Clap following one another diſtinctly enough: But there the *Ecchoes* of a *Muſical Note*, which was longer than a Clap, were ſo confuſed, that they ſeemed one continued *Long Sound*: which makes me think, that the *Eccho* in ſome Vaults, is nothing elſe but the *Sound* toſſed betwixt the Side Walls, and betwixt the Top and Bottom. This alſo makes me conjecture, That the Reaſon why *ſtringed Muſical Inſtruments* give a greater and longer *Sound* to the *Strings* than if the *Strings* were fixt to a ſingle Board, may be this; becauſe the *Sound* is toſſed from ſide to ſide in the Belly of the *Inſtrument*.

The Doctrine of Sounds; by Narciſſus, *Biſhop of* Ferns *and* Leighlin. N. 156. p. 472. Nov. An. 1683.

XXII. I cannot better Explain the *Uſefulneſs* of this *Theory* of *Sounds*, than by making a Compariſon 'twixt the Faculty of *Seeing* and *Hearing* as to their Improvements. In order to which, I Obſerve, That *Viſion* is threefold, *Direct*, *Refracted*, and *Reflex'd*, anſwerable whereunto we have *Opticks*, *Dioptricks*, and *Catoptricks*.

In like manner *Hearing* may be Divided into *Direct*, *Refracted* and *Reflex'd*; where to anſwer three Parts of our Doctrine of *Accouſticks*, which are yet nameleſs, unleſs we call them *Acouſticks*, *Diacouſticks*, and *Catacouſticks* (or in another Senſe, but to as good Purpoſe) *Phonicks*, *Diaphonicks*, and *Cataphonicks*.

Direct Viſion has been Improved two ways.

1. *Ex parte Objecti*, by the Arts of *Producing*, *Conſerving*, and *Imitating*, and duly *Applying*, Light and Colours.

2. *Ex parte Organi vel Medii*, by making uſe of *Tubes without Glaſſes*, or, a Man's Cloſed Hand to look through. So likewiſe *Direct Hearing*, partly has, and partly may further receive great and notable Improvements, both *ex parte Objecti*, and *ex parte Organi vel Medii*.

1. As to the Object of *Hearing*, which is *Sound*, Improvement has been and may be made, both as to the *Begetting*; and as to the *Conveying* and *Propagating* (which is a kind of *Conſerving*) of *Sounds*.

1. As to the *Begetting* of *Sounds*. The Art of Imitating any *Sound* whether by Speaking (that is Pronouncing) any kind of Language (which really is an *Art*; and the *Art* of Speaking perhaps one of the greateſt) or by Whiſtling, or by Singing (which are allowed *Arts*) or by Hollowing, or Luring, (which the *Huntſman* and *Faulkner* would have to be an *Art* alſo) or by Imitating with the Mouth (or otherwiſe) the Voice of any Animal; as of Quails, Cats, and the like, or by repreſenting any Sound begotten by the Colliſion of Solid Bodies, or after any other manner; theſe are all *Improvements* of *Direct Hearing*, and may be *Improved*.

Moreover, the Skill to make all ſorts of *Muſical Inſtruments*, both Ancient, and Modern, whether *Wind Inſtruments* or *String'd*, or of any other Sort, whereof there are very many (as *Drums*, *Bells*, the *Syſtrum* of the *Egyptians* or the like) that *Beget* (and not only *Propagate*) *Sounds*: The Skill of Making theſe, I ſay, is an *Art*, that has much Improv'd *Direct Hearing*, and an *Harmonious Sound* exceeds a Single and Rude one, that is an *Immuſical Tune*: which *Art* is yet capable of farther Improvement. And I hope, That by the

Rules, which may happily be laid down, concerning the *Nature*, *Propagation*, and *Propportion* or *Adapting* of *Sounds*, a way may be ſound out, both to Improve *Muſical Inſtruments* already in uſe, and to Invent New Ones, that ſhall be more Sweet and Luſhious than any yet known. Beſides, that by the ſame means *Inſtruments* may be made, that ſhall Imitate any *Sound* in Nature, that is not *Articulate*; be it of Bird, Beaſt, or what thing elſe ſoever.

2. The *Conveying* and *Propagating* (which is a kind of *Conſerving*) of *Sounds*, is much helped by duly *Placing* the *Sonorous Body*, and alſo by the *Medium*.

For if the *Medium* be *Thin* and *Quieſcent*, and the *Sounding Body Placed* conveniently, the *Sound* will be eaſily and regularly *Propagated* and mightily *Conſerved*.

1. The *Medium* muſt be *Thin* and *Quieſcent*; Hence in a ſtill Evening, or the Dead of the Night (when the Wind ceaſes) a *Sound* is better ſent out, and to a greater Diſtance than otherwiſe.

2. The *Sonorious Body* muſt be *Placed* conveniently, *viz.* Near a *Smooth Wall*, either *Plain* or *Arched* (*Cycloidically* or *Elliptically*, rather than otherwiſe; though a *Circular* or any *Arch* will do; but not ſo well.) Hence in a *Church*, the nearer the *Preacher* ſtands to the *Wall* (and certainly it's much the beſt way to place *Pulpits* near the *Wall*) the better is he heard, eſpecially by thoſe who ſtand near the *Wall*, alſo, though at a greater Diſtance from the *Pulpit*, thoſe at the Remoteſt End of the Church, by laying their Ears ſomewhat cloſe to the Wall, may hear him eaſier than thoſe in the Middle.

Hence alſo do ariſe *Whiſpering Places*. For the Voice being applied to one End of an *Arch*, eaſily Rowls to the other. And indeed were the Motion and *Propagation* of *Sounds* but rightly underſtood, 'twould be no hard matter to contrive *Whiſpering Places* of infinite variety and uſe. And perhaps there could be no better or more pleaſant Hearing a *Conſort* of *Muſic*, than at ſuch a Place as this; where the *Sounds* Rowling long together before they come to the Ear, muſt needs Conſolidate and Imbody in one; which becomes a true *Compoſition* of *Sounds*, and is the very Life and Soul of *Conſort*.

2. If the *Sonorous Body* be placed near *Water*, the *Sound* will eaſily be *Convey'd*, yet mollify'd; as Experience teacheth us from a *Ring* of *Bells* near a River, and a great Gun ſhot off at Sea; which differ much in the Strength, and yet ſoftneſs and continuance, or *Propagation* of their *Sounds* from the ſame at Land; where the *Sound* is more Harſh and more Periſhing, or much ſooner Decays.

3. In a *Plain* a Voice may be heard at a far greater Diſtance, than in Uneven Ground: The Reaſon of all which, laſt nam'd *Phænomina* is the ſame, becauſe the *Sonorous Air* metting with little or no Reſiſtance upon a *Plane* (much leſs upon an *Arch'd*) ſmooth *Superficies*, eaſily Rowls along it, without being let or hindred in its Motion, and conſequently without having its Parts Disfigured, and put into another kind of Revolution, than what they had at the firſt *Begetting* of the *Sound*, which is the true Cauſe of its *Preſervation* or *Progreſſion*; and fails much when the *Air* paſſes over an Uneven Surface, according to the Degrees of its Inequality; and ſomewhat alſo, when it paſſes over the *Plane Superficies* of a Body, that is hard and reſiſting.

Wherefore the *ſmooth* Top of the *Water* (by reaſon of its yielding to the *Arched Air*, and gently riſing again with a kind of *Reſurge*, like to *Elaſticity* though it be not ſo; by which *Reſurge* it Quickens and Haſtens the *Motion* of the Air Rowling over it, and by its yielding preſerves it in its *Arched Cycloidical* or *Elliptical* Figure) the *ſmooth* Top of the *Water*, I ſay, for theſe Reaſons, and by theſe means, *Conveys* a *Sound* more entire, and to a greater Diſtance, than the *Plane Surface* of a piece of Ground, a Wall, or any other Solid Body whatever, can do.

2. The *Organ*, which is the *Ear*, is helpt much by *Placing* it near a Wall, (eſpecially at one end of an *Arch*, the *Sound* being *Begotten* at the other) or near the Surface of Water, or of the Earth; along which the *Sounds* are moſt eaſily and naturally Conveyed; as was before declared. And 'tis Incredible, how far a *Sound* made upon Earth (by the Trampling of a Troop of Horſes, for Example) may be heard in a ſtill Night, if a Man lays his Ear cloſe to the Ground in a large *Plane*.

Otacouſticks here come in for helping the *Ear*; which may be ſo contrived (by a right underſtanding the *Progreſſion* of *Sounds*, which is the Principal Thing to be known for the due regulating all ſuch kinds of Inſtruments) as that the *Sound* might enter the *Ear* without any *Refraction*.

1. *Refracted Viſion* (which is always made *ex parte Medii*) ariſes from the different *Denſity*, *Figure*, and *Magnitude* of the *Medium*; which is ſomewhat altered by the divers *Incidence* of the *Viſible Rays*, and ſo it is in *Refracted Hearing*, all theſe Cauſes concur to its *Production*; and ſome others to be hereafter conſidered.

Now as any Object (a Man for Example) ſeen through a Thickened Air, by *Refraction* appears greater than really he is: So likewiſe a *Sound* heard through the ſame Thickened part of the *Atmoſphere*, will be conſiderably vary'd from what it would ſeem to be, if heard through a Thinner *Medium*. And this I call a *Refracted Sound*.

Improvements of *Refracted Viſion* have been made, by Grinding or Blowing Glaſſes into a certain Figure, and placing them at due Diſtances; whereby the Object may be (as 'twas) enabled to ſend forth its Rays more Vigoroſly, and the *Viſive Faculty* Impowered the better to receive them. Thus,

1. A fine Glaſs Bubble, filled with Clear Water, and Placed before a Burning Candle or Lamp, does help it to dart forth its Rays to a Prodigious Length and Brightneſs.

2. The *Viſive Faculty* is much Helped.

1. By *Spectacles* and other Glaſſes which are made to Help the *Purblind* and *Weak Eyes*, to ſee, at any competent Diſtance.

2. By *Perſpective Glaſſes* and *Teleſcopes*, which Help the *Eye* to *See* Objects at a very great *Diſtance*, which otherwiſe would not be diſcernible.

3. By *Microſcopes* or *Magnifying Glaſſes*, which help the *Eye* to ſee *Near Objects*, that by reaſon of their ſmallneſs were *Inviſible* before.

4. By *Polyſcopes* or *Multiplying Glaſſes*, whereby one thing is repreſented to the *Eye* as many, whether in the ſame or different Shapes.

After the ſame manner, Inſtruments may be contrived for aſſiſting both the *Sonorous Body* to ſend forth its *Sound* more ſtrongly, and the *Acouſtick Faculty* to receive and diſcern it more eaſily and diſtinctly. And thus;

1. An Inſtrument may be Invented, that applied to the *Mouth*, (or any *Sonorous Body*) ſhall ſend forth the *Voice* Diſtinctly as to a prodigious *Diſtance* and *Loudneſs*. For if the *Stentoro-phonicon* (which is but a Rude and Inartificial Inſtrument,) does ſuch great Feats; what might be done with One compoſed according to the Rules of Art? whoſe *Make* ſhould comply with the Laws of *Sonorous Motion* which that does not.

2. There are ſome *Inſtruments*, and more ſuch may be Invented to help the *Ear*: As,

1. *Otacouſticks* (and better may be made) to help *Weak Ears* to hear at a reaſonable Diſtance alſo. Which would be as great a help to the *Infirmity* of *Old Age*, as the other Invention of *Spectacles* is, and perhaps greater; for as much as the *Hearing* what's ſpoken is of more daily uſe and concern to ſuch Men, than to be able to *Read* Books, or to view Pictures.

2. A ſort of *Otacouſticks* may be ſo contrived, as that they ſhall Recieve in *Sounds* made at a very great *Diſtance*, which otherwiſe would have been *Inaudible*: And theſe *Otacouſticks*, in ſome Reſpects, would be of greater uſe than *Perſpectives*.

1. In Time of *War* for diſcovering the *Enemy* at a good *Diſtance*, when he Marches or Lies Incamp'd behind a Mountain or Wood, or any ſuch Place of Shelter, which hinder the Sight from reaching very far.

2. At *Sea*, when in dark *Hazy Weather* the Air is too thick, or in *Stormy* Tempeſtuous *Weather*, the Waves Riſe too High, for the *Perſpectives* to be made uſe of.

3. In Dark Nights, when *Perſpectives* become almoſt Inſignificant; and yet at ſuch times, generally Soldiers take their March, when they would ſurprize their Enemies.

4. *Microphones*, or *Micracouſticks*, that is, *Magnifying Ear Inſtruments*, which may be Contrived after that manner, that they ſhall render the moſt *Minute Sound* in Nature diſtinctly Audible, by *Magnifying* it to an unconceiveable Loudneſs. By the help whereof we may hear the different Cries and Tones of the ſmalleſt *Animals*.

4. A *Polyphone*, or *Polyocouſtick*, ſo ordered that One *Sound* may be heard; either of the Same, or a Different *Note*. Inſomuch that who uſes this *Inſtrument*, he ſhall at the Sound of a *Single Viol* ſeem to hear a whole *Conſort*; and all True *Harmony*. By which means this *Inſtrument* has much the Advantage of the *Polyſcope*.

I have called it *Refracted Hearing*, becauſe made through a *Medium*, viz. Thick Air, or an Inſtrument, through which the *Sound* paſſing is broken or *Refracted*.

3. *Reflected Viſion* (which is always made *ex parte Objecti*) hath been Improv'd by the Invention of *Looking Glaſſes* and *Poliſh'd Metals*; whether *Plane Concave* or *Convex*, of ſeveral Figures, and Placed at Determinate Diſtances.

In like manner *Reflex'd Audition* (which is only made *ex parte Corporis Oppoſiti*) may be Improved by Contriving ſeveral ſorts of *Artificial Ecchoes*. For (ſpeaking in general) any *Sound* falling *Directly* or *Obliquely* upon any Denſe Body, of a ſmooth (whether Plane or Arch'd) Superficies, is beat back again and Reflected, or does *Eccho* more or leſs.

I ſay, (1.) *Falling Directly* or *Obliquely*; becauſe, if the *Sound* be ſent out and Propogated Parallel to the Surface of the Denſe Body, there will be no *Reflection* of *Sound*, no *Eccho*.

I ſay, (2.) Upon a Body of a *Smooth Superficies*; becauſe if the Surface of the *Corpus Obſtans* be Uneven, the Air by *Reverbeation* will be put out of its Regular Motion, and the *Sound* thereby broken and extinguiſh'd: So that, tho' in this caſe alſo the Air be beaten back again, yet *Sound* is not Reflected, nor is there any *Eccho*.

I ſay, (3.) It does *Eccho More* or *Leſs*, to ſhew, that when all things are, as is before deſcrib'd, there is ſtill an *Ecchoing*, though it be not always Heard, either becauſe the *Direct Sound* is too Week to be beaten quite back again to him that made it; or that it does Return home to him, but ſo weak, that without the help of a good *Otacouſtick* it cannot be diſcerned; or that he ſtands in a wrong Place to receive the *Reflected Sound*, which paſſes over his Head, under his Feet, or to one ſide of him; which therefore may be Heard by a Man ſtanding in that place, where the *Reflected Sound*, will come, provided no interpos'd Body, does intercept it; but not by him that firſt made it.

Theſe *Ecchoes* (like *Reflected Viſion*) may be ſeveral ways *Produced*, as;

1. A *Plane Corpus Obſtans Reflects the Sound* back in its due *Tone* and *Loudneſs*; if allowance be made for the proportionable Decreaſe of the *Sound* according to its Diſtance.

2. A *Convex Corpus Obſtance Repels* the *Sound* (inſenſibly) Smaller; but ſomewhat quicker (though weaker) than otherwiſe it would be.

3. A *Concave Corpus Obſtans Ecchoes* back the *Sound* (inſenſibly,) Bigger, Slower, (though Stronger) and alſo *Inverted*; but never according to the order of Words. Nor do I think it poſſible for the Art of Man to Contrive a ſingle *Eccho*, that ſhall Invert the *Sound* and Repeat backwards; becauſe then the Words laſt ſpoken, that is, which do laſt occur to the *Corpus Obſtans*, muſt firſt be *Repell'd*; which cannot be. For where in the mean time ſhould the firſt Words hang and be conceal'd or lie dormant? Or how, after ſuch a Pauſe be Reviv'd and Animated again into Motion? Yet in Complicated or Compound *Ecchoes*, where many Receive from one another, I know not whether ſomething that way may not be done.

From this Determinate *Concavity* or *Archedneſs* of theſe *Reflecting Bodies* it comes to paſs, that ſome of them from a certain Diſtance or Poſiture, will *Eccho* back but one Determinate *Note*, and from no other Place will they *Reverberate* any; becauſe of the undue Poſition of the *Sounding Body*. Such an one (as I remember) is the *Vault* in *Merton Colledge* in *Oxford*.

4. *The Echoing Body*, being Removed farther off, *Reflects* more of the *Sound*, than when nearer. And this is the Reaſon, why ſome *Echoes* Repeat but one Syllable, ſome one Word, and ſome many

5. *Echoing Bodies* may be ſo contriv'd and Plac'd, as that *Reflecting* the *Sound* from one to the other, either *Directly* and *Mutually*, or *Obliquely* and by Succeſſion, out of one *Sound* ſhall many *Echoes* be begotten; which in the firſt Caſe will be altogether and ſomewhat Involv'd or Swallow'd up of each other; and thereby Confuſed (as a Face in Looking-Glaſſes obverted;) in the other they will be Diſtinct, Separate and Succeeding one another, as moſt *Multiple Echoes* do.

Moreover, a *Multiple Echo* may be made, by ſo Placing the *Eccboing Bodies*, at Unequal Diſtances, that they *Reflect* all one way, and not One on the Other; by which means a *Manifold Succeſſive Sound* will be heard (not without aſtoniſhment;) One Clap of the Hands like many; One *Ha* like a Laughter; One ſingle Word like many of the ſame *Tone* and *Accent*; and ſo one *Viol* like many of the ſame kind, Imitating each other.

Furthermore, *Echoing Bodies* may be ſo ordered, that from any one *Sound* given, they ſhall produce many *Echoes* different both as to their *Tone* and *Intention*. By this means a *Muſical Room* may be ſo Contrived, that not only One *Inſtrument*, played on in it ſhall ſeem many of the ſame Sort and Size; but even a *Conſort* of (ſomewhat) Different ones; only by Placing certain *Echoing Bodies* ſo, as that any *Note* (played) ſhall be Returned by them in 3*ds*, 5*th*, and 8*ths*, which is poſſible to be done otherwiſe than was mentioned before in *Refracted Audition*.

I have been thus large, that I might give you a little Proſpect into the Excellency and Uſefulneſs of *Acouſticks*, and that thereby I might excite others to bend their Thoughts, towards the making of *Experiments* for the Compleating this (yet very Imperfect though Noble) *Science*; a *Specimen* whereof I will give you in theſe three *Problems*.

Prob. I.] *To make the leaſt Sound (by the help of Inſtruments) as Loud as the Greateſt; a Whiſper to become as loud as the Shot of a Cannon.*

By the help of this *Problem* the moſt minute Sounds in Nature may be Clearly and Diſtinctly heard.

Prop. II.] *To Propagate any (the leaſt) Sound to the greateſt Diſtance.*

By the help hereof any *Sound* may be Conveyed to any, and therefore heard at any *Diſtance*, (I muſt add, within a certain, though very large Sphere.)

Moreover by this means a *Weather-Cock* may be ſo contrived, as that with an Ordinary Blaſt of Wind it ſhall Cry (or Whiſtle) Loud enough to be heard many Leagues. Which happily may be found of ſome Uſe, not only for Pilots in mighty Tempeſtuous Weather, when *Light Houſes* are rendred almoſt uſeleſs: But alſo for the Meaſuring the Strength of Winds, if allowance be made for their Different Moiſture. For I conceive, That the more Dry any Wind is, the Louder it will Whiſtle, *cæteris paribus*; I ſay, *cæteris paribus*, becauſe, beſides the Strength and Dryneſs of *Winds* or Breath, there are a great many other things (hereafter to be conſider'd) that Concur to the Increaſe of *Magnifying* of *Sounds*, begotten by them in an Inſtrument expoſed to their Violence, or Blown into.

Prob. III.] *That a Sound may be convey'd from one Extream to the other (or from one Diſtant Place to another) ſo as not to be heard in the Middle.*

By the help of this *Problem* a Man may talk to his Friend at a very conſiderable Diſtance, ſo that thoſe in the Middle Space ſhall hear nothing of what paſſed betwixt them.

Fig. 197. I ſhall here Add a *Semiplane* of an *Acouſtick* or *Phonical Sphere*, as an Attempt to explicate the great Principle in this *Science*, which is the *Progreſſion* of *Sounds:*

You are to conceive this (Rude) *Semiplane* as *Parallel* to the *Horizon*; for, if it be *Perpendicular* thereunto, I ſuppoſe the upper *Extremity* will be no longer Circular, but *Hyperbolical*, and the lower part of it ſuited to a greater Circle of the Earth. So that the whole *Phonical Sphere* (if I may ſo call it) will be a ſolid *Hyperbola*, ſtanding upon a *Concave Spherical Baſe.* I ſpeak this concerning *Sounds* made (as uſually they are) nigh the *Earth*, and whoſe *Sonorous Medium* has a free paſſage every way. For if they are Generated High in the Air, or Directed one way, the Caſe will be different; which is partly Deſigned in the Inequality of the Draught.

XXIII. A Paper, *of leſs General Uſe*, Omitted, *viz.*

Carriages. n. 161. p. 666. E*Xperiments to be made, relating to* Carriages; *propoſed by Sir* William Petty.

XXIV. Accounts *of Books* and Additions, Omitted.

n. 32. p. 626. 1. D*E vi Percuſſionis*, Joh. Alphonſ. Borelli : Bononiæ, 1667. *in* 4to.

n. 73. p. 2210. 2. *De Motionibus à* Gravaiate *dependentibus Liber*, Jo. Alphonſi Borelli, *in Academia* Piſana *Matheſeos* Profeſſoris, Regio Julio 1670. *in* 4to.

n. 67. p. 2057. 3. *Dialogi Phyſici, quorum Primus de Lumine*; *Secundus & Tertius de* Vi Percuſſionis & Motu; *Quartus de Humoris Elevatione per Canaliculum*; *Quintus & Sextus de Variis Selectis. Auth.* Honor. Fabry, *S. Jeſu.* Lugduni Galliarum, 1669. *in* 8vo.

n. 54. p. 1086. n. 61. p. 2005. n. 76. p. 2286. n. 87. p. 5074. 4. Mechanica; *ſive de* Motu, *Tractatus Geometricus*; *Auth.* Joa. Wallis, *S. S. Th. D.* Londini, 1670. 1671. *in* 4to. *The* Author *here makes ſome* Additions *to* Prop. I. Chap. XV. p. 753. *concerning the* Center *of* Gravity *of the* Hyperbola.

n. 61. p. 2008. 5. *Exercitationes Mechanicæ*, Alexandri Marchetti. Piſis, 1669. *in* 4to.

n. 82. p. 4050. 6. *De Reſiſtentia Solidorum*, Alexandri Marchetti *in* Piſana *Academia Phil. Prof.* Florentiæ. 1665. *in* 4to.

n. 73. p. 2213. 7. *Hypotheſis, Phyſica nova, ſive Theoria* Motus Concreti, *una cum Theoria* Motus Abſtracti. *Auth.* Gothfredo Gulielmo Lebnitio. *J. V. D.* Lond.
n. 74. p. 2227. 1621 *in* 12°. *Of this Book Dr,* Wallis *here gives his Opinion.*

8. *La Statique, ou la Science, de* Forces Mouvantes *par le P.* Ignace Gaſton Pardies. *S. J. à* Paris 1673. *in* 12° *The Firſt part being of* Local Motion. *Printed at* Paris, 1670. *was* Engliſhed *and Printed at* London, *the ſame Year, in* 12°. n. 94. p. 6042 n. 65. p. 2010

9. Chriſtiani Hugenii Zulichemii *Horologium Oſcillatorium.* Pariſiis, 1673 *in* Fol. n. 95. p. 6068.

10. *A Diſcourſe made before the* Royal Society *concerning the Uſe of* Duplicate Proportion *in ſundry Important Particulars; together with a* New Hypotheſis *of* Elaſtique *or* Springy Bodies: *By Sir* William Petty. n. 109. p. 209.

11. *Triaté de la* Percuſſion *ou* Choq *des* Corps, *&c. par M.* Mariotte, *de l'*Academie Royal *des* Sciences. *A* Paris, 1673. *in* 12°. n. 134. p. 859.

12. *Philoſophiæ Naturalis Principia Mathematica. Authore* Iſ. Newton *Lond. in 4to.* n. 186. p. 291. n. 226. p. 445.

13. *Traité de* Mouvement *des* Eaux *& des autres* Corps Fluids *par feu; M.* Mariotte. *à* Paris. 1686, *in 8vo.* n. 181. p. 191.

14. *Mechanick Exerciſes; or,* The Doctrine of *Handy Works. By Mr.* Joſ. Moxon. Lond. 1677. *in 4to.* n. 138. p. 967. n. 139. p. 987.

15. *The* Speaking-Trumpet, *as it hath been Contrived, and Publiſhed, by Sir* Samuel Moreland; *together with its* Uſes *both at* Sea *and* Land. Lond. 1671. n. 79. p. 3056

CHAP. VI.

Hydroſtaticks. Hydraulicks.

To Weigh Water, or other Fluids; by . . . . . . A New Aræometer; *by M.* Homberg. n. 262. p. 530. *Fig.* 192.

I. 1. TAke a *Viol* with a very *Narrow Body*, and, when it is almoſt full the Water is to be Dropt into it, drop by drop, 'till it can hold no more. Then *Weigh* it exactly, and deduct the *Weight* of the *Empty Viol.*

2. A Is a Glaſs Bottle like a little *Matracium*, of which the *Neck* B C is ſo ſmall that a Drop of Water therein takes up the Space of 5 or 6 Lines, near that *Neck* is a little Capiller Tube D, about 6 Lines long, and Parallel to the *Neck* B C; The Opening B is a little dilated, in the Faſhion of a Tunnel, for pouring more eaſily the Liquors into the Bottle, and the little Tube D is for giving a way to the Air contained in that Veſſel to go out, when the Liquor is poured in at B; the Point C is a little Mark at the ſame height, as the end of the little Tube D.

When we fill the Veſſel, we pour the Liquors into it, by the Opening B, until it goes out by the little Tube D, and, if the Height of the Liquor is even to the Mark C, 'tis well; if it is Lower, we muſt fill more to that Point; if it is Higher, we muſt ſtrike ſoftly upon the Opening B, till the Overplus of the Liquor be even to the Point C in the *Neck* of the Bottle. By that means we have always exactly the ſame *Volume* of *Liquor*, and we can know how the ſame *Volume* of the ſeveral Liquors weighs more one than another preciſely. But we muſt conſider the Variation of the Weather, when we compare the Weight of a Liquor which we weigh in Summer time, with the Weight of another which we have weighed in the Winter; for the ſame Liquor, being more Rarified in the Hot time, and condenſed in the Cold, the ſame Volume of it will be more Weighty in Cold Weather than in Warm.

A New Eſſay Inſtrument; by M. Boyle. n. 24. p. 447. n. 115. p. 329. *Fig.* 193.

II. 1. Many Years ago I made uſe of a little *Glaſs Inſtrument*, conſiſting of a *Bubble*, and furniſhed with a *Long* and *Slender Stem* to compare the *Specifick Gravities* of *Different Liquors* by its more or leſs *Sinking* in *Them*: And I have ſince employed it to diſcover the *Specifick Gravities* of *Solid, ſeveral appended*, by its being more or leſs depreſſed by them in the *ſame Liquor*. For 'tis clearly deducible from the Grounds of *Hydroſtaticks*, that any Solid Body Heavier than Water, loſes in the Water as much of the Weight it had in Air, as Water of equal Bulk to the Immerſed Solid would weigh in the Air; and conſequently, ſince Gold is by far the moſt Ponderous of Metals, a piece of Gold and one of Equal Weight of Copper, Braſs, or any other Metal, being propoſed, the Gold muſt be leſs in bulk, than the Copper or Braſs. And by this means, if both of them be weighed in the Water, the Gold muſt loſe in that Liquor leſs of its former Weight than the Braſs or Copper; becauſe the baſer Metal, as well as the Gold, grows lighter by the weight of a Bulk of Water equal to it; and the baſer Metal being the more Voluminous, the correſpondent Water muſt weigh more than that which is correſpondent to the Gold, Whence I concluded, that the *Floating Inſtrument* abovementioned would be made to ſink deeper by an Ounce, for Inſtance of Gold, hanging at it under Water, than by an Ounce of Braſs, or any other Metal, which, by reaſon of its greater Bulk than Gold, loſing more of its weight by the Immerſion, muſt needs retain leſs, and ſo have leſs power to Depreſs the Inſtrument 'twas faſtened to. Which Concluſion will alſo hold (though the Diſparity be not ſo great and Conſpicuous) in reference to other Metals, as Lead and Tin, that differ in *Spdcifick Gravity*.

This *Inſtrument* may be of *Glaſs*, *Copper*, *Silver*, or almoſt any other Solid *Body*, that is, or may be made, fit to *Float* on the Water, with a *Guinea, &c. Hanging* at it, and of a Texture cloſe enough to keep out the Water. It conſiſts of three Parts: The *Ball* or Globulous part; the *Stem* or Pipe; and that which *Holds* the *Coin*.

Fig. 194.

The *Ball* or Round part of BCDE (if of *Metal*) conſiſts of two thin Concave Plates, exactly Sodered together in the middle; and at the diſtanteſt parts from the Commiſſure, there ought to be left two oppoſite *Holes*, one in each Plate, for the two other parts of the Inſtrument. This middle part, though for Brevities ſake we name it the *Ball*, ſhould not be exactly Round; but of any Shape that ſhall be found fit ro make the *Inſtrument* keep to its Erect poſture ſteadily in the Water. It muſt contain as much Air as may ſerve to keep the whole *Inſtrument* when loaded, from Sinking beneath the top of the *Stem*.

The *Stem* A B is to be Soldered on to the *Ball* at the uppermoſt of the two mention'd *Holes*. It may be either Hollow or Solid: But it ought to be made very ſlender; that the different Depreſſions of the Inſtrument in the Water may be the more Notable. And, for theſame reaſon, it ought not to be too Short, eſpecially if it be to be applied to other Uſes than the Examining of *Guineys*.

At the Undermoſt of the two *Holes* in the *Ball*, is Inſerted and Solder'd the undermoſt part of the *Inſtrument*, which I call the *Screws*, or the *Stirrup*. The *Screw* F is a very ſhort Piece of Braſs with a broad Slit in it, capable of receiving the edge of the *Guinea*, which with one turn or two of a ſmall and ſlight lateral Screw may be kept faſt in it, and readily, the Operation being ended, taken out again. The *Stirrup* G is made of a piece of Wire, that, a little beneath the bottom of the *Ball*, is bent round, ſo as to ſtand Horizontally, that the *Guinea* may be laid on it. Fig. 195.

It would be convenient, that the *undermoſt Stem* and the *Screw* be made by it ſelf, that it may be at pleaſure thruſt upon the Stem, and taken off again. For, by this means, if the *Ball* of the *Inſtrument* be made large enough, you may have room to put on for Ballaſt, as occaſion ſhall require, one, two, or three flat and round pieces of Copper, Lead, *&c.* with each of them a Hole in the middle fitted to the Size of the Stem, ſo that they may be put on as near the Lower part of the *Ball* as you think fit, and then the *Screw* may be thruſt on after them, not only to take hold of the *Coin* or Metalline mixture to be *Examin'd*, but to ſupport the thin Plates. Fig. 196

To adjuſt this *Inſtrument* for the uſe of examining *Guineas*; which are by far the moſt uſual *Gold Coins* that paſs in *England*, you muſt by the help of the *Stirrup* or *Screw*, Hang, at the bottom of it, a piece of that *Coin* which you know to be Genuine and (having carefully ſtopt the Orifice of the Stem (if it be a Pipe) that no Water may get in at it, Immerſe the *Inſtrument* leiſurely and perpendicularly into a Veſſel full of clean Water, 'till it be Depreſt almoſt to the top of the Stem, and then; letting it alone; if, being Settled, it continue in the ſame Station and Poſture, your work is done, if it Emerge, you muſt add a little weight to it, either by putting into the *Stem*, if it be Hollow, ſome Duſt Shot, Filings of Lead, or ſome other Minute and Heavy Body, or elſe by putting on the ſhort Stem abovementioned, that comes out beneath the *Ball*, a flat, round and perforated piece of Lead, of Weight ſufficient to enable the *Guinea* to Depreſs the weight as low as 'tis deſired: But, if it ſink quite under Water, you muſt lighten it either with a File, or by ſcraping or grating off a little of the Ballaſt Plate abovementioned; or, if you have put any Weight into the Cavity to poize it, by taking out ſome of that, till you have made it Light enough: This being done. a Mark, H is to be made juſt at the place where the Surface of the Water touches the *Stem*, and then taking out your *Inſtrument*, ſubſtitute, in the place of your *Guinea*, a little round Plate of Braſs, of the ſame Weight, or a Grain or two heavier, in the Air; and, putting the *Inſtrument* into the Water, as before, ſuffer it to Settle, and make another Mark I, at the Interſection of the *Stem* and the Horizontal Surface of the Water. Fig. 193. Fig. 194.

There may (though 'tis like there very ſeldom will) happen a Caſe, wherein, though the Principle, our *Inſtrument* is framed on, will hold good, yet the Practical Application may be Unſecure. For, if a *Falſifier of Money* have the Skill, by *Waſhing* or otherwiſe, to take off much of the Quantity or Subſtance of the *Guinea*, without altering or impairing either the *Figure* or *Stamp*, the piece of Coin will not be able to Depreſs our Inſtrument to the Uſual Mark, and may thereby make it to be judged *Counterfeit*, when 'tis indeed but too *Light*. But it preſently ſhews, that the propoſed *Guinea*, if it be not *Counterfeit*, is otherwiſe *Abuſed*; and, though it does not clearly determine, whether that likewiſe proceed from the want of *Specifick Gravity* in the Metal, or from the Coins having been *Waſhed* or otherwiſe fraudulently *Leſſened*; yet it probably reſolves the doubt, becauſe, if the want of Weight appear by the *Inſtrument* to be very great, as it uſually does, where the piece has been Robbed of ſome of its Subſtance, 'tis a ſtrong Preſumption, that 'tis rather *Waſhed*, *&c.* than *Counterfeited*. However, it will be ſure to prompt him that uſes it, to employ the Ballance, which will preſently aſſiſt him to reſolve his Doubt. For if the Suſpected Coin have in the Air its due Weight, 'twill argue that the great Lightneſs of it in the Water proceeds from it's not being of the requiſite fineneſs; and, if it want much of its due Weight in the Air, 'tis very probable, that 'tis *Waſhed*, *&c.* rather than of another Metal than *Gold*

Any other kind of *Gold Coin*, that is near about the Weight of a *Guinea*, may be Examined by our *Inſtrument* after the manner above deliver'd. If the *Coin* be Heavier than a *Guinea*, as is a twenty Shilling piece of *Broad Gold*, the *Ballaſt*, whether internal or external, of the *Inſtrument*, muſt be taken off, that ſo Heavy a *Coin* may not quite ſink it. But, if it be Lighter than a *Guinea*, one may add as much *Gold* (of the ſame *Alloy*) beaten into thin Plates, as with the Coin propoſed, will make up in the Air the Weight of a *Guinea*. For then this Aggregate, being examined as if it were a *Guinea*, will diſcover in the Water, whether the *Coin* be *Right* or *Counterfeit*.

This *Inſtrument* may be alſo made to ſerve to examine ſome ſorts of *White Money*, leſs Heavy than Half-Crowns. And, becauſe it may be uſeful to know in General, what Coins may, and what may not, be Examined by this or that particular *Inſtrument* propoſed, I ſhall here add a general way that is not difficult for finding this out; namely, firſt by Weighing the piece of *Gold* or *Silver* in the Air, and afterwards in the Water, and Subſtracting the latter from the former, to obtain the Difference of the two Weights: And next by Weighing alſo in the Air and in the Water a piece of Copper or Braſs, if this be the likelieſt to be employed in *Counterfeiting* the *Coin*, and obſerving likewiſe the Difference between thoſe Weights. For, the leſſer of theſe Differences being Subtracted from the greater, the remains will ſhew, how much the true piece of Coin will out-weigh the other in the Water, and conſequently, if ſo many Grains, as this reſidue amounts to, being Added to the Weight of the Lighter Metal, do make a ſufficiently manifeſt Depreſſion of it below the Mark it would ſtay at without that Addition, one may probably Conclude, that the Difference between a True and Counterfeit piece of Coin propoſed will be diſcoverable by the *Inſtrument*. But it may be Expedient, for thoſe that have frequent Occaſions to Examine Various ſorts of Coin, to have a ſeveral *Inſtrument* adjuſted for each of them, to ſave themſelves ſome Pains and Trouble.

With this *Inſtrument*, Pure *Tin* may be certainly Diſtinguiſh'd from ſuch as is Adulterated. For, as *Gold*, being the Heavieſt of Metals, cannot be Allay'd by any other that will not Depreſs our *Inſtrument* leſs than *Gold* can do; ſo *Tin*, being the Lighteſt of Metals, cannot be mixed with any other that will not Sink it Lower than unmix'd *Tin*, (ſtill ſuppoſing the Weight to be the ſame in the Air.)

After the ſame manner may *Pewter* be Compared and Examin'd. For, having once obſerv'd how much the *Inſtrument* is Depreſt by a piece of two, three, or four Drams, or even an Ounce Weight of *Pewter*, which is known to be good, and to contain ſuch a proportion of Lead in reference to the Tin, if you load the *Inſtrument* with an equally Heavy piece of any other Maſs of *Pewter* propounded, if the *Inſtrument* Sink deeper, 'twill be a ſign that the former Proportion of Lead may be very probably argued to exceed in the mixture; I ſay probably, becauſe perhaps 'tis poſſible to Embaſe *Pewter* by Mixing not only Lead, but other Mineral Subſtances, whoſe *Specifick Gravity* is not well known: But yet I ſay very probably, becauſe the Addition of too much Lead is the moſt Gainful way of Adulterating *Pewter*.

This *Inſtrument* may alſo aſſiſt us, to make ſuch an Eſtimate as will not much Deceive us of the *Fineneſs* of *Gold* and its differing *Allays* with Silver, or ſome other determinate Metal.

In order to this, the *Inſtrument* may be fitted to ſink to the tip of the Pipe with ſome determinate weight of the *Fineſt Gold*, as of 24 *Carats*, as they call that which is moſt *Pure* and *Fine*. But 'twill be convenient, that this Metal, in the Air, be juſt an Ounce, or half an Ounce, or ſome ſuch Determinate Weight, that is commodiouſly Diviſible into many aliquot Parts. Then you may make a Mixture that contains a known proportion of the Metal wherewith you *Allay* the *Gold*; as if it hold 19 or 15 parts of *Gold*, and one of *Silver*; and, letting the *Inſtrument* ſettle in the Water, Mark the place where the Surface of the Water cuts the *Stem* or *Pipe*. And then putting in another Mixture, wherein the Silver has a new and greater Proportion to the *Gold*; as if the former be an 18*th* or a 14*th* part of the Latter, you may Obſerve, how much leſs than before this Depreſſes the *Inſtrument*, and ſo you may proceed with as many Mixtures or Degrees of *Allays* as you think fit, or can be Diſtinguiſh'd conveniently on the *Stem*; being always careful, that, whatever be the Proportion of the two Ingredients, the Weight of the Maſs in the Air be juſt the ſame with that of the *Pure Gold*, which we may have lately ſuppoſed to be an Ounce, or half an Ounce.

By the ſame Method may be Examined the differing *Allays* of *Pure Silver*, upon the Admixture of ſuch and ſuch determinate Proportions of Copper, or any other Metal Lighter in Specie than *Silver*; and by the ſame way, with a ſlight Variation, 'twill not be difficult to Eſtimate, how much divers *Coins*, whether of *Silver* or *Gold*, are more or leſs Embaſ'd by the known Ignobler Metal that is mix'd in the piece propoſed. Theſe Eſtimates (which may be made without much Trouble) will come nearer the Truth, not only than the Eſtimates wont to be made by the *Touch-Stone*, but perhaps too, than ſome of thoſe that divers make with Trouble, Inconvenience, and Charge.

It may be alſo Employ'd to Examine other *Mixtures* beſides *Allay'd Coins*, and that if the *Inſtrument* be adjuſted to an Ounce, for inſtance, of Pure Copper, it may help Men to make an Eſtimate of the *Allay* of *Tin*, or the Quantity of it that is often times added to *Copper*, to make different Sorts of *Bell Metal*, and of thoſe *Metalline Specula*, whether Plain or Concave, that are call'd Steel Glaſſes, as alſo of Solders conſiſting of certain Proportions of Silver and Braſs, or Copper; in all which, and divers others, the Diſcovery of the Proportion of the Ingredients may, on ſome Occaſions, be Uſeful to Tradeſmen, as well as deſirable to *Virtuoſi*. And though I have Obſerved, that by Mixture, Tin and Copper acquire a *Specifick Gravity* ſomewhat differing from what their Ingredients promiſe; yet, ſince the *Inſtrument* is to be fitted for ſuch Eſtimates, not by Calculation, but by Trials, the Eſtimates may be made near enough to the Truth.

Further Conſidered; by . . . n. 116. p. 553.

2. Long ſince I took Notice, how Light and Silver-like the Pewter was which deſcended to us; but, as ſoon as, to follow the Faſhion, we Changed it, the Weight and the very Colour was altered; and is in every Change more and more Embaſed. And, if our *Silver-Smiths* hold on their Degrading Mixtures, I ſhall Queſtion, whether our Silver-Plate may not ſhortly come down to approach our Fore-Fathers Pewter: I mean, in the Country, where 'tis never or ſeldom Tried.

The Weight of Water in Water. by Mr. Boyle. n. 50 p. 6001.

III. A Glaſs Bubble, of about the Bigneſs of a Pullet's Egg, was purpoſely blown at the Flame of a Lamp, with a ſomewhat long Stem turn'd up at the End, that it might the more conveniently be broken off. This Bubble being very well heated to Rarifie the Air, and thereby drive out a good part of it, was nimbly Sealed at the End, and, by the help of the Figure of the Stem, was by a convenient Weight of Lead depreſſed under Water, the Lead and Glaſs being tied by a String to one Scale of a good Ballance, in whoſe other there was put ſo much Weight, as ſufficed to Counterpoiſe the Bubble, as it hung freely in the midſt of the Water. Then with a long Iron Forceps I carefully broke off the Seal'd End of the Bubble under Water, ſo as no Bubble of Air appear'd to Emerge or Eſcape through the Water, but the Liquor by the Weight of the Atmoſphere ſprung into the unrepleniſh'd part of the Glaſs Bubble, and fill'd the whole Cavity about half full and preſently; as I foretold, the Bubble ſubſided, and made the Scale it was faſtened to Preponderate ſo much, that there needed 4 Drachms and 38 Grains to reduce the Ballance to an *Æquilibrium*. Then, taking out the Bubble with the Water in't, we did, by the help of the Flame of a Candle, warily applied, drive out the Water (which otherwiſe is not eaſily excluded at a very narrow Stem) into a Glaſs Counterpoiſed before; and we found it, as we expected, to Weigh about 4 Drachms and 30 Grains, beſides ſome little that remained in the Egg, and ſome ſmall matter that may have been Rarify'd into Vapours, which added to the Piece of Glaſs that was broken off under Water and loſt there, might very well amount to 7 or 8 Grains. By which it appears not only, that Water hath ſome Weight in Water, but then it Weighs very near or altogether as much in Water, as the ſelf ſame Portion of a Liquor would Weigh in the Air. We Repeated the Experiment with another Seal'd Bubble as big as a great Hen Egg, with like Succeſs.

The Preſſure of Water in great Depths; by a Perſon of Honour. n. 193. p. 504.

IV. *Apr.* 7. 1680. Being off of *Pantalara* near *Sicily* in a Calm, I let down a *Bottle* 70 *Fathom*, ſtopp'd with an Excellent good tender Cork, well Fitted, and the Cork came up in the Bottle $\frac{3}{4}$ full of Salt Water. The Bottle was again fitted with an Excellent good Cork, but of a Woodineſs or Hardneſs as ſome Corks are, with the which, being let down in like manner, the Cork continued in its Place, but as it were Bruiſed, and the Bottle as before, about $\frac{3}{4}$ full of Salt Water: Whereupon I took a good Ox Bladder, and bound it fourfold over the Month of the Bottle without any Cork at all, only I put a piece of Leather to keep the Glaſs from cutting the Bladder; and ſo ordered, it was let down as before, but taken up without any Water, or the leaſt Moiſture in it.

May 18. 1680. Being in a Stark Calm ſome Leagues diſtant from the Coaſt of *South Spain*, off the great Hills of *Granada*, we took a Bottle and clapt a Leather on the mouth of it, tying over that a ſingle part of the Bladder, the which we let down 75 *Fathom*, but it came up again Entire; We then made a Hole in the Leather, about the Bigneſs of a large Pea, and let the ſame down again 75 *Fathom*, but it came up perforated in the Vacant place where the Leather had the Hole in it, and almoſt full of Water; we then bound over another part of the Bladder ſingle, and let it down but 30 *Fathom*, but it came up whole and entire; whereupon immediately we let it down 50 *Fathom*, but it came up broke and full of Water. Then, we again fitted the Bottle with the ſaid perforated piece of Leather and a Double Bladder; and let it down 50 *Fathom*, but it again came up Entire: So again, immediately we let it down 75 *Fathom*, but then it came up broken and full of Water.

June 24. 1680. Being in $39\frac{1}{4}$ Degrees of *Latitude*, and by the Ships Account 150 Leagues Weſtward of *Portugal*, I cauſed a *Florence Flask* to be well ſtopped with a Bladder over the Mouth of it, and Lower'd it down 30 *Fathom*, but it was taken up broken. Whereupon, imagining that the roughneſs of the Leads Halling ſo tender a Body ſo violently through the Water might be the breaking thereof, I cauſed another *Flask* in like manner to be fitted, and cloſe by it I tied likewiſe another *Flask*, ſo as to be born with the Mouth downwards, as were the other, but which was not Stopp'd; and theſe I cauſed to be taken up when they had been but 10 *Fathom* under Water, and found them both Entire; but the Open *Flask* almoſt full of Water; the which being emtied, were both let down again and taken up at 20 *Fathom*, when the Open *Flask* was entire, tho' full of Water, but the Other broken to pieces.

By Dr. Oliver. n. 204. p. 908.

Jun. 8. 1693. In the *Bay* of *Biſcay*, when we had 100 *Fathom* of Water, we took a Quart Glaſs Bottle ſtopt with a large Cork: And, Faſtening it to our Plumbing-Rope with a Lead at the End, we ſunk it to the *Bottom* of the Sea, which as ſoon as we perceived, we drew it up again, and found the Cork quite preſſed through the Neck of the Bottle into its Cavity, and the Bottle full of Salt Sea-Water. We Repeated our *Experiment* with another Bottle and Cork in the ſame manner as before, but, the Cork being not ſound, the Sea-Water ſoaked through it, and the Bottle was half full of Water; ſo the Cork remained in the Mouth of the Bottle, not preſſed down at all. We repeated our *Experiment* a third time in 90 *Fathom* of Water, with a very ſound Cork, and much larger than the Mouth of the Bottle. We beat it in with a Hammer as far as it would go, leaving about an Inch of the Cork above the Mouth of the Bottle. The Cork at this *Trial* was preſſed down only into the Neck, and became Level with the Mouth of the Bottle: But I really believe, had we had 10 or 20 *Fathoms* of Water more, it would have ſucceeded as at our firſt *Tryal*.

The Weight of divers Bodies try'd by the Direction of the Phil. Society at Oxford. n. 169. p. 926. Mar. An. 1685.

V. 1. The following Bodies were poured gently into a Veſſel of well ſeaſoned Oak, whoſe Concave was an exact *Cubick Foot*. Thoſe in the Twelve firſt Experiments were Weighed in *Scales* turning with two Ounces, but the laſt Seven were Weighed in *Scales* turning with one Ounce. The *Pounds* and *Ounces* here mentioned are *Averdupois*.

		℔.	℥.
1	A Foot of *Wheat* (worth 6 *s.* a Buſhel.)	47	8
2	*Wheat* of the beſt ſort (worth 6 *s.* 4 *d.* a Buſhel.	48	4
3	The ſame ſort of *Wheat* meaſured the ſecond time.	48	2
	Both ſorts were *Red Lammas Wheat* of the laſt Year		
4	*White Oats* of the laſt Year.	29	8
	The beſt ſort of *Oats* were 2*d.* in a Buſhel better than theſe.		
5	*Blue Peaſe* (of the laſt Year,) and much Worm-eaten.	49	12
6	*White Peaſe* of the laſt Year but one.	50	8
7	*Barley* of the laſt Year: (the beſt ſort ſells for 1 *s.* 6 *d.* in a Quarter more than this;)	41	2
8	*Malt* of the laſt Year's *Barley*, made two Months before.	30	4
9	Field *Beans* of the laſt Year but one.	50	8
10	*Wheaten Meal* (unſifted.)	31	0
11	*Rye Meal* (unſifted.)	28	4
12	*Pump Water.*	62	8
13	*Bay Salt.*	54	1
14	White Sea *Salt.*	43	12
15	*Sand.*	85	4
16	*Newcaſtle Coal.*	67	12
17	*Pit Coal* from *Wedneſbury* 63, but this is very uncertain in the filling the Interſtices between the greater pieces.	63	9
18	*Gravel.*	109	5
19	Wood-*Aſhes.*	58	5

Pump-

2 *The* Specifick Gravities *of ſeveral Bodies, by the Direction of the* Phil. Society *at* Oxford. *Ib. p.* 927.

Pump-Water	1000
Fir Dry	546
Elm Dry.	600
Cedar Dry.	613
Walnut-Tree Dry.	631
Crab-Tree meanly Dry	765
Aſh meanly Dry, and of the Out-ſide lax part of the Tree.	734
Aſh more Dry, about the Heart.	845
Maple Dry.	755
Yew of a Knot or Root 16 Years Old.	760
Beech meanly Dry.	854
Oak very Dry almoſt Worm-eaten.	753
Oak of the Out-ſide ſappy part Fell'd a Year ſince.	870
Oak Dry; but of a very ſound cloſe Texture.	929
The *Same* tryed another Time.	932
Logwood.	913
Claret.	993
Moil Cyder, not Clear.	1017
Sea-Water ſettled Clear.	1028
College Plain *Ale* the ſame.	1028
Urine.	1030
Milk.	1031
Box the ſame.	1031
Redwood the ſame.	1031
Sack.	1033
Beer Vinegar.	1034
Pitch.	1150
Pit-Coal of *Staffordſhire.*	1240
Speckled Wood of *Virginia*	1313
Lignum Vitæ.	1327
Stone Bottle.	1777
Ivory.	1826
Alabaſter.	1872
Brick.	1979
Heddington Stone, the Soft Lax kind.	2029
Burford Stone, an old Dry Piece.	2049
Paving Stone, a hard Sort from about *Blaidon.*	2460
Flint.	2542
Glaſs of a Quart Bottle.	2666
Black Italian Marble.	2704
White Italian Marble tryed twice.	2707
White Italian Marble of another ſort, of a viſible Cloſer Texture.	2718
Block *Tin.*	7321
Copper.	8843
Lead.	11345
Quick Silver.	14019
Quick Silver.	13593

The laſt *Experiment* was tried with another quantity of *Quick-Silver*, which had been uſed in Water in the preceeding *Experiment* : However, I rather truſt the laſt, for that I found a ſmall miſtake (though herein the Calculation allowed for) in the Weight of the *Glaſs* containing the *Quick-Silver*, in the Trial before.

The *Solids* here mentioned, were Examined *Hydroſtatically* by Weighing them in Air and Water; but the *Fluids*, by Weighing an equal Portion of each in a Glaſs holding about a Quart. The Numbers ſhew the Proportion of *Gravity* of equal Portions of theſe Bodies : But if of theſe Bodies we take Portions equally Heavy, their *Magnitudes* will be *reciprocally* proportional to their correſpondent Numbers : *e. g.* a Cubic Foot of *Water* is to a Cubic Foot of *Alabaſter* in *Gravity* as 1000 to 1872; but a Pound Weight of *Water*, is to a pound Weight of *Alabaſter* in *Magnitude* as, 1872 to 1000. So that, knowing by the former Table, the Weight of a Cubic Foot of *Water*, and by this proportion in *Gravity* betwixt *Water* and *Alabaſter* ; we may by the Rule of Three find the *Weight* of a Cubic Foot of *Alabaſter*, and ſo of any other of theſe Bodies ; or we may know their *Magnitude* by knowing their *Gravity*. So that, an irregular piece or quantity of theſe Bodies being offered, 'tis but *Weighing* them, and we may know their juſt *Magnitude* without farther trouble.

3.
By Mr. J. C.
n. 199. p 904.
Apr. An. 1693

Pump-Water	1000
Cork.	237
Saſſafras Wood.	482
Juniper Wood (Dry.)	556
Plum-Tree (Dry.)	663
Maſtic.	849
Santalum Citrinum.	809
Santalum Album.	1041
Santalum Rubrum.	1128
Ebony.	1177
Lignum Rhodium.	1125
Lignum Aſphaltum.	1179
Aloes.	1177
Succinum Pellucidum.	1065
Succinum Pingue	1087
Jet.	1238
The top part of a *Rhinoceros Horn.*	1242
The Top part of an *Ox Horn.*	1840
The (Blade) *Bone* of an *Ox.*	1656
An *Humane Calculus.*	1240
Another *Calculus Humanus.*	1433
Another *Calculus.*	1664
Brimſtone, ſuch as commonly Sold.	1811
Borax	1720

A

A Spotted *Factitious Marble.*	1842
A *Gally-Pot.*	1928
Oyſter Shell.	2092
Murex Shell.	2590
Lapis Manati.	2270
Selenitis.	2322
Wood *Petrified* in *Lough Neagh.*	2341
Onyx Stone.	2510
Turcois-Stone.	2508
Engliſh Agat.	2512
Grammatias Lapis.	2515
A *Cornelian.*	2568
Corallachates.	2605
Talc.	2657
Coral.	2689
Hyacinth (Spurious.)	2631
Jaſper (Spurious.)	2666
A Pellucide *Pebble.*	2641
Rock Cryſtal.	2659
Cryſtallum Diſdiaclaſticum.	2704
A Red *Paſte.*	2842
Lapis Nephriticus.	2894
Lapis Amiantus from *Wales.*	2913
Lapis Lazuli.	3054
An *Hone.*	3288
Sardachates.	3598
A *Granat.*	3978
A Golden *Marcaſite.*	4589
A Blue *Slate* with ſhining Particles.	3500
A *Mineral Stone,* yielding 1 part in 160 *Metal.*	2650
The *Metal* thence Extracted.	8500
The (reputed) *Silver Ore* of *Wales.*	7464
The *Metal* thence Extracted.	11087
Biſmuth.	9859
Spelter.	7065
Spelter Solder.	8362
Iron of a Key.	7643
Steel.	7852
Caſt *Braſs.*	8100
Wrought *Braſs.*	8280
Hammer'd *Braſs.*	8349
A Falſe *Guinea.*	9075
A True *Guinea.*	18888
Sterling Silver.	10535
A *Braſs* Half Crown.	9468

Electrum a *Britiſh Coin.*	12071
A *Gold Coin* of *Barbary.*	17548
A *Gold Medal* from *Morocco.*	18420
A *Mentz Gold Ducat.*	18261
A *Gold Coin* of *Alexander's.*	18893
A *Gold Medal* of Queen *Mary's.*	19100
A *Gold Medal* of Queen *Elizabeth's.*	19125
A *Medal* eſteemed to be near *Fine Gold.*	19636

The Different Weight of ſeveral Liquors in Winter *and* Summer; *by M.* Homberg. n. 262. p. 530. *Vid, Sup.* §. I. 2. Mar. An. 1700.

VI. M. *Homberg*, has given us the following Table of the *Various Weights* of ſome more Uſual Liquors in the *Coldeſt* Time, and in the *Hotteſt*.

	In Summer.			*In Winter.*		
	℥	ʒ	gr.	℥	ʒ	gr
The *Aræometer* full of *Mercury.*	11	0	6	11	0	32
Oil of *Tartar.*	1	3	8	1	3	31
Spirit of *Urine.*	1	0	32	1	0	43
Oil of *Vitirol.*	1	3	58	1	4	4
Spirit of *Nitre.*	1	1	40	1	1	70
Spirit of *Salt.*	1	0	39	1	0	47
Aqua Fortis.	1	1	38	1	1	55
Vinegar.	0	7	55	0	7	60
Spirit of Wine.	0	6	47	0	6	61
River Water	0	7	53	0	7	57
Diſtilled Water.	0	7	50	0	7	54
This Empty *Aræometer* Weighs				0	1	28

Experiments about the Superficial Figures of Fluids, *eſpecially Liquors Contiguous to other Liquors. and their Reflective Powers; by Mr.* Boyle. n. 131. p. 775. Jan. An. 1676.

VII. 1. Having poured a ſtrongly *Alkalizat Menſtruum* (I uſed that made of *Fix'd Nitre*, diſſolved by the moiſture of a Cellar) into a Pipe of Glaſs, ſealed at one end, and not full a quarter of an Inch in Bore; that the Cavity, which in a greater depth would ſeem leſs deep, might be the more conſpicuous: We gently poured on it ſome highly Dephlegm'd *Spirit* of *Wine*, which we knew would not Mix with it, but ſwim above it, and preſently, as we had gueſs'd, we found the *Figure* of the *Surface* of the Lower Liquor Changed, and the Cavity quite deſtroyed; the *Surface* that ſeemed as it were, common to the two *Contiguous Liquors*, appearing Flat or Horizontal. And ſuch a Level Superficies we had, by putting theſe two *Liquors* together in a much Wider Glaſs.

2. We found alſo, that by Employing *Oyl* of *Turpentine*, inſtead of the *Spirit* of *Wine*, the Liquor did almoſt totally Loſe its Cavity.

3. But, if, inſtead of *Deliquated Tartar*, we put common Water into the Pipe, we found this Liquor to retain its Concave Surface, though we put to it ſome *Oil* of *Turpentine*, and left it to reſt upon the Water a good while.

4. Having provided ſome pure *Oil* of the *Gum* of *Guaiacum*, and poured a little of it into a ſlender Pipe, we found the Upper Superficies of it to be Concave, almoſt, if not altogether, like that which Water would have had in the ſame Pipe. But, when I put a little Water upon this *Oil*, it preſently changed the Figure of its Surface, which became viſibly, though not very much, Protuberant or Convex.

5. Having put ſome *Oil* of *Tartar* into the ſlender Pipe, and put ſome Drops of the *Oil* of *Guaiacum* to it, we found, that this Liquor did not manifeſtly alter the Concave Figure of the Surface of the Liquor *Alcali*, as the *Oil* of *Turpentine* had done: And, having for Curioſities ſake warily poured a little Water upon the *Oil* of *Guaiacum*, I found, as I had reaſon to ſuſpect, that the upper Superficies of it Changed preſently from a Concave Figure to a Convex, ſo that this *Oil* in the midſt of the other two Liquors appeared like a little red Cylinder; which, inſtead of having Circular Baſes, was Protuberant at both ends; but more at that which touched the *Oil* of *Tartar*.

6. I put ſome *Eſſential Oil* (as *Chymiſts* call it) of *Cloves* into a new ſlender Pipe, and, having obſerved it to be ſomewhat Concave at the Top, where it was Contiguous to the Air, we cauſed a little Common *Water* (perhaps a quarter of a Spoonful or leſs) to be put to it, and found, as we expected, the Surface of this *Oil* alſo to be Tumid. And in regard this Liquor, as well as the forementioned *Oil* of *Guaiacum*, though it were ſo heavy as to ſink into Water, would not do ſo in *Deliquated Salt* of *Tartar*, we did, into another ſlender Pipe, put firſt ſome of this laſt named Liquor, then ſome of the *Aromatic Oil*, and laſtly, a little Common Water; by which means we found, that the Little *Cylinder* of *Oil*, did, like that of the *Oil* of *Guiacum*, appear Convex at both ends, but, was unlike it in one Circumſtance, that the *Oil* of *Cloves* appear'd more Convex at the upper end where 'twas Contiguous to the *Water*, than at the Lower, that leaned upon the Surface of the *Oil* of *Tartar*.

7. Having taken a little ſlender Glaſs, that was much longer, but of the like Bore with the former, we put into it a ſmall quantity of *Quick-Silver*, and, having taken notice how the Upper Superficies ſwelled in the middle above the Level of the Parts where it touched the Glaſs, we poured ſome Water upon it, and found a Manifeſt and Conſiderable Depreſſion of the Surface, though the Protuberance were not quite Suppreſſed.

8. This *Phænomenon*, having been for greater ſecurity ſeveral times Repeated, ſometimes it ſeem'd, that when the *Aqueous Cylinder* was much longer, the Depreſſion of the *Mercurial* Surface was ſomewhat greater. But this did not ſo conſtantly happen; But we often obſerved, that, though a very little Water ſufficed by its Contact to make, in the Judgment of the Eye, a manifeſt Abatement of the Protuberance of the *Quick-Silver*; yet it had not the ſame effect on that Ponderous Fluid, that it had, when being Increaſed almoſt as high as the length of the Pipe would permit, a greater Weight of it was

Incumbent on the *Mercury*, for then I manifeſtly perceived, and ſhewed to others, that the Surface of the *Quick Silver* being Depreſſed almoſt to a Level in thoſe Parts of it that were near rhe inſide of the Glaſs, there was about the Middle of the Surface an Elevation of *Mercurial Matter*, that appeared to be rather more than a half Globe, and was to the Height of its full Semidiameter, raiſed above the reſt of the *Mercurial* Surface, and in that State it continued, as long as I thought fit to let it do ſo. And, leſt this Tryal ſhould Impoſe upon me, I cauſed it to be more than once Repeated; and, the better to confirm it, I afterwards cauſed the Incumbent *Water* to be Little by little ſuck'd up, and found as I expected, that, when the Incumbent *Water* began to be too much ſhorten'd, the little Teat or Segment of a Sphere, lately mentioned, began to be ſomewhat Flatned; and Subſided more and more as the *Water* was further taken off.

9. Having conveyed into one of our *Pneumatical Receivers*, a Couple of ſuch ſlender Pipes as have been already deſcribed, one of them Furniſhed with *Common Water*, and the other with *Quick Silver*, we cauſed the Common Air to be diligently Pump'd out, without obſerving any Senſible Change in the Concave Figure of the *Water*: But, as for the *Quick Silver*, I knew not what to conclude about it. For, having Repeated the Tryal twice or thrice, the *Mercury* ſometimes ſeem'd manifeſtly to Swell, to be more Protuberant upon the Exhauſtation of the *Receiver*, than when it was put in, eſpecially when its Figure was attentively Viewed, and the External Air, that was Pump'd out but ſlowly, was ſuffer'd to Re-enter with all convenient Celerity. But that which yet kept me doubtful was, that I obſerved, That, upon the diligent withdrawing of the Air's Preſſure upon the *Quick Silver*, there diſcloſed themſelves ſome little Bubbles, which, I fear'd, we had not been able to free it altogether from, and which might be ſuſpected to have ſome Intereſt in the *Phænomenon*. We alſo convey'd into our *Receiver*, a clear *Cymical Oil* that was heavier than *Water*, and, whilſt it was Contiguous to it, had not a Concave but a Convex Surface, and, having placed the Pipe furniſh'd with both Liquors in the *Pneumatical Receiver*, we Pump'd out the Air, without finding that the *Oil* ſenſibly altered its Protuberant Surface, as neither did the Water loſe the Concave Figure of it's upper Surface.

10. I took *Fix'd Nitre*, (or which is Analogous to it, *Salt* of *Tartar*) reſolved *per Deliquium* into a Tranſparent Liquor, and having filled a clear Viol half full with this, I poured on it a convenient quantity of *Vinous Spirit* exactly *Rectified*, that there might be no Phlegm to occaſion an Union betwixt the two Liquors, which ought, as ours did, to retain Diſtinct Surfaces, and ſpeedily regain them, though the Glaſs were well ſhaken. Then, having found by a Tryal formerly mentioned, that Common *Oil* of *Turpentine*, if employed in a Competent Quantity, will not totally (and much leſs will readily) Diſſolve in *Spirit* of *Wine*, and alſo having Obſerved (what may ſeem ſomewhat ſtrange) that if this *Spirit* of *Wine* be exquiſitely Dephlegm'd, the *Oil*, though a *Chymical* One, will not Swim on it, but Sink in it; I warily let fall ſome drops of *Oil* into the *Spirit*, and had the pleaſure to ſee, as I expected, that they fell towards the bottom of the Glaſs, till their Deſcent was ſtop'd by

the Horizontal (for it was not Concave) Surface of the *Akhalizat Liquor* of *Fix'd Nitre*. And, becauſe my deſign was chiefly to obſerve the Superficial Figure of a Fluid Encompaſſed by other Fluids without touching any Solid Body, I ſhall here take notice of the chief *Phænomena* that were produced of that kind, without Staying to Enquire into the Cauſes or the Conſequences of them.

1. If the *O'y Drops* were but ſmall, they ſeem'd to the Eye exactly enough *Spherical*. For, the *Oil* differing but very little in the *Specifick Gravity* from the *Spirit* of *Wine*, the Drops did but juſt touch the Surface of the ſubjacent *Alcali*; and, the ſame Drops being but ſmall, their own Weight was not great enough viſibly to Depreſs them, and hinder that Roundneſs which the Preſſure of the Ambient *Spirit*, or their own Viſcoſity, endeavoured to give them.

2. If an Aggregate of Drops were conſiderably bigger than thoſe newly mentioned, as if it had about a third Part of an Inch in Diameter, it would then manifeſtly lean upon the *Alcalizat Liquor* as upon a Floor, and appear ſomewhat *Elliptical*, (for ſome little part of the bottom was a Plain;) the Weight of the upper parts Depreſſing the Drops, and making the Horizontal Diameter ſomewhat longer than the Tranſverſe.

3. If a yet greater Portion of *Oil* were let fall upon the Heavy Liquor, it would for a pretty while appear in the form of a ſomewhat Imperfect *Hemiſphere*, or ſome other large Section of a *Sphere*, the lower part being cut off; (as if a Globe were divided by a Plain) by the Horizontal Surface of the *Deliquated Salt*.

4. But, if the Quantity of *Oil* were not too great, 'twas pretty to obſerve, that, though at firſt putting in, it did perhaps ſpread itſelf over the Subjacent Liquor, and lie as it were flat upon it; yet, by little and little, (for 'twas but ſlowly) it would by the Action of the Ambient, concuring with it's own Tenacity, be raiſed above the Surface of the Fluid *Nitre*, and be Reduced to the Figure, either of half a Globe, or of a greater Segment of a Globe, or even of an Imperfect Ellipſis, according to the Bulk or Weight of the *Oil*.

5. Though theſe Globules, or Portions of *Oil*, did oftentimes readily mingle, when they touched one another, yet divers times alſo we obſerved, that, having warily approach'd them, we were able to make them touch without Mingling, inſomuch that we have with pleaſure made them ſo far bear againſt one another's Surfaces, as manifeſtly to preſs them inwards, though being parted they would preſently Reſume their former Figure. But, in caſe any of theſe *Oily Portions* came by a more Preſſing contact to be United, they would then alter the Figures they had whilſt ſeparate, and take another ſuitable to the Bulk of the Aggregate.

6. When a large Portion of *Oil* reſted upon the *Saline Liquors*, if then the Ambient *Spirit* was moderately and warily Agitated, 'twas not unpleaſant to obſerve the various Figurations, which the Convex and Protuberant part of the mutilated Globe would be put into by the Shakes, without any Viſible Solution of Continuity, or conſiderable Motion of the whole Body, which would very quickly recover its former Figure. Though, if the Agitation were too

ſtrong, ſome Portions would be quite broken off, and preſently turned into Little Globes.

11. I tried to produce another *Phænomenon*, that would not have been unpleaſant, by putting together in a ſomewhat large Veſſel, with other Liquors, two *Oils*, (whereof one if I miſtake not, was from *Turpentine*) which firſt by reaſon of the Oleaginous Nature wherein they agreed, might exactly mingle and make a Compounded Liquor; and then, by reaſon of there being one Heavier, and the other Lighter in *Specie* than Water, might by this Liquor be again ſeparated, and Include betwixt them the Liquor that had Divided them. But I found that the *Oils* being once united would not be eaſily parted, but according to the Prevalency of the Lighter or Heavier Ingredient, in the mixture, the Compounded *Oil*, would almoſt totally either Emerge to the top of the Water, or Lie beneath at the Bottom of it; I ſay, almoſt totally, becauſe ſome Part of rhe *Oil*, which was not perhaps all uniformly Mix'd, did not keep in a Body with the reſt; but either was ſeparated from the Maſs in the form of Globules, or elſe ſticking to the ſide of the Glaſs, had the other part of its Superficies, which was Contiguous to the Water, very *Variouſly Figured*, according as the Bulk and Degree of Gravity of the adhering *Oil*, and other Circumſtances happen'd to Determine.

Theſe are ſome of the *Phænomena* I obſerved in *Oil* of *Turpentine*, when 'twas invironed only with Fluids; but, if it were permitted to be Contiguous to the Inſide of the Glaſs, and ſo to faſten part of its Surface to a Solid, the greater part of the Surface, which remained expoſed to one or both of the Contiguous Liquors, would partly by their Action, and partly by the Gravity of the *Oil* itſelf, be put into *Figures* ſo *Various*, and ſometimes ſo *Extravagant*, that 'twas much more pleaſant to behold them, than it would be eaſy to deſcribe them.

12. *Confining Fluids* may have Diſtinct Surfaces, without having, at leaſt in many Poſitions, *Refractions* differing enough, or *Reflexions* ſtrong enough, to make the Plain that Diſterminates them, obvious to the Eye. Thus, when the *Oil* of *Tartar*, or *Nitrous Alcali*, that I employed, happen'd to be very Clear and Colourleſs, I have more than once made highly *Rectified Spirit* of *Wine* Float upon it ſo, that in moſt Poſitions the Vial ſeem'd to have in it but one uniform Liquor; the Plain that divided the two Fluids being unapt to be diſcerned, but in a Poſition wherein the Rays of Light paſſing thence to the Eye, fell very Obliquely on it; and indeed, when there was no little Duſt or other Feculency, ſwimming upon the Surface of the *Oil* of *Tartar*; I had ſometimes much ado to convince Ordinary Spectators, that the Vial, in two diſtinct Regions of it, contained two *Unſociable Liquors*.

13. We took a *Deliquated Alcali*, made of *Nitre* and *Tartar*, and deeply Tinged with *Cochineel*; and, that the Liquors might not only be Heterogeneous, but as differing in *Gravity* and *Denſity*, as we could make them, we poured on it a peculiar kind of *Oil* lighter than *Spirit* of *Wine*, and holding the Plain where the two Liquors were Contiguous in a convenient Poſition, in reſpect of the Light and the Eye, I obſerved it to make a ſtrangely vivid *Reflexion* of the Incident Beams of Light; So that this Phyſical Surface which

was flat, look'd almoſt, for 'twas not ſo *Specular*, like that of *Quick-Silver*; and, when I kept it 'till Night, and conſidered it by the Light of a Candle, the bright Figure of the Flame was ſtrongly Reflected almoſt as from a Cloſe Specular Body; which tempted me to ſuſpect, that there might be ſomething elſe than the bare ſmoothneſs of the Surface of the *Alkalizat Liquor* to produce ſo briſk a *Reflection*; and the rather, becauſe I did not Obſerve, that the remains of the ſame Tinged *Alcali*, which I kept in another Glaſs, nor a Portion of the ſame *Oil*, which I had alſo by me in a ſeparate Vial, did either of them afford ſo Vivid a *Reflection* from its Surface; tho' I did the leſs wonder at this, becauſe of the great Diſpoſition to *Reflect Light*, which I had formerly the Curioſity to Obſerve in the forementiоn'd *Oil*, when I joyned it with other Liquors. I ſhall add, that looking on this Liquor, as a Body which, though it have all the neceſſary Quantities of an *Oil*, does in regard of its Origin, and ſome Properties I have found in it, differ from common *Chymical Oils*; I was invited the more to obſerve its *Phenomena* in reference to *Reflection*, and I found among other Things (not pertinent to this Place,) *Firſt*, That the Confining Plain often mentioned between the Tinged *Alcali*, and this *Liquor*, did not appear Red itſelf, nor communicate that Colour to the Image of the Flame of a Candle Reflected from it. *Secondly*, That when I warily ſhook the Vial, which contained the two Liquors, the uppermoſt would be reduced into a ſeeming Froth, conſiſting of a great number of Imperfectly Globular Bodies, which after a while would make a kind of a Rude Phyſical Plain; which, though neither very Horizontal, nor ſenſibly Smooth, would at its upper Superficies, ſend back the *Incident Light* with more Briskneſs than one would expect; and, when the ſeeming Froth conſiſted of ſmaller Particles, theſe, when they were of a certain Size, and conveniently Placed, in reference to the Flame of a Candle and the Eye, would (as more than one Trial informed me) Reflect the *Incident Light* ſo many ways, and ſo viſibly, that they ſeemed, for multitude and ſplendor, like little Sparkling Corpuſcles of Poliſhed *Silver*; or almoſt like theſe Gliſtering ones, that appear when a clean Plate of *Copper* is firſt Immerſed into a much allayed Solution of good *Silver*, made in *Aqua Fortis*. *Thirdly*, That though pure *Spirit* of *Wine* be ſo thin a Lipuor, and our *Oil* is nevertheleſs, ſo Light as to Swim upon it, yet I found the Confining Surface very ſtrongly *Reflexive*.

I have alſo found, that ſome other *Eſſential Oils* (as *Chymiſts* call thoſe that are Diſtilled with Water in *Limbecks*) and particularly an Unſophiſticated *Oil* of *Lemons*, did with our *Tinged Alcali* afford moſt of the ſame Phænomena; but not ſo Briſk a *Reflection*: I ſay, moſt, chiefly becauſe with *Spirit* of *Wine*, their *Subtile Oils*, as I formerly noted, will readily be confounded: Though our Anomalous *Oil* be unſociable with it.

14. In Cold Weather we took *Eſſential Oil* of *Anniſeeds*, whoſe Property it n. 132. p. 799. is to Coagulate in ſuch Weather, and, having in a gentle warmth brought it to be Fluid, we poured it into a ſlender Vial more than half filled with *Common Water*, that had been alſo a little warmed, that the *Oil* might not be too haſtily reduced to its former State. This Oil being Lighter than ſo much *Water*, and, being poured on in a convenient Quantity, had its Upper Surface

ſomewhat Concave, as that of the Water was; but the Lower Surface, Surrounded by the Water, was very Convex, appearing almoſt (for it was not perfectly) of the Figure of a great Portion of a *Sphere*. This being done, the Vial was ſtopt, and ſuffered to reſt for ſome time in a Cold place, by which means the *Water* continuing Fluid as before, the *Oil* of *Anniſeeds* was, as I expected, found *Coagulated* in a Form approaching to that it had whilſt in a Fluid State; I ſay approaching, becauſe it was not eaſy to diſcern the exact Figure in the Vial I was fain to make uſe of: And I ſuſpected, that the *Oil* grown Conſiſtent was become leſs Convex than before. But 'twas worth Obſerving, how great a Difference there was between the dull *Reflection* it made when it was *Coagulated*, and the fine *Reflection* it had made whilſt 'twas a Liquor. The later of which *Reflections* brought into my mind, how Vivid the *Reflective Power* of ſome *Fluids* is in Compariſon of that of the Generality of *Solid Bodies*.

15. Having obſerved, That *Quick-Silver*, and Rectified *Oleum Petræ*, are, the Former of them the Heavieſt, and the Latter the Lighteſt, of all the Viſible Fluids that are yet known to me; I put ſome (Diſtill'd) *Quick Silver* into a ſmall Vial, and held it in ſuch a poſture, That the *Incident Light* was ſtrongly remitted to my Eye, I then ſlowly put to it ſome *Petroleum*, that being well Rectified was very clear, and obſerved, that, as this Liquor covered the *Quick Silver*, there was at the Imaginary Plain, where they both confined, a Brisker *Reflection* than the *Quick Silver* alone had given before. On this occaſion it will not be amiſs to take notice, that either the Surface of the Air itſelf, as thin and yielding a Fluid as it is, or the Surface of a Solid contiguous to included Air, or ſome interpoſed *Subtile Matter*, may *Reflect* the *Incident* Beams of *Light* more ſtrongly than moſt Men would expect. To this purpoſe, I remember, that a Curious Perſon having one Day brought me a couple of Rarities, which he told me were two pieces of a Solid, but tranſparent Body, that he had caſually found; in one of which there was a *Pearl*, Large, Round, and Orient, and in the other a leſs Perfect One: One of them was opened, and that which had appeared a *Pearl* was found to be but a Cavity, that contained no groſſer Subſtance than Air. And I have by me, a well ſhap'd piece of Glaſs of a good Thickneſs, with an Aerial Bubble in the middle, which by ſome Qualities particularly its Pear-like ſhape and Vivid *Reflection*, does not ill reſemble a fair, though not *Orient Pearl*. But, in ſuchlike Obſervations, the Poſition of the Eye, and that wherein the Body receives the Beams of Light, may be very conſiderable. For I have by me a Small Stone (with which I have puzzled a Skilful Jeweller to determine what kind of *Gem* it was) that being laid flat upon ones Hand, or a piece of Paper, and look'd on Directly downwards, looks almoſt like a piece of Common Glaſs, and is tranſparent: But, if the Eye be ſo Placed, that the *Incident* Beams of *Light*, by whoſe *Reflection* it's ſeen, fall with a convenient degree of Obliquity upon the Stone, it makes an exceeding pretty ſhew, ſometimes appearing like a fine *Opal*, and ſometimes not very unlike an *Orient Pearl*.

16. We made a competent Quantity of a *Reſinous* or *Gummous Subſtance*, that looked like High Coloured *Amber*, but was eaſy to melt. This we put into a deep round Glaſs with a wide Mouth, and held it by the Fire-ſide in a moderate Warmth, 'till it was brought into a Fluid State; then we Tranſferr'd it into one of our *Pneumatical Receivers*, where we preſumed, that this Temporary Liquor, would, as well as Liquors that are conſtantly ſuch, diſcloſe *Aerial Bubbles*, when the Preſſure of the Air was withdrawn from it; and accordingly having cauſed the Air to be Pumped out by degrees, we found, that ſtore of Bubbles appeared at the Top of the Liquor, and made there a copious Froth, many of them being by reaſon of the Viſcoſity of the Fluid, very large, and divers of them, becauſe of the Nature and Texture of it, and the Thinneſs of the Films, being adorned with the Colours of the Rainbow, whoſe Vividneſs made them pleaſant to behold, and ſuggeſted to us ſome *Optical Conſiderations*. But, notwithſtanding this Froth, I cauſed the Pumping to be continued, that thoſe Bubbles that had moſt of Common Air in them, and which therefore are wont to Riſe firſt, might get to the Top, and the Subſequent Bubbles might meet with more reſiſtance from the Liquor ſtill tending to grow Cold, and ſo might be the more Expanded, and yet kept from Emerging by the Concretion of the Reſinous Subſtance; and anſwerable to this we found, That, when this Subſtance had reſumed its conſiſtent form, there were intercepted, between the Upper and the Lower Surfaces of it, ſome Bubbles that were not ſmall, which yet had a conſiderable *Reflection*, notwithſtanding the ſmall Quantity of the groſſer Particles of the Air, that may be ſuppoſed to be contained in Bubbles ſo very much Expanded.

17 'Tis taken for granted, That the *Falling Drops* of *Rain* are *Spherical*; yet their Deſcent is ſo ſwift, that I fear 'tis rather *Suppoſed* then *Obſerved* that their Figure is *Spherical*; which will be the more queſtionable, if it be true, which is vulgarly thought, That *Hail* is but *Rain Frozen* in its paſſage through the *Air*. For 'tis evident, That the Grains of *Hail* are frequently of other Figures than truly Orbicular. But the *Surface* of *Water* may have Differing Figures, according as 'tis totally Encompaſs'd with Heterogeneous Fluids, or as 'tis only in ſome places Contiguous to one or more of them. In the former caſe we found it not ſo eaſy to make an Obſervation, becauſe, we know not of any two Liquors (ſetting *Mercury* aſide) that will not Mingle either with One Another, or with Water. We therefore catiuouſly convey'd into ſome *Chymical Oil* of *Cloves* ſome Portions of *Common Water* of differing Bigneſſes, taking care, as far as we could, that they might not touch one another; by which means the *Oil* being Tranſparent, and yet ſomewhat coloured, 'twas eaſy to obſerve, that the ſmaller Portions of *Water* were ſo near totally Invironed with the *Oil*, that they were reduced into almoſt Perfect *Globes*; Thoſe Portions, that were ſomewhat bigger (as about twice the bigneſs of a Pea,) would be of a Figure ſomewhat approaching to that of an *Ellipſis* (for 'twas not the ſame) and thoſe Portions that were yet ſomewhat Larger, tho' they ſeemed to be ſunk almoſt totally beneath the *Oil*; yet, they held to it by a ſmall Portion of themſelves, whoſe Surface was eaſily enough diſtinguiſhable from rhat of the *Oil*. Theſe larger Portions of *Immerſed Water* being

almoſt wholly Inviron'd with the other Liquor, were by it reduced into a Round Figure, which was ordinarily ſomewhat *Elliptical*, but more Depreſſed in the Middle than that Figure requires.

18. Having into a Slender Pipe, of that ſort that has been deſcribed before, put a little *Oil* of *Cloves*, and upon this ſome *Oil* of *Turpentine*, that ſo the *Water* might both above and beneath be touched by Heterogeneous Liquors, I obſerved not the *Oil* of *Cloves* to be very manifeſtly Tumid at the top, nor the Lower Surface of the *Oil* of *Turpentine* (for the Upper was Concave) to be very Convex: For ſomewhat Convex it was Downwards. And, from this 'twill be eaſy to conclude, the Figure of the Cylindrical Portion of *Water* intercepted between theſe two Oils.

19. I took *Oil* of *Anniſeeds*, thaw'd by a gentle warmth, and *Common Water*, and, having put them together in a convenient ſhap'd Glaſs, they were ſuffered to ſtand in a cold Place, 'till the *Oil* was coagulated; which done, it was Parted from the *Water*, and by the Roughneſs of its Superficies manifeſted, as I expected, that, when its Parts were no longer agitated and kept eaſily Diſplaceable, by the Subtile permeating matter, or whatever other Agent or Cauſe it were to which it owed its *Fluidity*, when the Contiguous *Water* grew unable to inflect, or otherwiſe Place them after the manner requiſite to conſtitute a *Smooth Surface*. And, what happen'd to that part of the *Oil's* Surface that was touched by the *Water*, happen'd alſo to that which was contiguous to the *Air*, ſave that the *Aſperity* of the laſt named Surface was differing from the other, which, whether it were an accidental or conſtant *Phænomenon*, farther Trial muſt determine. But I have often obſerved, That the Upper Surface of *Oil* of *Anniſeeds*, when this Liquor comes to be Coagulated by the Cold Air, was far enough from being *Smooth*, being variouſly Aſperated by many Flaky Particles, ſome of which lay with their broad, and others with their edged Parts upwards.

20. An Inequality and Ruggedneſs of Superficies I have alſo obſerved in *Water*, when, having covered it with *Chymical Oil* of *Juniper*, and expoſed it in very cold Weather, though the *Oil* continued *Fluid*, yet the *Water* being Frozen had no longer a *Smooth Superficies*, as whilſt in its Liquid ſtate 'twas contiguous to the *Oil*. And the like Inequality, and rather a greater, we obſerved in the Surface of *Water* Frozen, which had *Chymical Oil* of *Turpentine* Swimming over it, yet a no leſs, if not a much greater, Roughneſs may be oftentimes obſerved in the Surfaces of divers Liquors that abound with *Water*, when, thoſe Liquors being Frozen, their Surfaces have an immediate Contact with the Air. I ſhall here add, that, having purpoſely cauſed a Strong and Blood-red Decoction of the Soot of Wood to be expoſed in a large Glaſs in a very Cold Night, I was more pleaſed than ſurprized, to find in the Morning a Cake of Ice, that was curiouſly Figured, being full of large Flakes ſhap'd almoſt like the broad Blades of Daggers, but neatly Fringed at the Edges. But that which I chiefly mention theſe Figures for, is, that they ſeem to be as it were Imboſt, being both to the Eye and the Touch Raiſed above the Horizontal Plain or Level of the other Ice.

21. I have ſometimes obſerved the like *Phænomenon* in one and the ſame Liquor, and particularly not long ſince in Froſty Weather, on a Vial where I had long kept *Oil* of *Vitriol*, I perceived that the Cold had reduced far the greateſt part of the *Menſtruum* into a Conſiſtent Maſs, whoſe upper Surface was very Rugged and oddly Figur'd, though it lay covered all over with a pretty deal of high coloured Liquor, that was not Frozen or Coagulated, nor ſeem'd to be diſpoſed to be ſo, at leaſt, in that Degree of Cold.

22. This may be alſo obſerv'd in the beſt ſort of what the *Chymiſts* call *Regulus Martis Stellatus*, where the Figure of a Star, or a Figure ſomewhat like that of the Decoction of the Soot lately mentioned, will frequently appear Imboſt upon the Upper Superficies of the *Regulus*; and ſuch a Raiſed Figure I have ſeen on a Maſs of *Regulus* made of *Antimony* without *Mars*. But, if to thoſe two Bodies *Copper* be alſo Skillfully added, the Superficies will be often times adorned with new Figures according to the Circumſtances; tho' the moſt uſual I took notice of was that of a Net, that ſeemed to cover the Surface of the Compounded *Regulus*. But this is not ſo conſtant, but that I have by me a Maſs of the Conical Figure, conſiſting of two very Contiguous, but eaſily ſuperable Parts, whereof the Lowermoſt, which abounds more in *Metal*, hath its upper Surface covered with Round Protuberances, in Shape and Bigneſs not unlike to ſmall Peaſe cut in two; and theſe are ſo really Imboſt and Elevated above the reſt of the Superficies, that the other part of the Cone, which is of a more Scorious Nature, has in its Lower Surface, which exactly fits the Upper of the *Regulus*, Cavities, for Number, Shape and Bigneſs, anſwering to the Protuberances lately mentioned, which Argues that the *Regulus* cooled firſt with that Inequality of Surface we have deſcribed, and that the Lighter and more Recrementitious Subſtance, continuing longer Fluid, had thereby Opportunity to accommodate itſelf to the Superficial Figure of the *Regulus*, on which it firſt leaned, and afterwards Coagulated.

Why Bodies Diſſolved Swim in Menſtrua Specifically Lighter than themſelves: by Mr. Will. Molyneux. n. 181. p. 88. *May* An. 1686.

VIII. 1. My Brother, Mr. *Tho. Molyneux* (in the *Nouvelles de la Republique de Lettres*) has given this Reaſon for the *Phænomenon, viz.* That the *Internal Motion* of the Parts of the Liquor does keep up the Particles of the *Diſſolved Solid*, for they, being ſo very minute, are moveable by the leaſt Force Imaginable, and the Action of the *Particles* of the *Menſtruum* is ſufficient to Drive the Atomes of the *Diſſolved Solid Body* from place to place; and conſequently, notwithſtanding their *Gravity*, they do not *Sink in the Liquor Lighter than themſelves.*

But I conceive another Account may be given of this *Appearance*, and that the Prime Law of *Hydroſtaticks* is a little deficient. 'Tis true indeed, if we conſider only the *Specifick Gravity* of a Liquor, and the *Specifick Gravity* of a Solid Particle Floating therein, the forementioned Rule is exact; but in *Sinking* there is requiſite a *Separation* of the Parts of the Liquor by the *Sinking* Body; and there being a Natural Inclination in the Parts of all Liquors to *Union* ariſing from an Agreement or *Congruity* of their Parts, there is a Reſiſtance therein to any thing that Separates this Conjunction: Now, unleſs a Body have Weight enough to overcome this *Congruity* or Union of Parts, ſuch a Body will Float in a Liquor *Specifically Lighter* than itſelf. But that a Heavy

Body, as *Mercury* or *Iron*, may have its Parts reduced to that Minuteneſs that their *Gravity* or Tendency Downwards, is not ſtrong enough to Separate the *Cohesion* or Union of Parts of a Liquor, will be manifeſt, if we conſider, that the Reſiſtance made by the Medium to a *Falling* Body, is according to the Superficies of the Body: But, as the Body Decreaſes in Bulk, its Superficies does not proportionably Decreaſe; thus a Sphere of an Inch Diameter has not *eight* times leſs Superficies than a Sphere of two Inches Diameter, tho' it have *eight* times leſs bulk, and conſequently paſſing through a Medium, as ſuppoſe Air or Water, the Sphere of an Inch Diameter, is, proportionably to its Bulk, more *Reſiſted*, than a Sphere of two Inches Diameter in proportion to its Bulk, and hence it will come to paſs, that at laſt a Body may be reduced to that Minuteneſs, that its Gravity *Preſſing* Downwards (which is according to its Bulk) may be leſs than the *Reſiſtance* of the Medium, which operates on the Surface of the Body; ſeeing, as I ſaid before, the Surfaces of Bodies do not Decreaſe ſo faſt as their Bulks, theſe Decreaſing in a Triplicate, but thoſe in a Duplicate *Ratio* of the Bodies Diameters.

But, becauſe I have ſaid that the forementioned Law of *Hydroſtatiks* is a little Defective, I deſire to explain myſelf a little further in that point. In Weights *Falling* through the Air, were Gravity only conſidered, the Proportions of their Deſcents would be exactly as *Galileo* has Demonſtrated; but it is allowed by all, that the *Reſiſtance* of the Air, not being conſidered in thoſe Demonſtrations, they are not *Mathematically* True in Practice, but that really there is ſomething of that Proportion hindred by the Air's *Reſiſtance*. Now, what is this leſs than to ſay, that the *Reſiſtance* of the Air takes off ſome of the Operation of Gravity, or is able to withſtand or oppoſe part of its Action? And if ſo, what ſhall we ſay, were an *Iron* Sphere let through a Medium of Water? Surely, the Proportions of its Deſcents would be much more Diſturbed herein, as Water is much more ſolid and difficult to be Separated or paſſed through than Air, and conſequently we muſt needs grant, that more of the Operation of Gravity is taken off or reſiſted by this Oppoſition of the Water, than that of the Air. And if ſo, ſurely there may be a certain Degree of Gravity, that may be quite taken off by the *Reſiſtance* of the Water: Were a Piſtol Bullet let fall through the Air, it would Deſcend imperceptibly nigh the Proportions that *Galileo* has aſſigned, but were a ſingle Grain of Sand ſo let fall, it would be much hindred in its Courſe, and half of this Grain would be more obſtructed; what ſhall we then ſay of the Ten Thouſandth part, or of a part of the Ten Thouſandth Millionth of this, and again of the Infinite Subdiviſions of that, 'till at laſt we come to a Part that would be wholly *Reſiſted*, or kept up; ſuch as I conceive the Minute Particles of a Body *Diſſolved* in a *Menſtruum*.

On this Account, 'tis, I ſay, that the forementioned Principle of *Hydroſtaticks* is a little Defective; for it conſiders not the Natural *Congruity* of the Parts of a Liquor, whereby they deſire as 'twere to Unite and Keep together, juſt as we ſee two drops of Water on a dry Board being brought together, do jump and coaleſce, and therefore Liquors have an Innate Power of *Reſiſting* a certain Degree of Force that would *Separate* them; ſuch as I ſup-

poſe the Degree of Gravity, in the moſt minute Particles of a Body *Diſſolved* in a *Menſtruum*.

The forementioned Rule holds true to the moſt nice Senſe in great Bodies, but in thoſe that are by many Millions of Diviſions ſmaller, it ſeems to Fail.

I would not however be thought wholly to Reject my *Brother's Solution* of this *Problem*; for certainly that *Motion* (whatſoever it is) in a *Menſtruum*, which is able to diſſolve ſuch a Solid Body as Iron, that is, which is able to diſturb the Cloſe and ſtrong Coheſion of the Parts of Iron, may very well be ſuppoſed ſufficient to diſturb or keep up theſe Parts from Reſting in the bottom of the Veſſel, wherein the *Solution* was made; and certainly no better account can poſſibly be given of ſuch *Solutions*, than by ſuppoſing ſuch an *Internal Motion* in the parts of the *Menſtruum* Inſinuating themſelves into the Solid Body, and looſening its parts. But I leave to others to conſider what Kind of *Motion* and Peculiar Conformation of Parts is requiſite both in the *Menſtruum* and in the Diſſolved Body, that a Solution may reſult from their Commixture.

2. Tho' Liquors Conſiſt of Parts United, and tho' this Union be eaſily Deſtroyed, yet of Neceſſity it requires ſome degree of Force for Effecting it; Yet this *Property* ought not to be rely'd on as the Sole Cauſe of this *Appearance*; For, in this *Solution* of the *Problem*, We firſt Suppoſe the Minute Particles of a Heavy Body Rais'd, and then give the Reaſon of their not Sinking; Whereas 'tis not to be queſtioned, but that that Force which Raiſed them, is the ſame that Keeps them from Falling to the bottom.

Conſider'd; by Mr. Tho. Molyneux. *Ibid. p.* 93.

IX. Sir *Sam* Moreland undertakes to Demonſtrate, (contrary to the Common and Received Opinion through *England* and all *Europe*.)

1. That he will Force Water 60 Foot high with treble the Weight that ſhall Raiſe it 20 Foot, and ſo proportionably, *in infinitum*.

2. That by how much Wider the Barrel is, in which the Forcer Works, than the Pipe through which the Water is forced up, by ſo much is the *Engine* Preſſed with Unneceſſary Weight.

An Undertaking for Raiſing of Water; by Sir Sam. Moreland. n. 102. *p.* 25. Apr. An. 1674.

X. 1. Elapſa nuper Æſtate, *Ann.* 1684. in manus incidit Tractatus quidam inſcriptus *Sipho Wurtemburgicus*, ſive Sipho Inverſus Cruribus æque-altis Fluens & Refluens hactenus Inauditus: de hac *Machina* magnifica prædicat Author, ſed Lectorem orat ut ignoſcat Patrono ejus Sereniſſimo quod Myſterium Structuræ ejus ſibi ſervet. Hæc dum Legerem in mentem venit quo modo inſtrui poſſit *Sipho*, ut quæ de *Wurtemburgico* illo narrantur præſtaret. Habens ergo in manibus *Siphonem* quendam Vitreum, erexi eum ſupra duo Vaſcula quantum potui perpendiculariter; dumque in eo ſitu ſiſteretur, affudi in unum ex Vaſculis Aquam, donec Orificium *Siphonis* paululum ſuperaret, & ſtatim in alterum Vas ut expectabam effluebat Aqua: tunc Evacuato illo Vaſe in quod primum Aquam Infuderam in alterum Effudi, & immediate Aqua iſta Refluebat in prius Vas. Licet non auſim Artificium hujus *Siphonis* mei cum illo *Wurtemburgico* comparare tamen, ſi Utilitatem ſpectemus, cum

A Siphon *performing the ſame things with the* Sipho Wurtemburgicus; *by Mr.* J. Davis. n. 167. *p.* 846. Nov. An. 1684.

eo certare poſſe (præſertim ſi addatur ei Inſtrumentum quoddam, modo quodam peculiari à me excogitato,) non multum dubito.

By Dr. Papin. Ibid. p. 847.

2. In Tractatu de *Siphone Wurtemburgico*, qui *Stutgardiæ* Authore D°. *Salomone Reiſelio* nuper Editus eſt, magna quædam atque inaudita, ſi & Utilitatem & Raritatem & Artificium ſpectes, de novo illo *Siphone* prædicantur: ipſius autem Proprietates Characteriſticæ proponuntur his Verbis.

1. *Ut Orificia Crurum Duorum* Siphonis *ſui horizontaliter ſita Labris inſerantur; cum in Veterum inventis Crus longius infra Labrum ſeu Æquilibrium deſcendat ſemper.*

2. *Ut Orificiis vel partim vel dimidium Aqua repletis, Effluat tamen Aqua ſuper Montem ducta; cum in reliquis* Siphonibus *totum Orificium Aqua adimpleri ſeu immergi Aquæ debeat.*

3. *Ut in Siccitate diuturna quieſcens* Machina *tamen effectum producat Affluente iterum Aqua.*

4. *Ut Lumine ſeu Orificio alterutro Aperto, altero vero poſt Horas demum aut Diem, ſeu per Epiſtomium, ſeu Conum recluſo, Effluat tamen Aqua; cum in aliis utraque Simul Lumina Aperiri debeant.*

5. *Ut in Horizontali Linea poſitis Orificiis, & æqualibus quoad Altitudinem Cruribus exundet Liquor; cum in* Portæ *Aliorumve* Machinis *in æqualia debeant Crura eſſe, & Perpendiculum majus.*

6. *Ut ab utroque Labro in utrumque Labrum infuſa Aſcendat & Effluat Aqua; cum in Veteribus non niſi per unicum nempe Longius, Crus Effluat, nunquam Refluat.*

Hæc ſunt ipſius verba *Authoris*, qua vero ratione, quibuſve auxiliis tanti effectus produci poſſint, ne verbum quidem: Me itaque juſſit *Regia Societas* Machinam conſtruere, quæ eadem illa in *Libro* deſcripta Phænomena exhiberet. Rem quidem tribus variis modis exequutus ſum; ne vero tædioſus nimium videar, ſequentem methodum utpote facillimam inſtar omnium fore arbitror.

Fig. 197. A A Sunt duo Vaſa Metallica quibus duæ *Siphonis* extremitates inſeruntur. BCDEDCB eſt *Sipho*, cujus *Lumina* BB, In eadem Horizontali Linea diſponenda ſunt. F eſt Tubulus foramini in ſuperiori *Siphonis* parte adferruminatus, diligenterque obturandus, poſtquam *Sipho* Aqua exacte Repletus fuerit: Jam manifeſtum eſt Aquam in partibus CD, contentam, Aeri Externo ingreſſum prohibere ne ad ſuperiorem *Siphonis* partem E penetret: *Sipho* igitur Aqua ſemper plenus (modo debitam Altitudinem non excedat,) effectum ſuum certiſſimè producet ſtatim atque Aqua in Vaſibus A A, contenta alterutrius Orificii B, partem aliquam Replebit; quumque Ambobus Orificiis Aqua partim Repletis, in utroque Vaſe A ad eandem Horizontalem Lineam Superficies Aquæ pervenerit, ſi alterutri Vaſi vel tantillum Aquæ infuderis, pars ejus per *Siphonem* ſtatim in alterum Vas deferetur, eademque ratione cætera in *Libro* deſcripta *Phænomena* exhiberi poterunt.

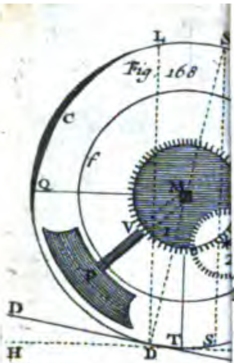


Fig. 168

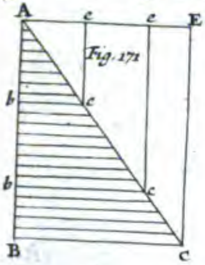
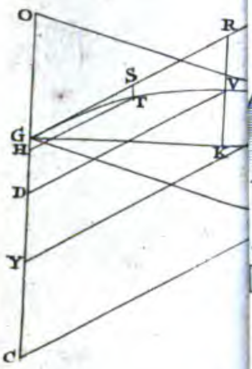
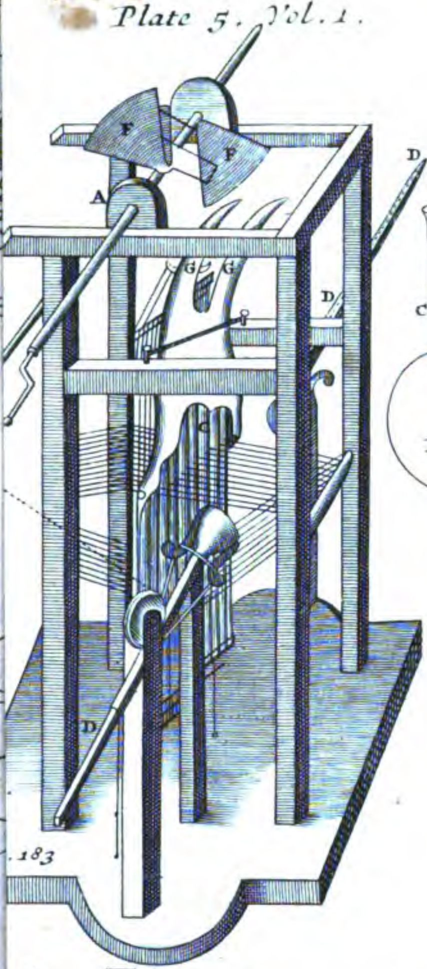


Fig. 171





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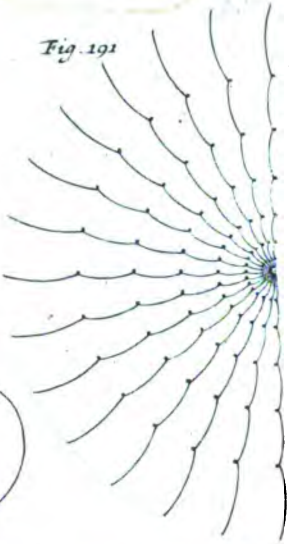


Fig. 191

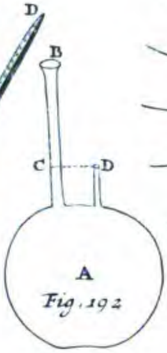


Fig. 192

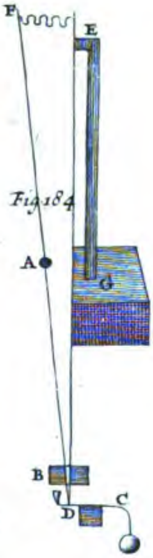


Fig. 184

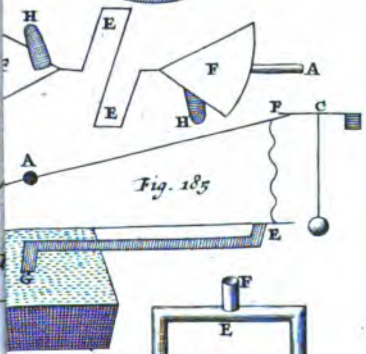


Fig. 185

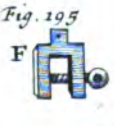


Fig. 195

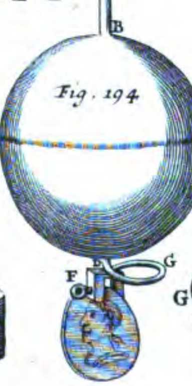


Fig. 194

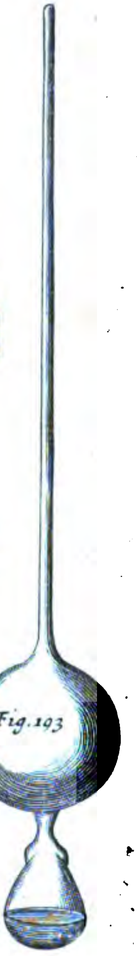


Fig. 193

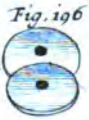


Fig. 196

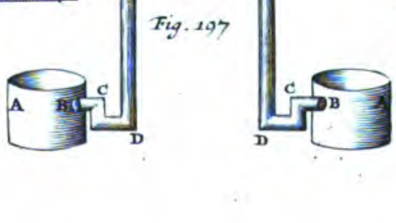
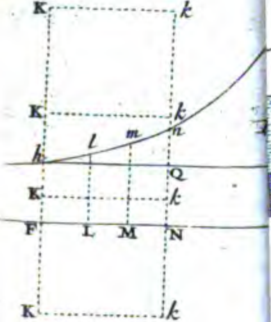


Fig. 197



3. Ne diutius *Sereniſſimæ Societatis* deſiderium fruſtrarer, aut morarer, fateor Excellentiſſimi D^{ni}. D^{ris}. *Papin Siphonem* ipſiſſimum eſſe *Wurtemburgicum*, etiam cum Recurvatione Pedum factum; neque aliud eſſe Myſterium, ut *Inventor* ipſe ſcripſit, in Summitate, quam quod per Infundibulum debeat Impleri, ſine qua Impletione non poteſt transfluere. Quod proximè Typis amplius confirmabitur, quia longum nimis eſſet & tædioſum hic omnia ſcribere.

By D. Salomon Reiſelius *himſelf.*
n. 178. *p.* 1272.
Dec. An. 1685.

Aſt, ut in præſenti oſtendam me hactenus nonnihil laboraſſe circa *Siphonis* effectus; ecce inter Experimentandum, hoc quoque mihi occurrit; ut didicerim, quomodo in *Vertice* vel ad *Latera* poſſit Effluere; quod hactenus multi promiſere, vix quiſquam efficit.

XI. 1. A A, Is a great Glaſs made like a Tumbler, but much bigger, and laid upon the Chimney Board, B B.

C C, Is the *Engine* like a ſmall *Rock*, that doth conſtantly *Spout* out Water by the two Holes D D: This *Rock* is kept at a diſtance from the bottom of the *Glaſs* A A; ſo that it may plainly be ſeen that it cannot Receive any Water by Subterraneal Tubes.

A New way of Raiſing Water, Ænigmatically propoſed; by Dr. Papin.
n. 173. p. 1093.
July An. 1685.
Fig. 198.

E E, Is a factitious *Coral*, reaching from the Center of the *Rock* C C, to the Center of the *Crown* F F.

F F, Is a *Crown* bearing upon the Aperture of the *Glaſs* A A, and holding the *Rock* C C, Suſpended at a conſiderable Diſtance from the Bottom.

G G, A Glaſs Open at both ends, apply'd to the *Rock* C C, to keep the Water upon it from falling down.

The Water in this Engine runs conſtantly H H, Two *Shells* to receive the Water from the *Jetto's*.

Solv'd: by Dr. Nath. Vincent.
n. 177. *p.* 1238.
Dec. An. 1686.

2. Within the *Rock* C C, there may be a Veſſel placed, which ſhall be made like the Body of a pair of *Bellows*, or thoſe *Puffs* heretofore uſed by *Barbers*, which being filled with Water, a piece of *Clockwork* put under it may produce the *Jetto's*; the Water being received into the *Shell* H H, and running thence into the hollow of the *Coral* E E; may be thereby conveyed into the Follicular Cavity in the ſame quantity it is Ejected from the two Emerging Tubes; and it will Circulate according to the going of the *Clock-work*.

3. I conceive that the Air is forced into the *Outer Glaſs* at the bottom thereof. That it then paſſes up between the two *Glaſſes*.

By Mr. R. A.
Ib. p. 1239.

That the *Outer Glaſs* or Caſe being cloſe Luted at the Head or *Crown* to which the *Inner Glaſs* is hung by the *Coral*, the Air is forced into the Mouth of the *Inner Glaſs*.

That the Air ſo forced preſſing on the Surface of the Water that covers the *Rock*, forces the Water to riſe through thoſe two extream Parts that are not at all Clogg'd, or covered with Water.

4. ABDE, ſignifies a Cylindrical Veſſel, Cloſed on every ſide, and Divided into two Rooms by the Floor E F.

G L M H, Is another Cylindrical Veſſel within that Upper Room, Cemented with its Mouth downwards to the Floor, and full of Water up to the Surface I K; the upper part thereof G I K H, being full of Air.

In a Letter Subſcribed, W. Tenon.
n. 178. *p.* 1254.
Fig. 199.
Dec. An. 1685.

QO, RP, Two Pipes, open above and below, and let through the Upper Room into this Veſſel, and reaching almoſt down to the Floor EF.

V W, a Pipe open above and below, and let into the Upper Room. Theſe Pipes muſt be cloſe joyned round about them to the Floors C D, and GH.

XY, Two little Hemiſpherical Bladders prepared with Oil or ſome Oily Subſtance (as Butter and Turpentine) againſt Water, and Cemented with their Mouths upward to the Floor E F, underneath.

α, β, Two Valves Opening out of the Upper Room in the Bladders.

γ, δ, Two other Valves Opening out of the Bladders into the Inner Veſſel above.

NZ, A *Pendulum* playing upon the Centre N, and having two Battledoor Arms *a*, *b*, to ſqueeze alternately the Bladders which reſt upon them.

Let the Upper Room be filled with Water at the Pipe VW, and if the *Pendulum* be made to play by *Clock-work*, the Bladders will perpetually Pump it thence into the Inner Veſſel, and the Compreſt Air G I K H in the Upper part of that Veſſel preſſing upon the Surface of the Water I K, will force it thence into the Pipes O Q, P R, out of which Spouting with a perpetual even Stream into the Spoons S T, it will run down by the Pipe W V, into the Upper Room again, the *Pendulum* will play moſt eaſily when the Upper Room is filled to the Top of the Pipe W V. Inſtead of the Bladders may be other Contrivances, as of Suckers or little Organ Bellows, playing alternately with two Leaves about an Axis in the middle.

By Dr. Papin. Ibid. p. 1274. Fig. 200.

5. AA, Is the *Great Tumbler*, that muſt have ſome little hole in the bottom as I.

ILL, A ſlender Pipe hidden by the Chimney Board BB, whereby the *Tumbler* AA, hath Communication with the *Pump* or *Bellows* MM.

MM, Some kind of *Pump* or *Bellows* well ſhut, and having no other Aperture, but through the Pipe ILL. Theſe are put in ſome ſecret Place where a Body may Play the ſame, and not be ſeen.

NN, A ſlender Pipe, that makes a Communication between the *Glaſs* AA, and the *Crown* FF; this Pipe reacheth near to the cover of the *Crown*, that the Water contained in it may not run down by that Aperture.

EE. the *Factitious Coral*, Hollow within, Shut at the Bottom and Open at the Top.

DD, D D, Two crooked Pipes Soldered to the Sides of the *Coral* E E, ſo that the Water running down the *Coral* may Spout out at the Holes DD.

OO, A Pipe hidden in the *Coral* EE, paſſing through the bottom of the ſame where it muſt be well Solder'd, and reaching near to the bottom of the *Rock* CC.

PP, A Pipe to convey the Water from the *Glaſs* GG, into the *Rock* CC; this Pipe is well Solder'd to the *Cover* of the ſaid *Rock*.

Q, A *Valve* working by a Spring at the bottom of the Pipe P P, to keep the Water, that gets in that Way, from returning back.

R, Another *Valve* at the top of the Pipe OO, that the Water getting up that way, may not fall through the ſame.

Now it is plain, that the *Rock* CC, being filled partly with Water, partly with Air; if we open the *Bellows* MM, the Air from the *Crown* F F, muſt run through the Pipe NN, into the *Tumbler* AA: And thence through the Pipe ILL, into M M to fill the Vacuity made therein: The Air in the *Crown* FF, being thus Rarified, gives liberty to the Air in the *Rock* C C, to Rarify too, by Driving the Water through the Pipe OO. The Water being got up into the *Crown* F F, runs down the *Coral* E E, and through the Croked Pipes D D, DD, Spouts out at their upper Apertures, and from the *Shells* HH, falls upon the *Rock* CC: If we come afterwards to ſhut the *Bellows* MM, the Air got into their Vacuity, muſt run back into the *Tumbler* AA, and preſs upon the Water at the Top of the *Rock* C C: But the Air in the ſaid *Rock* having been Rarified, its Spring is not ſufficient to Reſiſt this Preſſure, and ſo the Water is forced into the ſaid *Rock* through the Pipe PP: And by thus opening and ſhutting the *Bellows* M M, the Water muſt conſtantly circulate by the Ways aforeſaid.

The Uſe of this Contrivance.

As for the Uſes this Way for *Raiſing Water* may be applied to, this I do conceive: The Glaſſes, being meerly to conceal the Secret, muſt be left out; and there may be made ſeveral Receptacles above one another to receive the Raiſed Water, ſo as doth the *Crown* F F: And there ſhould be as many *Bellows* to Communicate every one with one *Receptacle*: Theſe *Bellows* ſhould be moved by an Axis, ſo that, when the *Firſt* is open, the *Second* ſhould be ſhut: the *Third* open, the *Fourth* ſhut; and ſo forth, alternatively; which may be eaſily done: By this means, the Firſt or Loweſt *Receptacle* would give the neceſſary Supply of Water to the *Second*, the *Second* to the *Third*, and the *Third* to the *Fourth*, &c. 'till the Water would be *Raiſed* to the intended Height; ſuch Receptacles might eaſily be ſet at 12 or 15 Foot above one another, and ſo but few of them might *Raiſe Water* to a conſiderable Height, as well as ordinary *Pumps* do: But this new Way would have this advantage, that in the Ordinary Pumps the ſtrength to be applied lieth near the Water to be Raiſed, but by this Contrivance the Stream of a River may be applied to draw Water out of a Mine far diſtant from it. By the ſame way the Stream of the *Thames* might keep conſtant *Water-Works* in *Windſor-Caſtle*, as eaſy almoſt as in the Loweſt Fields: The River *Seine* might do the ſame at St. *Germain*, and perhaps at *Verſailles* too, notwithſtanding the great Diſtance. For it is to be obſerved, That the Pipes of Communication between the *Bellows* and the *Engine*, being meerly for the Conveying of the Air, which moves very ſwiftly, they may be ſlender enough, and ſo contain but a ſmall Quantity of Air to be Rarified; and beſides, they will not be ſubject to Burſt or Leak, ſince the Preſſure they bear, being all external to the Pipe, will rather ſtrengthen than break the ſame. From whence it follows, That the ſaid Pipes need not be ſtrong, but may be made at very ſmall Charges. It is alſo to be obſerved, That thoſe *Bellows* which are open, have the Air in them very much Rarified, ſo that the outward Air lieth heavy upon (to ſhut) them, by which means the Motion of the *Engine* muſt be help'd in Lifting up the oppoſite *Bellows*, that are to be opened: And this Obſervation may anſwer the greateſt Difficulty that might be objected againſt this Contrivance: So that I don't queſtion but this way for *Raiſing Water*, may on ſeveral Occaſions be of great Advantage.

Further Explained. *Fig.* 201.

A B, A B, Are ſeveral *Receptacles* ſet above one another, which muſt be well Shut and Soldered every where.

CDD, CDD, Are two ſlender Pipes, whereby the Firſt and Third *Receptacles* have a Communication with the *Pump* HH.

EFF, E F F, Two other ſlender Pipes, whereby the Second and Fourth *Receptacles* have a Communication with the *Pump* I I.

HH, I I, Two *Pumps* whoſe Plugs are ſo moved by the Axis L L, that when one goeth down the other goeth up.

M M, A Wheel faſtened to the Axis L L, that it may be moved by the Stream of a River.

N O, P Q, N O, P Q, Are big Pipes for the Water to go up, from a Lower into a Higher *Receptacle.*

O, Q, O, Q, Are *Valves* fitted to the Top of the aforeſaid Pipes that the Water may not go down through the ſame.

Now it is plain, that, when the *Plug* in the *Pump* HH, is going up, the Air comes in through the Pipes C DD, and ſo it is Rarified in the Firſt and Third *Receptacles* marked A, A: And by that means the Water may be Driven up into the ſaid *Receptacles* through the Pipes N O, becauſe at the ſame time the *Plug* in the *Pump* II, going down, cauſeth the Air to return to its Ordinary Preſſure in the Second and Fourth *Riceptacles*, that it may be able to drive up the Water through the ſaid Pipes N O, and the Loweſt Pipe draws the Water that lies open to the Air. By the ſame reaſon when the *Plug* in the *Pump* I I, goeth up, the Air muſt come in through the Pipes E F F: And ſo it is Rarified in the Second and Fourth *Receptacles* marked B, B, and by that means the Water may be driven up into the ſaid *Receptacles* through the Pipes P Q, P Q, becauſe at the ſame time the *Plug* in the *Pump* H H, going down, cauſes the Air to return to its Ordinary Preſſure in the Firſt and Third *Receptacles*, ſo that it is able to Drive up the Water through the ſaid Pipes P Q.

Several Objections, made by M. Nuis, *Anſwered; by* *Dr.* Papin. n. 186. p. 263. Jan. An. 1687.

6. 1. To keep the *Receptacles* from being fill'd too much, the Water may be let out by Inſerting into each a Crooked Pipe, reaching a pretty way downwards, and having its Lower Aperture ſhut up with a Valve, whereby the Water may run out when the *Receptacle* ſhall be fill'd to a certain Height: And I may add, to prevent new Difficulties, that, leſt the *Pumps* ſhould be fill'd to much, a Valve may be made that ſhall open as ſoon as the Air in the *Pump* ſhould be more Compreſt than the Outward Air: So the Air getting in through any Pores would be conſtantly let out.

2. I have not poſitively promiſed a good Succeſs, but for *Windſor* and St. *Germain*; but when I ſpoke of *Verſailles* I uſed the Word *perhaps*, thereby ſhewing, that, before any one ſhould go about ſuch great Undertaking, he ſhould Reflect upon it more than I would then do, not having Occaſion for ſuch Work. But I now make the following Computation.

Let the Diſtance of *Verſailles*, as *M. Nuis* ſuppoſeth, be 12000 Foot, and the Capacity of each *Receptacle* be about one half of a Cubic Foot: I might make the Wheel with the Axis to make their *Revolution* in one Minute of Time, and ſo Order all Things that the Air under the Aſcending *Plugs* might come to be Rarified to ſuch a degree, that by its Elaſticity it might not Counterpoiſe more than 7 Foot of Water; but at the ſame time the Air in the *Receptacles* A, A, B, B, would, even in its greateſt Dilatation, be able to Counterpoiſe 17 Foot: So it is plain, that the Air will be driven from the *Receptacles* into the *Pumps* by a Strength equivalent to 10 Foot of Water: Now, if we compute the * *Velocity* of Air driven by ſuch a Preſſure: We ſhall find that the ſaid *Velocity* will be about 740 Foot in a Second: ſo that in half a Minute, during which the Plug goeth up, this Air might paſs above 22000 Foot, although it were not Rarified at all; but being Rarified, as we do ſuppoſe it to be, it might go a great deal further.

* *Vid. Sup. Cap.* V. §. XII.

I muſt now take notice, that according to the Honourable Mr. *Boyle's* Experiments, the *Rarefaction* of the *Air* is much leſſer than Mr. *Nuis* takes it to be: For the Water contained in the Pipe N O, is ſo far from cauſing the Air to fill up a ſpace four times bigger, that it will not Extend itſelf to a Space once bigger than before; Conſidering therefore the *Velocity* of the Air, and the ſmall Dilatation it doth ſuffer, if any one will take the trouble to Compute, he will find, that if the *Pumps* have in Diameter the Diagonal of a Square Foot, and the ſame Height: And if the ſmall Tubes of Communication be made of $\frac{1}{9}$ part of an Inch in Diameter; ſo that being 12000 Foot long, they may contain about one Cubic Foot of Air; that would be more than Sufficient to make the neceſſary *Rarefaction* in the *Receptacles*.

But for the good Succeſs of the *Engine* it is not enough to make the Air paſs from the *Receptacles* into the *Pumps*, it muſt alſo Return from the *Pumps* into the *Receptacles*: Now for this Intent it would be neceſſary to ſet the *Receptacles* but five Foot above one another; ſo, to drive the Water up the Pipe N O, it would be enough that the Air in the *Receptacle* B, ſhould Preſs with a ſtrength Equivalent to 23 Foot of Water: For it is plain, that five Foot in the Pipe N O, together with a Preſſure Equivalent to 17 Foot, which I have ſuppoſed to be in the Upper *Receptacle* A, will make but 22 Foot in all; and therefore 23 Foot Preſſing in the *Receptacle* B, muſt prevail, and cauſe the Water to Aſcend: Now the Preſſure in the *Receptacle* being but 23 Foot, and the Air in the *Pump* Returning to it's Ordinary Preſſure, which is about 33 Foot, it is plain that the Air going back to the *Receptacle*, will be Driven by Strength Equivalent to 10 Foot, as well as it had been in coming from the *Receptacle* towards the *Pump*: And ſo the Bigneſs aſſigned for the *Communication Pipes* will alſo prove more than ſufficient to this Effect.

From what I have been ſaying, it is plain, That in great Diſtances there ſhould be made as many *Pumps* as *Receptacles*, as hath been already propounded: And for to Raiſe Water but 60 Foot High, there ſhould be required 13 or 14 *Receptacles*, and as many *Pumps* of the Bigneſs aforeſaid. Some People may take this for a great Difficulty. But I Anſwer, That in this *Engine*, this is not ſo much as it ſeems at Firſt; becauſe, the Preſſure being all from without, there is no need of any great ſtrength to reſiſt it, and ſo the Metal for the *Pump* will Coſt but little: There may alſo be found Occaſions where to make ſo good uſe of them, that ſuch an *Engine* as I have Deſcribed would in a Years time ſave Labour enough to pay for many *Pumps*, ſince it might every hour Raiſe about 1800 Pounds of Water to the height of 60 Foot: Mean while I dont pretend to have given here the beſt Proportion for the Bigneſs of every part of the *Engine*; but it may be, by Altering the Capacity of the *Pumps*, of the *Pipes*, or of the *Receptacles*, a much more conſiderable Effect might be Produced.

3. The Water doth a not any time Aſcend higher than from a Lower *Receptacle* into the next Upper *Receptacle*; which Height is but 12 Foot: So that it is plain enough, that the Preſſure of the Air may be ſufficient to Drive it up. It is Indifferent, whether it be by Rarefaction or otherwiſe that the Water comes into the *Receptacle* A; it is enough that the Water is there, and that the Air Preſſes upon it with ſuch a Strength as will prevail againſt all that oppoſeth it.

4. Though the Uſe of the Pipes be meerly for Conveying of Air: They may nevertheleſs be eaſily fill'd with Water when need Requires, and ſo the Defects in them may as well be found out as in Pipes that are uſed for the Conveying of Water.

An Engine for Raiſing Water, by the Help of Fire; by Mr. Tho. Savery. n. 253. *p.* 228. *Fig.* 202. June An. 1699.

XII. A, The *Furnace*,
B, The *Boyler*.
CC, Two *Cocks*, which convey the *Steam* by turns to the *Veſſels* D D.
D D, The *Veſſels* which receive the Water from the Bottom, in order to Diſcharge it again at the Top.
E E E E, *Valves*.
F F, *Cocks* which keep up the Water, while the *Valves* on Occaſion are Cleans'd
G, The *Force* Pipe.
H, The *Sucking* Pipe.
I, The *Water*

An Hydraulique *Engine; by* .... n. 128. *p.* 679. *Fig.* 203. An. 1675.

XIII. This *Engine* is a *Cheſt* of Copper A, pierced with many Holes above B B, and holds within it the Body of a *Pump* E F M, whoſe *Sucker* D E, is Raiſed and Abaſed by two *Levers* C, O; Theſe *Levers* having each of them two Arms, and each Arm being fitted to be laid hold on by both Hands of a Man. Each *Lever* is pierced in the middle by a Mortaiſe *a a*, in which an Iron Nail, which paſſes through the Handle of the *Sucker*, turns when the *Sucker* is raiſed or lower'd. Near the Body of the *Pump* there is a *Copper Pot* I H K, joined to it by the Tube G, and having another Tube K N L, which in N may be Turned every way.

To make this *Engine* Play, Water is pour'd upon the *Cheſt* to enter in at the Holes that are in the cover thereof. This Water is drawn into the Body of the *Pump* at the Hole F, at the time when the *Sucker* is Raiſed; and when the ſame is let down, the *Valve* of the ſame Hole F ſhuts, and forces the Water to paſs through the Hole M into the Tube G, of which the *Valve* H being lifted up, the Water enters into the *Pot*, and filling the Bottom, it enters through the Hole K, into the Tube K N L, in ſuch a manner, that when the Water is Higher than the Tube K N L, and the Hole of the Tube G is ſhut by the *Valve* H, the Air Incloſed in the *Pot* hath no iſſue, and it comes to paſs, that, when you Continue to make the Water Enter into the *Pot* by the Tube G, which is much thicker than the Aperture of the end L, at which it muſt iſſue, it muſt needs be, that the Surplus of the Water that enters into the *Pot*, and exceeds that which at the ſame time iſſues thro' the ſmall end of the *Jet*, compreſſes the Air to find place in the *Pot*: which makes that, whilſt the *Sucker* is Raiſed again, to make new Water to Enter into the Body of the *Pump*, the Air which has been *Compreſs'd* in the *Pot*, Drives the Surplus of the Water by the Force of its *Spring*, mean time that a new *Compreſſion* of the *Sucker* makes new Water to enter, and cauſes alſo a new *Compreſſion* of Air. And thus the Courſe of the Water, which iſſues by the *Jet*, is always entertained in the ſame State.

A cheap Pump; by Mr. Conyers. n. 136. *p.* 888. June An. 1677.

XIV. A A, The Body of a Square Taper *Pump*, made of Oak, Elm, or Deal Planks; with a *Valve* at bottom *a a*.

B B, The *Bucket*, in the midſt of which there is a *Valve b*, not viſible in the Figure, being concealed by the Sides of the Leather *b b*.

CCC, The Iron to raiſe the *Bucket*.

DD, The Wood at the bottom of the *Bucket* containing the *Valve*.

Fig. 204.

EE, The *Handle* for Raiſing the *Bucket*, to be managed by fewer Hands than Ordinary *Pumps* are; which may be Altered ſo as to employ a Horſe, or Mill, or other ſuch like way, more Advantageous than that of this Handle managed by the Strength of Men.

FF, a Square *Taper-Box* with Holes in the Sides, and open at the Bottom; into the narrower part of which is encloſed the narrower End of the Body of the *Pump*.

GG, An Additional *Bucket* of a larger Dimenſion to be placed on the Iron Work of the *Pump* about H, when it ſhall be needful to Lengthen the Taper of of your *Pump*, and thereby to Raiſe the Water more forcibly to a greater Height.

II, The *Spout* of the *Pump*, to caſt out the Water of the ſame breadth with the ſide of the *Pump*.

KK, The Iron or Wooden Work ſet off, or bent back (if need be) and placed at the back of this *Pump* for the eaſier and more capacious Motion of the *Pump Handle*, in which it moves.

This *Pump* was by me Contrived in 1673. when the *New Canal* of *Fleet-River* in *London* was Enlarged: It was found to Raiſe at leaſt twice as much Water proportionably as thoſe of the ſame, or rather bigger Bore, that were firſt made uſe of and caſt by. It was 8½ Foot long, and 1 Foot 8 Inches broad at the top, and about 8 Inches broad at the Bottom, where it is inſerted in the *Box*, and did caſt out 8 Gallons at a *Stroke*, and 21 *Strokes* being made in one *Minute*, there were Delivered about 169 Gallons in a *Minutes* time; whence it is eaſy to Compute, what Quantity is thrown out in an *Hour*. This kind of *Pump* may by the ſame Contrivance, be made of a Tree Bored through with a Taper-Bore; and a Basket may be uſed at the bottom of the *Pump* inſtead of the *Box-Colender*.

Phil. Coll. n. 2. p. 35.

XV. Papers *of leſs General Uſe, (Extracted from a Book of* Jo. Alph. Borellius *de Motu Animalium,)* Omitted.

1. A *Way how a Man may* Swim *under* Water, *and* Breath *by the Help of a Bag about his Head.*

2. *Another Way of* Breathing *under* Water *by the help of a Leathern Pipe kept open by Wreathed Wires, and extended by the* Swimmer's *Head to the top of the Water.*

3. *A way to make a* Submarine Veſſel *accommodated with Ways to* Row *it; and to make it* Riſe *and* Sink *in the Water.*

XVI. Accounts *of Books* Omitted.

n. 8. p. 145. n. 10. p. 173.

1. HYdroſtatical Paradoxes, *made out by New* Experiments, *(for the moſt part Phyſical and Eaſy) by the Honourable* R. Boyle *Eſq;*

n. 226. p. 481.

2. Recueil de diverſes Pieces touchant quelques *Nouvelles Machines*, &c. Par. *D. Papin. M. D. A Caſſel.* 1695. in 8*vo*.

CHAP. IX.

Geography. Navigation.

A new Place for the firſt Meridian propos'd; by a Profeſſor of Math. at Seville. n. 118. p. 425. Oct. An. 1675.

I. L*Ongitudo Terreſtris* eſt Arcus Æquinoctialis ab uno ad alium Meridianum interceptus ſive Temporis Spatium, quod per Æquinoctialem numeratur, inter duo Loca; quare conſonum fuerit, *Longitudinis Principium* in ipſo Æquinoctiali conſtituere. Inſuper cum Circulus ille Æquinoctialis *Globum* in Borealem & Auſtralem partem dividat, ſi detur hujus *Primarii Meridiani* Fixatio in eo, erit inter Boreales & Auſtrales Æquitas atque Conformitas. Deinceps, oportet, ad Præciſionem, Locum hujus *Primarii Meridiani* eſſe parvum, ut *Longitudinis* Numeratio exactius exprimatur: non ut

aliqui, qui omnes *Inſulas Fortunatas* pro hujus *Principio* aſſumebant, & Diſtantiam duorum Graduum, inter earum aliquas, non notabant; quod certe abſurdum nimis erat. Iterum, Præcautio alia eſt habenda, ut *Primarius Meridianus* non confundatur cum Terris & Locorum Imaginibus in *Globo* vel *Mappa* exaratis, quod fiet ſi per Medias Terras Tranſeat: Et ſi præcipuas Terræ Partes dividat, ut *Americam*, *Africam*, & *Europam*, per Maria tranſiens, erit eo aptior & convenientior in *Globi Tarraquei* repræſentatione. Quas omnes memoratas cauſas conſiderans, Inveni, quod Natura (nihil fruſtra ſuppeditans) poſuit ſub ipſo *Æquinoctiali Circulo* Inſulam quandam prope *Braſiliam*, olim *Abroxos*, nominatam, quæ Inſula diſtat à *Teneriffæ Pico* Grad. 9. Occidentaliores, & ab *Uraniburgo* Grad. 42. Occidentaliores; in qua inveniuntur omnia ad *Primarium Meridianum* conducenda, ut à me æſtimatur.

M. Hugen's *Inſtructions for finding the* Longitude *with Pendulum Watches; Enlarged by* . . . .
n. 47. p. 937.
May An. 1669.

II. 1. Thoſe that intend to make uſe of *Pendulum Watches* at *Sea*, muſt have two of them at leſt; that, if one of them ſhould by miſhap or neglect come to ſtop, or (being by length of time become foul) need to be made clean, there may likely always remain one in Motion.

2. The *Watches* on *Shipboard* are to be Hung in a Cloſe Place, where they may be freeſt from Moiſture or Duſt, and out of Danger of being diſordered by Knocking or Touching.

3. Before the *Watches* be brought on *Shipboard*, 'tis convenient they be adjuſted to a *Middle* or *Mean Day*, the Uſe of them being then moſt eaſy.

To adjuſt the Watches.

4. Here take Notice, That the Sun paſſeth the 12 Signs, or makes one Entire Revolution in the *Ecliptick* in 365 Days, 5^{h}. 48′. or thereabout: And that thoſe Days, reckoned from Noon to Noon, are of different Lengths; as is known to all that are verſed in *Aſtronomy*. Now between the Longeſt and the Shorteſt of thoſe Days, a Day may be taken of ſuch a Length, as 365 ſuch Days, 5^{h}. 49′. *&c.* make up, or are Equal to that Revolution, And this is called the *Equal* or *Mean Day*, according to which the *Watches* are to be Set; and therefore the Hour or Minute ſhewed by the *Watches*, though they be perfectly Juſt and Equal, muſt needs differ almoſt continually from thoſe that are ſhew'd by the *Sun*, or are Reckoned according to it's Motion. But this Difference is Regular, and is otherwiſe called the *Æquation*; which is accounted from the firſt of *February* in the following *Table*.

	January		*February*		*March.*		*April.*		*May.*		*June.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	6	10	0	0	4	46	14	23	19	25	16	24
2	5	47	0	2	5	03	14	39	19	28	16	13
3	5	24	0	4	5	21	14	55	19	29	16	01
4	5	02	0	8	5	39	15	10	19	29	15	49
5	4	41	0	12	5	57	15	25	19	29	15	37
6	4	21	0	16	6	15	15	39	19	28	15	24
7	4	02	0	21	6	33	15	53	19	26	15	11
8	3	44	0	26	6	51	16	07	19	24	14	58
9	2	27	0	32	7	09	16	21	19	21	14	45
10	3	11	0	40	7	27	16	34	19	18	14	32
11	2	55	0	48	7	45	16	47	19	15	14	19
12	2	39	0	57	8	03	16	59	19	11	14	06
13	2	23	1	06	8	22	17	11	19	07	13	53
14	2	07	1	16	8	41	17	22	19	02	13	40
15	1	52	1	26	9	01	17	33	18	57	13	27
16	1	38	1	37	9	21	17	43	18	51	13	15
17	1	25	1	49	9	41	17	53	18	45	13	03
18	1	13	2	02	10	01	18	03	18	39	12	52
19	1	02	2	15	10	21	18	13	18	33	12	41
20	0	51	2	28	10	40	18	23	18	26	12	30
21	0	41	2	42	10	59	18	32	18	18	12	19
22	0	32	2	56	11	18	18	39	18	10	12	08
23	0	24	3	11	11	37	18	46	18	01	11	58
24	0	18	3	26	11	56	18	53	17	51	11	48
25	0	13	3	41	12	15	18	59	17	41	11	38
26	0	9	3	56	12	34	19	04	17	30	11	28
27	0	6	4	12	12	53	19	09	17	19	11	18
28	0	3	4	29	13	12	19	14	17	08	11	09
29	0	1			13	31	19	18	16	57	11	00
30	0	0			13	49	19	22	16	46	10	52
31	0	0			14	06			16	35		

July.

	July.		*August.*		*Septemb*		*October.*		*Novemb*		*Decemb.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	10	45	11	07	19	41	29	16	31	13	21	14
2	10	38	11	16	20	01	29	30	31	05	20	44
3	10	31	11	25	20	22	29	43	30	53	20	14
4	10	25	11	36	20	43	29	56	30	43	19	44
5	10	19	11	48	21	04	30	09	30	32	19	14
6	10	13	12	01	21	25	30	22	30	20	18	44
7	10	07	12	14	21	47	30	34	30	08	18	14
8	10	02	12	28	22	09	30	45	29	55	17	44
9	9	58	12	42	22	31	30	55	29	40	17	14
10	9	54	12	57	23	52	31	04	29	23	16	44
11	9	51	13	12	23	13	31	12	29	06	16	14
12	9	49	13	27	23	33	31	19	28	48	15	44
13	9	47	13	43	23	53	31	26	28	30	15	14
14	9	46	13	59	24	13	31	32	28	11	14	43
15	9	46	14	16	24	33	31	38	27	51	14	12
16	9	46	14	33	24	53	31	43	27	30	13	41
17	9	47	14	50	25	13	31	47	27	08	13	10
18	9	49	15	08	25	33	31	50	26	45	12	40
19	9	52	15	26	25	52	31	53	26	22	12	10
20	9	56	15	45	26	11	31	55	25	58	11	40
21	10	00	16	04	26	30	31	55	25	34	11	10
22	10	04	16	23	26	49	31	55	25	10	10	40
23	10	08	16	42	27	08	31	55	24	45	10	10
24	10	13	17	01	27	26	31	54	24	20	9	41
25	10	18	17	21	27	43	31	52	23	55	9	13
26	10	23	17	41	28	00	31	50	23	30	8	45
27	10	28	18	01	28	16	31	47	23	04	8	17
28	10	34	18	21	28	32	31	43	22	38	7	50
29	10	41	18	41	28	47	31	37	22	11	7	23
30	10	49	19	01	29	02	31	30	21	43	6	58
31	10	58	19	21			31	22			6	34

By the help of the foregoing *Table* you will always know what a Clock it is by the *Sun* preciſely, and conſequently, whether the *Watches* have been ſet to the right Meaſure of the *Mean Day*, or no; uſing the *Table* as follows.

When you firſt Set your *Watch* by the *Sun*, you are to Subduct from the Time obſerved by the *Sun*, the *Æquation* adjoyned to that Day of the Month in the *Table*, and to Set the *Watches* to the remaining Hours, Minutes and Seconds; that is, the *Watches* are to be Set ſo much Slower than the Time of the *Sun*, as (in the *Table*) is the *Æquation* of that Day; ſo that the *Æquation* of the Day Added to the Time of the Clock, is the True Time by the *Sun*. And when after ſome Days, you deſire to know by the *Watch* the Time by the *Sun*, you are to Add to the Time ſhewed by the *Watch*, the *Æquation* of that Day; and the Aggregate ſhall be the Time by the *Sun*, if the *Watch* hath been perfectly well Adjuſted after the Meaſure of the *Mean Days*; for the doing of which, this will be a convenient Way;

Draw a *Meridian Line* upon a Floor, and then hang two Plummets, each by a ſmall Thread or Wire, directly over the ſaid *Meridian*, at the diſtance of ſome two Foot or more one from the other, as the ſmallneſs of the Thread will admit. When the Middle of the *Sun* (the Eye being placed ſo, as to bring both the Threads into one Line) appears to be in the ſame Line exactly (for the better and more ſecure diſcerning whereof, you muſt be furniſh'd with a Glaſs of a Dark Colour, or ſomewhat Black'd with the Smoak of a Candle,) you are then immediately to Set the *Watch*, not preciſely to the Hour of 12, but by ſo much leſs as is the *Æquation* of that Day; *e. g.* If it were the 12*th* of *March*, the *Æquation* of that Day being by the Table, $8'. 3''$. Theſe are to be Subducted from 12 Hours, and the Remainder will be $11^h. 51'. 57''$. to which Hours, Minutes, and Seconds, you are to Set the *Index* of the *Watch* reſpectively: Then after ſome Days you are to obſerve again in the ſame manner, and likewiſe to note the Hour, Minute, and Second of the *Watch*; to which you are to add the *Æquation* of theſe Days, taken out of the Table; and if the Aggregate do juſt make 12 Hours, the *Watch* is Adjuſted to the Right Meaſure; but if it differ, you are to Divide the Minutes and Seconds of that Difference by the Number of the Days between both the Obſervations to get the Daily Difference.

Let us ſuppoſe this Second Obſervation to have been made the 20*th* of *March*, *viz.* Eight Days after the firſt, and finding that the Middle of the *Sun*, being ſeen in the Meridian in the ſame Line with the two Threads, as before,

The *Watch* Points, ——————————— $11^h. 51' 07''$.
The *Æquation* of the 20*th* of *March*, by the *Table*, is ——— 00 10 40
Which being Added to the Time, ſhow'd by the *Watch*, gives — 12 01 47

If this had been juſt 12 Hours, the *Watch* would have been well adjuſted, but being $1'. 47''$. more than 12, it hath gone ſo much too faſt in eight Days. And theſe $1'. 47''$. that is $107''$. being divided by 8, there come 13⅜ Seconds for the Difference of every 24 Hours; which Difference being known, if you want time, or have no mind to take the Pains to Adjuſt the *Watch* to its Right Meaſure, (this being not neceſſary, ſince you may bring it thus on *Ship-Board*) note only the Daily Difference, and regulate yourſelf accordingly. But if you will Adjuſt it better, you muſt Remove the leſs Weight of the *Pendulum* a little downwards, which will make it go Slower, and then you muſt begin a-new to obſerve by the *Sun*, as before. If it had gone too Slow, you muſt have Removed the mentioned Weight ſomewhat upwards. And this is of that Importance in the finding out of *Longitudes*, that if it be not Obſerved, you may ſometimes in the ſpace of Three Months miſreckon 7 Degrees, and more yet, (without any fault in the *Watches*;) which under the *Tropicks* will amount to above 400 *Engliſh Miles*.

The *Watch* may alſo be Adjuſted *on Board*, when a Ship Rides at Anchor, thus: In the Morning, when the *Sun* is juſt half above the *Horizon*, Note what Hour, Minute, and Second, the *Watch* points at, if it be going; if not, ſet it a going, and put the *Indexes* at what Hour, Minute, and Second you pleaſe. Let them go till *Sun Set*, and when the Body of the *Sun* is juſt half under the *Horizon*, ſee what Hour, Minute, and Second, the *Indexes* of the *Watch* point at, and note them too; and reckon, how many Hours, *&c.* are paſſed by the *Watch* between the one and the other. Then take the half of that Number, and add it to the Hours, *&c.* of the Morning Obſervation, and you ſhall have the Hours, *&c.* which the *Watch* did ſhow, when the *Sun* was in the *South*; whereunto Add the *Æquation* in the Table belonging to that Day, and note the Sum. Then, ſome Days being paſs'd, (the more the better,) you are to do juſt the ſame: And if the Hour of this laſt Day be the ſame that was noted before, your *Watch* is well adjuſted; but if it be more or leſs; the Difference divided by the Number elapſed between the two Obſervations, will give the Daily Difference. And if you will, you may let it Reſt there or otherwiſe, Removing the Leſſer Weight of the *Pendulum*, you may Adjuſt it better. You may alſo, inſtead of the *Sun's Riſing* and *Setting*, take two Equal Altitudes of the *Sun*, before, and after Noon, and having noted the time given by the *Watches* at the time of both the Obſervations, proceed with it in the ſame manner, as was juſt now directed for Obſerving the *Sun* in the *Horizon*. In either of which ways there may be ſome Errour, cauſed by the *Sun's Refraction*, which is conſiderable, and therefore needs not to be taken notice of.

To find by them the Longitude *at Sea.*

5. Give to each of the *Watches* a Name, or a Mark, as A, B, C; and before you ſet *Sail*, ſet them to the Time obſerved by the *Sun* in the place where you are, and whence you are departing, allowing for the *Æquation* of the Day whereon you make your Obſervation; which Day you are to note, if the *Watches* be not well adjuſted; otherwiſe it is not neceſſary.

Then afterwards being at *Sea*, and deſiring to know the *Longitude* of the Place where you are; that is, How many Degrees the Meridian of that Place is more Eaſterly or Weſterly, than the Meridian of that Place where you did ſet the *Watches*; you muſt obſerve by the *Sun* or *Stars*, what Time of the Day it is, as preciſely as is poſſible, and Note at the ſame time, to what Hour, Minutes and Seconds the *Watches* do point, (which Time, if the *Watches* be not ſet to the Right Meaſure, is by the known Daily Difference to be Adjuſted,) adding thereunto the *Æquation* of the preſent Day, which gives you the time of the Day, ſhewed by the Sun, at the Place where the *Watches* were ſet: And, if this time of the Day be the ſame with that obſerved where you are, then you are under the ſame Meridian with the Place where the *Watches* were ſet by the *Sun*; but, if the time of the Day, obſerved where you are, be greater than that ſhewed by the *Watches*, you may be aſſured, that you are come under a more Eaſterly Meridian; and if leſs you are come under a more Weſterly. And Counting for every Hour of Difference of Time, 15 Degrees of *Longitude*, and for every Minute, 15 Minutes, or ¼ of a Degree, you ſhall then know, how many Degrees, Minutes, *&c.* the ſaid *Meridians* do Differ from one another. *E. g.* Suppoſe the *Watches* A, B, C, were Set at the Place, whence you parted, on the 20*th* of *February*, to the time of Day obſerved by the *Sun*, abating the *Æquation* of the 20*th* of *February*, (*viz.* 2′ 28″.) and ſuppoſe that the *Watch* A, be ſet to its Right Meaſure, but that B goes every Day 7″. too Slow, and C every Day 12″ to Faſt. Some Days after, ſuppoſe the 5*th* of *May*, deſiring to know the *Longitude* of the Place where you are at *Sea*;

You Obſerve the Time of the Day there to be	05^{h}	18′	10″.
And you find the *Watch* A to point at	02	06	00
But the *Watch* B to point at	1	57	22
Going too Slow by 7″. every Day, which makes in 74 Days, (*viz.* From the 20*th* of *Febr.* to the 5*th* of *May*;)	00	08	38
Which being added to its own Time, gives the ſame with that of the *Watch* A, *viz.*	02	06	00
You find alſo the *Watch* C to point at	02	20	48
Going 12″. too Faſt every Day, which makes in 74 Days	00	14	48
Which being Subducted from its own Time, gives again	02	06	00
The Time of the Day therefore by the *Watches* being	02	06	00
Add thereunto the *Æquation* of the 5*th* of *May*	00	19	29
And ſo you have for the Time of Day at the Place where the *Watches* were ſet	02	25	29
But the Time Obſerved being	05	18	10
Exceeds this by	02	52	41
Wherefore the *Meridian* of the Place, where you are *May* 5. is more Eaſterly, than the Place where the *Watches* were Set by	02	52	41
Which being reduced to Degrees, reckoning 15 Degrees for an Hour comes to	43°	10′	15″.

'Tis true, that from the ſame Reckoning it may be concluded, that you are 180 Degrees more Eaſterly, which happens, becauſe the *Hour Index* goes round in the ſpace of 12 Hours in the *Watches*; but the Difference is ſo great, that one cannot be deceived in it; elſe the *Watch* might be ſo made, that the *Index* ſhall go round about once in 24 Hours.

To find the Time of the Day at Sea.

6. Since that for finding the *Longitude*, the Time of the Day, at the Place where you are, muſt be known, (as hath been ſaid above) you muſt have a

care to Obſerve that Time as preciſely as poſſible. For every Minute of Time, that you miſreckon, makes ¼ Degree in *Longitude*, which amounts, near the *Æquator*, to above 15 *Engliſh Miles*, but leſs elſewhere. Wherefore to find the Time of the Day with certainty, the beſt way is to Obſerve the *Sun's Altitude* when it is in the Eaſt or Weſt, (the nearer the better :) for, being there, its *Altitude* Changes in a ſhort Time more ſenſibly than before or after ; and thus from the Height of the Pole and the Declination of the Sun the Hour may be Calculated.

7. At the *Riſing* and *Setting* of the *Sun*, when it is half above the *Horizon*, *An Eaſier Way.* mark the Time of the Day, which the *Watches* then ſhew ; and, though ye have in the mean time ſailed on, it is not Conſiderable. Then Reckon by the *Watches* what Time is elapſed between them, and Add the half thereof to the Time of the *Riſing*, and you ſhall have the Time by the *Watches*, when the *Sun* was at *South*, to which is to be added the *Æquation* of the preſent Day by the Table. And, if this together makes 12 Hours, then was the Ship at Noon under the ſame *Meridian*, where the *Watches* were ſet with the *Sun*. But, if the Sum be more than 12, then was ſhe at Noon under a more *Weſterly Meridian* ; and, if leſs, then under a more *Eaſterly* ; and that by as many Times 15 *Deg.* as that Sum Exceeds or comes ſhort Hours of 12 ; as the Calculation thereof hath been already deliver'd.

Suppoſe, *e. g.* that the *Watches* A and B, as before, were ſet with the *Sun* at the Place whence you parted, the 20*th* of *February* ; and the *Indexes* ſet to the Hour, Min. and Sec. ſhewed by the *Sun*, abating the *Æquation* of that day, *viz.* 2′ 20″ ; the *Watch* A being Reduced to the Right Meaſure, and B going too ſlow by 7″ a day. Afterwards on the 22*d* of *May*, deſiring to know the *Longitude* of the Place to which you are come, you Obſerve in the

Morning the *Sun* half above the *Horizon*, when the *Watch* points at	2h	30′	10″
And in the *Evening*, the *Sun* being half under the *Horizon*, when the ſame *Watch* points at	3	8	40
To find the Time Elapſed between them, Subducting the Time of the *Riſing*	2	30	10
From	12	0	0
There Remains	9	29	50
Adding thereunto the Time of the *Setting*	3	8	40
You have for the Time Elapſed between the Obſervations	12	38	30
Whereof the Half	6	19	15
Being Added to the Time of *Riſing*	2	30	10
You have the Time by the *Watch* A, when ☉ was in the *South*	8	49	25
And after the ſame manner you are to ſeek the Time by the *Watch* B, when the *Sun* was in the *South* ; which Let be	8	38	48
But this *Watch* going 7″ a day too ſlow, it is Retarded in 91 days (from the 20*th* of *Febr.* to the 22*d* of *May*)	0	10	37
Which therefore Added to the ſaid Time gives	8	49	25
That is the ſame Time given by the *Watch* A. Now Adding to this Time of the *Watches*, the *Æquation* of the 22*d* of *May*	0	18	10
You have	9	7	35

Which is the ſame Time of the Day with that of the Place, where the *Watches* were ſet when the *Sun* was in the ſame *Meridian* with the Ship, or where the Ship was at Noon. h. ′ ″

The Difference is—————————— 2. 52 25

Wherefore this laſt *Meridian* is by ſo much more *Eaſterly*, than the firſt, which being reduced to Degrees, (as hath been formerly directed (make—————————— ° ′ ″ 43 6 15

'Tis manifeſt, that by this way you find preciſely enough the *Longitude* of the Place, where you were at Noon, or the Time of the Sun's being in the South: Which although it differs from the *Longitude* of the Place, where you are when you obſerve the *Setting* of the *Sun*; yet you may Eſtimate near enough, how much you have advanc'd, or chang'd the *Longitude* in theſe few Hours, by the *Log-Line*, or other Ordinary Practices of *Reckoning* the Ship's Way; or (which is the ſurer Way) by the Degrees paſſed in 24 Hours by a former day's Obſervation.

You may alſo, inſtead of Obſerving the *Sun's Riſing* and *Setting*, Obſerve the *Setting* firſt, and then the next Morning the *Riſing*; marking at both Times the Time ſhew'd by the *Watches*; and find thence, afer the ſame manner as before, the *Longitude* of the Place where the Ship was at Midnight.

Finally, You may alſo, inſtead of the *Riſing* and *Setting* of the *Sun*, Obſerve before and after Noon two *Equal Altitudes* of the *Sun*, Noting the Time ſhown by the *Watches*, and Reckoning in the ſame manner, as hath been ſaid of the *Riſing* and *Setting*: Yet it is to be conſider'd, that the *Altitudes* of the *Sun* are beſt taken, when it is about Eaſt and Weſt, as hath been already intimated. But note that in Sailing North and South you make not the Obſervations at the Sun's *Riſing* and *Setting*, but at its being due *Eaſt* and *Weſt*.

8. But you may put the Rule here preſcribed in Practice, by taking two *Equal Altitudes* of ſome known *Star*, that Riſeth high above the Horizon. For you ſhall thence according to the mentioned Rule, know at what time by the *Watches* the Star hath been in the South, and ſo the *Right Aſcenſion* of that Star being known, as alſo the *Right Aſcenſion* of the *Sun*, you may thence eaſily Calculate, what Time it then was: Which, being compared with the Time of the *Watches*, as before, ſhall give the *Longitude* of the Place where you were, when you had the Star in the Meridian.

9. If the *Watches* that have gone exactly for a while, ſhould come to differ from one another (as in length of time it may well happen, that the one or other fail a Minute, more or leſs;) in that caſe it will be beſt to Reckon by that which goes faſteſt; unleſs you perceive an apparent cauſe, why it goes too faſt (as it may happen when the *Cheeks* retain not their proper Figures) ſeeing it is not ſo eaſy for theſe *Pendulum Watches* to move faſter than at firſt, as it is to go ſlower. For the *Wire*, on which the *Pendulum* hangs, may perhaps by the violent Agitation of the Ship, come to ſtretch a little, but it cannot grow ſhorter; and the little weight of the *Pendulum* may perhaps ſlip downward, but cannot get up Higher.

If it ſhould be ſaid, that upon any Foulneſs the *Watch* will go faſter by reaſon of the ſhorter Vibrations of the *Pendulum*, it is to be conſidered, that this is only True when the *Watches* have no *Cheeks*, but when they have them 'tis not ſo. n. 48. p. 976.

10. When you get Sight of any known Country, Iſland or Coaſt, be ſure to Note the *Longitude* thereof as exactly as you can by the help of the Rules here preſcribed. Firſt, thereby to Correct the *Sea-Maps*, after the *Longitude* of a Place ſhall have been found at divers times to be the ſame, ſo that you doubt no more of it. For all *Maps* are very Defective as to the Situation of Places in reſpect of Eaſt and Weſt, chiefly where Seas are interpoſed. Secondly, to be able always to know in the Proſecution of your Journey, how far yon have ſailed from any Place to the Eaſt or Weſt. And, if by any Notable Miſchance or Careleſſneſs all the *Watches* ſhould come to ſtand ſtill, yet you may at any Place, whereof the *Longitude* is certainly known, ſet them a Going again, and Adjuſt them there by the *Sun*, and ſo reckon the *Longitudes* from that ſame Meridian. For you are to know, that you are not at all obliged to put one certain Meridian of any known Place as a Beginning of the *Longitude Reckoning*; this happening only in *Maps*, or Tables of *Longitude*: As when you take for that purpoſe the *Meridian* of the *Pico* in *Teneriffe*, or that of the Iſlands of *Corvo* and *Flores* (the moſt Weſterly of the *Azores*) or any others. Yet it were very fit, that all *Geographers* agreed and pitched upon one and the ſame *Firſt Meridian*, that ſo all Places might be known by the ſame Degrees as well of *Longitude* as *Latitude*; though in Voyaging, it is ſufficient to obſerve only the Difference of *Longitudes*, beginning to Reckon from the *Meridian* of any Place, you pleaſe, as if it were the *Firſt*. n. 47. p. 951.

11. If it happen that being at *Sea* all the *Watches* ſtop, you muſt as ſpeedily as is poſſible, ſet them a Moving again, that you may know how much you Advance from that Place towards the *Eaſt* or *Weſt*.

The Journal for the Watches.

12. The *Watches* being diſtinguiſhed by Marks, as A, B, or the like, every Day about Noon, or when moſt conveniently you can, Obſerve the Time of the Day by the *Sun*, or by the *Stars* at Night, and ſubduct thence the Minutes and Seconds, that are adjoined to that Day in the Table, and write the Remainder down in the Paper, wherein 9 Columns or more are marked, placing them in the Second Column, having placed the Day of the Month in the Firſt; and at the ſame time write down the Hours, Minutes, and Seconds, of each *Watch* in a diſtinct Column, all Oppoſite one to another. Then in another Column write down the Difference between the Time taken by Obſervation, and that given by the *Watches*, or one of them. Then one Column for the *Latitude*: One for the *Longitude* by the Ordinary Way of *Reckoning*; Another for the *Longitude* taken from the Difference between the Time found by Obſervation, and that given by the *Watches*: And at laſt a large Column to Note the Accidents, that befall the *Watches*, &c.

The Succeſs of Pendulum Watches; by Maj. Holmes. n. 1. p. 13.

2. Major *Holmes* having left the Coaſt of *Guinea*, and being come to the Iſle of St. *Thomas* under the *Line*, he adjuſted his *Watches* there, and put to Sea, and Sailed Weſtward, 7 or 800 Leagues, without Changing his Courſe;

after which finding the Wind favourable, he ſteered towards the Coaſt of *Africk* N. N. E. But, having Sailed upon that Line, a matter of two or three hundred Leagues, the Maſters of the other Ships under his Conduct, apprehending that they ſhould want Water, before they could reach that Coaſt, did propoſe to him to ſteer their Courſe to *Barbadoes*, to ſupply themſelves with Water there. Whereupon the ſaid *Major*, having called the Maſter and Pilots together, and caus'd them to produce their Journals and Calculations, it was found that thoſe Pilots did differ in their Reckoning from that of the *Major*, one of them about 80 Leagues, another about an hundred, and the third more; but the *Major* judging by his *Pendulum Watches* that they were only ſome thirty Leagues diſtant from the Iſle of *Fuego*, which is one of the Iſles of *Cape Verde*, and that they might reach it next Day, and having a great Confidence in the ſaid *Watches*, reſolved to ſteer their Courſe thither, and having given Order ſo to do they got the very next Day about Noon a Sight of the ſaid Iſle of *Fuego*, finding themſelves to Sail directly upon it, and ſo arrived that Afternoon as he had ſaid.

M. Hugens being informed of this Succeſs, wrote to *Paris* to this effect; Ib. *p.* 14. I did not imagine that the *Watches* of this Firſt Structure would ſucceed ſo well, and I had reſerved my main Hopes for the New Ones. But, ſeeing that thoſe have already ſerved ſo ſucceſsfully, and that the other are yet more juſt and exact, I have the more Reaſon to believe, that the Invention of *Longitudes* will come to its Perfection. In the mean time I ſhall tell you, that the States did receive my Propoſition, when I deſired of them a Patent for theſe New *Watches*, and the Recompence ſet apart for the Invention in Caſe of Succeſs; and that without any difficulty they have granted my Requeſt, commanding me to bring one of theſe *Watches* into their Aſſembly, to explicate unto them the Invention, and the Application thereof to the *Longitudes*, which I have done to their Contentment.

III. 1. Inveni tandem Modum *Lunæ Locum* ſciendi, exili quodam Inſtrumentiolo tantum adjutus ad unum vel duo Scrupula; &, quod mirum eſt, nec *Refractiones*, nec *Parallaxes* meis Obſervationibus obſunt, quia Ingenioſa Methodus his Tricis me liberat. Poſſum hac Methodo *Terra Marique* uti; ideo Tabulis *Lunæ* Correctis non amplius Modus deſideratas *Locorum Longitudines* captandi, in omnibus *Terræ Mariſque* Locis ignorabitur.

Longitudes from the Moon's Places; by Math. Profeſſor at Seville. n. 118. *p.* 427.

2. Quod de Inſtrumentiolo ſuo ſcribit *Profeſſor Hiſpalenſis*, Fidem meam (quod bonâ ipſius Veniâ dictum velim) ſuperat. Privari enim *Luna* nec *Refractione*, nec *Parallaxi*, in Horizonte noſtro poteſt, niſi ad *Zenith* aliquando poſſet pertingere; eouſque enim extenduntur, ejuſque Locum implicant *Refractiones*: Deſinit in Nonageſimo Gradu *Eclipticæ* ſemel tantum de Die *Longitudinis Parallaxis*, ſed *Latitudinis* non perinde. Nec ſatis capio, quomodo fabricari Inſtrumentum poſſit, quod unà cum *Parallaxi Refractionem*, cujus Incrementum longè diverſam habet rationem, conſideret.

Conſider'd by Mr. Flamſteed. Ib. *p.* 431.

IV. The

Longitudes *from Lunar Occultations; by* M. Halley. n. 181. p. 87.

IV. The Obſervation of *Lunar Occultations* is of Singular Uſe to determine the *Longitude* of Places, eſpecially thoſe that are far Remote.

Longitudes *by the Revolution of* Jupiter *upon his Axis; by M.* Caſſini. n. 82. p. 4042.

V. The *Revolution* of *Jupiter* upon his *Axis* being the ſwifteſt, and the moſt Regular Motion that is hitherto known in the Heavens, a Traveller alone, even without having any Correſpondence with other Obſervers, may make uſe of it to find the *Longitudes* of the moſt Remote Places of the Earth.

Longitudes *by the Satellite Eclipſes; by M.* Hevelius. n. 78. p. 3030.

VI. 1. Utrum æque bene *Jovialium Eclipſes* ad inveſtigandam *Differentiam Meridianorum* conducant, ut quidem Occultationes Fixarum à *Luna*, habeo fere cur dubitem; præprimis ob minus Tardum *Jovialium* Motum, utut etiam accuratiori Tubo peragantur.

By M. Borelli. n. 128. p. 691.

2. The *Eclipſes* of the *Satellites* of *Jupiter*, which happen almoſt every Day, afford a fair Way for eſtabliſhing the *Longitudes* over all the Earth. For, beſides that theſe *Eclipſes* are very frequent, the *Emerſion* and *Immerſion* of theſe *Satellites*, eſpecially in the Shadow of *Jupiter*, is ſo Momentany and ſo Senſible, that they may be obſerved with the greateſt Exactneſs, being altogether Exempt from thoſe Eſſential Inconveniences that accompany the *Eclipſes* of the *Sun* and *Moon*, which alſo are Rare, and whoſe *Beginning* and *End* are always doubtful by reaſon of a certain *Ambiguous Light*.

The *Longitude* of Places at *Sea*, *Capes*, *Promontories*, and divers *Iſlands*, being once exactly known by this Means, would doubtleſs be of Great Help, and conſiderable Uſe to *Navigation*.

By Mr. Flamſteed. *n.* 151. p. 322.

3. The *Eclipſes* of *Jupiter's Satellites* have been eſteemed, and certainly are a much better Expedient for the diſcovery of the *Longitude* than any yet known, by reaſon that they happen frequently, and are eaſily obſervable with a Teleſcope of 12 Foot, or for need with one of eight.

n. 154. p. 404.
n. 165. p. 760.

The *Longitude* might be alſo attained by Obſervations of the *Moon*, if we had Tables that would anſwer her Motions exactly; but after 2000 Years experience (for we have ſome Obſervations of *Eclipſes* much Ancienter) we find the beſt Tables extant erring ſometimes 12 Minutes or more in her apparent Place, which would cauſe a fault of half an Hour, or 7½ *deg.* in the *Longitude* deduced by comparing her *Place* in the Heavens with that given by the *Tables*. I undervalue not this Method, for I have made it my Buſineſs, and have ſucceeded in it, to get a large Stock of good *Lunar Obſervations* in order to the Correction of Her *Theory*, and as a ground-Work for better *Tables*; but, if we ſhould happily attain what we ſeek, yet the Calculation will be ſo Perplexed and Tedious, that it will be found much more Inconvenient and difficult than that I propoſe by obſerving the *Eclipſes* of *Jupiter's Satellites*, which however at preſent I muſt prefer. For I am perſuaded, that the *Eclipſes* of the *Firſt* will ſcarcely be found above 4 Minutes of Time different from my Calculations, and I hope it will ſcarce ever be found to Err ſo much. But, if the ſame *Eclipſe* may be obſerved in two Diſtant Places at the ſame time, or compared with an Obſervation of the ſame *Satellite*, made within a Week elſewhere, the *Difference* of *Meridians* will be had ſomething better, than by comparing two Obſervations of the ſame *Phaſis* of a *Lunar Eclipſe*, made in Diſtant Places. For, whereas it is ſomewhat difficult by reaſon of the *Penumbra* to determine the True Time of the Application of either of the *Moon's* Limbs to the Shadow, the *Satellite Eclipſes*, eſpecially thoſe of the *Firſt*, are almoſt Momentany.

And, whereas there can rarely happen 4 *Eclipſes* of the *Moon* Viſible the ſame Year, thoſe of the *Satellites* happen ſo frequently that there are more of them Viſible in one Year than we Count Days in it, though the Planet *Jupiter* lie hid under the *Sun*'s Rays every Year a whole Month together.

I know our *Navigators* will Object againſt this Method, that it is difficult to Practice at *Sea*, becauſe Long Teleſcopes are required, which the Motion of the *Ship* will not permit them to manage abroad. But, if it be not Practicable at *Sea*, they cannot deny but that it is at *Land*; and that the *True Longitude* of Remote Coaſts from us is the Firſt thing deſired for the Correction of their Charts : Let them attempt theſe Firſt, and I doubt not but the Succeſs will encourage them ſo much, that they will readily find means to put it in Practice at *Sea* ; That the *French* have uſed this Method ſucceſsfully both in *Denmark* and in their *Own Country* ; That a Teleſcope of 14 Foot Long at moſt, or for need one of 8 Foot, with broad *Eye Glaſſes*, will be ſufficient for this purpoſe ; That the Difficulty cannot be known 'till it be tryed, and that Uſe renders many things eaſy, which our firſt Thoughts conceived Impracticable.

If it be Required to know whether any of thoſe *Eclipſes* which are Inviſible with us, be Viſible in any other Given Place, Convert the *Difference* of *Meridians* betwixt it and *London* into Time; and, if the Place Lie to the *Eaſt* of *London*, Add it to, if to the *Weſt*, Subtract it from, the Time of the Appearance at *London*, the Sum or Difference accordingly ſhall be the True Time of the *Eclipſe* under that *Meridian*, at which, if *Jupiter* be above the Horizon, and the *Sun* beneath it, the *Eclipſe* is there Viſible, otherways not.

Or, By the help of the *Ephemerides* of the *Planet*'s Places and a *Terreſtrial Globe*, the ſpace on it in which any of theſe *Eclipſes* will be Viſible may be found thus.

Firſt ſeek the true Places of the *Sun* and *Jupiter* with his *Latitude* in the *Ephemerides*, whereby you may find their Declinations and Right Aſcenſions, either by the Vulgar Tables, or the Globe itſelf, exactly enough for this Method.

Bring *London* on the *Globe* to the *Meridian*, and, detaining it there, note what *Deg*. of the *Æquator* is cut by it. From this Subtract the Time of the *Eclipſe* after Noon converted into *deg*. and *min*. the Remainder ſhews you the *Longitude* of that *Meridian* on the Earth, where it is then Noon when the *Satellite* is *Eclipſed*, which I therefore call the *Meridional Longitude* of the *Eclipſe*. Bring this *Meridional Longitude* under the *Meridian*, and Elevate the nearer Pole to the *Sun* as much as is his *Declination*, keep the *Globe* in this Poſition, and, if *Jupiter* be in *Conſequence* of the *Sun*, Draw a

Line of the *Globe* along the *Eaſtern Horizon*, it paſſes over all thoſe Places where the *Sun* is Setting at that Time, but, if *Jupiter* be in *Antecedence* of the *Sun*, Draw the ſaid Line on the *Globe* by the *Weſtern* Edge of the *Horizon*, it paſſes over all thoſe Places where the *Sun* is then Riſing. *Jupiter* being in *Conſequence* of the *Sun*, Add the Difference of *His* and the *Sun*'s Right Aſcenſions to the *Meridional Longitude* afore-mentioned, bring the *Deg.* of the *Æquator* anſwering their Sum under the Meridian, Raiſe the *Pole* next *Jupiter* equal to his Declination, and, detaining the *Globe* in this Poſition, Draw a Line again to the *Eaſtern Horizon*, the Space intercepted betwixt This and the Line of the *Sun*'s Setting, before Deſcribed on the *Globe*, comprehends all thoſe places on the Earth where this *Eclipſe* is ſeen from *Sun Setting* till *Jupiter* is *Set*. But, if *Jupiter* were in *Antecedence* of the *Sun*, Subtract the difference of His and the *Sun*'s Right Aſcenſions from the *Meridional Longitude*, Set the *Degree* of the Æquator anſwering the Remainder under the *Meridian*, and Elevate the *Pole* next *Jupiter* equal to his Declination. Keeping the *Globe* in this poſition, Draw a Line by the *Weſtern* Edge of the *Horizon*, the Space Included betwixt This and the Line of the *Sun's* Riſing contains all thoſe places on the Earth, where this *Eclipſe* is Viſible betwixt *Jupiter's Riſing* and *Sun-riſe*.

When any *Eclipſe* of theſe is obſerved, the Difference betwixt the Noted Time and that given by the *Tables* ſhall be the *Difference* of *Meridians* betwixt the Place of the Obſervation and *London*.

As the *Sun* Removes from the Conjunction of *Jupiter*, the *Ingreſſes* of the *Satellites* into his Shadow become Obſervable. When he is about 30° from it the *Emerſions* of the *Fourth*, and at 60° of the *Third*, begin to be ſeen betwixt the Shadow and Body continuing ſo till the *Sun* be arrived within 60° of the Oppoſition of *Jupiter*, when the *Emerſions* of the *Third* fall behind his Body, but the *Emerſions* of the *Fourth* continue Viſible till he be leſs than 30°, Diſtant from ☍, at which Time they alſo are hid behind him, all the Appearances being made really to the Right Hand or in Antecedence of *Jupiter*, tho' with Inverting Teleſcopes, they appear on the contrary, to the Left.

After the Oppoſition of the *Sun* and *Jupiter* we begin to ſee the *Immerſions* of the *Satellites* from the Shadow now on the Left Hand, or in Conſequence of *Jupiter*, but through Inverting Glaſſes on the Right, when the *Sun* is near 30 *deg.* from the Oppoſition, the *Ingreſſes* of the *Fourth*, when 60 *deg.* from it, of the *Third*, begin to be obſervable betwixt the Body and Shadow, continuing ſo till the *Sun* arrive at the ſame or rather within ſomething a wider Diſtance for the Conjunction of *Jupiter*.

4. Reſtauratæ *Geographiæ* Fundamenta, Hac Methodo facillimâ ac nullo fere Inſtrumentorum apparatu præſtandâ, ſed quæ minime Fallat Jaccantur. Quæ huc pertinent præcepta *Aſtronomice* Doctos latere non poſſunt ; unicum monere non abs re erit, nempe, Tubo octo vel etiam ſeptem pedum, hoc eſt, facile portatili, momenta harum *Eclipſium* ſatis diſtinctè obſervari poſſe, præſertim in exterioribus *Satellitibus*, ſi modo Lentis Objectivæ Apertura 2½ vel 3 pollices pateat. Sic enim Radiorum maxima copia ad oculum Refracta perveniet, unde Minimæ hæ Stellulæ in vicinia *Jovis* conſpici poſſint, quæ

By Mr. Halley. n. 191. *p.* 435.

alias Luce ejus nimiâ obfuſcarentur; ac, quamvis Coloribus tingantur, ac *Jovis* Limbus parum nitidus videatur, tamen, cum de momento amiſſæ vel recuperatæ Lucis unice agatur, ſufficit eas Lumine quantum fieri poſſit auctas in oculos certius incurrere.

n. 214. p. 237. The *Eclipſes* of the *Firſt Satellite* of *Jupiter* are found by the *Royal Academy* at *Paris*, in Aſcertaining the Geographical Site of the Principal Ports of *France*, almoſt *Inſtantaneous*, and with good Teleſcopes diſcernible almoſt to the very Oppoſition of *Jupiter* to the *Sun*. So that, could the *Satellites* be Obſerved with Teleſcopes Manageable on *Shipboard*, a *Ship* at *Sea* might be enabled to find the Meridian ſhe was in to a very great Exactneſs, beyond what we can yet Hope to do by the *Moon*, tho' She ſeem to afford us the only Means Practicable for the *Seamen*. However, before *Saylors* can make uſe of the Art of *Finding* the *Longitude*, it will be requiſite that the *Coaſt of the whole Ocean* be firſt Laid down Truly, for which Work this Method by the *Satellites* is moſt Appoſite.

Long. and Lat. of Derby; *by Mr.* Flamſteed. n. 55. p. 1107, 1106. n. 111. p. 237.

VII. The *Longitude* of *Derby* from *London* W. is 5 or 6 *min.* the *Latitude* 52° 57′ or 59′.

Lat. of Ection. n. 76. p. 2272.

VIII. *Ectoniæ*, in Com. *Northamptoniano*, *Lat.* 52° 15′.

Long. and Lat. of Townley. n. 127. p. 664.

IX. *Townleii* in Com. *Lanc.* Latitudo Obſervata (ut ſcribit D. *Townleius*) 53° 44′; *Longitudo* à Meridiano *Londinenſi* 9. circiter ſcr. hor. ad Occaſum.

Lat. of Tredagh. n. 164. p. 749.

X. The *Latitude* of *Tredagh* in *Ireland* is 53° 40′.

Long. of Oxford *and* Dantzick; *by Mr.* Halley. n. 129. p. 724. *vid. ſup. cap.* IV. § LXII.

XI. Having carefully conſidered the *Parallaxes* of the *Moon* in the Obſervations of the Occultation of *Mars*, *Aug.* 21. 1676. at *Dantzick* and *Greenwich*, I find from the *Immerſion* the *Difference* of *Meridians* between *Greenwich* and *Oxford* 4′ 57″, between *Greenwich* and *Dantzick* 1ʰ 14′ 50″: By the *Emerſion* the *Firſt* of theſe *Differences* is found 4′ 59″, the latter 1ʰ 14′ 41″; which near Agreement ſhews the Exactneſs of the Obſervations.

Long. of Paris; *by M.* Caſſini. n. 117. p. 390. *vid. ſup. cap.* IV. § XLV.

XII. 1. Obſervationes D. *Flamſteedii* circa *Lunæ* Eclipſin *Jul.* 7. ſt. n. 1675. cum noſtris in *Regio Obſervatorio* habitis magna cum voluptate contuli. Ex iis quippe *Differentiam Meridianorum*, quam olim, noſtrarum Obſervationum Collatione, Minutorum 11 definieram, nunc decem novis Comparationibus eandem intra pauca Secunda video confirmari.

Obſervationes D. *Flamſtedii.*	h.	′	″	Obſervationes *Noſtræ*	h.	′	″	*Differentia Merid.*	′	″
Pentadactyl. Tectus	1	55	15	Idem ſeu *Seleucns.*	2	06	15		11	00
Porphyrites Tectus.	2	02	20	Idem ſeu *Ariſtarchus.*	2	12	40		10	20
Sinæ Limbus Primus.	2	05	30	Ejuſdem ſeu *Tychonis*	2	16	30		11	00
				vel*		16	25		10	55
Ætnæ Limb. Primus.	2	06	00	Ejuſ. ſeu *Corpernici.*	2	16	30		10	30
				vel*		16	40		10	40
Besbici Limbus Prior	2	23	05	Ejuſ. ſeu *Manil* Med.	2	34	15	Min. quam	11	05
Herminius Tectus.	2	26	03	Ad eund. ſeu *Dionyſ.*	2	36	15	Maj. quam	10	12
Tetigit Limbum Primum *Corocondometis.*	2	39	30	Ejuſ. ſeu *Palud. Som.*	2	50	20		10	50
Tetig. *Palud. Mæotid.*	2	45	00	Eand. ſeu *Mare Caſp.*	2	55	20		10	20
				vel*		55	40		10	40
Mæotis tota Tecta.	2	50	40	Eadem.	3	01	10		10	30
Immerſio.	2	56	55	*Immerſio.*	3	07	45		10	50
				vel*	3	07	40		10	45

Tota hæc * denotat D. *Caſſini* peculiarem Exiſtimationem, in reliquis cum DD. *Picardo* & *Romero* conſentit.

2. Medium *Eclipſeos* Lunaris, *Jan.* 1. ſt. n. 1675. deductum eſt ex Comparatione *Initii* & *Finis* —— 3 20 00 (h. ′ ″) n. 123. p. 562.

Duarum *æqualium Phaſium* —— 3 20 15 vid. ſup. Cap. IV. §. XLVI.

Ex D. *Flamſtedii* Obſervationibus Medium *Eclipſis* pari modo eruatur. Is quippe $2^{h}\ 29'\ 30''$. Diſtantiam Cuſpidum obſervavit $17'\ 16''$ & $3^{h}\ 52'\ 45''$, *Eclipſi* decreſcente, Diſtantiam obſervavit $18'\ 57''$, uno ſcil. Minuto $41''$ Majorem. Itaque *Medium Eclipſis* propius eſt poſteriori Obſervationi quam priori. Medium Tempus inter utramque Obſervationem fuit $3^{h}\ 11'\ 7''$. Tardius igitur aliquanto deducitur hinc *Eclipſis Medium*; unde *Differentia Meridianorum* proveniret Minor $9'$; quod minime convenit Obſervationibus Certioribus *Eclipſis* præcedentis Æſtivæ, ex quibus illam deduxi Min. 10¾. Prior Obſervatio noſtra cum Priori D. *Flamſtedii*, aliquanto tardiore, comparata, *Differentiam Meridionorum* exhibet Majorem $8'\ 35''$. Poſterior noſtra, tardior Obſervatione poſteriori D. *Flamſtedii*, *Differentiam Meridianorum* exhiberet Minorem $9'\ 40''$.

	h.	′	″
Finis à D. *Flamſtedio* Exiſtimatus	4	07	15
Et à *Nobis.*	4	15	25
Differentiam Meridianorum inferret	0	08	10
Initium à D. *Halleio Londini* Obſervatum	2	16	00
Cum Obſervato à *Nobis*	2	24	35
Differentia Meridianorum faceret	0	08	35

Ex hac igitur *Eclipſi Differentia Meridianorum* erueretur duobus circiter Minutis minor quam ex *Eclipſi* Æſtatis præcedentis, quam tamen huic longè præfero; non ſolum ſpectatâ majori Facilitate Determinandi Tempora Appulſuum & Emerſionum in ea *Eclipſi Totali*, quam in hac *Partiali*;

tiali; verum etiam ob Aeris ſerenitatem, qua utique æqualiter uſi fuimus in ea *Eclipſi*, cum in hac *Pariſiis* Cœlum ſereniſſimum, *Londini* fuit ſubnubilum. Priori itaque ſtandum cenſeo, donec per Obſervationes *Immerſionum* & *Emerſionum Satellitum Jovis*, quos ad hanc rem exiſtimo maxime Idoneos, rem ſcrupuloſius determinemus.

Ib. p. 564.

By Mr. Flamſteed. Ib. p. 565.

3. *Differentiæ Meridianorum*, ab Eclipſi *Lunæ Junii* 27. 1675. *Londini* & *Pariſiis* Obſervata deductæ, vix fidere poſſum, quippe licet Tempora Phaſium à vobis Obſervatarum accuratiſſimè determinata credam,. Ego, cum amplior non ſuppeteret, Quadrante uſus fui 20 tantum Digitorum Radio, ad Horologium Corrigendum, quippe Nuda duntaxat habuit Pinnacidia; & propterea de Momento Phaſis alicujus certior eſſe vix potui quam ad unum Minutum Horarium. Noviſſimam *Eclipſin Dec.* 22. 1675. Inſtructior obſervavi; cum tamen mihi Aer ſubnubilus extiterit, & propter Obliquam *Lunæ* in Umbram Terræ Incidentiam tardiſſimus fuerit ejus ad Maculas Appulſus, minus apta fuit hæc *Eclipſis* huic Negotio. Incerta igitur inter duo minuta Horaria manet etiamnum *Meridianorum* noſtrorum *Differentia*, quam tamen nullus dubito Nos pro Votis aliquando determinaturos eſſe.

Long. of Straſborg *and* Paris; *by* M. Bullialdus. n. 125. p. 610. *vid. ſup. Cap.* IV § XLVI.

XIII. Meridianus *Pariſienſis* ab *Argentoracenſi* diſtat 22′ 48′. ex *Fine Eclipſeos Lunaris Jan.* 1. ſt. n. 1676. Diſtat autem ex hac *Eclipſi Meridiannus Pariſienſis* à *Londinenſi* ad Ortum 6′ 38″, qui ex Obſervatione *Eclipſ. Jul.* 7. 1675. apparuit 10′, ut etiam in *Eclipſ. Jan.* 11. ejuſdem Anni.

Long. of Avignon; *by Mr.* Halley. *Ph. Col.* n. 5. p. 126.

XIV. Avignon is 19′ 40″, or 4° 50′ to the Eaſtward of *London*.

Long. & Lat. of ſeveral Places in France. n. 163. p. 718, 719, 710. *vid. ſup. Cap.* IV. § XXXIV

XV. *M. Caſſini* having Compared together the Obſervations of the *Solar Eclipſes* of *July* 12. *ſt. n.* 1684. and made ſuch Reductions as the Parallax requires, lays down the *Longitudes* from *Paris* to

Aix in *Province* 14′ E. The *Lat.* by M. *Gautier* is 43° 30′.
Avignon 8½ E.
Lyons 8′, or 13′. E.
Roſes 4′ E. The *Lat.* by M. *Chaſſeles* 42° 10′.
Honfleur 7′ W.
Pau, 11′. W. The *Lat.* by *P. Richaud* 43° 30′.

Long. of Lisbon. n. 146. p. 151. *vid. ſup. Cap.* IV. § XLIX.

XVI. Mr. *Jacols* an *Engliſh* Merchant reſiding at *Lisbon*, informed Mr. *Flamſteed* that he Obſerved the *Beginning* of the *Lunar Eclipſe*, Feb. 11. 168½ there at 8h 31′. *p. m.* which gives the *Difference* of the *Meridians* betwixt the *Obſervatory* at *Greenwich*, and *Lisbon*, 41½ *Minutes* of Time, or 10° 22′. conſiderably Different from our *Mapps* and *Sea-Charts*.

Lat. of Madrid; *by* E. *of* Sandwich. n. 22. p. 390.

XVII. The *Earl* of *Sandwich* Eſteem'd, by the *Sun's* Altitude in the *Solſtice*, and by other Meridian Altitudes, the *Latitude* of *Madrid* to be 40° 10′; which differs conſiderably from that aſſigned by others, The General Chart of *Europe* giving to it 41° 30′, the General Map of *Spain* 40° 27′, and a large Provincial Map of *Caſtile* 40° 38′.

XVII.

XVIII. 1. Per plures Congruentes Obſervationes *Lunares* & aliorum Planetarum, inveni *Diſtantiam* Civitatis *Hiſpalenſis Longitudinariam* eſſe ab *Uraniburgo* 90′, ſive $1\frac{1}{2}$h, vel intra 2′ Differentem.

2. Videat D. *Profeſſor* quomodo *Merediani Hiſpalenſis* ab *Uraniburgico* Interſtitium Scrupulorum 90′ conſtituerit: Deliquii enim *Lunaris* Obſervationes *Jan.* $\frac{1}{11}$. 1675. *Londini Medium* ponunt 7h $11'\frac{1}{2}$ *p m.* cui Annotationes *Pariſinæ* conſentiunt; dicti *Profeſſoris* Obſervationes *Medium Hiſpali* ſtatuunt 6h 47′: Noſtrorum ergo *Meridianorum Differentiæ* $24'\frac{1}{2}$: At Nos inter & *Uraniburgum* non intercedunt niſi Minuta 52′. Eſt igitur *Meridianorum Differentia* nonniſi 1h $16'\frac{1}{2}$ inter *Hiſpalim* & *Uraniburgum.* Vereor tamen, annon Oculis nudis D. *Profeſſoris* factæ fuerint Obſervationes: *Incidentiæ* quippe & *Emerſionis* Tempora faciunt 1h 5′; cum *Noſtræ*, *Pariſinæ*, *Hevelianæque* Obſervationes non faciant ea Tempora plus quam 1h $1'\frac{1}{2}$, forſan aliquanto minus.

Long. of Seville *and* Uraniburg; *by* .... Math. Profeſſor *at* Seville. n. 118. p. 427. *By Mr.* Flamſteed. *ib.* p. 431. *vid. ſup. Cap.* IV. § XLIV.

XIX. *Differentia Meridianorum Hafniæ* & *Pariſiorum*, Obſervationibus *Jovialium*, reperta eſt à D. *Picard*, 0h 41′ 40″.

Long. of Copenhagen; *by M.* Picard. n. 146. p. 145.

XX. *An.* 1680. *Oct.* 23. *St. v. S. Joſ. Ponthia*, and *Marco AntonioCellio*, with a Teleſcope of 25 Palms, Obſerved the Total *Immerſion* of the *Firſt Satellite* in *Jupiter's* Shadow at *Rome*, at 10h 7′ 53″; *p. m.* which in our *Obſervatory* here I noted at 9h 15′ 41″, whoſe Difference is the *Difference* of our *Meridians* = 52′ 12″, or 13° 03′. Again, *Jan.* 28 1685. S. *Francis Blanchini* Obſerved the Total *Immerſion* of the *Firſt* at *Rome*, at 11h $19'\frac{1}{4}$ which I ſaw not here, but my Numbers give at 10h $27'\frac{1}{4}$: Therefore the *Difference* of *Meridians* is $52'\frac{1}{2}$, and *Rome* lies ſo much more *Eaſterly* than the *Obſervatory* at *Greenwich*; agreeing with the former Obſervation.

The Noble *Tycho* judged therefore not much amiſs, when he placed *Uraniburg* and *Rome* under the ſame *Meridian*; for by ſeveral Obſervations of *Satellite Eclipſes* it is Evident, that the *Difference* of *Meridians* betwixt *Uraniburg* and our *Obſervatory* is 51′ 10″ of Time, ſo that *Rome* lies only one *Minute* of Time, or $\frac{1}{4}$ of a *Deg.* to the *Eaſt* of *Uraniburg*.

Long. of Rome *and* Uraniburg; *by Mr.* Flamſteed. n. 177. p. 121.

XXI. 1. *Dantzick* is by many and undoubted Obſervations proved to be 1 15′ 30″. more *Eaſterly* than *London*.

2. *An.* 1683. Die ipſo *Solſtitii Æſtivi* 21 *Junii* ſt. n. *Gedani*, Altitudo *Solis* in Meridie fuit 59° 7′, Quadrante quidem parvulo Orich. ſed tamen ſatis accurato. Die vero *Æquinoctii Autumnalis* Altitudo *Solis* in Meridie reperta eſt 35° 27′.

Long. of Dantzick; *by Mr.* Halley. *Ph. Col.* n. 5. p. 124 *Lat. of* Dantzick; *by M.* Hevelius. n. 151. p. 330. n. 154. p. 424.

XXII. The *Longitude* of *Nuremburg* has been formerly ſtated 11° from *London*, and ſince found to be ſo by Obſervations of the *Eclipſe* of the *Sun* *July* 2ᵈ 1684. which made it $44'\frac{1}{2}$ of Time.

Long. of Nuremburg; *by* .... n. 182. p. 147.

Long. of Moſcua Leipſick and Aleppo; by . . . . n. 192. *p.* 452. *vid. ſup. Cap.* V. §. LIII.

XXIII. The *Duration* of the *Lunar Eclipſe Apr.* 5. 1688. is made by M. *Timmerman* from 7ʰ 38′, about 10ʰ 45′, which agrees within 8 or 10 *Minutes* with our *Tables*, that never err ſenſibly in the continuance of *Eclipſes*; and ſo much ought to be allowed to an Obſerver not ſufficiently inſtructed to diſtinguiſh the *Penumbra* from the true Shadow, though a ſmall *Teleſcope* were uſed in this Obſervation. Let us conclude then, that the *End* was at 10ʰ 40′ at *Moſco*. We do not find that this *Eclipſe* was obſerved at *London*: However this defect is in good part ſupplied by an Obſervation thereof made at *Leipſick*, by M. *Gotfrid Kirck*, and publiſhed in his *Ephemerides* for the Year 1689; Where the *End* is determined at 8ʰ 54′ *p. m.* Hence *Moſco* will be 1ʰ 46′ to the *Eaſtward* of *Leipſick*; and the *Difference* of *Meridians* between *London* and *Leipſick* being already determin'd 49′, it will follow that *Moſco* is 2ʰ 35′ to the *Eaſt* of *London*, or 38° 45′ of Longitude, which from other Accounts we find to be very near that of the City of *Aleppo* in *Syria*.

n. 181. *p.* 86. *it is* 52 *Min.*

Lat. of ſeveral Places in Ruſſia. ib. p. 454.

By the ſame hand we have procured the Latitudes of the following Places, obſerv'd, as 'tis ſaid, with a large Quadrant.

Moſco	55°	34′
Tereſlaw	57	44
Wologda	59	19
Woſtak	61	15
Arch-Angel	64	30

Latitudes of ſome Remarkable Places; by Mr. Francis Vernon. n. 124. *p.* 582. Jan. An. 1675.

XXIV. I have been as curious as I could in taking the Latitudes of ſome Remarkable Places: As I find them I ſhall give them you.

Athens	38°	05′	*Patras*	38°	40′
Corinth	38	14	*Delphos*	38	50
Sparta	37	10	*Thebes*	38	22
Corone	37	02	*Negropont* or *Chalcis*	38	31

Latitudes of Conſtantinople *and* Rhodes; *directed to A. B.* Uſher; *by Mr.* Greaves. n. 178. *p.* 1295. Dec. An. 1685.

XXV. Upon Intimation of your *Grace*'s Deſires, and upon Importunity of ſome Learned Men, having finiſhed a Table, as a Key to your *Grace*'s exquiſite Diſquiſition, touching *Aſia properly ſo called*; I thought my ſelf obliged to give both you and them a reaſon, why in the ſituation of *Byzantium* and the Iſland *Rhodes*, (which two Eminent Places I have made the *παραπέγματα* and Bounds of the *Chart*,) I diſſent from the Traditions of the *Ancients*, and from the Tables of our *Late* and beſt *Geographers*; and conſequently, diſſenting in theſe, have been neceſſitated to alter the Latitudes (if not Longitudes): of moſt of the remarkable Cities of this diſcourſe. And firſt for Byzantium, the received Latitude of it by *Appianus*, *Mercator*, *Ortelius*, *Maginus*, and ſome others, is 43° 5′. And this alſo we find in the *Baſil Edition* of *Ptolemy's Geography*, procured by *Eraſmus* out of a Greek MS. of *Pettichius*. The ſame likewiſe is confirmed by another choice MS. in Greek of the moſt Learned and Judicious Mr. *Selden*, to whom for this Favour and ſeveral others I ſtand obliged. And as much is expreſſed in the late *Edition* of *Ptolemy* by *Bertius*, compared and corrected by *Sylburgius*, with a Manuſcript out of the *Palatine Library*. Wherefore it cannot be doubted, having ſuch a croud of Witneſſes, but that *Ptolemy* aſſigned to *Byzantium*, as our beſt *Modern Geographers* have done, the Latitude of 43° 5′. And this will farther appear, not only out of his *Geography*, where it is often expreſſed, but alſo out of his Μεγάλη Σύνταξις, or *Almageſt* as the *Arabians* term it, where deſcribing the Parallel paſſing διὰ Βυζαντίου, he aſſigns to it 43° 5′. What was the Opinion concerning *Byzantium* of *Strabo* preceding *Ptolemy*, or of *Hipparchus* preceding *Strabo*, or of *Eratoſthenes* Ancienter, and it may be Accurater than all of them, (for *Strabo Lib.* 2. calls him τελεώτατον πραγματευσάμενον περὶ τ γεωγραφίας) though *Tully* (*Lib. Ep. Ad Att.*) makes *Hipparchus* often reprehend *Eratoſthenes* as *Ptolemy* after him doth *Marinus*, their Writings not being now extant, (unleſs thoſe of *Strabo*) cannot be determined by us. But as for *Strabo*, in our Inquiry we can expect little Satisfaction; for his Deſcription of Places having more of the *Hiſtorian* and *Philoſopher*, (both which he hath performed with ſingular Gravity and Judgment) than the Exactneſs of a *Mathematician*, who ſtrictly reſpects the Poſition of Places, without Inquiſition after their Nature, Qualities and Inhabitants, (though the beſt *Geography* would be a Mixture of them all, as *Abulfeda*, an *Arabian Prince* in his *Rectification of Countries* above 300 Years ſince hath done;) I ſay for theſe Reaſons we can expect little Satisfaction from *Strabo*, and leſs may we hope for from *Dionyſius Afer*, *Arrianus*, *Stephanus Byzantius*, and others. Wherefore, next having recourſe to the *Arabians*, who in *Geography* deſerved the ſecond Place after the *Græcians*, I find in *Naſſir Eddin* the Latitude of *Byzantium*, which he terms *Buzantiya*, *Conſtantiniya*, to be 45°, and in *Uleg Beg's Aſtronomical Tables* the ſame to be expreſſed. *Abulfeda* chiefly follows four Principal Authors as his Guides, in the compiling of his *Geographical Tables*, thoſe are, *Alfaras*, *Albiruny*, *Hon Saiid Almagraby*, laſtly *Ptolemy*, whoſe *Geography* he terms a *Deſcription of the Quadrant*, (or the fourth Part of the Earth) Inhabited; and all theſe, according to his Aſſertion, place *Byzantium* in 45° of Latitude. And here it may juſtly be wondred, how this Difference ſhould ariſe between the *Greek Copies* of *Ptolemy*, and thoſe Tranſlated into *Arabick* by the command of *Almamon*, the Learned *Caliph* of *Babylon*; for *Abulfeda* expreſly relates, that *Ptolemy* was firſt intepreted in his Time, that is, in the Computation of *Almecinus* in *Erpenius*'s *Edition*, and of *Emir Cond* a *Perſian Hiſtoriographer*, more than 800 Years ſince: Concerning which *Abulfeda* writes thus, *This Book* (diſcourſing of *Ptolemy's Geography*) *was tranſlated out of the Grecian Language into the Arabick for Almamon*: And in this I find, (by three fair MSS of *Abulfeda*) *Byzantium* to be conſtantly Placed in 45°. and as conſtantly in the *Greek Copies* in 43° 5′. But in the πρόχειροι Κάνονες of *Chryſococca*, out of the *Perſian Tables*, made about the Year 1346. in *Scaliger*'s Calculation) it is Placed in 45°. To reconcile the Difference between the *Greeks* and *Arabians* may ſeem impoſſible, for the common Refuge of flying to the Corruption of Numbers by Tranſcribers, and laying the Fault on them which ſometimes is the Author's, will not help us in this particular; ſeeing the *Greek Copies* agree among themſelves, and the *Arabick Copies* amongſt themſelves. The beſt way to end the Diſpute will be, to give Credit concerning the *Latitude* of *Byzantium*, neither to the *Greeks* nor *Arabians*. And that I have reaſon for this Aſſertion, appears by ſeveral Obſervations of mine at *Conſtantinople*, with a Braſs Sextant of above 4 Foot *Radius*. Where taking, in the *Summer Solſtice*, the Meridian Altitude of the *Sun* without uſing any προσταφαίρεσεις for the Parallax and Refraction, (which at that time was not neceſſary,) I found the *Latitude* to be 41° 6′. And in this *Latitude* in the *Chart* I have placed *Byzantium*, and not in that either of the *Greeks* or *Arabians*. From which Obſervation, being of ſingular Uſe in the *Rectification* of *Geography*, it will follow by way of Corollary, that all *Maps* for the *North East* of *Europe*, and of *Aſia*, adjoining upon the *Boſphorus Thracius*, the *Pontus Euxinus*, and much farther, are to be Corrected; and conſequently the Situation of moſt Cities in *Aſia, properly ſo called*, are to be brought more Southerly than thoſe of *Ptolemy* by almoſt two intire Degrees, and than thoſe of the *Arabians* by almoſt four.

Concerning *Rhodes*, it may be preſumed, that, having been the Mother and Nurſe of ſo many Eminent Mathematicians, and having long flouriſhed in Navigation, by the Direction of theſe, and by the Vicinity of the *Phœnicians*, they could not be Ignorant of the preciſe *Latitude* of their Country, and that from them *Ptolemy* might receive a true Information. Though it cannot be denyed, but that *Ptolemy* in Places remoter from *Alexandria* hath much erred. I ſhall only inſtance in our Own Country, where he ſituates Λονδίνιον, that is *London*, in 54° of *Latitude*; and the τὸ μέσον or the middle of the *Iſle* of *Wight*, (which in the printed Copies is falſely termed 'Οὐίκλησις, but in the MSS. rightly 'Οὐῆκτις,) in 52° and 20′ of *Latitude*. Whereas *London* is certainly known to have for the Altitude of the *Pole*, or *Latitude* of the Place, only 51° and 32′, and the middle of the *Iſle* of *Wight* not to exceed 50° and ſome Minutes.

But in my Judgment *Ptolemy* is very excuſable in theſe and the like Errors, of ſeveral other Places far diſtant from *Alexandria*; ſeeing he muſt for their Poſition neceſſarily have depended either upon Relations of *Travellers*, or Obſervations of *Mariners*, or upon the *Longitude* of the *Day* meaſured in thoſe times by *Clepſydræ*: All which how uncertain they are, and ſubject unto Error, if ſome Celeſtial Obſervations be not joyned with them, and thoſe exactly taken with large Inſtruments, (in which kind the *Ancients* had not many, and *Our Times*, (excepting *Tycho Brahe*, and ſome of the *Arabians*) but a few,) I ſay no Man, that hath converſed with *Modern Travellers* and *Navigators*, can be ignorant. Wherefore to excuſe thoſe Errors of his (or rather of others fathered by him) with a greater Abſurdity, by aſſerting the *Poles* of the *World* ſince his time to have Changed their ſite, and conſequently all Countries their *Latitudes*, as *Mariana* the Maſter of *Copernicus*, and others after him have Imagined: Or elſe to Charge *Ptolemy*, being ſo excellent an Artiſt, with Ignorance, and that even of his own Country, as *Cluverius* hath done, (from which my Obſervations at *Alexandria*, and

Memphis, may Vindicate him,) the former were too great a Stupidity, and the latter too great a Preſumption. But to return to *Rhodes*: An Iſland (in *Euſtathius's Comment* upon *Dionyſius's* Περιήγησις) of 920 Furlongs Circuit, where according to *Ptolemy* the Parallel paſſing διὰ Ῥόδου, hath 36° of *Latitude*, and ſo hath *Lindos*, and Ἰηλυσσὸς the chief Cities of the Iſland; the ſame is confirmed by the MS. but where the Printed Copy and *Euſtathius* read Ἰηλυσσός, which *Mercator* renders *Talyſſus*, the M S. renders Ιαλισός. *Abulfeda* in ſome Copies ſituates the Iſland *Rhodes*, (for he mentions no Cities there) in the *Latitude* of 37 *Deg*. and 40 *Min*. and the *Geography* of *Said Ibn Aly Algiorgany*, commended by *Gilbertus Gaulmyn*, in 37°, if it be not by a Tranſpoſition in the MS. of the Numerical Letters in *Arabick* 37 for 36, which by reaſon of their Similitude are often confounded, in *Arabick* MSS. By my Obſervations under the Walls of the City *Rhodes*, with a fair Braſs *Aſtrolable* of *Gemma Friſius*, containing 14 Inches in the Diameter, I found the *Latitude* to be 37° and 50′. A larger Inſtrument I durſt not adventure to carry on Shore in a Place of ſo much Jealouſie. And this *Latitude* in the Chart I have aſſigned to the City *Rhodes*, (from the Iſland ſo denominated, upon which on the North Eaſt ſide it ſtands ſituated) better agreeing with the *Arabians* than with *Ptolemy*, whom I know not how to excuſe.

Long. and Lat. Cape of Good Hope. n. 185. p. 253.

XXVI. In the ſecond Book of the *Voyage de Siam des Peres Jeſuites*, are related two Obſervations of the *Satellites* of *Jupiter*, capable, if well made, to aſcertain the *Longitude of the Cape of Good Hope*. The firſt was there made *June* 2d *ſt. n.* 1685, when at 11h 29′ 20″. the *Firſt* or Innermoſt *Satellite* touched the *Weſtern Edge* of *Jupiter*, and at 11h 30′ 50″ it appeared no more: This Obſervation is ſaid to be made with an excellent Teleſcope of 12 Foot. The other was on *June* the 4*th* following *ſt. n.* when the *Emerſion* of the ſame *Satellite* was obſerved at 9h 37′ 40″. from which Latter is concluded, that the *Longitude* of the *Cape* is 18° to the *Eaſt* of *Paris*, for that the ſaid *Emerſion*, according to the *Calculus* of *Caſſini*, in the *Meridian* of *Paris*, ought to have happen'd at 8h 26′. This ſame *Emerſion* is computed by Mr. *Flamſteed*, at 8h 19′, at *London*, that is 3 *min*. later then by *S. Caſſini*; and conſidering that neither is verified by Obſervation in *Europe*, the *Longitude* hence deduced is doubtful at leaſt 3 *min*. if this had been the only Obſervation. But the former being conſidered will yet ſhew that there is a much greater Doubt ſtill remaining: For from certain Aſtronomical Principles the *Parallax* of the *Orb*, or Difference between the Place of *Jupiter* ſeen from the *Sun* and *Earth* was, at the time of the firſt Obſervation, 9° 9′; Which Arch that *Satellite* moves in 1h 6′. and the utmoſt Duration of an *Eclipſe* hereof in this Poſition of *Jupiter* being ſcarce 2h 20′. (as appears by the accurate Obſervations of M. *Caſſini* and Mr. *Flamſteed*) it will follow, that from the *Immerſion* behind *Jupiter's* Weſtern Edge to the *Emerſion* out of the Shadow, there could not be full 3h 26′. Wherefore the *Emerſion* out of the Shadow, on *June* 2*d*, ought according to the time of *Immerſion*, to be at 14h 56′, at the lateſt at the *Cape*; which by

Mr. *Flamſteed's Calculus* was at *London* $13^h\ 51'$. or according to *S. Caſſini* at $13^h\ 58'$. at *Paris.* Hence the Longitude of the Cape will be found but 14 *deg.* and a half at moſt to the *Eaſt* of *Paris*; ſo that theſe 2 Obſervations will differ in the reſult about a quarter of an Hour which is a little too much. However there are ſome Reaſons that ſeem to argue for this Latter Longitude rather than the former; for it is much eaſier to obſerve what becomes of a Luminous Object that appears, than to wait upon the firſt Appearance of a Star Eclipſed: And it is probable that the *Satellite* might in the Latter time be ſeveral Minutes *Emerged* out of the Shadow, when they might firſt perceive it; but they could not but ſee the Application to the Body of *Jupiter* in the Former, if we may ſoppoſe their Teleſcopes ſo good as they are ſaid to be. And that the *Cape of Good Hope* is not more than an Hour to the *Eaſt* of *Paris*, is proved by the conſtant Conſent of our *Navigators*, who find by their Reckonings that the Iſland of St. *Helena* is about 22 or 23 *deg.* of *Longitude* to the *Weſtward* of the *Cape*: (and that Sailing both backwards and forwards, 'tis the ſame, which takes away the Objection of Currents) Now by Accurate Obſervations made at St. *Helena*, and compared with others made in *Europe* at the ſame time, the *Longitude* of that Iſle is certainly about $8\frac{1}{2}$ *deg.* to the *Weſt* of *Paris*; it follows therefore that the *Cape* cannot be much more than 14 or 15 *deg.* to the *Eaſt* of *Paris*; and undoubtedly it muſt be leſs than 18°, for 3 *deg.* is much too great an Error to be committed in ſo ſhort a Diſtance of Sailing.

Long. of St. Helena. *ib.*

The Long. of Madagaſca; *by* Mr. Flamſteed. n. 143. *p.* 15. *vid. ſup. Cap.* IV.§.XLVIII

XXVII. Mr. *Thomas Heathcot* was Chirurgeon to a Ship, which, *Aug.* 19. 1681. lay at the bottom of a deep Bay on the Weſtern Shore of *Madagaſcar*, and that part which the *Portugueſe* and our *Maps* call the *Terra del Gada*; He had then with him on Shore, a Quadrant of two Foot Radius, and a Teleſcope of 9 Foot, but no Clock; to ſupply which Defect, he made a *Pendulum* of a String and a Bullet 39 Inches long, that each ſingle Vibration might anſwer a Second of Time. Waiting the *Beginning* of the *Eclipſe* with his Glaſs, as ſoon as He ſaw the True Shadow enter on the *Moon's* Limb, he cauſed his Friends who aſſiſted him, to make the Pendulum Vibrate, and count its Vibrations; of which they had numbred 140 = 2′ 20″ of Time, when he took the height of *Procyon* (then *Eaſt* of the Meridian) 25° 39′. The next day he obſerved the *Sun's* Meridional Height with the ſame Quadrant, whence he found the Latitude of the Place 19° 29′. *South*: hence the time when he took the height of *Procyon* is found $4^h\ 51'$ *mane*, and ſubſtracting the 2′ 20″, paſt ſince the obſerved *Beginning* of the *Eclipſe*, its
True *Beginning* was at ———————————— $4^h\ 48'\ 40''$
Which at the *Obſervatory* here I noted at ———————— 1 50 40
therefore this part of *Madagaſcar* is more *Eaſterly* ———— 2 58 00
or 44° 30′, which our *Maps* make 52°; that is $7\frac{1}{2}$ *Deg.* more Remote from it than it really is.

The Long. and Lat. of Balla-

XXVIII. Taking the Obſervations of the *Occultation* of the *Bull's Eye*, *Oct.* 28. 1680. under the Examination of a *Calculus*, I find that at $8^h\ 6'$

or the *Immerſion* at *London*, the true Place of the *Moon* correct by *Parallax* was ♊ 4° 32′ 24″ but at 16^h 00′. at *Ballaſore Road* (in the *Lat.* of 21° 20′. N. and about 20 Miles E. S. E. from the Town) the True Place of the *Moon* was ♊ 5° 54′. that is 10° 21′ 36″. more than at 8^h 6′. at *London*: Now according to the Moon's Velocity at that Time, ſhe paſſed an Arch of 1° 21′ 36″. in 2^h 8′ 40″. of time, ſo then at 10^h 14′ 40″. at *London*, the *Moon* was in the ſame Place as at 16^h 00″. at *Ballaſore Road*, whence the Difference of Longitude will be 5^h 45′ 20″. or 86° 20′. *Ballaſore* being ſo much to the *Eaſtward* of *London*.

ſore, *in* India; *by Mr.* Edm. Halley, *Ph. Col.* n. 5. p. 124. *vid. ſup. Cap.* IV.§ LXVI. Feb. An. 1682.

2. By the Calculation of the *Immerſion* of the *Bull's Eye*, *Dec.* 1680. I find that at the 14^h 49′. at *Ballaſore* the *Moon's* true Place was ♊ 6° 30′ 30″. and at 7^h 46′ 12″. the Correct Time of the *Immerſion* at *Dantzick*, the true Place was ♊ 4° 55′ 11″. that is 1° 35′ 20″. ſhort of the Place deduced from the Obſervation at *Ballaſore Road*, which make in Time 2^h 32′ 40″. whence it follows, that 10^h 18′ 52″. at *Dantzick* make 14^h 49′. at *Ballaſore Road*, and the Difference of Longitude 4^h 30′ 8″. and *Dantzick* being 1^h 15′ 30″. more *Eaſterly* than *London*, *Ballaſore Road* will be from *London* 5^h 45′ 38″. or 86° 24′. and the ſame Difference of *Meridians* will be found 86° 14′. If you make uſe of the *Immerſion* at *Dantzick*.

Vid. ſup. Cap. IV.§.LXVII.

3. For farther Confirmation hereof, Mr. *Benj. Harry*, being aſhore at *Ballaſore* Town, obſerved with very great Care and Exactneſs, *Nov.* 18, 1680. that at 9 h 13. the Star which *Tycho* calls, *in Cotyla dextra* Aquarii *duarum præcedens*, (and which was then in *Aquarius* 28° 52′. and *Lat* 2° 46′. *N.*) was in a Right Line with the *Cuſps* of the *Moon*, then near the firſt Quarter. The Star's Place is confirmed by the Agreement of *Hevelius's* Obſerſervations with thoſe of *Tycho*, and the *Theory* of the *Moon* cannot be conſiderably faulty in that Part of the *Orb*, it falling preciſely on her greateſt *Equation*: wherefore by the Theory and Numbers of *Horrox*, the true Place of the *Moon* at 2^h 53′. at *London* is found ♒ 29° 22′ 10″. but at 9^h 13′. at *Ballaſore*, her Place was in ♒ 29° 41′ 17″. that is, 19′ 7″. more than at *London*, which in Time gives 36′, ſo that 3^h 29′ at *London*, was 9^h 13′ at *Ballaſore*, and the Difference of Long, 5^h 44′ or 86° 00′. preciſely, which the *Dutch Maps* make full out 99°. And the *French Maps* of *Sanſon*, pretending to correct them, have made them 5° worſe, and the Error 18° compleatly. What then is to be thought of the Deſcriptions of thoſe Places which have been but ſeldom Viſited?

XXIX. Differentiam Longitudinum *Cantonem* inter & *Pariſios* deduxi 7^h 23′. ex Exitu *Mercurii* & *Solis* Diſco *Cantoni* & *Norimbergæ* Obſervato, & ex *Eclipſibus Lunæ* Obſervatis *Norimbergæ* & *Pariſiis*.

Long. of Canton; *by M.* Caſſini. n. 245. p. 371. IV.§.XCVIII

XXX. Ex Altitudine Meridiana Maxima Stellæ Polaris à *P. P. S. J.* Obſervata Die 31 *Dec.* 1694. Correcto Inſtrumento 42° 16′ 50″. Suppoſita Refractione 1′ 17″. & Diſtantia Stellæ Polaris à *Polo* tunc temporis 2° 19′ 57″. eruitur *Altitudo Poli* 39° 54′ 56″.

Lat. & Long. of Pekin; *by M.* Ja. Caſſini. n. 237. p. 53.

Ex ejuſdem *Stellæ Polaris* Altitudine Meridiana Minima, obſervata Diebus 7, 8, 13. *Maii*, 1695. Correcto Inſtrumento 37° 36′ 40″. Suppoſita Refractione 1′ 28″. & Diſtantia *Stellæ Polaris* à *Polo* 2° 19′ 50″. Eruitur *Altitudo Poli* 39° 55′ 2″.

	°	′	″
Neglecta Refractione Altitudo maxima *Stellæ Polaris* deducta ex Obſervatione 31 *Dec.* præcedentis fuiſſet ſub Initium *Maii* ——	42	16	43
Et Altitudo ejuſdem minima tunc fuit ——	37	36	40
Quare Differentia Altitudinum ——	04	40	03
Et Diſtantia *Stellæ Polaris* à *Polo* ——	02	20	01½
Et *Altitudo Poli* apparens ——	39	56	41½
Ad hanc *Altitudinem* apparentem Refractio ex mea Tabula eſt	00	01	10
Quare *Altitudo Poli* in *Regia Pekinenſi* ——	39	55	31½

Pro *Longitudine Pekinenſis* Urbis Obſervata eſt Immerſio *Primi Jovis Satellitis* in *Jovis* Umbram *Die* 18 *Jan.* 1695. $12^{h}\ 51'\ 14''$.

Tabulæ noſtræ eo Die hanc *Immerſionem* repræſentant $5^{h}\ 18'\ 49''$. Obſervationes autem eodem Menſe habitæ in *Obſervatorio Regio Pariſienſi* oſtendunt Tabulas retardaſſe tunc Temporis 2′ 30″.

Quare fuit illa *Immerſio Pariſiis* $5^{h}\ 16'\ 19''$. Itaque *Differentia Meridianorum* inter *Pekinum* & Urbem *Pariſienſem* erit $7^{h}\ 34'\ 55''$.

Cum autem ex aliis Obſervationibus olim deducta fuerit eadem *Meridianorum Differentia* $7^{h}\ 36'$. ſumi poterit $7^{h}\ 35'\frac{1}{2}$.

Lat. of St. Salvadore. n. 105. *p.* 91.

XXXI. St. *Salvadore* in *Braſil* is in the *Southern Latitude* of 12° 47′.

Lat. of Bridge-Town. n. 189. *p.* 370.

XXXII. *Bridge-Town* in *Barbadoes* is in the *Northern Latitude* of 12° 58′.

A Deſcription of Nova Zembla; *by* M. Nich. Witſen. n. 101. *p.* 3. *Fig.* 205. Mar. An. 1674.

XXXIII. 1. I herewith ſend you what I have received out of *Muſcovy*, which is a *New Map* of *Nova Zembla* and *Weigats*, as it hath been diſcover'd by the expreſs Order of the *Czar*; and drawn by a Painter, call'd *Panelapoetski*, who ſent it me from *Moſco* for a Preſent; by which it appears, that *Nova Zembla* is not an *Iſland*, as hitherto it hath been believed to be; and that the *Mare Glaciale* is not a Sea, but a *Sinus*, or Bay, the Waters whereof are Freſh. Which is the ſame with what the *Tartars* do alſo aſſure us, who have taſted theſe very Waters in the midſt of the *Sinus*. The *Samojeds*, as well as the *Tartars*, do unanimouſly affirm, that paſſing on the back of *Nova Zembla*, at a conſiderable Diſtance from the Shore, *Navigators* may well paſs as far as *Japan*. And 'tis a great Fault in the *Engliſh* and *Dutch*, that, ſeeking to get to *Japan* on the South-ſide of *Nova Zembla*, they have almoſt always paſs'd the *Weigats*.

The Letter O in the great River *Oby* marks the Place of a Cataract or Fall of Waters. The Letter K denotes the Conjunction of *Zembla* with the *Continent*. The River marked L runs from *China*, called *Kitaie*: which is not every where Navigable, by Reaſon of the *Rocks*, and other Inconveniences that Obſtruct the paſſing of Veſſels. *Weigats* it ſelf is very difficult to paſs, becauſe of the great Quantity of *Ice* continually falling into it out of the River *Oby*, whereby that ſtreight Paſſage is ſtopped up. The *Sa-*

mojeds

jects go every Year a Fiſhing upon the ſaid Freſh Sea, and that on *Nova Zembla*'s ſide.

n. 193. p. 494.

2. I formerly thought *Nova Zembla* had been a *Continent* : But I have ſince been better informed, and retracted that Error. And, whereas the late M. *Voſſius* would needs perſuade himſelf, as well as he did others, to their Ruin, that there was a Paſſage to *Japan* by the *North*, and that the *Tartarian* Countries behind *Nova Zembla* did decline immediately towards the South; I did always oppoſe it; and think I can even demonſtrate the Impoſſibility thereof. So that what we wrote, to encourage Mariners to that Attempt, was even directing them to the Point of Death, as it afterwards enſued.

A Map of France; *by M.* Picard, *and M.* De La Hire. n. 236. p. 443. *Fig.* 20. Mar. An. 1697.

XXXIV. What is noted with a ſingle Line, is exactly copied from the *Map*, which *M. Sanſon*, one of the moſt Illuſtrious *Geographers* of this Age, preſented to the *Dauphin*, *An.* 1679. The Names of Cities, whoſe Situation is alſo taken from this *Map*, are written in *Italian* Characters ; the Correction of the Poſition of Coaſts (which is deduced from the Obſervations which were made to that End) is marked with a Stroke a little ſhadow'd towards the *Sea*, as is commonly done ; and the Names of Cities, whoſe Situation is corrected, are ſet down in *Roman* Characters.

The Degrees of *Latitude* are marked on both ſides of the Border, and the Degrees of *Longitude* in the ſame Border, above and below ; but the Diviſion of them begins at the *Meridian* that paſſes through the *Obſervatory* at *Paris*, by going to *Eaſt* and *Weſt*, and not at the *Meridian* of the Iſle of *Fer*, as hath been eſtabliſhed, becauſe we do not exactly know the Situation of this Iſland in reſpect of the *Obſervatory*.

What a Compleat Treatiſe of Navigation *ſhould contain; by* Sir W. Petty, n. 198. p. 657. Mar. An. 1693.

XXXV. 1. What *Arithmetick* in whole Numbers and Fractions, as alſo in Decimals and Logarithms, is neceſſary for the ſame ? And what Books are beſt for Teaching ſo much thereof ? 2. What Vulgar, Practical, Mechanical *Geometry*, performable by the Scale and Compaſs, is ſufficient ? 3. What *Trigonometry*, Right Lined and Spherical, will ſuffice ? 4. How many *Stars* are to be be known ? 5. What *Inſtruments* are beſt for Uſe at *Sea*, with the Conſtruction of them, and the Manner of uſing them ? 6. The whole Skill of the *Magnet*, as to the Directive Virtues thereof, and all the Accidents that may befall it ? 7. The *Hydrography* of the *Globe* of the Earth; the *Perſpective* of the Coaſts, and the Deſcription of the under-water-bottom of the *Sea*, 8. The Knowledge of Winds and *Meteors*, ſo far as the ſame is attainable. 9. The Hiſtory and Skill of all ſorts of *Fiſhings*. 10. The Art of *Medicine* and *Chirurgery*, peculiar for the *Sea*. 11. The *Common Laws* of the *Admiralty*, and *Juriſdiction* of the *Sea*. 12. The ſeveral *Victuallings* and *Cloathings* fit for Seamen. 13. The whole Science of *Ebbing* and *Flowing*, as alſo of Currents and Eddies at Sea. 14, *Dromometry*, and the Meaſures of a Ship's Motions at Sea. 15. The *Building* of *Ships* of all ſorts, with the ſeveral Riggings and Sails for each Species, and the Uſe of all the Parts and Motions of a Ship. 16, *Naval Oeconomy*, according to the ſeveral Voyages and Countries. 17. The Art of *Coning*, *Rowing*, and *Sailing*, of all the ſeveral ſorts of Veſſels.

ſels. 18. The *Gunnery*, Fireworks and other Armatures peculiar to Sea and to Sea-Fights. 19. The Art of *Loading* and *Unloading* the chief Commodities to the beſt Advantage. 20. The Art of *weighing ſunk Ships* and Goods, as alſo of *Diving* for ſunk Goods in deep Wates. 21. The General *Philoſophy* of the *Motion* and Figures of the *Air*, the *Sea*, and of Seaſons; of *Timber*, *Iron*, *Hemp*, *Brimſtone*, *Tallow*, &c. And of their ſeveral uſes in *Naval* Affairs. 22. An Account of 5 or 6 of the beſt *Navies* of *Europe*, with that of the *Arſenals*, *Magazines*, *Yards*, *Docks*, &c. 23. An Account of all the *Shipping* able to croſs the *Seas* belonging to each Kingdom and State of *Europe*. 24. An Account of all the chief *Commercial* Parts of the World; with mention of what *Commodities* are originally carried from, and ultimately to, any of them. 25. An Account of the chief *Sea-Fights*, and all other *Naval Expeditions* and Exploits, relating to *War*, *Trade*, or *Diſcovery*, which have happened in this laſt Century. 26. Of the moſt advantageous Uſe of *Teleſcopes* for ſeveral Purpoſes at *Sea*. 27. Of the ſeveral *Depths* of the *Sea*, and *Heights* of the *Atmoſphere*. 28. The Art of making *Sea-water Freſh* and *Potable*, and fit for all Uſes in Food and Phyſick at Sea.

The Collection of Secants, and the true Diviſion of the Meridian in the Sea-Chart; by Dr. Wallis. n. 176. p. 1193. *Fig.* 206. *Nov. An.* 1685.

XXXVI. 1. Though it be well known, that, in the *Terreſtrial Globe*, all the *Meridians* meet at the *Pole*, (as E P, E P,) whereby the Parallels to the Equator, as they be nearer to the Pole, do continually decreaſe.

2. And hereby a Degree of *Longitude* in ſuch Parallels is leſs than a Degree of *Longitude* in the Equator, or a Degree of *Latitude*:

3. And that in ſuch Proportion, as is the Co-Sine of *Latitude* (which is the Semidiameter of ſuch Parallel) to the Radius of the *Globe*, or of the Equator.

4. Yet hath it been thought fit (for ſome Reaſons) to repreſent theſe *Meridians*, in the *Sea-Chart*, by parallel ſtraight Lines; as E p, Ep.

5. Whereby, each Parallel to the Equator (as L A) was repreſented in the *Sea-Chart*, (as la) as equal to the Equator E E; and a Deg. of *Longitude* therein, as large as in the Equator.

6. By this means, each Degree of *Longitude* in ſuch *Parallels* was increaſed, beyond its juſt proportion, at ſuch rate as the Equator (or its Radius) is greater than ſuch *Parallel*, (or the Radius thereof.)

7. But, in the Old *Sea-Charts*, the Degrees of *Latitude* were yet repreſented (as they are in themſelves) equal to each other; and to thoſe of the Equator.

8. Hereby, amongſt many other Inconveniencies, (as Mr. *Ed. Wright* obſerves, in his *Correction of Errors of Navigation*, firſt publiſhed in the Year 1599, the Repreſentation of the Places remote from the Equator was ſo Diſtorted in thoſe Charts, as that (for Inſtance) an *Iſland* in the *Latitude* of 60 Degrees, (where the Radius of the *Parallel* is but half ſo great as that of the Equator) would have its Length (from *Eaſt* to *Weſt*) in Compariſon of Breadth (from *North* to *South*) repreſented in a double Proportion of what indeed it is.

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The Collection of Secants, and the true Division of the Meridian in the Sea-Chart; by Dr. Wallis, n. 276, p. 1193. *Fig.* 206. Nov. An. 1685.

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9. For rectifying this in ſome Meaſure, (and of ſome other Inconveniencies) Mr. *Wright* adviſeth, that (the *Meridians* remaining Parallel, as before) the Degrees of the *Latitude* remote from the Equator, ſhould at each Parallel, be protracted in like Proportion with thoſe of *Longitude*.

10. That is; As the Co-Sine of *Latitude*, (which is the Semidiameter of the Parallel) to the Radius of the *Globe*, (which is that of the Equator:) So ſhould be a Degree of *Latitude* (which is every where equal to a Degree of *Longitude* in the Equator,) to ſuch Degree of *Latitude* ſo protracted (at ſuch Diſtance from the Equator;) and ſo to be repreſented in the Chart.

11. That is, every where, in ſuch Proportion, as is the reſpective Secant (for ſuch *Latitude*) to the Radius. For, as the Co-Sine, to the Radius; ſo is the Radius, to the Secant of the ſame Arch or Angle; or Σ : R :: R : ſ. *Fig.* 207.

12. So that (by this means) the Poſition of each Parallel in the *Chart* ſhould be at ſuch Diſtance from the Equator, compared with ſo many Equinoctial Degrees or Minutes, (as are thoſe of *Latitude*) as are all the Secants (taken at equal Diſtances in the Arch) to ſo many times the Radius.

13. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the Spherical Surface (ſuppoſing the Eye at the Centre) on the Concave Surface of a *Cylinder* erected at Right Angles to the Plain of the Equator.

14. And the Diviſion of *Meridians*, repreſented by the Surface of a *Cylinder* erected (on the Arch of *Latitude*) at Right Angles to the Plain of the *Meridian* (or a Portion thereof.) The Altitude of ſuch Projection (or Portion of ſuch Cylindrick Surface) being (at each Point of ſuch Circular Baſe) equal to the Secant (of *Latitude*) anſwering to ſuch Point. *Fig.* 208.

15. This Projection (or Portion of the Cylindrick Surface) if expanded into a Plain, will be the ſame with a plain Figure, whoſe Baſe is equal to a Quadrantal Arch extended (or a Portion thereof) on which (as Ordinates) are erected Perpendiculars equal to the Secants, anſwering to the reſpective Points of the Arch ſo extended: The leaſt of which (anſwering to the Equinoctial) is equal to the Radius; and the reſt continually increaſing, 'till (at the *Pole*) it be infinite. *Fig.* 209.

16. So that, as ERSL, (a Figure of Secants erected at Right Angles on EL, the Arch of *Latitude* extended) to ERRL, (a Rectangle on the ſame Baſe, whoſe Altitude ER is equal to the Radius; ſo is EL (an Arch of the Equator equal to that of *Latitude*,) to the Diſtance of ſuch Parallel, (in the *Chart*) from the Equator.

17. For finding this Diſtance, anſwering to each Degree and *Minute* of *Latitude*, Mr. *Wright* (as the moſt obvious way) adds all the Secants (as they are found calculated in the *Trigonometrical Canon* from the Beginning to the Deg. or Min. of Latitude propoſed.

18. The Sum of all which, except the Greateſt, (anſwering to the Figure Inſcribed) is too little: The Summ of all, except the Leaſt, (anſwering to the Circumſcribed) is too Great; (which is that he follows:) And it would be nearer to the Truth than either, if (Omitting all theſe) we take the Intermediates; for *Min.* $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$. *&c.* or (the double of theſe) *Min.* 1, 3, 5, 7, *&c.* Which yet (becauſe on the Convex-ſide of the Curve) would be ſomewhat too Little.

19. But any of theſe ways are exact enough for the Uſe intended, as creating no ſenſible Difference in the *Chart*.

20. If we would be more exact; Mr. *Oughtred* directs, (and ſo had Mr. *Wright* done before him) to divide the Arch into Parts yet ſmaller than Minutes, and calculate Secants ſuiting thereunto.

21. Since the *Arithmetick of Infinites* introduced, and (in purſuance thereof) the Doctrine of *Infinite Series*; (for ſuch Caſes as would not, without them, come to a determinate Proportion;) Methods have been found for ſquaring ſome ſuch Figures.

Fig. 207. 22. In order to a *Quadrature* for this Figure of Secants (by an Infinite Series fitted thereunto) put we, for the *Radius* of a Circle, R; the Right Sine of an Arch or Angle, S; the Verſed Sine, V; the Co Sine, (or Sine of the Complement) $\Sigma = R - V = \sqrt{Rq - Sq}$; the Secant, ſ; the Tangent, T.

23. Then is $\Sigma : R :: R :$ ſ. That is, $\Sigma)\,R^2\,($ſ $= \frac{R^2}{\Sigma}$; the Secant.

24. And $\Sigma : S :: R : T$. That is, $\Sigma)\,S\,R\,(T = \frac{SR}{\Sigma}$; the Tangent.

Fig. 210. 25. Now, if we ſuppoſe the Radius CP, divided into equal Parts, and each of them $= \frac{1}{\infty} R$;) and on theſe, to be erected the Co-Sines of Latitude L A.

26. Then are the Sines of Latitude in Arithmetical Progreſſion.

27. And the Secants anſwering thereunto, L ſ $= \frac{R^2}{\Sigma}$.

28. But theſe Secants, (anſwering to the Right Sines in Arithmetical Progreſſion,) are not thoſe that ſtand at equal Diſtances on the Quadrantal Arch extended. *Fig.* 209.

29. But, ſtanding at Unequal Diſtances (on the ſame extended Arch;) Namely, on thoſe Points thereof, whoſe Right Sines (whilſt it was a Curve) are in Arithmetical Progreſſion. As *Fig.* 211. *Fig.* 211.

30. To find therefore the Magnitude of R E L ſ, *Fig.* 209. which is the ſame with that of *Fig.* 211, (ſuppoſing E L of the ſame Length in both; however the Number of Secants therein may be unequal) we are to conſider the Secants, though at Unequal Diſtances, *Fig.* 211. to be the ſame with thoſe of Equal Diſtances in *Fig.* 210. anſwering to Sines in Arithmetical Progreſſion.

31. Now, theſe Intervals, (or Portions of the Baſe) in *Fig.* 211. are the ſame with the Intercepted Arches, (or Portions of the Arch) in *Fig.* 210. For this Baſe is but that Arch extended.

32. And theſe Arches, (in Parts infinitely ſmall) are to be reputed equivalent to the Portions of their reſpective Tangents intercepted between the ſame Ordinates. As in *Fig.* 210, 212. *Fig.* 212.

33. That is, Equivalent to the Portions of the Tangents of Latitude.

34. And theſe Portions of Tangents are to the Equal Intervals in the Baſe, as the Tangent (of Latitude) to its Sine.

35. To find therefore the true Magnitude of the Parallelograms (or Segments of the Figure; we muſt either protract the Equal Segments of the Baſe, *Fig.* 210. (in ſuch Proportion as are the reſpective Tangents to the Sine) to make them Equal to thoſe of *Fig.* 211.

36. Or elſe, (which is equivalent) retaining the equal Intervals of *Fig.* 210, protract the Secants in the ſame Proportion. (For either way the Intercepted Rectangle or Parallelograms will be equally increaſed) as LM. *Fig.* 212.

37. Namely, as the Sine (of *Latiude*) to its Tangents; ſo is the Secant, to a Fourth; which is to ſtand (on the Radius equally divided) inſtead of that Secant. *Fig.* 212.

$$S:\frac{SR}{\Sigma}(::\Sigma:R.)::\frac{R^2}{\Sigma}:\frac{R^3}{\Sigma^2=R^2-S^2}=LM.$$

38. Which therefore are as the Ordinates in (what I call *Arith. Infin. Prop.* 104.) *Reciproca Secundanorum*: Suppoſing Σ^2 to be Squares in the Order of Secundans.

39. This (becauſe of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progreſſion) is Reduced (by Diviſion) into this *Infinite Series*

$$R:+\frac{S^2}{R}+\frac{S^4}{R^3}+\frac{S^6}{R^5}, \&c.$$

40. That is, (putting $R = 1$,) $1 + S^2 + S^4 + S^6$, &c.

41. Then, (according to the *Arithmetick of Infinites*) we are to interpret S, ſucceſſively by 1 S, 2 S, 3 S, &c. till we come to S, the greateſt. Which therefore repreſents the Number of all.

And, becauſe the firſt Member doth repreſent a Series of Equals, the ſecond of Secundans; the third of Quartans, &c. Therefore the firſt Member is to be multiplied by S; the ſecond by $\frac{1}{2}$ S; the third by $\frac{1}{5}$ S; the fourth by $\frac{1}{7}$ S; &c.

43. Which makes the Aggregate, $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$, &c. = E C L M.

44. This, (becauſe S is always leſs than $R = 1$) may be ſo far continued, till ſome *Power* of S become ſo ſmall, as that it (and all which follow it) may be ſafely neglected.

45. Now, (to fit this to the *Sea-Chart*, according to Mr. *Wright*'s Deſign) having the propoſed Parallel (of *Latitude*) given; we are to find (by the *Trigonometrical Canon*) the Sine of ſuch *Latitude*; and take Equal to it, C L = S. And by this find the Magnitude of ECLM, *Fig.* 212. that is, of REL ſ, *Fig.* 211. that is, of REL ſ, *Fig.* 209. And then, as RRLE (or ſo many times the Radius) to REL ſ, (the Aggregate of all the Secants;) ſo muſt be a like Arch of the Æquator (equal to the Latitude propoſed;) to the Diſtance of ſuch Parallel, (repreſenting the Latitude in the *Chart*) from the Æquator: Which is the Thing required.

46. The ſame may be obtained in like manner, by taking the Verſed Sines in Arithmetical Progreſſion. For, if the Right Sines (as here) beginning at the Æquator, be in Arithmetical Progreſſion, as, 1, 2, 3, &c. Then will

the Verſed Sines, beginning at the *Pole*, (as being their Complements to the Radius) be ſo alſo.

47. The ſame may be applied in like Manner, (though that be not the preſent Buſineſs) to the Aggregate of *Tangents*, (anſwering to the Arch divided into equal Parts.

48. For thoſe anſwering to the Radius ſo divided are $\frac{SR}{\Sigma}$; (taking S in Arithmetical Progreſſion.)

49. And then Inlarging the Baſe (as in *Fig.* 211.) or the Tangent (as in *Fig.* 212.) in Proportion of the Tangent to the Sine.

$$S : \frac{SR}{\Sigma} \; (: : \Sigma : R) : : \frac{SR}{\Sigma} : \frac{SR^2}{\Sigma^2} = \frac{SR^3}{R^2 - S^2}$$

50. We have by Diviſion this Series, $S + \frac{S^3}{R^2} + \frac{S^5}{R^4} + \frac{S^7}{R^6} + \frac{S^9}{R^8}$, &c

51. That is, (putting $R = 1$) $S + S^3 + S^5 + S^7 + S^9$, &c.

52. Which (multiplying the reſpective Members by $\frac{1}{2}S$, $\frac{1}{4}S$, $\frac{1}{6}S$, $\frac{1}{8}S$, $\frac{1}{10}S$, &c.) becomes $\frac{1}{2}S^2 + \frac{1}{4}S^4 + \frac{1}{6}S^6 + \frac{1}{8}S^8 + \frac{1}{10}S^{10}$, &c.

Which is the Aggregate of Tangents to the Arch whoſe Right Sine is S.

53. And this Method may be a Pattern for the like *Proceſs* in other Caſes of like Nature.

Two Problems in Navigation, *propos'd; by Mr.* Nich. Mercator. n. 13. p. 215. Jun. An. 1666.

XXXVII. The Line of *Artificial Tangents*, or the *Logarithmical Tangent Line*, beginning at 45°. and taking every half Deg. for a whole one, is found to agree pretty near with the *Meridian-Line* of the *Sea-Chart*, they both growing, as it were, after the ſame proportion. But the Table of *Meridional* Degrees being calculated only to every ſexageſimal Minute of a Degree, ſhews ſome ſmall Difference from the *ſaid Logarithmical Tangent-line*. Hence it may be doubted, whether the Difference do not ariſe from that little Error which is committed by calculating the Table of *Meridional Degrees* only to every Minute.

But, if a certain Rule could be produced, by which the Agreement or Diſagreement of the ſaid two Lines might be ſhewn, the *Helix* or *Spiral* Line of the *Ship's Courſe* would be reduced to a more preciſe Exactneſs than ever was pretended by any.

The ſame Rule would alſo diſcover a far eaſier way of making *Logarithms*, than ever was practiſed or known; and therefore might ſerve, whenever there ſhould be Occaſion, to entend the *Logarithms* beyond the Number of Places that are yet extant.

Moreover, ſuch a Rule would enable Men to draw the *Meridian Line* Geometrically, that is, without Tables or Scales; which indeed might alſo be done by ſetting of the *Secants* of every whole or half Degree, if there were not this Inconveniency in it,(which is not in my Rule :) That a Line compoſed of ſo many ſmall Parts, would be ſubject to many Errors, eſpecially in a ſmall Compaſs.

The ſame Rule will alſo ſerve to find the Courſe and Diſtance between two Places aſſigned, as far as Practice ſhall require it; and that without any Table of *Meridional Parts*, and yet with as much Eaſe and Exactneſs.

And, ſeeing all theſe things do depend on the Solution of this Queſtion, *Whether the Artificial Tangent Line be the True Meridian Line*; it is therefore that I undertake, by God's Aſſiſtance, to reſolve the ſaid Queſtion. And, to let the World know the Readineſs and Confidence I have to make good this Undertaking, I am willing to lay a Wager againſt any one or more Perſons that have a Mind to engage, for ſo much as another Invention of mine (which is of leſs Subtilty, but of a far greater Benefit to the Publick) may be worth to the *Inventor*.

As for the great Advantage, that all *Merchants*, *Mariners*, and conſequently the *Common-Wealth*, may receive from this other *Invention*, it is, in my Judgment, highly valuable, ſeeing it will oftentimes make a *Ship* ſail, though, according to the common way of *Sailing*, the Wind be quite contrary, and yet as near to the Place intended, as if the Wind had been favourable: Or, if you will, it will enable one to gain ſomething in the Intended Way, whether the Wind be good or no, (except only when you go directly *South* or *North*) but the Advantage will be moſt where there is moſt need of it) that is, when the Wind is contrary: So that one may very often gain a fifth, fourth, third Part, or more of the Intended Voyage; according as it is longer or ſhorter, *viz.* always more in a longer Voyage, where the Gain is more conſiderable, and more welcome, not only by ſaving Time, but alſo Victuals, Water, Fuel, Mens Health, and ſo much Room in the *Ship*.

The Analogy of Logarithmick Tangents to the Meridian Line *Demonſtrated*; by Mr. Edm. Halley. n. 219. p. 202. Feb. An. 1696.

XXXVIII. It was firſt diſcovered by chance, and, as far as I can learn, firſt publiſhed by Mr. *Henry Bond*, as an Addition to *Norwood's Epitome of Navigation*, about 50 Years ſince, that the *Meridian Line was analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.*

For the Demonſtration of that *Propoſition*, it is requiſite to premiſe theſe four *Lemmata*.

Lemma 1. *In the Stereographick Projection of the Sphere upon the Plain of the Æquinoctial, the Diſtances from the Centre*, which in this Caſe is the Pole, *are laid down by the Tangents of half thoſe Diſtances*; that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3. 20.

Lemma 2. *In the Stereographick Projection, the Angles, under which the Circles interſect each other, are in all Caſes equal to the Spherical Angles they repreſent*; which is a very valuable *Property* of this Projection.

Fig. 213.

Demonſt. Let EPBL be any great Circle of the *Sphere*, E the Eye placed in its Circumference; C its Centre, P any Point thereof; and let FCO be ſuppoſed a Plain erected at Right Angles to the Circle EPBL, on which FCO we deſign the Sphere to be Projected. Draw EP croſſing the Plain FCO in *p*, and *p* ſhall be the Point P projected. To the Point P draw the Tangent APG, and on any Point thereof, as A, Erect a Perpendicular AD, at right Angles to the Plain EPBL, and draw the Lines PD, AC, DC; and the Angle APD ſhall be equal to the Spherical Angle contained between the

Plains APC, DPC. Draw alſo AE, DE, interſecting the Plain FCO in the Points a and d; and join $a\,d$, $p\,d$: I ſay, the Triangle $a\,d\,p$, is ſimilar to the Triangle ADP, and the Angle $a\,p\,d$ equal to the Angle APD. Draw PL, AK, parallel to FO, and by reaſon of the Parallels $a\,p$ will be to $a\,d$, as AK, to AD: But, (by *Eucl.* 3. 32.) in the Triangle AKP, the Angle AKP = LPE, is alſo equal to APK = EPG: wherefore the Sides AK, AP, are equal, and 'twill be as $a\,p$, to ad; ſo AP to AD. Whence the Angles DAP, $d\,a\,p$, being Right, the Angle APD, will be equal to the Angle $a\,p\,d$, that is, the Spherical Angle is equal to that on the *Projection*, and that in all Caſes. Q. E. D.

This *Lemma* I lately received from Mr. *Ab. de Moivre*, though I ſince underſtand from Dr *Hook*, that he long ago produced the ſame thing before the *Society*. However, the *Demonſtration*, and the reſt of the Diſcourſe is my own.

Lemma 3. *On the Globe, the Rhumb Lines make equal Angles with every Meridian*, and, by the 'foregoing Lemma, they muſt likewiſe make *equal Angles with the Meridians in the Stereographick Projection on the Plain of the Equator*: They are therefore in that *Projection*, *Proportional Spirals* about the Pole-point.

Fig. 214. Lemma 4. *In the Proportional Spiral it is a known Property, that the Angles* BPC, *or the Arches* BD, *are Exponents of the Rationes of* BP *to* PC,: For, if the Arch BD be divided into inumerable equal parts, Right Lines drawn from them to the Centre P, ſhall divide the Curve B c c C into an Infinity of *Proportionals* between PD and PC, whoſe Number is equal to all the Points d, d, in the Arch BD: Whence, and by what I have delivered concerning the *Conſtruction* of *Logarithms*, it follows, that, as BD to B d, or, as the Angle BPC. to the Angle BP c, ſo is the *Logarithm* of the *Ratio* of PB to PC, to the *Logarithm* of the *Ratio* of PB to P c. vid. ſup. Cap. 1. §XXVIII.

From theſe *Lemmata* our Propoſition is very clearly *Demonſtrated*: For, by the *Firſt* PB, P c, PC, are the Tangents of half the Complements of the Latitudes in the *Stereographick Projection*: And, by the *Laſt* of them, the Differences of Longitude, or Angles at the Pole between them, are *Logarithms* of the *Rationes* of thoſe Tangents one to the other. But the *Nautical Meridian Line* is no more than a Table of the *Longitudes*, anſwering to each Minute of *Latitude* on the *Rhumb Line*, making an Angle of 45 Degrees with the *Meridian*. Wherefore the *Meridian Line* is no other than a *Scale of Logarithmick Tangents of the Half Complements of the Latitudes*. Q. E. D.

Corol. 1. Becauſe that in every Point of any *Rhumb-Line* the Difference of *Latitudes* is to the *Departure*, as the Radius to the Tangent of the Angle that *Rhumb* makes with the Meridian; and thoſe equal *Departures* are every where to the *Differences* of *Longitude*, as the Radius to the Secant of the *Latitude*; it follows that the *Differences* of *Longitude* are on any *Rhumb*, *Logarithms* of the ſame Tangents, but of differing Species; being proportioned to one another, as are the Tangents of the Angles made with the *Meridian*.

Coroll. 2. Hence any Scale of Logarithm Tangents (as thoſe of the Vulgar Tables made after *Brigg's* Form ; or thoſe made to *Napier's*, or any other Form whatſoever) is a Table of the Differences of Longitude, to the ſeveral Latitudes, upon ſome determinate *Rhumb* or other ; and therefore, as the Tangent of the Angle of ſuch Rhumb, to the Tangent of any other Rhumb : So the Difference of the *Logarithms* of any two Tangents, to the Difference of Longitude on the propoſed Rhumb, intercepted between the two Latitudes, of whoſe half Complements you took the Logarithm Tangents.

Now the *Momentary Augment*, or *Fluxion* of the *Tangent-line* at 45 Degrees, is exactly double to the *Fluxion* of the Arch of the Circle, (as may eaſily be proved) and the Tangent of 45 being equal to the Radius, the *Fluxion* alſo of the *Logarithm Tangent* will be double to that of the Arch, if the *Logarithm* be of *Napier's* Form : But, for *Brigg's* Form, it will be as the ſame doubled Arch multiplied into 0,43429, *&c.* or divided by 2,30258, *&c.* yet this muſt be underſtood only of the Addition of an Indiviſible Arch, for it ceaſes to be true, if the Arch have any determinate Magnitude.

Hence it appears, that, if one Minute be ſuppoſed Unity, the Length of the Arch of one Minute being 0,000290888208665721596154, *&c.* in parts of the Radius, the Proportion will be as Unity to 2,908882, *&c.* So Radius to the Tangent of 71° 1′ 42″, whoſe *Logarithm* is 10,46372611720718325204, *&c.* and under that Angle is the *Meridian Interſected* by that *Rhumb-line*, on which the Differences of *Napier's* Logarithm Tangents of the Half Complements of the Latitudes are the true Differences of Longitude, eſtimated in Minutes and Parts, taking the firſt 4 Figures for Integers. But for *Vlacq's* Table we muſt ſay,

As 2302585, *&c.* to 2908882, *&c.* ſo Radius to 1,2633114387244569212, *&c.* which is the Tangent of 51° 38′ 9″, and its Logarithm 10,101510428507720941162, *&c.* wherefore in the *Rhumb-line*, which makes an Angle of 51° 38′ 9″ with the Meridian, *Vlacq's Logarithm Tangents* are the true Differences of Longitude. And this, compared with our *Second Corollary*, may ſuffice for the Uſe of the Tables already computed.

But, if a Table of *Logarithm Tangents* be made by Extraction of the Root of the *Infinitesimal Power*, whoſe Index is the Length of the *Arch* you put for Unity, (as for the Minutes the 0,0002908882*th*, *&c. Power*) which we will call *a*; ſuch a *Scale* of Tangents ſhall be the true *Meridian Line*, or Summ of all the Secants taken infinitely many. Here the Reader is deſired to have Recourſe to my little Treatiſe of *Logarithms*, that I may not nead to repeat it. By what is there delivered ir will follow, that putting *t* for the Exceſs or Defect of any Tangent above or under the Radius or Tangent of 45°; the *Logarithm* of the *Ratio* of Radius to ſuch Tangent will be *vid. ſup. cap.* I. §. XXVIII.

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 - \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \text{ \&c.}$$

when the *Arch* is greater than 45, or

$$\frac{1}{m} \text{into } t + \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 + \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \text{ \&c.}$$

when it is leſs than 45°. And by the ſame Doctrine putting T for the Tangent of any Arch, and t for the Difference thereof from the Tangent of another Arch, the Logarithm of their *Ratio* will be $\frac{1}{m}$ into $\frac{t}{T}+\frac{tt}{2TT}+\frac{t^3}{3T^3}+\frac{t^4}{4T^4}+\frac{t^5}{5T^5}$, &c. When T is the greater Term; or, $\frac{1}{m}$ into $\frac{t}{T}-\frac{t^2}{2T^2}+\frac{t^3}{3T^3}-\frac{t^4}{4T^4}+\frac{t^5}{5T^5}$, &c. when T is the Leſſer Term.

And, if m be ſuppoſed, 0,0002908882, &c. $= a$, its reciprocal $\frac{r}{a}$ will be 3437,7467707849392526, &c. which multiplied into the aforeſaid *Series* ſhall give preciſely the Difference of the Meridional Parts, between the two Latitudes to whoſe half Complements the aſſumed Tangents belong. Nor is it material from whether Pole you eſtimate the Complements, whether the Elevated or Depreſſed; the Tangents being to one another in the ſame *Ratio* as their Complements, but inverted.

In the ſame Diſcourſe I alſo ſhewed, that the Series may be made to converge twice as ſwift, all the even Powers being omitted; and that, putting τ for the Summ of the two Tangents, the ſame *Logarithm* would be $\frac{2}{m}$ or $\frac{2r}{a}$ into $\frac{t}{\tau}+\frac{t^3}{3\tau^3}+\frac{t^5}{5\tau^5}+\frac{t^7}{7\tau^7}+\frac{t^9}{9\tau^9}$, &c. but the Ratio of τ to t, or of the Summ of two Tangents to their Difference, is the ſame as that of the Sine of the Summ of the Arches, to the Sine of their Difference. Wherefore, if S be put for the *Sine Complement*, of the *Middle Latitude*, and s for the Sine of half the Difference of *Latitudes*, the ſame *Series* will be $\frac{2r}{a}$ into $\frac{s}{S}+\frac{s^3}{3S^3}+\frac{s^5}{5S^5}+\frac{s^7}{7S^7}+\frac{s^9}{9S^9}$, &c. wherein as the Differences of *Latitude* are ſmaller, fewer Steps will ſuffice. And, if the *Equator* be put for the Middle Latitude, and conſequently $S=R$, and s the *Sine* of the *Latitude*, the *Meridional Parts* reckoned from the *Equator* will be $\frac{s}{a}+\frac{s^3}{3rra}+\frac{s^5}{5r^4a}+\frac{s^7}{7r^6a}$, &c. which is Co-incident with Dr. *Wallis's*
vid. §. XXXVI *Solution*. And this ſame *Series*, being half the *Logarithm* of the *Ratio* of $R+s$ to $R-s$, that is of the *Verſed Sines* of the Diſtances from both Poles, does agree with what Dr. *Barrow* had ſhewn in his XI. *Lecture*.

The ſame *Ratio* of *r* to *s* may be expreſſed alſo by that of the Summ of the Co-ſines of the two *Latitudes*, to the *Sine* of their Difference: As likewiſe by that of the *Sine* of the Summ of the two *Latitudes*, to the Difference of their Co-ſines: Or by that of the *Verſed Sine* of the Summ of the *Co-Latitudes*, to the Difference of the *Sines* of the *Latitudes*: Or as the ſame Difference of the *Sines* of the *Latitudes*, to the *Verſed Sine* of the Difference of the *Latitudes*; all which are in the ſame *Ratio* of the Co-ſine of the *Middle Latitude*, to the *Sine* of half the Difference of the *Latitudes*. As it were eaſy to demonſtrate, if the Reader were not ſuppoſed capable to do it himſelf, upon the bare Inſpection of a Scheme duly repreſenting theſe Lines.

This Variety of *Expreſſion* of the ſame *Ratio* I thought not fit to be omitted, becauſe by help of the *Rationality* of the *Sines* of 30°, in all Caſes where the Summ or Difference of the *Latitudes* is 30°, 60°, 90°, 120°, or 150°; ſome one of them will exhibit a ſimple *Series*, wherein great Part of the Labour will be ſaved. Bur the former ſeems for all Uſes the moſt convenient, whether we deſign to make the whole *Meridian-Lines*, or any Part thereof, *viz.* $\frac{2r}{a}$ into $\frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}$, &c. wherein *a* is the Length of any Arch, which you deſign ſhall be the Integer or *Unity* in your *Meridional Parts* (whether it be a Minute, League or a Degree, or any other,) S the Co-ſine of the *Middle-Latitude*, and *s* the *Sine* of half the Difference of *Latitudes*; but, the *Secants* being the Reciprocals of the Co-ſine $\frac{s}{S}$ will be equal to $\frac{ſs}{rr}$ putting *ſ* for the *Secant* of the *Middle-Latitude*; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2ſs}{ra}$ This multiplied by $\frac{ſs}{3SS}$ that is by $\frac{ſſss}{3rrrr}$, will give the ſecond Step; and that again by $\frac{3ſſss}{5rrrr}$, the third Step; and ſo forward till you have compleated as many Places as you deſire. But, the *Squares* of the *Sines* being in the ſame *Ratio* with the *Verſed Sines* of the double Arches, we may inſtead of $\frac{ss}{3SS}$ aſſume for our Multiplicator $\frac{v}{3V}$, or the Verſed-ſine of the Difference of the *Latitudes* divided by thrice the Verſed ſine of the Summ of the *Co-Latitudes*, &c. which is the utmoſt *Compendium* I can think of for this Purpoſe, and the ſame *Series* will become $\frac{2sr}{aS}$ into $1 + \frac{v}{3V} + \frac{v^2}{5V^2} + \frac{v^3}{7V3} + \frac{v^4}{9V^4}$: Hereby we are enabled to eſtimate the Default of the Method of making the *Meridional Line*, by the continual Addition of the Secants of Equi-different Arches, which, as the Differences of thoſe Arches are ſmaller, does ſtill nearer and nearer approach the Truth. If we aſſume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common Difference of a Rank of *Arches* : It will be in all Caſes, as the Arch of one Minute, to its Chord; ſo the Secant of the middle Latitude, to the Firſt Step of our Series. This by Reaſon of the near Equality between *a* and 2 *s*, which are to one another in the *Ratio* of Unity to 1 — 0,00000000352566457713, *&c.* will not differ from the Secant *ſ*, but in the *9th* Figure; being leſs than it in that Proportion. The next Step being + $\frac{2ſ^3s^3}{3arr}$ will be Equal to the Cube of the Secant of the *Middle Latitude* multiplied into $\frac{2sss}{3arr}$ = 0,0000000070513290871 5; which therefore, unleſs the Secant exceed Ten Times Radius, can never amount to 1 in the Fifth Place. Theſe two Steps ſuffice to make the *Meridian Line*, or *Logarithm Tangent*, to far more Places than any of Natural Secants yet extant, are computed to; but, if the Third Step be required it will be found to be + *ſs* into $\frac{2s^5}{5ar^4}$ = 0,00000000000000089498; by all which it appears that Mr. *Wright*'s Table does no where exceed the true *Meridian Parts* by fully half a Minute; which ſmall Difference ariſes by his having added continually the Secants 1′, 2′, 3′, *&c.* inſtead of $0\frac{1}{2}'$, $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, *&c.* But, as it is, it is abundantly ſufficient for *Nautical* Uſes. That in Sir *Jonas Moor*'s *New Syſtem of the Mathematicks* is much nearer the Truth, but the Difference from *Wright* is ſcarce ſenſible, till you exceed thoſe Latitudes where *Navigation* ceaſes to be practicable, the one exceeding the Truth about half a Minute, the other being a very ſmall Matter deficient therefrom.

For an Example eaſy to be imitated by whoſo pleaſes, I have added the the true *Meridional Parts* to the firſt and laſt Minutes of the *Quadrant.*

The *Firſt Minute.* 1,00000000141026586 2178.

The *Second,*——— 2,00000000564106380 6707.

The *Laſt*, or 89° 59′. 30374,963431141422 8643, and not 32348, 5279 as Mr. *Wright* has it, by the Addition of the Secants of every whole Minute : Nor 30249,8 as Mr. *Oughtred*'s Rule makes it, by adding the Secants of every half Minute. Nor 30364,3 as Sir *Jonas Moor* had concluded it by I know not what Method, though in the reſt of his Table he follows *Oughtred.*

The ſame may be deduced independently from the *Arch* it ſelf. For if the Latitude from the *Equator* be eſtimated by the Length of its *Arch* A, Radius being Unity, and the *Arch* put for an Integer be *a*, as before; the *Meridional Parts* anſwering to that *Latitude* will be $\frac{1}{a}$ into A, + $\frac{1}{6}$ A ^{3} + $\frac{1}{24}$ A ^{5} +

$\frac{\frac{1}{84}}{60}$ A^7 or $\frac{61}{5040}$ A^7 + $\frac{\frac{11}{2830}}{1386}$ A^9 or $\frac{1385}{362880}$ A^9 *&c* which Couverges much ſwiſter than any of the former *Series*, and beſides has the Advantage of A increaſing in Arithmetical Progreſſion, which would be of great Eaſe, if any ſhould undertake *de novo* to make the Logarithm Tangents, or the *Meridian-line* to any more Places than now we have them. The Logarithm Tangent to the Arch of 45° + $\frac{1}{2}$ A being no other than the aforeſaid Series $A + \frac{1}{6}A^3 + \frac{1}{24}A^5$, *&c.* in *Napier*'s Form, or the ſame multiplied into 0,43429, *&c.* for *Brigg*'s.

But, becauſe all theſe *Series* towards the latter End of the Quadrant do *converge* exceeding ſlowly, ſo as to render this Method almoſt uſeleſs, or at leaſt very tedious: It will be convenient to apply ſome other Arts, by aſſuming the Secants of ſome intermediate Latitudes; and you may for s, or the *Sine* of n, the Arch of half the Difference of *Latitudes*, ſubſtitute $a - \frac{1}{6}a^3 + \frac{1}{120}a^5 - \frac{1}{5040}a^7 + \frac{1}{362880}a^9$, *&c*, according to Mr. *Newton*'s Rule for giving the Sine from the *Arch*; and, if a be no more than a Degree, a very few Steps will ſuffice for all the Accuracy that can be deſired.

And, if a be commenſurable to a, that is, if it be a certain Number of thoſe Arches which you make your Integer, then will $\frac{a}{a}$ be that Number which if we call n, the Parts of the Meridional Line will be found to be

$$\frac{sn.}{r}\text{ into}\left\{\begin{array}{l} 1+\frac{ss\,aa}{3r^4}+\frac{s^4a^4}{5r^8}+\frac{s^6a^6}{7r^{12}},\ \&c. \\[6pt] -\frac{aa}{6rr}-\frac{ss\,a^4}{6r^6}-\frac{s^4a^6}{6r^{10}},\ \&c. \\[6pt] +\frac{1}{120}\frac{a^4}{r^4}+\frac{13s^2a^6}{360r^8},\ \&c \\[6pt] -\frac{1a^6}{5040r^6},\ \&c,\end{array}\right.$$

In this the Firſt two Steps are generally ſufficient for Nautical Uſes, eſpecially when neither of the Latitudes exceed 60 Degrees, and the Differences of Latitudes do not paſs 30 Degrees.

To conclude, I ſhall only add, that Unity being *Radius*, the Co-ſine of the Arch A, according to the ſame Rules of Mr. *Newton*, will be $1 - \frac{1}{2}A^2 + \frac{1}{24}A^4 - \frac{1}{720}A^6 + \frac{1}{40320}A^8 - \frac{1}{3628800}A^{10}$, *&c.* from which and the former *Series* exhibiting the *Sine* by the Arch, by Diviſion it is eaſie to conclude, that the *Natural Tangent* to the Arch A is $A + \frac{1}{3}A^3 + \frac{2}{15}A^5 + \frac{17}{315}A^7 + \frac{62}{2835}A^9$, *&c.* and the *Natural Secant* to the ſame Arch $1 + \frac{1}{2}A^2 + \frac{5}{24}A^4 + \frac{61}{720}A^6 + \frac{227}{8064}A^8$, *&c.* And from the *Arithmetick of Infinites*, the Number of theſe *Secants* being the Arch A, it follows that the Sum Total of all the *Infinite Secants* on that Arch is $A + \frac{1}{6}A^3 + \frac{1}{24}A^5 + \frac{61}{5040}A^7 + \frac{277}{72576}A^9$, *&c.* the which, by what foregoes, is the Logarithm Tangent of *Napier*'s Form, for the Arch of $45° + \frac{1}{2}A$, as before.

And collecting the Infinite Summ of all the *Natural Tangents* on the ſaid Arch A, there will ariſe $\frac{1}{2}AA + \frac{1}{12}A^4 + \frac{1}{45}A^6 + \frac{17}{2520}A^8 + \frac{31}{14175}A^{10}$, *&c.* which will be found to be the Logarithm of the Secant of the ſame Arch A.

To find the Variation of the Compaſs at Sea. by . . . . n. 24. p. 435. Apr. An. 1667.

XXXIX. The Height of the Pole, and the *Sun*'s Declination being known, a large Ring-Dial, truly wrought, having a Box with a Compaſs or Needle fixt to its Meridian below, may go as near as any other Inſtrument, to ſhew the Variation of the *Needle* at *Sea*. For, when it is ſet to the juſt Hour and Minute of the Day, the Meridian of it ſtands juſt in its due Place; and ſo ſhews how far the *Needle* varies from it, as exactly as the largeneſs of the Card will permit.

But, becauſe theſe Dials are ſo rarely juſt, *&c.* though they may be uſed and taken Notice of, yet they are not to be relied on. The Thing therefore is to be performed, as followeth:

Find out the *Sun*'s *Azimuthal* Diſtance from the Meridian ſome Hours before or after Noon, and then its *Magnetical Azimuth*, or Diſtance from the Meridian pointed at by the *Needle*, and the Difference of thoſe two Diſtances is the Variation of the *Needle.*

To find the *Sun*'s true *Azimuth*, or by how many Degrees, *&c.* of the Horizon, it is diſtant from the Meridian: Its Declination, its Altitude, and the Elevation of the Pole, muſt all three be known; and thence the true *Azimuth* may be eaſily calculated. The true *Azimuth* of the *Sun* being thus found, and the *Magnetical Azimuth* of it, according to your *Needle*, being obſerved, ſubtract the leſſer Number from the greater, and the Remainder is the Variation of the *Needle.* If the *Magnetical Azimuth* be leſs than the other, then the Variation is towards the ſame ſide of the Meridian, where the *Sun* is; if greater, on the other.

To obſerve the *Sun's Azimuth* by the *Needle*, and the *Needle's* Variation, to Degrees, any *Needle* long enough to afford upon a Card under it a Circle divided into Degrees, put into a ſquare Box, after the ordinary manner of *Clinatories*, will ſerve turn; by placing the Box ſo, as the Sun may ſhine upon any two oppoſite ſides of it, at the ſame time when the Sun's Height, &c. are taken. For then the *Needle's* Diſtance from the Diameter of the Circle on the Card, that is parallel to thoſe Sides, is the *Magnetick Azimuth* required.

The ſame may be done with an ordinary *Sea-Compaſs*, ſo it have a Circle towards the Limb of the Card divided into Degrees, by faſtening a ſmall Thread, Lute-ſtring or Wire (not of Iron) ſo upon it, as to paſs juſt over the Centre of that Circle: and placing a ſtrait Piece of Wood or Braſs-wire perpendicular on the Edge of the Box at the end of the Thread, and turning it to the Sun till the ſhadow of it fall juſt upon the Thread; then obſerve what Degrees of the Circle on the Card the Thread cuts, by looking plum upon it; and that is the Sun's *Magnetical Azimuth*.

But to have the *Variation* to *Degrees* and *Minutes*, (which is moſt deſireable) then the Obſervation laſt mentioned muſt be made with a *Quadrant*, *Sextant*, or ſome ſuch other Inſtrument, ſo large as to admit of the Diviſion of a *Degree* into *Minutes*: which will require the Radius to be about 3 Foot; the larger the better. If a *Quadrant*, then, it being laid flat, and the ſquare Box with the *Needle* placed upon it, move the *Quadrant* to and again, till that ſide of it, on which the Box is placed, lie parallel to the *Needle* when at quiet: Then the Sight of the *Quadrant* being ſlid along the Limb of it, till the Sun ſhine on both its ſides at the ſame time, the Mid-line, that divides equally the Sight, when the Sun ſhines upon it through the Slit, will mark the *Degree* and *Minute* of the *Sun's Magnetical Azimuth*. All which is eaſie to be put in Practice.

To find this Variation by the Stars, is ſo eaſie, that every Maſter can do it.

A Caution for Obſerving the Variation at Sea; *by Mr.* Edm. Halley, n. 195. p. 571. Oct. An. 1692.

XL. It is a received Error, in the Practice of obſerving the Variation at *Sea*, to take it by the Amplitude of the *Riſing* and *Setting Sun*, when his Centre appears in the Viſible Horizon, whereas he ought to be obſerved when his Under-Limb is ſtill above the Horizon about $\frac{2}{3}$ of his Diameter, or 20 Minutes, upon the Score of the *Refraction*, and the Height of the Eye of the *Obſerver* above the Surface of the *Sea*: Or elſe they are to work the *Amplitude* as they do the *Azimuth*, reckoning the *Sun's* Diſtance from the *Zenith* 90° 36′.

This, though it be of little Conſequence near the Equinoctial, will make a great Error in *High Latitudes*, where the *Sun* riſes and ſets obliquely.

A Caution to Seamen *bound up the* Engliſh Channel; *by* Mr. Halley, n. 247. p. 725. Nov. An. 1700.

XLI. The *Latitudes* of the *Lizard* and *Scilly*, are laid down too far Northerly by near 5 Leagues: For, from undoubted Obſervation, the *Lizard* lies in 49° 55′. the middle of *Scilly* due *Weſt* therefrom, and the South Part thereof neareſt 49° 50′. Whereas in moſt Charts and Books of

Navigation they are laid down to the *Northward* of 50°. and in ſome full 50° 10′. Nor was this without a good Effect, as long as the *Variation* continued *Eaſterly*, as it was when the *Charts* were made. But ſince it is become conſiderably *Weſterly*, (as it has been ever ſince the Year *1657.*) and is at preſent about 7½ *Deg.* all Ships ſtanding in, out of the Ocean, *Eaſt* by the *Compaſs*, go two thirds of a Point to the *Northward* of their true Courſe, and in every 80 Miles they ſail, alter their *Latitude* about 10′. So that if they miſs an Obſervation for two or three Days, and do not allow for this *Variation*, they fail not to fall to the *Northward* of their Expectation, eſpecially if they reckon *Scilly* in above 50°, and to run up the *Briſtol Channel*, not without great Danger of all, and the Loſs of many of them. This has been by ſome attributed to the *Indraught* of St. *George's Channel*: But, the *Variation* being allowed, it hath been found, that the ſaid *Indraught* is not ſenſible. It is therefore recommended to all Maſters of Ships, that they ſteer two Watches E. by S. for one E. which will exactly keep their Parallel; as alſo, that they come in, out of the Sea, on a Parallel not more *Northerly*, than 49° 40′. which will bring them fair by the *Lizard*.

XLII. Papers of leſs General Uſe, omitted.

Pendulum Watches, n. 118. p. 440. *vid. ſup. Cap.* V. *Sect.* V.

n. 128. *p.* 710.

n. 129. p. 742.

Mr. *Oldenburg* having publiſhed from *Journal de Scavans*, an Account of M. *Hugen's* Portable *Watches*, Dr. *Hook*, in the Poſtſcript to his *Deſcription of Helioſcopes*, complains of it, for not having taken Notice, *That this Invention was firſt found out by an* Engliſhman, *and long ſince publiſhed to the World.* To this Mr. *Oldenburg* anſwers, by relating the *Plain Truth of the Matter*: Whereupon, Dr. *Hook*, in a Poſtſcript to his *Lampas*, further complains, and reflects on Mr. *Oldenburg's* Integrity and Faithfulneſs in his *Management of the Intelligence of the Royal Society.* This gave Occaſion to the *Council* of that Society to declare, *That Mr.* Oldenburg *had carried himſelf Faithfully and Honeſtly and given no juſt Cauſe of ſuch Reflections*; To which Mr. *Oldenburg* adds Part of a Letter from Mr. *Hugens* to him, *Offering, (if Mr.* Oldenburg *believes a Patent in* England *might be worth ſomething) all he might there pretend to.* So that if Mr. *Oldenburg* had a Deſire to take out a Patent, it was for no other Contrivance than Mr. *Hugen's*.

XLIII. Account of Books and Emendations omitted.

n. 231. p. 670. 1. Volumen Primum *Geographorum Gr.* Minorum. *Oxon.* in 8*vo.*

n. 231. p. 671. 2. *Dionyſii* Periegeſis, Græce & Latine, cum Scholis Gr. tam Editis quam Ineditis. Cura *Edw. Thwaits*, M. A. *Oxon.* in 8*vo.*

n. 91. p. 5172. 3. *Bernhardi Vareni*, M. D. *Geographia* Generalis; aucta & illuſtrata ab *Iſaaco Newtono*, R. S. S. *Cantab.* 1672. in 8*vo.*

n. 231. p. 666. 4. *Philippi Cluverii* Introductio in Univerſam *Geographiam*, tam Veterem quam Novam: Tabulis *Geographicis* 46. ac Notis olim ornata à *Joanne Bunone*; jam vero locupletata Additamentis & Annotationibus *Jo. Frid. Hekelii* & *Jo. Reiskii*. *Amſt.* 1697. in 4*to.*

5. *Geography Anatomiz'd*; or a Compleat *Geographical Grammar*. Being a ſhort and exact *Analyſis* of the whole Body of *Modern Geography*, after a New and Curious Method; by *Pat. Gordon*, M. A. F. R. S. The *Second Edition.* n. 256. p. 335.

6. An Account of the Meaſure of a *Degree* of a *Great Circle* of the *Earth*; by M. *Picart*. *Paris*. 1671. *Fol.* Tranſlated into *Engliſh*; by Mr. *Waller*, *R. S. Sec. Lond.* 1687. *This Book is here abridg'd; and the Sum of the whole amounts in ſhort to this,* M. Picart *meaſured on a plain and ſtraight Ground a ſpace of* 5663 Toiſes, *to ſerve for the Firſt* Baſis *to divers Triangles; by which he hath concluded the* Length *of a* Meridian Line *equivalent to a* Degree *of* Latitude, *to be* 57060 Toiſes *or* Fathoms, *that is,* 28½ Leagues *and* 60 Toiſes. n. 112. p. 261. n. 124. p. 596. n. 189. p. 376.

7. The Seaman's Practice; by Mr. *Richard Norwood*, *Lond.* 1636. in 4to. *The Meaſure of a* Degree *is here extracted from that Book. Mr.* Norwood, *An.* 1635. *having actually meaſured, for the moſt Part, the Way from* York *to* London, *and having obſerved the Meridian Altitudes of the* Sun *in both Places, he found the* Difference *of* Latitude *to be* 2° 28'. *and the* Diſtance *of their* Parallels 905751. Engliſh *Feet; and therefore one* Degree of a Great Circle *is* 367196 *Feet, or* Numero Rotundo 367200 *Feet, which is equal to* 69½ Engliſh Miles, and 14 Poles; *Whereas the* French *make it no more than* 365000 *ſuch Feet.* n. 126. p. 636.

8. Longitude found; by *Hen. Bond*, Sen. *Lond.* 1676. in 4to. *A Miſtake in that Book is here corrected.* n. 95. p. 6065. n. 130. p. 774.

9. A Book publiſhed by Mr. *Jo. Moxon*, deſcribing a new ſort of Terreſtrial Globes, invented by the E. of *Caſtlemain*. n. 139. p. 9883. Ph. Col. n. 1. p. 43.

10. The *Engliſh Atlas. Oxford*, for *Moſes Pitt.* 1680. *Fol.* Ph. Col. n. 28. p. 39.

11. A new Map of *England*, full 6 Foot Square, wherein *Computed* and *Meaſured* Miles are entered in Figures; by Mr. *Jo. Adams*. n. 135. p. 886.

12. A large and curious Map of *Great Tartary*; by M. *Nich. Witſon*. n. 193. p. 492.

CHAP. VIII.

Architecture. Ship-Building.

Stones fit for Building; by . . . . *n.* 53. *p.* 6010. Apr. An. 1673.

I. THERE is a ſort of grey *Freeſtone* at *Paris*, every where on the South-ſide of the River *Sein*, which is of a reaſonable courſe Greet, and ſo ſoft when firſt taken out of the *Quarry*, that 'tis dreſt and hewn with broad ſharp Axes, almoſt as eaſily as dried Clay, but grows harder and harder in the Air; 'tis very durable, and exceeding fit for *Building*. The *Portland* Stone is of a fine Chalky Greet, fit for all curious hewn and carved Work, though not fit for Water or Fire. On the contrary, the *Freeſtone* in *Kent*, of a whitiſh grey Colour, laſts well in Air and Water; the Greet thereof leſs fine and chalky than that of *Portland*. The *Derbyſhire Freeſtone*, though it endure the fierceſt Fire, is yet brittle, and ſo unfit for fine and curious Workmanſhip.

The Choice and Charges of Slate for covering Houſes; by Mr. Sam. Colepreſs. *n.* 50. *p.* 1009. Aug. An. 1669

II. 1. Take the thin cleft Stone, Slat or Shindle, and ſo knock it againſt any hard Matter, as to make it yield a Sound; if the Sound be good, and clear, that ſort of Stone is not crazy, but firm and good. Or,

If in hewing it does not break before the Edge of the *Sects*, (the Hewing Inſtrument of the *Slatters*) you may not much doubt of the Firmneſs of the Slate. But,

3. If after it hath been exactly weighed (and the Accompt thereof laid by) it be put, and for 2, 4, or 8 Hours left to remain all under Water in a Veſſel; and afterward taken up and wiped very clean with Cloths, if then it weigh more than before, 'tis of that kind which imbibes Water, and therefore not ſo fit to endure any conſiderable time without rotting the Lathes and Timber.

4. Theſe Stones may be pretty well gueſſed at, whether they be of a cloſe or looſe Texture, by their Colour: For the over blackiſh Blue is apteſt to take in Water; but the lighter Blue is always the firmeſt and cloſeſt. To which may be added the Touch; for a good Stone feels ſomewhat hard and rough; whereas an open Stone feels very ſmooth, and as it were Oily.

5. Place your Stone long-ways perpendicular in the midſt of a Veſſel of Water, (no matter how ſhallow the Water be, ſo it exceed half a Foot depth;) and be ſure, the upper un-immerſed Part of the Stone be not accidentally wetted by the Hand, or otherwiſe; and ſo let it remain a Day, or half a Day, or leſs. If it be a good firm Stone, it will not draw (as they ſpeak)

ſpeak) Water more than half an Inch above the Level of the Water, and that perhaps but at the Edges only, the Parts of which might be ſomewhat looſened in the Hewing. But a bad Stone will draw Water up to the very top, be the Stone as long as it will, all over.

As for the Charge of covering Houſes with Slate, they may be thus computed.

	s.	d.
1000 of *Efford Small Blue* at the Ship's ſide in *Plymouth* Harbour ——	5	6
1000 of *Efford Large Blue* ——	9	9
1000 of *Can Pelmel* ——	7	0
1000 of *Small Blue* of other Quarries ——	4	0
1000 of *Large Blue* ——	8	0
3000 of *Small Blue*, accompted two Tuns in Carriage by Water.		
1000 of *Large Blue*, 1 Tun.		
3000 of *Small* will cover 1 *Pole* of Work at the 5 Pin Plain.		
Every *Pole* of Work is either 6 Foot broad and 14 up, on both ſides, or 168 Foot in Length, and one in Breadth.		
3000 of *Large*, will cover two *Poles* of plain Work.		
Hewing of all ſorts of plain *Pelmel per* 1000 ——	1	6
Pinning per 1000, 8 *d. Pins per* 1000. 8 *d.* ——	1	4

Three Buſhels (*Wincheſter* Meaſure) of good *Lime*, will take 6 Buſhels of freſh Water Sand, and ſerve to lay on one *Pole* of Work; though much leſs may ſerve the Turn.

300 of Lathes to every *Pole* of Work.

1000 of Lathe Nails to every 3000 of Lathes.

An able Workman may { Lath one *Pole* of Work
Lay on 2000 or more of *Slate*
Hew 1500 plain
Pin 4000 } by the Day.

Chequer-work conſiſts in Angles, Circles, and Semicircles, *&c.* which require no common Skill and Time in hewing and laying.

It is worthy Obſervation, That, if a ſide Wall happen to take Wet by the beating of the Weather, or the like, when nothing elſe will cure it, our *Kerſeying* with *Slate* (which is much uſed in the curious Fronts of Houſes, eſpecially in Towns) will quickly remedy it.

We have ſome ſorts, which by the Conjectures of the more Experienced *Helliers*, (or Coverers with Slate), have continued on Houſes ſeveral Hundreds of Years, and yet as firm as when firſt put up.

III. The *Cuſtom* of *Felling Timber* here in the *South* of *England*, differs from that of *Staffordſhire*, in the Time of *Felling*, and manner of *Barking*. It is *Felled* here in the *Spring*, as ſoon as the *Sap* is found to be fully up, by the Trees putting out, and then *Barked* after the Trees are proſtrate, the *Sap* yet remaining in the Bodies of them: Whereas there it is firſt *Bark'd*, (in the Spring as here) but before it is *Felled*, the Trees yet living and ſtanding all the *Summer*, and not *Felled* till the following *Winter*, when the *Sap* is fully in Repoſe.

The beſt Time of Felling of Timber; by Dr. Rob. Plot. n. 192. p. 455. Jan. An. 1691.

In the *Spring* Seaſon, and ſome time after, all Trees are pregnant and ſpend themſelves (as Animals do in their reſpective Offsprings) in the Production of Leaves and Fruits, and ſo become weaker than at other times in the Year; their Cavities and Pores being then turgid with Juices, or Sap, which (the Trees being felled at that time) ſtill remain in the Pores, having no manner of Means of being otherwiſe ſpent, and there putrifie; not only leaving the Tree full of thoſe Cavities which render the *Timber* weak; but, *2dly*, breeding a Worm, as both *Pliny* and Mr. *Evelyn* teſtifie, that will ſo exceedingly prejudice it, that it becomes altogether unfit for ſtrong Incumbencies, or other robuſt Uſes. *3ly*, All Timber fell'd at this Time of the Year, whether the Juices putrifie, or otherwiſe ſweat forth, or dry away, is not only ſubject to rift and gape, but will ſhrink ſo conſiderably, that a Piece of ſuch *Timber* of a Foot ſquare will uſually ſhrink in the Breadth of ¼ of an Inch; than which, ſays *Vegetius*, nothing is more pernicious, if us'd for the *Building* of *Ships*. To which, *4thly*, the Firſt and Greateſt *Roman* Emperor, *Julius Cæſar*, adds, that tho' *Ships* may be made of ſuch moiſt Timber, felled in the Spring, yet they will certainly be Sluggs, not near ſo good Sailers as Ships made of Timber felled later in the Year.

In all which Circumſtances, I find, moſt of the *Ancients* ſo very well agree, that none of them adviſe the felling of *Timber*, for any ſort of Uſe, before *Autumn*, at ſooneſt; others, not till the Trees have born their Fruit; which, ſays *Theophraſtus*, muſt always be proportionably later, as their Fruits are ripe later in the Year. A third ſort, not till *Mid-winter*; not till *November*, ſays *Palladius*, nay, not till the *Winter Solſtice*, ſays the wiſe *Cato*; and then too in the Decreaſe or *Wane* of the *Moon*, between the 15*th* and 23*d* Day of her Age, ſays *Vegetius*, or rather, according to *Columella*, between the 20*th* and the *New-Moon*. In general, ſays *Theophraſtus*, the *Oak* muſt be fell'd very late in the *Winter*, not till *December*, as the Emperor *Conſtantine Pogonatus* poſitively aſſerts, the *Moon* too being then under the Earth, as 'tis for the moſt part in the Day time in the firſt Part of its Decreaſe. And the felling of *Oak* within thoſe Limits they call *Tempeſtiva Ceſura*, *Felling Timber* in Seaſon, which they all unanimouſly pronounce (if thus felled) will neither ſhrink, warp, nor cleave, nor admit of Decay, in many Years; it being tough as *Horn*, and the whole Tree in a manner (as *Theophraſtus* aſſerts) as hard and firm as the Heart; with whom alſo agrees our Countryman Mr. *Evelyn*, if you fell not *Oak* (ſays he) till the Sap is in Repoſe, as 'tis commonly about *November* and *December*, after the Froſt has well nipped them, the very *Saplings* thus cut will continue without Decay, as long as the Heart of the Tree.

And the Reaſon of this is given in ſhort by *Vitruvius*, *quia Aeris* Hyberni *vis comprimit & conſolidat Arbores*, becauſe the *Winter* Air doth cloſe the Pores, and ſo conſequently conſolidates all Trees, by which means, the *Oak*, (as he and *Pliny* both expreſs it) will acquire a ſort of *Eternity* in its Duration; and much more will it ſo, if it be Bark'd in the Spring, and left ſtanding all the Summer, expoſed to the Sun and Wind, as is uſual in *Staffordſhire*, and the adjacent Countries, whereby they find, by long Experience, the Trunks of their Trees ſo dried and harden'd, that the ſappy Part, in a manner, becomes as firm and durable as the Heart itſelf.

Which way of *Barking* and *Felling* of *Timber*, tho' it were unknown to the Ancients (as perhaps it is to all the World beſides theſe few Counties) yet they ſeem not unacquainted with the Rationality of the Practice. The great *Vitruvius* prefers the *Timber* on the *South-ſide* the *Apennine* (where it winds about, and incloſes *Tuſcany* and *Campania*, and ſtrongly reflects the conſtant *Heat* of the *Sun* upon it, as it were from a Concave) incomparably before that which grows upon the *North-ſide* of the ſame Hill, in the ſhady moiſt Grounds: Of which his Opinion he renders us this Reaſon, for that the Sun does not only lick up the ſuperfluous Moiſture of the *Earth*, whence the *Trees* are ſupplied in ſuch ſhady Places with too great a Quantity, but in great meaſure exhales the remaining Juices (after the Production of Leaves and Fruits) out of the *Trees* themſelves, rendring the Timber of them the more cloſe, ſubſtantial and durable; which certainly it would do alſo much more effectually, if the *Bark* were taken off in the Spring of the Year, as is accuſtomed in *Staffordſhire*, where the People are content to uſe this Method in their Proviſion of Timber, though but for private Uſes.

Much rather then ſhould it be done in ſo publick a Concern as the *Building* of *Ships*, where tough and ſolid Timber is much more neceſſary than in ordinary *Buildings*. There is indeed an *Act* of *Parliament*, 1 *Jac.* 1. *Chap.* 22. which forbids the *Felling* of *Timber* for ordinary Uſes (in conſideration of the *Tan*) at any other time but between the 1ſt of *April*, and the laſt of *June*, when the Sap is up, and the *Bark* will run; made on Suppoſition, (I gueſs) that ſhould they have admitted *Felling Timber* in any other Seaſon, the *Tanners* would have wanted a Supply of *Bark*. To which I readily anſwer, That I fear the *Legiſlators* that preſſed the making that Act, were ignorant that the *Bark* might be taken off in the Spring, and that the Tree would notwithſtanding live and flouriſh till the *Winter* following, as I have ſeen many in *Staffordſhire*: So that though the Tree be not *Fell'd* till the *Winter Solſtice*, or *January* following, yet the *Tanner* is not at all defeated of his *Tan*, but has it here in as due Seaſon, as in any of the Southern Countries. The *Legiſlators*, I ſay, were ignorant of this, otherwiſe they would never have made an Act ſo pernicious to the whole Kindom, as *Felling Timber* at this Seaſon is, for the ſake of a few *Tanners*.

But notwithſtanding this Ignorance, yet then they were ſo wiſe as to except in that Act the *Timber* to be uſed in building of Ships, which may be *Fell'd* in *Winter*, or any other time; as I am told all the ancient Timber remaining in the *Royal Sovereign* was, it being ſtill ſo hard, that 'tis no eaſy matter to drive a Nail into it.

'Tis true ineeed, that the *barking* and *peeling* the Tree ſtanding is ſomewhat more troubleſome, and therefore ſomewhat more chargeable, than when they are proſtrate; and that 'tis likely, People therefore have uſually fell'd their Timber, as well for *Shipping* as other Uſes, in the Spring of the Year, for the ſake of the more eaſy and cheap *Barking* it only, rather than any thing elſe. 'Tis too true, that *Timber* is harder to *Fell* in the *Winter*, it being now ſo compact and firm, that the Ax will not make ſo great Impreſſion as it doth in the *Spring*, which will alſo increaſe the Price of the *Felling* ſome ſmall Matter, and its *Sawing* afterwards; but how inconſiderable theſe things are in compariſon of the great Good of this manner of *Felling*, I think is ſelf-evident.

The greateſt Objection, that I can foreſee will be urged here in the *South* againſt this Practice, is, that if the *Timber* be not *fell'd* till *Mid-winter*, or *January*, where it grows in *Copſes* and *Woods*, they cannot perhaps incloſe their young *Sprigs* ſo ſoon as ſome may imagine needful, and therefore will be backward to fell their Timber (ſo growing) at that Seaſon. To which I anſwer, That the *Timber* ſo fell'd in *Woods* or *Copſes* may be eaſily carried off before the ſecond Spring, and ſo the Prejudice ſmall, and the firſt it muſt be there, wherever it is fell'd. But *Secondly*, That which will quite remove this inconſiderable Difficulty is, that perhaps it may be expedient that no Timber whatſoever growing in *Woods* or *Copſes*, be at all bought in the *King's Yards*, for that *Timber* growing in ſuch ſhady Places; and ſo fenced from the *Sun* and *Wind*, as *Timber* in *Woods* for the moſt part is, cannot be ſo good as that which comes from an expoſed Situation, ſuch as it uſually has in *Foreſts*, *Parks*, *Hedge-rows*, and open Fields; where too it is indifferent at leaſt, if not better, for the Proprietor, that it be fell'd in *Winter*, (when the Graſs and Corn is gone) than in the *Spring* it ſelf: and the Officers deſigned for that purpoſe may buy all their *Timber* under ſuch Conditions as to be fell'd in *Winter*, enjoining the Proprietor to take off the Bark in the Spring in due Time, making him ſome ſmall Allowance for the Trouble he will have in peeling it ſtanding.

The Difference of Timber *in different* Countries, *and Fell'd at* different Seaſons; *by M.* Ant. Van. Leuwenhock. n. 213. *p.* 224. Sep. An. 1694.

IV. It is the common Opinion, That *Timber* which is fell'd in *Winter*, is ſtronger and more laſting, as being more cloſe and firm than that which is fell'd in *Summer*: But, M. *Leuwenhock's* Sentiment is, That there is no Difference, except in the *Bark*, and outermoſt Ring of the Wood, which in the *Summer* are ſofter, and ſo more eaſily pierced by the *Worm*; *Wood* conſiſting of hollow Pipes, which in the *Summer* and *Winter* both, are full of Moiſture, they do not ſhrink in the *Winter*, and therefore the *Wood* cannot be cloſer at one time than another, for otherwiſe it would be full of Cracks and Clefts. The ſudden and unexpected rotting of ſome Timber, he conceives to proceed from ſome inward Decay in the Tree before it was fell'd; having obſerved all Trees to begin to decay at firſt in the Midſt, or Heart of the Tree, tho' poſſibly the Tree may ſtand and grow for near an Hundred Years afterwards, and increaſe in Bigneſs all along.

2. He ſays, he was once of Opinion, that Trees growing in good Ground, but increaſing ſlowly, were the beſt and ſtrongeſt Timber; and that thoſe Trees, which in few Years grew large, were the ſofteſt and brittleſt; the contrary to which, upon Enquiry of experienced Workmen, he found to be true, and inſtances of an *Elm* of 80 Years Growth, which was 11 Foot in Circumference, and proved excellent tough Timber.

3. The *Age* of *Trees* is to be known by the Number of Rings to be ſeen when the *Tree* is cut athwart, in each of which Rings is one Circle of large open *Pipes*; now the fewer of theſe large *Pipes* the ſtronger the *Timber* is: wherefore by Conſequence theſe *Trees* that make the largeſt Growth in a Year, muſt be the cloſer and ſtronger, and therefore thoſe *Trees* that grow in warm Countries grow faſteſt, and are the beſt and tougheſt Timber; which he confirms by *Riga* and *Dantzick* Oak, which is of ſlow Growth, and proves ſpongy and brittle *Timber*, whereas the contrary is obſervable in *Engliſh* and *French* Oak, which grows faſter, and is excellent *Timber*.

The Bridge at St. Eſprit *in* France; *by Dr.* Tankred Robinſon, *n.* 160. *p.* 584. *Fig.* 215.

V. 1. This Famous *Roman Bridge* at *Pont St. Eſprit*, is very crooked, bowing in many Places, and making ſeveral unequal Angles, eſpecially in thoſe Places where the Torrent runs ſtrongeſt, as where the *Turret* ſtands. 4. In which Place the Angle is moſt unequal, and the greateſt; the *Arches* are very wide, and have their Feet ſecured by two *Pedeſtals* that encompaſs them. Both theſe *Pedeſtals* have their ſeveral Degrees or Ranks of *Jettings* out, like ſo many Rows of Stairs or Steps, the lowermoſt Order puſhing out moſt, the others being leſs, and going gradually more in; the ſecond or uppermoſt *Pedeſtal* is much leſs than the firſt or lowermoſt, being built a little within its Lines of *Circumference*; 1, 2. Between the great *Arches* there are *Windows*, or, (as it were) ſmall *Arches*; 3. that come down to the very Plane of the ſecond, or uppermoſt *Pedeſtal* dividing the Feet of the great Arches. From this my rude Deſcription it appears to me, that the *Romans* have here contrived all poſſible ways to break gradually the mighty Force of the *Roſne*, and to render its Paſſage eaſy, and inoffenſive to the Feet of the great *Arches*; for here we ſee ſo many ſeveral *Paliſadoes* and *Sluices*, as may be ſufficient to defend this wonderful Fabrick againſt all Storms of the *Torrent*; the ſeveral Ranks of *Stairs* jetting from the *Pedeſtals* (for the moſt part Triangularly built, and faced well with Free-ſtone) oppoſing and breaking the Stream ſeverally, I mean, not altogether, or at the ſame time, by reaſon of their various Inequalities in ſtanding out: In caſe the Flood ſhould ſwell ſo high (as it frequently does) as to cover both the *Pedeſtals*, then the ſmall Arches, dividing the Feet of the great ones, help to convey the Water through, which otherwiſe might endanger the great *Arches*.

By Dr. Liſter. *ib. p.* 585.

2. That which ſeems the Foot of the *Arch* is an *Horizontal Arch* gradually contracted, every Stone being of vaſt Length and Wedge-like, laid level with the Water. This I ſpeak by Memory.

Compared with ſome *other* Bridges; *by Dr.* Tank. Robinſon. n. 163. *p.* 712.

3. The ſtately *Modern Bridge* at *Avignon* hath yielded in many Places to the extream Rapidity and Violence of the *Rhoſne*. Its Fall in my Opinion may be aſcribed to three Defects. Firſt, It was not ſo multangular, as that at St. *Eſprit*: Secondly, It wanted in three or four Places, the little *Arches* dividing the Feet of the great ones, and in thoſe Parts it hath ſuffered moſt; for where thoſe uſeful Sluices are, there I obſerved the Bridge to ſtand ſtill the moſt intire. Thirdly, The *Pedeſtals* (or as you very properly call them *Horizontal Arches*) were not ſo Geometrically and exactly laid, as thoſe of *Pont St. Eſprit*, there Jettings out were few, and they not gradually contracted; ſo that the Force of the Stream muſt be greater upon the Fabrick.

Though the *Tiber* be not ſo ſwift as the *Rhoſne*, yet it is ſubject to greater Inundations, as many Inſcriptions aſſure us. No River ever had ſo many Bridges built with that Magnificence and Art, as this; and tho' they were more pompous and rich in rare Stones, in Sculpture, &c. than that I formerly ſent you a Draught of from *Montpelier*; yet they had the like Proviſion for their Security and Preſervation, and their Deſign was much the ſame; which may be ſeen at *Rome* this very Day at the old *Pons Milvius* (now *Ponte Nolle*) near the *Via Flaminia*, in the Marble Remains of the *Pons Æmilius*, repaired with rich Materials by *Antoninus Pius*) on the Side of the *Ripa*, or *Traſtavere*, near the Root of the *Aventine Hill*, where firſt the *Pons Sublicius* ſtood; as alſo in the *Pons Fabritius* and the *Ceſtius*, that leads over to the *Inſula Tiberina*; in all which there are ſtill very fair Marks of the old *Roman* Structure and *Deſign*; and if that prodigious City had not been knocked ſo oft to pieces by *barbarous Sackers*, we might have had ſtill as clear Proofs from the other Bridges. *viz.* the *Pons Triumphalis*, the *Senaectius*, &c. But *Gothiſh* and *Northern Torrents* broke all before them.

A Bridge *without any Pillar under it; from the Journal of the* Phil. Society *of* Oxford. *ib. p.* 714. *Fig.* 216.

VI. A *Timber Bridge* may be built 70 Foot long, or ſomewhat more, without any *Pillar* under it, which may be uſeful in ſome Places where *Pillars* cannot be conveniently built, after this manner; AC, and BO are Beams 28 Foot long, and AB is 32 Foot long. Under the Angles are ſet two large *Braces* EL, and SR. At each End is a Wall, on which are laid two *Beams* BH, and AD, each 20 Foot long; under theſe two are two *Braces* DE, and RH. There may alſo be Braces at the End of the Arches, that may lie obliquely croſs the Bridge. It may be laid with Planks and Railed. Behind the Walls are *Cauſeys* FD, and AN. The Length of the Bridge CMO, is 70 Foot; the Height KM is 19 Foot.

An Aqueduct *near* Verſailles. N. 171. *p.* 1016. May. An. 1615.

VII. 1. The *Aqueduct* which is to be made near *Maintenon*, for the carrying the River *Bure* to *Verſailles*, will have in Length 7000 Fathom; 462 whereof will be 35 Fathom and 4 Foot high, the reſt will be lower, according to the Difference of the Ground, but no leſs than 5 Foot and 6 Inches high. There will be to the ſaid *Aqueduct* 861 *Arches*, which, where they are higheſt, will have 12 Fathom in Breadth, and 8 Fathom in Thickneſs, diminiſhing to 14 Foot at the Top. The other *Arches* will be leſſer in Breadth, as well as Thickneſs, acccording to the Nature of the Ground. The ſaid *Aqueduct* will have 15 Inches Fall to every Thouſand Fathom in Length; ſo that for the 7000 Fathom, there will be 8 Foot 8 Inches Fall. The River is to paſs by *Maintenon, le Parc Eſpernon, Gajeran, Rambouillet, les Eſſars, le Perrey, Cognieres*, nnd from thence to *Verſailles*. There are 14000 Soldiers that work there, under the Command of the Marqueſs *d'Uxelles*, with three Commiſſaries of War for their Conduct.

N. 176. *p.* 1206.

2. A *Magazine* for the Waters upon the Moutain *Monthoron*, is already cut, which will have 2200 Perches of *Surface* (each Perch being 8 *French* Foot) and 12 Foot in Depth. In another Place, much lower, will be another *Magazine*, to receive the Waters of many *Fools*, the moſt part of which,

as yet have no Water in them. In the Valley of *Buc* will be an *Aqueduct*, the Middle whereof will be raiſed 22 Fathom high, for conveying the Pools of *Sarle*, which it's ſaid contain much Water, though there be nothing but Rain to fill them; this *Aqueduct* is 300 Fathom long, and paſſes thro' two Mountains which have been cut through upon that Account. The Valley alſo on both ſides of the *Aqueduct* is raiſed 11 Fathom high, to make Paſſages.

An *Aqueduct* is alſo making near the Tower of *Stone*, (where the Mills raiſe the Water) which will now paſs without Force to the Top of the Mountain; and there be part of it diſtributed into ſeveral great *Ciſterns*, which are making about *Marli*, for that Place.

The Elevation of the *Aqueduct* of *Maintenon* is now ſet forth at but 2560 Fathom; whereas it was deſigned to be carried on more than 8000 Fathom, and the Remainder will be made of Earth, which muſt be brought thither: This Opinion prevails, in regard it gives a quicker Diſpatch, tho' it may be doubted, it will not be for the Better.

Theſe 2560 Fathoms contain 242 *Arcades*, whoſe *Aperture* is 6 Fathom and $\frac{1}{2}$, and the *Face* of each *Pillar* ſuſtaining the *Arches*, 4 Fathom; there will be then on the ſide of *Maintenon* 33 *Single Arches*, afterwards 71 *Double* ones; (as having one *Arch* upon another) then 46 *Treble* ones; which will generally be 216 Foot 6 Inches high, (*viz.* up to the Floor of the *Channel*) afterwards 72 *Double* ones; then 20 *Single*, which will reach to the Mount of *Earth*, that is to be 50 Foot high.

From the Ground up to the *Second Arcade* are 16 Fathom, from the *Second* to the *Third*, or upper *Arcade*, are 14 Fathom, (which *Arcades* are Double in Number to thoſe they ſtand upon) and 6 Fathom 6 Inches more, to the Floor of the *Channel*, which will at leaſt be 7 Foot high, beſides the *Parapet*.

The *Pillars* by the Ground are 8 Fathom thick, but, what with the *Slopes* and *Shortnings*, which are made in every Story, the Top, where the *Channel* goes, will be but 20 Foot broad. There will likewiſe be at each Pillar a *Buttreſs* jetting out one Fathom, and two Fathom wide.

The intelligent Obſerver, tho' well skill'd in Things of this Nature, as being no Stranger to the *Writings* of the Ancients, or the famous Ruins and Remainders of their *Fabricks* in *Italy*, and other Places, profeſſes himſelf ſurpriz'd with the Greatneſs of this Undertaking at *Verſailles*, and *Maintenon*; for the Magnificence of the Deſign, the Number of Labourers, the Exceſſiveneſs of the Expence, and the admirable Beauty of the Work.

A very large Stone Chimney, with a peculiar ſort of Arch-Work; by Dr. J. Wallis. n. 166. p. 800. *Fig.* 217.

VIII. Having been lately at *Edgecot* in *Northamptonſhire*, at the Houſe of *Tobias Clancey* Eſq. he ſhewed me in an Ancient Kitchen (now diſuſed) two *Chimneys*, vaſtly large, of *Stone-Work*: Which I took the more Notice of, becauſe of a peculiar Way of *Arch-Work* in the Front of them; whereby, without the Advantage of a *Diſcharge* of *Timber* (which is uſual, in ſuch Caſes, to defend the *Arch-Work* from being overburden'd,) an *Arch* of maſſy Stone, (in each of them) ſuſtains itſelf at a great Length, tho' almoſt upon a Flat, being very little raiſ'd in the Middle. Over this Arch (aftet ſome walling interpoſed) there is another Arch (to defend the former) more raiſed from the Flat. The Dimenſions of all, I have thought fit here to ſubjoin.

A B. The Breadth between the *Jambs*, from Inſide to Inſide, 18 Foot.

C D. The Depth of the *Stones* in the Lower Arch, 22 Inches ; locked one into another, with a crooked Joint.

D E. The Diſtance in Walling, between the Arches, 2 Foot and 7 Inches.

E F. The Depth of the *Stones* for the Upper Arch, 15 Inches : With a ſtreight Joynt.

G H. The Place of two vaſt *Tunnels* of Stone.

K. A *Window* between them.

A new Kind of Stairs ; by M. Weighelius. n. 74. p. 2212.

IX. M. *Weighelius* hath lately invented an odd *Bridge*, or kind of Stairs, by which a Man ſhall deſcend, and yet really be raiſed upward ; and going as 'tere upon a Plain, ſhall from a lower, by gently ſubſiding, arrive to an upper Story.

Preſerving of Ships from being Wormeaten ; by . . . . . n. 11. p. 190. Apr. An. 1666.

X. In the *Indian Seas*, there is a kind of ſmall *Worms* that faſten themſelves to the *Timber* of the Ships, and ſo pierce them, that they take Water every where ; or if they do not altogether pierce them through, they ſo weaken the Wood, that it is almoſt impoſſible to repair them. Some have imployed *Deal Hair* and *Lime*, &c. and therewith Lined their Ships ; but beſides that, this does not altogether affright the *Worms*, it retards much the Ship's Courſe. The *Portugals* ſcorch their Ships, in ſo much that in the *Quick Works* there is made a Coaly Cruſt of about an Inch thick. But as this is dangerous, it happening not ſeldom that the whole Veſſel is burnt ; ſo the Reaſon why the *Worms* eat not through *Portugal* Ships, is conceived to be the exceeding Hardneſs of the *Timber* employed by them. There is *Holland* a in Man that pretends to have found an admirable Secret to remedy this Evil. And a very worthy Perſon in *London*, ſuggeſts the *Pitch*, drawn out of *Sea-Coals*, for a good Remedy to ſcare away thoſe noyſome *Inſects*.

An Account of Lead Sheathing ; by Mr. J. Bulteel. n. 100. p. 6192. Jan. An. 1674.

XI. Some ſew Years ſince, Sir *Phil. Howard* and Major *Watſon*, with great Charge and Induſtry found out a new Way, by a Manufacture of our own, to preſerve the Hulls of Ships from the Worms, *&c.* which is much ſmoother and conſequently better for Sailing, and more cheap and durable than the Way of *Boards*, *Pitch*, *Tar*, *Roſin*, *Brimſtone*, or any *Sheathing* or *Graving* hitherto uſed. The *King* and *Parliament* being ſatisfied, upon Examination, of the great Benefit that might redound thereby to his *Majeſty* and Subjects in general, for the Inventors Incouragement to make the ſame publick, were pleaſed, almoſt 4 Years ſince, to grant them an *Act* of *Parliament* for the ſole Uſe of this their Invention, with Penalty and Prohibition to all others. In Proſecution whereof, Experiments have been made upon ſeveral of his *Majeſty's*

Majeſty's Ships, *viz.* the *Phœnix*, done three Years ago, has made two Voyages into the *Streights*, &c. and when ſhe was lately taken into the *Dock* at *Woolwich* to be repaired, upon View of the *Maſter Shipwright* and others, her Sheathing was found to be in as good Condition, as at the firſt doing ; and the Ship ſo tight during the whole time, that they were forced to heave in Water to keep her ſweet. The *Dreadnought*, a Third Rate, done in *June*, 1671 ; the *Henrietta*, *Lyon*, and *Mary*, all three of the Third Rate, and done a Year and half ſince, being lately laid on Ground at *Sheerneſs* and *Portſmouth*, are found to be all in as good Condition, and the Sheathing to continue as firm and as well as at the firſt doing ; as the *Maſter-Builder* and *Aſſiſtant* at *Portſmouth*, and others, have certified.

The *Bread Rooms* alſo of ſome of theſe, and many others of his *Majeſty's* Ships, have been Lined within, almoſt in the ſame manner the *Sheathing* is without ; which has prov'd a great Preſervation of the Bread, as ſeveral of the *Purſers* and *Officers* of the ſaid Ships have Certified ; and by Reaſon of its Duration muſt be much cheaper and better than *Tin*, which is ſo liable to ruſt, or any Way yet uſed.

Alſo the Lead itſelf (which is the Principal thing uſed herein) they make ſo Cloſe preſſed, Smooth, and Equal, or of what Thickneſs or Thinneſs deſired, that great Uſe may be made thereof about ſeveral other things relating to *Shipping*.

XII. *A Paper of leſs General Uſe* Omitted, *viz.*

Directions for Inquiries concerning *Stones* and other *Materials* for the Uſe of *Building*. n. 93. p. 6010.

XIII. Accounts of *Books* omitted.

1. *Vitruvius* done into *Engliſh* ; by Mr. *Chr. Waſe*. n. 72. p. 2190.

Les dix Livres d'*Architecture* de *Vitruve*, corrigez, & traduits nouvellement en *Francois*, avec des Notes & des Figures ; par *Claude Perrault*. *Paris* 1673. in Folio. n. 112. p. 279.

2. Cours d'*Architecture*, enſeigne dans l'*Academy Royale* d'*Architecture*, premiere Partie ; par M. *Francois Blondel*. a *Paris*, 1675, in Fol. n. 112. p. 549.

3. *Raphaelis Fabretti Urbinatis* de Aquis & Aquæductibus Veteris *Romæ*, Diſſertationes tres. *Romæ* 1680. in 4*to*. n. 155. p. 466.

4. Modern Fortification, *&c.* by Sir *Jonas Moor*. 1673. in 8*vo*. n. 95. p. 6071.

5. Novelle Maniere de *Fortifier* les Places ; par M. *Blondel*. *Hague*, 1684. n. 158. p. 586.

6. *Marci Meibomii* de Fabrica *Triremium* Liber. *Amſterlodami* 1671. in 4*to*. n. 79. p. 3071.

7. *Scheeps-Boow en Beſtier*, that is, *Naval Architecture* and *Conduct* ; by *N. Witſen*. *Amſterdam*. 1671. in *Folio*. n. 77. p. 3006.

8. L'*Architecture Navale*, avec le *Routier* des *Indes Orientales* & *Occidentales* ; par le Sieur *Daſſiè*. à *Paris*. 1677. in 4*to*. n. 135. p. 879.

CHAP. IX.

Perſpective. Sculpture. Painting.

A Perſpective Inſtrument; *by* Sir Chr. Wren. n. 45. *p.* 898. *Fig.* 218. Mar. An. 1669.

I. A Is a ſmall Sight with a ſhort Arm R, which may be turned round about, and moved up and down the ſmall *Cylinder* C D, which is ſcrewed into the Piece ED, at D: This Piece ED moving round about the *Center* E; by which means the Sight may be removed either towards R, or F.

EF is a *Ruler* faſtened on the two *Rulers* GG, which *Rulers* ſerve both to keep the ſquare *Frame* S SS S perpendicular, and, by their ſliding through the ſquare Holes T T, they ſerve to ſtay the Sight, either farther from, or nearer to the ſaid Frame; on which Frame is ſtuck on with a little Wax the Paper OOOO, whereon the *Picture* is to be *drawn* by the *Pen* I. This *Pen* I is by a ſmall Braſs Handle V ſo fix'd to the *Ruler* HH, that the Point I may be kept very firm, ſo as always to touch the Paper. H H is a *Ruler*, that is always, by means of the ſmall Strings *a a a*, *b b b*, moved *Horizontally*, or Parallel to it ſelf; at the End of which is ſtuck a ſmall Pin, whoſe Head P is the Sight, which is to be moved up and down on the out Lines of any *Object*.

The Contrivance of the *Strings* is this. The two *Strings a a a*, *b b b*, are exactly of an equal Length. Two Ends of them are faſten'd into a ſmall Leaden Weight Q Q, which is moved in a Socket on the back ſide of the Frame, and ſerves exactly to counterpoiſe the *Rnler* H H, being of equal Weight with it. The other two Ends of them are faſten'd to two ſmall Pins H H, after they have rolled about the ſmall Pulleys N, M, M, L, L, K K; by means of which Pulleys, if the Pen I be taken hold of, and moved up and down the Paper, the Strings moving very eaſily, the Ruler will always remain in an Horizontal Poſition.

The Manner of uſing it is this: Set the *Inſtrument* upon a Table, and fix the Sight A, at what Height above the Table, and at what Diſtance from the Frame S S S S, you pleaſe. Then, looking through the Sight A, and holding the *Pen* I in your Hand, move the Head of the *Pin* P up and down the out Lines of the Object, and the Point I will deſcribe on the Paper O O O O, the Shape of the Object ſo traced.

A new way of Delineating by Parallel Viſual Rays, exactly

II. *Parallelogrammum Proſopographicum* eſt A B C D, Stylus Centralis H F, Calamus Deſignator L C, Index K A, ſive Regula oblonga, Plano *Parallelogrammi*, ope Clavi Striati ex ære, ſecundum Specimen E, ad Angulos rectos aptata. Huic Regulæ inſiguntur duæ Dioptræ P R, S V; in medio P R,

pertuſum

Obſerving the Symmetry; by Mr. St. Clare, n. 96. p. 6080. Fig. 219. July. An. 1673.

pertuſum eſt foramen O, in medio SV, erigitur Filum perpendiculare Regulæ R A, in cujus medio eſt Globus quidam parvus, per quem, & Foramen O, Radius ad oculum (quem inter delineandum non oportet eſſe fixum, ſed Libertum & Solutum) ab Objecto protenditur.

1. *Animadvertendum*, Radium per Foramen O, & Globulum, protenſum, ſemper fore perpendicularem Plano *Parallelogrammi*, ſive ejus Diametro, quæ ex Recta Linea extenſa per Stylum Deſignatorium L C, & Centrum fixum HF, & dictum Globulum parvum, in qua Linea ſemper verſatur iſte Globulus, qualiſcunque ſit *Parallelogrammi* motus.

2. *Not.* Planum Delineatorium ſenſibile ſuper quod volutatur apex L, Styli Pictorii LC, ad amuſſim deſcribentis Imaginem ad motum Indicis K A, & in quod infixus eſt Stylus Centralis H F, eſſe Q Y X T; Planum vero, merè Rationale, ſive *Mathematicum*, priori continuum, eſſe ε δ β γ.

3. *Not.* Omnes Radios ab Objecto per Globulum & Foramen O, protenſos ad Oculum, (in tot medii Diaphani punctis, duce Indice K A, collocatum, quot ſunt puncta in ſuperficie viſibili Objecti deſcribendi quæ ſunt infinita) ſemper fore ſibi invicem *Parallelos*.

Objicient forſan quidam, in Objectis longe diſſitis Dioptrarum nullum fore uſum. Quid vero illud noſtrâ intereſt, cum ad tollendas tantum in *Proſopographia* difficultates, quibus hactenus *Scheineri Parallelogrammum* laboboravit, hæc noſtra Methodus comparata eſt? Sæpius enim expertus ſum (licet ab hoc non eſt quod ſequius de illius *Inſtrumenti* præſtantia ab Artifice ſtatuatur) nequaquam inter Partes Ectypi In Plano eam eſſe *Symmetriam*, quæ inter Partes Prototypi diſſiti.

A Method of caſting Statues of an extraordinary Thinneſs; by M. John Weichard Valvaſor. n. 186. p. 259. *Fig.* 220. Jan. An. 1687.

III. I here ſend you a Method of *Caſting Statues* in *Metal*, in Obedience to the Commands of the *Royal Society*; It is as follows. Firſt, I form out of good Clay, that will endure the Fire, and not crack either in Drying or Burning, ſuch a Figure or *Statue* as I deſire to caſt; when this is well dry, I make, all over the Figure, little Holes of no great Depth (but both Size and Depth proportionate to the Bigneſs of the *Statue*) into which I let ſmall Pieces of *Metal*, and with ſome of the ſame *Clay* fix them firmly in the *Holes*; the Uſe of theſe Bits of *Metal*, *a a a a a*, is to keep the *Core* and *Mould* from touching one another, or falling together when the *Wax* runs out; and that they may remain conſtantly in the ſame fixt Poſture. This done, I ſcrape away with ſome proper Inſtrument, as much of the Clay in Thickneſs as I deſign for the Thickneſs of my *Statue*; and then laying it in a *Furnace*, I burn the *Core* till it be red hot: (by the *Core* is meant always the *Statue* firſt made in *Clay*.) When it is cold, I rub the *Core* all over with that ſort of *Earth* or Colour, which our *German Potters* uſe to colour the Joints of the Tiles, when they ſet *Stoves* of Tiles or (*Kachel-Oſens*;) this Colour much reſembles *Black Lead* which is uſed to *Deſign* on Paper, and eaſily wipes out with Bread, but it is not the ſame. This Colour I mix with Water, and daub all over the *Core*, becauſe the *Metal* is found to run freely upon it. There are other Subſtances proper for this Uſe, but I have always made uſe of this, eſpecially for thin *Statues*. This done, I lay upon the *Core* as much yellow Wax mixed

ed with Pitch or Roſin, as will make the Thickneſs of the intended Statue, which I form in the Wax, with all the Exactneſs poſſible.

Here note, that the Particles of *Metal* mentioned to be ſet into the *Core*, to keep it a Diſtance from the *Mould*, muſt be ſo ſet as to fall in with the Surface of the Wax exactly; and that the Reaſon of mixing Pitch or Roſin with the Wax is, becauſe that when it is burnt out, it makes a great Smoke, and that Smoke adhering to the *Mould*, occaſions the *Metal* to run more freely; as I have experienced it. Next I put all over upon the Surface of this *Statue* of Wax, little Pieces of Wax which I call the little *Channels c c c c c c*, (all which muſt be contrived ſo as to enter the great *Channels d d d.*) This done, I cover the *Core* and Wax all over with the ſame ſort of Clay, that will endure the Fire without cracking; and ſo I have my *Concave Statue* or *Mould* made. Upon this I lay the great *Channels* marked *d d d d*, both upright and tranſverſe, formed likewiſe in Wax, and placed according to Judgment, ſo as beſt to receive the Ends of the little *Channels c c c c c c*, for the more eaſy diſtribution of the *Metal*. The great *Channels* muſt all meet at the top of the *Statue*, ſo as come out by one Hole, as at E, where the Metal is to be poured in: It is alſo neceſſary to have a *Channel* or two to let out the Air as the *Metal* enters, as thoſe marked *f f*, and there muſt be a Hole or two left at the Foot, as *g g*, where the great *Channels* and *waxen Statue* joyn; and whereat when the Mould is burnt, the Wax as well of the *Statue* as of the *Channels* may run out. The great *Channels* being thus placed, the Mould muſt be again laid over with the ſame ſort of Clay; (I uſe conſtantly to bind about the Mould with Iron Wire, and then lay on more Clay) and when this Mould is well dry, then I heat it red-hot; as I did before the *Core*, ſo now both together.

I burn the *Core* firſt, that there may not need ſo ſtrong a Fire to burn the Mould as will melt the ſmall Bits of Metal: But for ſmall manageable *Statues*, of not above a Foot or two high, they may be both burnt together, and there is no need of the Holes *g g*, but the Mould may be inverted, and the Wax run out by the *Channels f f*, and E.

The Mould being thus Burnt, I ſtop with the ſame Clay the two Holes *g g*, and then I bury it in a Pit, and proceed as is uſual in *Caſting* of *Bells*, and the like; but Care muſt be taken, that the *Metal* be very well in Fuſion.

If it be a ſmall *Statue*, not above a Foot or two high, whoſe *Mould* may be managed in ones Hands; then I make me a *Concave Statue* of Wax, of the Thickneſs I deſire, and then place upon it all thoſe great and leſſer Channels, as afore: Which done, I put it all together, into a liquid Subſtance made of Plaiſter and Tile, or Brick-Duſt tempered with Water.

If the *Statue* be intended very thin, then I take Copper, and when it is well in Fuſion I mix with it a good quantity of *Zinc*, without obſerving any certain Proportion of Weight; the more *Zinc* the better the *Metal* Runs. I have ſometimes for ſmall and thin *Statues* put in above a third part of *Zinc*. I have found by Experience, that this Mineral makes the *Metal* run moſt freely, and gives it a fair Golden Colour.

The Statue being caſt, I take off the *Mould*, and cut off all the little *Channels*; all which, both great and ſmall, are filled with *Metal*, which may be kept for farther Uſe: In theſe there is much more *Metal* than in the whole Statue; for, if the Statue be very thin, there muſt be more and bigger *Channels*, and ſo, the cheaper the Statue, the more weighty the *Channels*, and the more *Metal* remaining.

To know the Quantity of *Metal* requiſite for any intended Work, I take a Lump of the ſame Mixture of Wax and Pitch, with which I make the *Mould* of my Statue; and, having weighed it, I make a *Mould* upon it, and caſt in the ſame a Lump of *Metal* of the ſame Size; which I weigh, and thereby compute the Proportion of the Weight of the *Metal* and *Wax*; and then, obſerving how many Pounds of Wax I uſe about the *Figure* and Channels, I can calculate to a ſmall matter how much *Metal* I need to melt.

Hitherto I have caſt no Statue above 9 Foot high, but I doubt not but I could, by the ſame Methods, caſt one of any Bigneſs deſired.

A Deſcription of ſome Simple Colours; by Mr. Rich. Waller. n. 179. p. 26, 30. Jan. An. 1686.

Simple Blues.

IV. 1. *Spaniſh White* is made of Chalk and Allum burnt together.

2. I take the *Lapis Armenius* to be the *Blue Bice* ſold in the Shops, for it is light and friable; formerly brought out of *Armenia*, now from the Silver *Mines* of *Germany*, called *Melochites*, in *High Dutch*, *Berghblaw*.

3. *Ultramarine* is made of the blueſt *Lapis Lazuli*, which is freeſt from Gold Veins, by Calcination.

4. *Smalt* is made of *Zaffer* and *Pot-Aſhes* calcined together in a Glaſs Furnace.

5. *Litmaſe* or *Litmoſe*, I ſuppoſe the Juice of a Plant.

6. *Indigo*, ſaid by *Pliny* to be brought from *India*; a kind of *Mud* adhering to the *Froth* about *Reeds*, and that when tried with a Coal, the true burns with a purple *Flame*, and ſmells of the *Sea*. *Linſchoten* ſays, it is called *Anil*, that it grows in *Cambaia*, and is a Plant like *Roſemary*, which is gathered and dried, then wetted with fair Water, and beaten to a Mud.

7. *Indian Ink*, its uſe is known to *Pliny*, tho' not its Compoſition; which is yet undiſcovered, except it ſhould be burnt *Rice*, as hath been thought.

Simple Yellows and Reds.

1. *Ceruſe* is the *Ruſt* of *Lead*, made by a vaporous Calcination; *Pliny* writes thus of it in *Cap.* 34. *Lib.* 18. *Ceruſe Pſimithium* is made in the Plumbers Shops, of ſmall Plates of *Lead* laid upon a Veſſel of ſtrong *Vinegar*, what falls into the *Vinegar* is taken out and dried in the *Sun*: and in *Cap.* 6. *Lib.* 35. He ſays it was made at *Rome* of burnt *Marble Flint* quenched in Vinegar.

2. *Maſticot* is a kind of improper *Calx* or *Tin*.

3. *Gutta Cambæ*, or *Cambodia*, the inſpiſſated Juice of a Plant, not well known; it comes from both the *Indies*. Some think it the Juice of *Euphorbium*; others *Scammony* or *Tithymal*; others *Ricinus*, others refer it to the greater *Cataputia*, *Eſula*, or the *Flowers* of the *Indian Ricinus*, and will have it coloured with *Turmerick*; as *Schroder*.

4. *Oker*, a kind of natural Earth. There are two ſorts thereof, the one *Native*, formerly brought out of *Africa*, now from *Dacia* and *Hungary*, and from many Places in *England*, eſpecially in the Foreſt of *Dean*: The other a factitious Subſtance of *Lead* burnt and quenched in Vinegar. In *Pliny's* Time it was made of *Rubrica*, or *Reddle* burnt.

5. *Orpiment*, a flat inflammable Mineral, juſtly ranked amongſt Poyſons for its extream corroſive Quality. *Pliny* ſays, it was dug up in *Syria* on the Surface of the *Earth*: and that the Emperor *Caligula* had hopes of getting *Gold* out of it; wherefore he cauſed 14 Pounds of it to be tried, which afforded him very good *Gold*, but in ſo ſmall a Proportion, that he loſt by the Tryal.

6. *Umber* is a Native Earth.

7. *Red-Lead*, a Colour unknown to the Ancients, made of *Litharge*, or burnt *Lead*, by a *Reverberatory Calcination*, or of *Ceruſe* put in a Platter over the Fire, which muſt be continually ſtirred till it has acquired a *Red-Lead* Colour. Dr. *Charleton de Foſ.*

8. *Burnt Oker* is the common *Yellow Oker* burnt in the open Fire.

9. *Cinnabar*, or *Vermilion*. There are two ſorts, *Native*, or the *Minium* of the *Ancients*, which is the Mineral that yields *Quickſilver*; whereof and of *Sulphur* it chiefly conſiſts; it is found in the Mines of *Iſtria*. This Colour was amongſt the *Ancient Romans* uſed to ſacred Purpoſes, and on Feſtivals *Jupiter*'s Face was painted therewith, as likewiſe the Bodies of thoſe that entred in Triumph. The *Factitious Cinnabar* is that which we now uſe, and is made by a Sublimation of *Mercury* and *Sulphur*.

10. *Carmin*, made of *Cochineel*.

11. *Lake*, thought to be an *Arabick* Word: It is made of *Flocks* Dyed, or Shavings of Scarlet Cloth, or of the *Cochineel Inſect*, or elſe of *Kermes-Berries*, their Tincture being extracted with a Lye of *Pot-Aſhes*, and then precipitated with a Solution of *Roch-Allom*. After the ſame Manner a *Lake* may be made of any Plant or Flower. There is another ſort of *Lake* made of *Gum-lac*, by extracting its Tincture with Urine.

12. *Sanguis Draconis* is the Gum of a Tree, which looks like dry'd Blood; 'tis brought out of ſeveral Places in the *Eaſt-Indies*.

13. *Engliſh Reddle*, or *Ruddle*, is found in many Places of *England*; amongſt the reſt, near *Witney* in *Oxfordſhire*.

14. *Lamp-black*, by *Pliny* thus deſcribed: 'Tis made of the Soot of Roſin, or Pitch burnt, Houſes being built on purpoſe for it, that keep in the Smoak.

To make China Varniſhes; *by* Dr. Will. Sherard. n. 262. *p.* 525. Mar. An. 1700.

V. This Way of making ſeveral *China Varniſhes* was firſt ſent from the *Jeſuits* in *China* to the Great Duke of *Tuſcany*.

Take of *Crude Varniſh* 60 Ounces, ordinary Water 60 Ounces, mix them well together till the Water diſappears, afterwards put this Matter into a wooden Veſſel 5 or 6 Palms long, and 2 or 3 broad, mix them with a wooden *Spatula*, for a whole Day in the *Summer*'s Sun, and for two in the *Winter*. It is afterwards kept in Earthen Veſſels with a Bladder over it, and cool. This is the *Varniſh prepared in the Sun*.

Boyling the Oyl of Wood.

Take 20 Ounces of the Oyl, called *Oyl of wood*, of that of the *Fruit* 10 Drams, give them 5 or 6 Boils, till it comes to be a little *Yellow*. Let it cool, and put to it 5 Drams of *Quick-lime* powdered.

To give the firſt Grounds called Camiſcia.

Take *Swines Blood* and *Quick-lime* powder'd, mix them well, lay this Mixture on the Wood, and, when it is dry, ſmooth it with Pumice-Stones.

Take

To make Black Varniſh.

Take of the *Varniſh* prepared in the Sun, 60 Ounces, *Stone-black Allum*, (ſuppoſed to be a ſort of *Copperas*) diſſolved in a little Water, 3 Drams; 70 Drams of *Lamp-Oyl*, call'd by the *Portugueze Azeite de Candea*. It is prepared in a wooden Veſſel, as the prepared *Varniſh*, obſerving to put in the *Lamp-Oyl* at twice.

Pitch-colour'd Varniſh.

Take of the Oyl of *Wood* crude, (called by the *Portugueze Azeite de Pao*) 40 Drams, of the Lamp-Oyl, called *de Candea*, Crude 40 Drams: It is prepared in the Sun in a wooden Veſſel, as the prepared Varniſh.

Red Varniſh.

Take 10 Drams of *Cinnabar*, 20 Drams of *Varniſh* prepared, a little Oyl *de Candea*, or Lamp-Oyl; mix them well.

Yellow Varniſh.

Take of the *Yellow Colour* 10 Drams, 30 Drams of the prepared *Varniſh*, with ſome *Lamp-Oyl*.

Musk-colour'd Varniſh.

Take of the *Red Varniſh* 10 Drams, of the *Black Varniſh* 4 Drams; mix them well.

An Examen of Pictures *pro-poſ'd; by M.* Colbert. n. 47. *p.* 953. May. An. 1669.

VI. M. *Colbert* being pleaſed ſome time ſince to viſit the *Academy Royal* for the Improvement of *Painting* and *Sculpture*, expreſſed himſelf to this Effect, That he thought it proper from time to time that the Works of the moſt *Excellent Painters* ſhould be *Examined*, and ſuch Obſervations made thereon as would inform others wherein the *Perfection* of a *Picture* conſiſts. Which hath been ever ſince practiſed among them, as the beſt Means to carry the Art of *Painting* to its higheſt Perfection; ſuch an *Examen* of the beſt *Pictures* diſcloſing many Secrets of that Art, for which there are no Rules, and opening a Door to debate many important Queſtions not hitherto treated of.

Wax Work, and a new kind of Maps in Low Relievo, *in* France; *by* …. n. 6. *p.* 99. Nov. An. 1665.

VII. Here is a Man who makes more lively Counterfeits of Nature in *Wax*, than ever I yet ſaw in *Painting*, having an extraordinary Addreſs in *Modelling* the *Figures*, and mixing the *Colours* and *Shadows*; making the Eyes ſo lively, that they ſeem to kill, excelling all things of this Art I ever beheld.

I have alſo ſeen a new kind of *Maps* in *Low Relievo*, or Sculpture; for Example, the Iſle of *Antibe*, upon a Square about 8 Foot, made of Boards, with a Frame like a *Picture*: There is repreſented the Sea, with Ships, and other Veſſels artificially made, with their Cannons, and other Tackle of Wood fix'd upon the Surface, after a new and moſt admirable manner. The Rocks about the Iſland exactly form'd, as they are upon the natural Place; and the Iſland itſelf with all its Inequalities, and Hills and Dales; the Town, the Forts, the little Houſes, Plat-form, and Cannons mounted; and even the Gardens, and Plat-forms of Trees, with their green Leaves ſtanding upright, as if they were growing, in their natural Colours; *in fine*, Men, Beaſts, and whatever you may imagine to have any Protuberancy above the Level of the Sea. This new delightful, and moſt inſtructive Form of a *Map*, or wooden Country, you are to look upon either Horizontally, or ſide-long, and it affords equally a very pleaſant Object.

To Colour Marble; by …. n. 7. p. 125. Dec. An. 1665.

VIII. Whether the Way mentioned by *Kircher* in his *Mundus Subterraneus* will ſucceed or not, is much doubted by ſome experienced Men: But 'tis cer-

tain that a *Stone-cutter* in *Oxford*, Mr. *Bird*, hath many Years ſince found out a Way of doing the ſame thing, in effect, that is there mentioned; and hath practiſed it for many Years. That is, he is able ſo to apply a *Colour* to the out-ſide of poliſhed *Marble*, as that it ſhall ſink a conſiderable *Depth* into the Body of the Stone, and there repreſent like Figures or Images as thoſe are on the out-ſide; deeper or ſhallower, according as he continues the Application a longer or leſſer while.

An Extraordinary Tincture given to a Stone; by Dr. Salomon Reiſel. n. 179. p. 22.

IX. *Aurifaber Stutgardianus*, qui & Gemmis & Metallicis Typis Nummorum Cudendorum Inſculpendis Artificioſus eſt, Nomine *Chriſtophorus Muller*, *An.* 1685. Aurum Aqua Regis ſolutum, Oleo Tartari præcipitatum atque edulcoratum, quod *Aurum fulminans* dicunt, dum in Scutella, quam Maturellam vocant, ex *Lapide Chalcedonico* coloris unici Pellucidi *Onychini*, ſeu Cornei, vitro pro Fuſione præparato Rubro mixtum & Aqua Fontana Imbutum tereret, ad facienda Encauſta ſeu *Smalta*; de quibus *Ant. Nerius*, vertente *Andrea Friſio*, egit *Lib. 6. Artis Vitrariæ*, invenit iterato tertium eodem Labore, quod Color Pulveris iſtius Puniceus, qui per dies aliquot ſiccatus in Vaſculo manſerat, quouſque inter terendum etiam ad *Marginem* effluxit, relictis tamen puris hinc inde Spatiis *Onychini* Coloris, duriſſimam hanc *Gemmam*, quæ Limam ſpernit, ita profunde penetraverit, non tantum in Scutella, ſed & ipſo Piſtello, & diſtrinxerit Maculis atque Circulis ſat ordinate ductis, ut Color hic neque ſimplici Aqua, neque Lixivia, vel acriori alio Liquore potuerit deleri, & quidem ſine Polituræ elegantioris Detrimento. Talis itaque Tinctura per repetitas Trituras dicti Pulveris tentata denuo aliquoties, in ſimilis Coloris alio Vaſculo, neque vero apparuit poſtea ut ante unquam. Sed hoc imprimis circa Tinctionem hujus Vaſculi obſervandum eſt, quod ſecundum Texturam *Gemmæ*, tam nudo quam armato Oculo, in Tincta Interna, & Sincera Externa parte Vaſis, notentur Fibræ ſeu ductus Circulares; juxta quos Bracteis Succi Lapidei Novi per Intervalla impoſitis, in ejuſmodi Mole excreviſſe credendum eſt; uti *Bezoar* aliique Lapides Laminis ſuper accreſcentibus augentur, & Ligna, in quorum ultimorum Trunco Circuli ſeu Annuli deſignant ſucci annui Numerum & Incrementa: adeo ut hic *Purpureus* ille *Color* Lineis pallidioribus & obſcurioribus, prout vel denſiores vel rariores Poras molliorem vel duriorem Texturam offendit, Circulares Ambitus circa Verticem aliquam, veluti circa Medullam, ſeu Cor ut appellant, aut Granum, aut Paleam in aliis Lapidibus & Lignis, ſignaverit; intermiſtis quoque hinc inde Maculis & Spatiis obſcurioribus. Veluti Illuſt. *Boyle Specim. de Orig. & Virt. Gemmarum.* Sect. 1. p. 22, 23. in *Adamante* & *Granatis* acies & commiſſuras tenuium Bractearum aut Planorum obſervavit; quod *Granum* Artifices, ſeu planam Contexturam non diſſimilem Fiſſilitati Ligni, vocant.

Jam vero Tingi poſſe quoque *Marmora* & *Alabaſtra* & *Oſſa* per Lixiviatos & acres Succos, hinc inde ſcriptum eſt: quod fortaſſis de *Gemmis* ſperandum eſt, quando *Rob. Boyle*, *Cit.* Sect. 2. p. 123. ex iis Tincturam manifeſtam extractam eſſe ſcribit, *alibi.* p. 43. & 190. per Vapores Minerales tinctos eſſe *Cryſtallos Petroſos*, atque *p.* 45. ipſum Sapphirum per Vapores ſubterraneos.

Cum denique ex Obſervatione noſtra manifeſtum ſit, revera *Tinctam* eſſe Gemmam *Chalcedoniam*, quamvis fortuito acciderit, neque repetito Proceſſu ſimile quid evenerit, merebitur tamen Meditationem, an ex Aſtrorum Fluxu aliave abſcondita potius vi venerit, & Tentamen, an ex Mixtura Salium & Succorum Acrium poſſit imitando produci ejuſmodi Tinctura, & quidem ſine Igne, ut Splendor & Pelluciditas Gemmæ non deſtruatur, Durities autem maneat, adeoque ipſa Gemmæ pretioſitas non tantum ſervetur; ſed & per Tincturam novam creſcat.

X. *Papers* omitted.

1. A Deſcription of *Scheiner's Stereographick Parallelogram*, and its *Imperfections* conſidered; by Mr. *J. St. Clare*, vid. ſup. Sect. II. n. 96. p. 6085.

2. A Table of *Simple* and *Mix'd Colours*, in *Latin*, *Greek*, *French*, and *Engliſh*: with a Specimen of each Colour prefix'd to its proper *Name*; by Mr. *Rich. Waller*. n. 179. p. 24, 29.

XI. *Accounts of Books* omitted.

n. 21. p. 383.

1. Entretiens ſur les Vies & ſur les Ouvrages des plus excellens *Peintres*, *Anciens* & *Modernes*; par M. *Felibien*. n. 39. p. 784.

2. An *Idea* of the *Perfection* of *Painting*: Originally written in *French*, by *Roland Freart*, Sieur *de Cambray*; and render'd *Engliſh* by *J. Evelin*, Eſq; F. R. S. *Lond.* 1668. in 8*vo*. n. 47. p. 954.

3. A General *Idea* of the *Art* of *Painting*, and Relation of Seven *Conferences* held at *Paris* in the *Academy Royal*, for the Improvement of the *Arts* of *Painting* and *Sculpture*. n. 86. p. 5048.

4. Optique de *Portraiture* & *Peinture*, contenant la *Perſpective*, *Speculative*, & *Pratique* Accomplie; *&c.* Par *Gregoire Huret*, , de l' *Academie Royale* de *Peinture* & *Sculpture*. *A Paris*, 1670. *in Fol.*

CHAP.

CHAP. X.

Muſick.

Of the Trembling of Conſonant Strings; by Dr. Wallis. n. 134. *p.* 839. Mar. An. 1677.

I. IT hath been long ſince obſerved, That if a *Viol-ſtring*, or *Lute-ſtring*, be touched with the Bow or Hand, another *String* on the ſame or another Inſtrument not far from it, (if an *Uniſon* to it, or an *Octave*, or the like) will at the ſame time tremble of its own Accord. But I can now add, That not the whole of that other String doth thus tremble, but the ſeveral Parts ſeverally, according as they are *Uniſons* to the whole, or the Parts of that String ſo ſtruck. For Inſtance, Suppoſing A C to be an upper *Octave* to $\alpha\gamma$, and thereſore an *Uniſon* to each Half of it, ſtopped at β. If, while $\alpha\gamma$ is open, A C be ſtruck; the two Halves of this other, that is, $\alpha\beta$, and $\beta\gamma$, will both tremble; but not the middle Point at β. Which will eaſily be ebſerved, if a little Bit of Paper be lightly wrapt about the String $\alpha\gamma$, and removed ſucceſſively from one End of the String to the other. Fig. 221.

Fig. 222. In like manner, If A D be an Upper *Twelfth* to $\alpha\delta$, and conſequently an *Uniſon* to its three Parts equally divided in β, γ; if $\alpha\delta$, being open, A D, be ſtruck, its three Parts $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, will ſeverally tremble, but not the Points $\beta\gamma$. In like manner, if A E, be a *double Octave* to $\alpha\epsilon$, the four Quarters of this will tremble when that is ſtruck, but not the Points β, γ, δ. So if A G, be a *Fifth* to $\alpha\eta$; and conſequently each Half of that ſtopped in D, an *Uniſon* to each third Part of this ſtopped in $\beta\gamma$; while that is ſtruck, each Part of this will tremble ſeverally, but not the Points β, γ; and while this is ſtruck, each of that will tremble, but not the Point D. The like will hold in leſſer *Concords*; but the leſs remarkably, as the Number of Diviſions increaſes. Fig. 223. Fig. 224.

This was firſt of all, (as I know of) diſcovered by Mr. *Will. Noble*, M. A. of *Merton College*; and by him ſhewed to ſome of our *Muſicians*, about three Years ſince; and after him by Mr. *Tho. Pigot*, A. B. of *Wadham College*, without knowing that Mr. *Noble* had diſcover'd it before. I add this further, (which I took Notice of upon Occaſion of making Tryal of the other,) that the ſame *String*, as $\alpha\gamma$, being ſtruck in the midſt of β, each Part ~~being *Uniſon* to the other, will give no *clear Sound* at all, but very confuſed~~. And not only ſo (which others have obſerved, that a *String* doth not ſound clear if ſtruck in the midſt;) but alſo, if $\alpha\delta$ be ſtruck at β, or γ, where one Part is an *Octave* to the other; and in like manner, if $\alpha\epsilon$ be ſtruck at β or δ; the one Part being a *double Octave* to the other. And ſo if $\alpha\zeta$ be ſtruck in γ or δ; the one Part being a *Fifth* to the other; and ſo in other like *Conſonant* Diviſions; but ſtill the leſs remarkable, as the Number of Divi- Fig. 225.

Fig. 226.

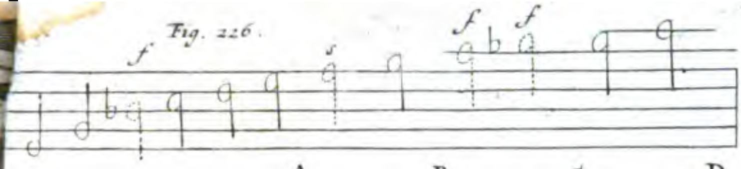


Fig. 231.

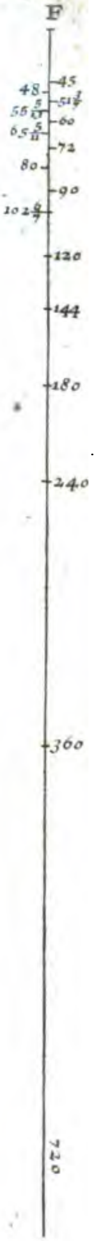


Fig. 227.

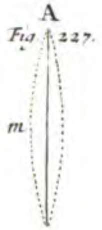


Fig. 228.

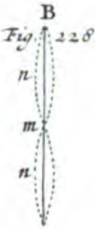


Fig. 229.

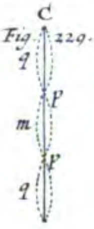


Fig. 230.

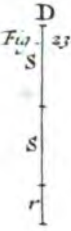
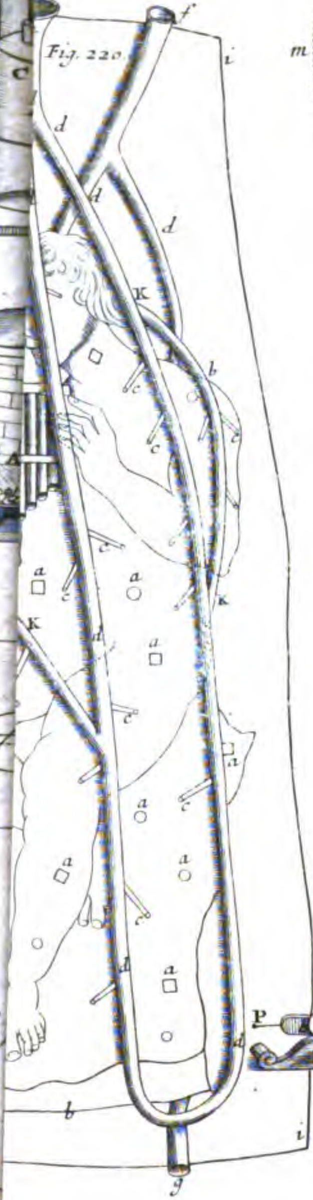


Fig. 220.



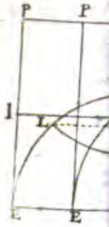


Fig. 208.

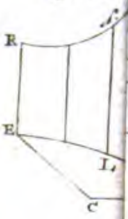


Fig. 221.

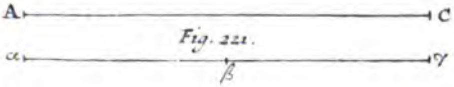


Fig. 222.

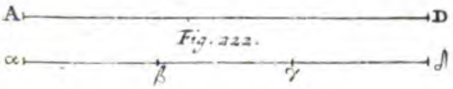


Fig. 223.

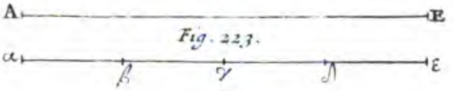


Fig. 224

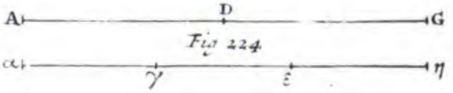


Fig. 225.

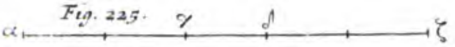
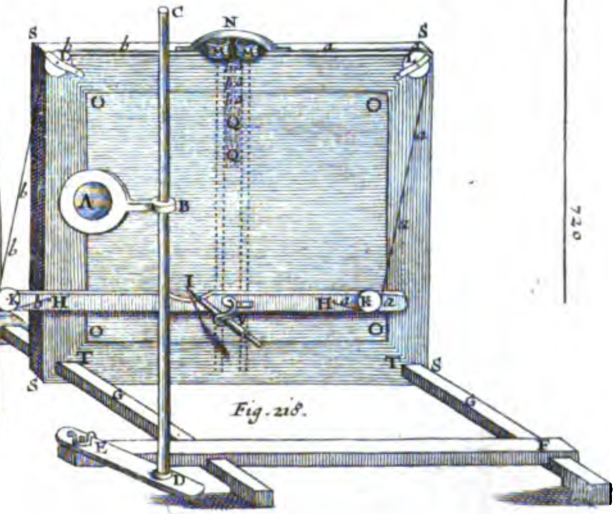


Fig. 218.



Diviſions encreaſeth. This and the former I judge to depend upon one and the ſame Cauſe, *viz.* the Contemporary Vibrations of the ſeveral *Uniſon* Parts, which make the one tremble at the Motion of the other: But, when ſtruck at the reſpective Points of Diviſions, the Sound is incongruous, by Reaſon the Point is diſturbed, which ſhould be at Reſt.

A *Lute-ſtring* or *Viol-ſtring* will alſo thus anſwer to a *Conſonant Note* in *Wind Inſtruments*: But not ſo remarkably to the *Wire-ſtrings* of an *Harpſicord*. And we feel the Wainſcot Seats, on which we ſit or lean, to tremble conſtantly at certain Notes on the *Organ*, or other *Wind Inſtruments*; as well as at the ſame Notes on a *Baſs Viol.* I have heard alſo (but cannot aver it) of a thin fine *Venice Glaſs* cracked with the ſtrong and laſting Sound of a *Trumpet*, or *Cornet*, (near it) ſounding an *Uniſon* or a *Conſonant Note* to that of the *Tone* or *Ting* of the *Glaſs*.

Concerning theſe *Phænomena*, an exquiſite Solution is given by Dr. *Narciſſus Marſh*, in Dr. *Plot's Natural Hiſtory* of *Oxfordſhire*. n. 135. p. 879.

The Defects of a Trumpet, and Trumpet-Marine; by Mr. Fran. Roberts. n. 195. p. 559 Fig. 226. Oct. An. 1692.

II. The *Extent* of the *Trumpet* cannot be ſtrictly determined; it reaches as *High* as the Strength of the Breath can force it: But by conſidering its *Notes* within the ordinary Compaſs of the Scale of *Muſick* (from *Double C-fa-ut* to *C-ſol-fa in alt*) the Nature of the higher Notes will plainly appear. Theſe are all ſet down in the Table; only take Notice, that the *Prick'd Notes* are imperfect, not exactly in Tune, but a little *Flatter* or *Sharper* than the Places where they ſtand, according as *f* or *s* is ſet over them.

Here we may make two Enquiries.

1. Whence it comes to paſs, that the *Trumpet* will perform no other *Notes* (in that Compaſs) but only thoſe in the Table, which are uſually called by *Muſicians Trumpet-Notes*.

2. What is the Reaſon that the *7th*, *11th*, *13th* and *14th Notes* are out of Tune, and the others exactly in Tune.

In this Matter we may receive ſome Light from the *Trumpet Marine*, an Inſtrument, though as unlike as poſſible to the *Trumpet* in its Frame, one being a *Wind-Inſtrument*, the other a *Monochord*, yet has a wonderful Agreement with it in its Effect.

The Sound is ſo like, as not to be eaſily diſtinguiſhed by the niceſt Ear, and, as it performs the very ſame *Notes*, ſo it has the ſame Defects as a *Trumpet*, for, if the *Strings* be ſtopt in any Part but ſuch as produces a *Trumpet-Note*, it yields a harſh and uncouth (not a *Muſical*) *Sound*.

Let us therefore proceed to our firſt Inquiry, and examine what is the Reaſon that the *Trumpet Marine* will perform no other but the *Trumpet-Notes*. It is a known *Experiment* of two *Uniſon Strings*, that ſtriking one of them moves the other, which probably proceeds from hence, that the Impulſes of the Air, which are made by one String, do more eaſily ſet another in Motion, which lies in a Diſpoſition to have its *Vibrations Synchronous* to them, than a Third, whoſe Motion would be croſs.

We may improve this a little farther, by obſerving that a String will move not only at the Striking of an *Uniſon*, but an *8th* or *12th*, though after a different Manner.

If

Fig. 227. If an *Uniſon* be ſtruck, it makes one intire Vibration in the whole String, and the Motion is moſt ſenſibly in the midſt at *m*, for there the Vibrations take the greateſt Scope.

Fig. 228. If an 8*th* is ſtruck, it makes two Vibrations; and the Point *m* is in a manner Quieſcent, and the moſt ſenſible Motion at *n*, *n*.

Fig. 229. If a 12*th* be ſtruck, then it makes three Vibrations; and the greateſt Motion at *q*, *m*, *q*; and hardly to be perceived at *p*, *p*. So that in ſhort, this *Experiment* holds, when any *Note* is ſtruck which is an *Uniſon* to half the String, and a 12*th* to a Third Part of it.

Fig. 230. In this Caſe, (the Vibrations of the equal parts of a String being ſynchronous) there is no Contrariety in the Motion to hinder each other, whereas it is otherwiſe, if a *Note* is *Uniſon* to S, that does not divide the String into equal Parts, for then the Vibrations of the Remainder *r*, not ſuiting with thoſe of the other Parts, immediately make a Confuſion in the whole.

Now in the *Trumpet-Marine*, you do not ſtop cloſe, as in other *Inſtruments*, but touch the String gently with your Thumb, whereby there is a mutual Concurrence of the upper and lower part of the String to produce the Sound. This is ſufficiently evident from that, That if any thing touches the String below the Stop, the Sound will be as effectually ſpoiled, as if it were laid upon that Part which is immediately ſtruck with the *Bow*. From hence therefore we may collect, that the *Trumpet-Marine* yields no *Muſical* Sound, but when the Stop makes the upper Part of the *String* an *Aliquot* of the Remainder, and conſequently of the whole: Otherwiſe, as we juſt now remarked, the Vibrations of the Parts will ſtop one another, and make a Sound ſuitable to their Motion, altogether confuſed.

Now, that theſe *Aliquot Parts* are the very Stops which produce the *Trumpet-Notes*, ſhall be plainly ſhewn in the treating of the ſecond Inquiry, *viz.* What is the Reaſon that the 7*th*, 11*th*, 13*th*, 14*th Notes* are out of Tune; and the reſt exactly in Tune?

All Writers of the Mathematical Part of *Muſick* agree,

That by Shortnning a String	Half	the Sound is raiſed	an Eight
	a Third Part		a Fifth
	a Fourth		a Fourth
	a Fifth		a Sharp Third
	a Sixth		a Flat Third.

From this Foundation all the other *Notes* are derived. The Flat and Sharp Sixth are to be the Flat and Sharp *Third* to the *Fourh*, and the 7*th* the like to the 5*th*: The Second to be a Fifth to the Fourth below, *&c.* By this Rule let us examine what Notes a *Monochord* fretted in its *Aliquot Parts* will produce.

Fig. 231. Suppoſe the *Monochord* F to conſiſt of 720 Parts, and its Tone *Double C-fa-ut*, the firſt *Note* in the *Table*; then Half of it will be 360, and a third Part 240, *&c.*

Now I ſay, *Fretting*, (or *Stopping* with the Thumb) at 360 muſt produce *C-fa-ut*; becauſe 360 being Half 720, the Sound will Riſe an *Eight* from *double C-fa-ut*. Again 360 being *C-fa-ut*, 240 muſt make *G-ſol-re-ut*, the Third Note in the Table; becauſe 240 being juſt a Third-part leſs than 360, the Sound will Riſe a *Fifth* from that Note. After the ſame manner proceeding Step by Step it will be evident that,

180	which is leſs than	240	by juſt	a Fourth	produces	*C-ſol-fa-ut*	the fourth	*Note* in the *Table.*
144		180		a Fifth		*E-la-mi*	fifth	
120		144		a Sixth		*G-ſol-re-ut*	6th	
90		180		Half		*C-ſol-fa*	8th	
80		120		a Third		*D-la-ſol*	9th	
72		90		a Fifth		*E-la*	10th	
60		90		a Third		*G-ſol-re-ut*	12th	
48		60		a Fifth		*B-fa-bi-mi*	15th	
45		90		Half		*C-ſol-fa*	16th	

By the ſame Reaſon,

100	which is leſs than	120	by juſt	a 6th	produces	*B-fa-bi-mi Flat*,
$67\frac{1}{2}$		90		a 4th		*F-fa-ut*
54		$67\frac{1}{2}$		a 5th		*A-la-mi-re*
50		100		Half		*B-fa-bi-mi Flat*,

And Conſequently,

$102\frac{6}{7}$	the 7th	*Note* in the Table is a little	*Flatter*	than	*B-fa-bi-mi Flat*,
$65\frac{5}{11}$	11th		*Sharper*		*F-fa-ut*,
$55\frac{5}{13}$	13th		*Flatter*		*A-la-mi-re*,
$51\frac{3}{7}$	14th		*Flatter*		*B-fa-bi-mi Flat*,

Which anſwers the Second Inquiry.

Now to apply this (in a few words) to the *Trumpet*, where the *Notes* are produced only by the Different Force of the *Breath*; it is reaſonable to imagine that the ſtrongeſt Blaſt *Raiſes* the Sound by breaking the Air within the *Tube* into the ſhorteſt Vibrations, but that no *Muſical* Sound will raiſe, unleſs they are ſuited to ſome *Aliquot Part*, and ſo by Reduplication exactly meaſure out the whole Length of the *Inſtrument*, as in *Fig.* 229. for otherwiſe a Remainder will cauſe the ſame Inconvenience in this Caſe, as in *Fig.* 230. To which if we add that a *Pipe*, being ſhortened according to the Proportions we even now diſcourſed of in a *String*, *Raiſes* the Sound in the ſame Degrees, it renders the Caſe of the *Trumpet* juſt the ſame with the *Monochord*.

For a *Corollary* to this Diſcourſe, we may obſerve that the Diſtances of the *Trumpet Notes*, Aſcending continually, decreaſed in Proportion of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *in infinitum*. For,

$$\text{The}\left\{\begin{array}{l}\text{Second}\\ \text{Third}\\ \text{Fourth, \&c.}\end{array}\right\}\begin{array}{l}\text{Note in the Ta-}\\ \text{ble differs from}\\ \text{the}\end{array}\left\{\begin{array}{l}\text{Firſt}\\ \text{Second}\\ \text{Third, \&c.}\end{array}\right\}\text{by}\left\{\begin{array}{c}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\end{array}\right\}\text{of the } \textit{String}\text{, \&c.}$$

The Diviſion of the Monochord; by Dr J. Wallis. n. 238. p. 80. Mar. An. 1698.

III. Any *String* or *Chord* of a *Muſical Inſtrument Open* (or at its full length) will Sound (what we call) an *Octave* (or *Diapaſon*) to that of the ſame *String* ſtopt in the Middle, or at half its Length. Hence it is that we commonly aſſign, to an *Octave*, the *Duple* Proportion (or that of 2 to 1) becauſe ſuch is the Proportion of Lengths (taken in the ſame *String*) which give thoſe Sounds. And (upon a like Account) we aſſign to a *Fifth* (or *Diapente*) the *Seſqui-alter* Proportion (or that of 3 to 2.) And to a *Fourth* (or *Diateſſeron*) the *Seſqui-tertian* (or that of 4 to 3.) And to a *Tone* (which is the Difference of a *Fourth* and *Fifth*) the *Seſqui-octave* (or that of 9 to 8 :) Becauſe Lengths (taken in the ſame *String*) in theſe Proportions, do give ſuch Sounds.

And (univerſally) whatever Proportion of Lengths (taken in the ſame *String* equally ſtretched) do give ſuch and ſuch Sounds ; ſuch Proportions (of Gravity) we aſſign to the Sounds ſo given.

But, when an *Eight* (or *Octave*) is ſaid (in common Speech) to conſiſt of 12 *Hemi-tones*, or 6 *Tones* ; this is not to be underſtood according to the Utmoſt Rigour of *Mathematical* Exactneſs, (of ſuch 6 *Tones*, as what they call the *Diazeutick Tone*, or that of *la*, *mi*, which is the Difference of a *Fourth* and *Fifth* ;) but, as exact enough for common Uſe. For 6 ſuch *Tones*, (that is, the Proportion of 9 to 8, 6 times repeated) is ſomewhat more than that of an *Octave*, (or the Proportion of 2 to 1) : And conſequently, ſuch an *Hemitone*, is ſomewhat more than the Twelfth-part of an *Eight* or *Octave*, or *Diapaſon*. But the Difference is ſo little, that the Ear can hardly diſtinguiſh it: And therefore (in common Speech) it is uſual ſo to ſpeak.

And, accordingly, when we are directed to take the Lengths (for what are called the 12 *Hemi-tones*) in *Geometrical* Proportion, it is to be underſtood (not to be ſo in the utmoſt Strictneſs, but) to be accurate enough for common Uſe ; for placing the *Frets* on the Neck of a *Viol*, or other *Muſical Inſtrument* ; wherein a greater Exactneſs is not thought neceſſary. And this is very convenient, becauſe (thus) the Change of the *Key* (upon altering the Seat of *mi*) gives no new Trouble, for this doth indifferently ſerve any *Key* ; and the Difference is ſo ſmall, as not to offend the Ear.

But thoſe who chuſe to treat of it with more Exactneſs go this way to work.

Preſuppoſing the Proportion for an *Octave* or *Dia-paſon*) to be that of 2 to 1 ; they divide this into two Proportions; not juſt Equal (for that would fall upon the *Surd* Numbers, as $\sqrt{2}$ to 1 ;) but near equal (ſo as to be expreſſed in ſmall Numbers.) In order to which, inſtead of taking 2 to 1, they take (the Double of theſe Numbers) 4 to 2 ; (which is the ſame Proportion as before; and interpoſe the Middle Number 3. And of theſe three Numbers, 4, 3, 2, that of 4 to 3, is the Proportion of a *Fourth* (or *Dia-teſſeron*.) And that of 3 to 2, the Proportion for a *Fifth* (or *Dia-*

pente.) And theſe two, put together, make up that of an *Octave* (or *Diapaſon,*) that of 4 to 2, (or 2 to 1.) And the Difference of thoſe two, that of a *Tone,* or 9 to 8. As will plainly appear in the ordinary Method of *Multiplying* and *Dividing Fractions,* That is, $\frac{4}{3}\times\frac{3}{2}=\frac{4}{2}=\frac{2}{1}$. And $\frac{4}{3})\,\frac{3}{2}\,(\frac{9}{8}$.

Thus in the common *Scale* (or *Gam-ut)* taking an *Octave,* in theſe Notes, *la, fa, ſol, la, mi, fa, ſol, la*; ſuppoſe from E to *e* (placing *mi, B-fa-b-mi,* which is called the *Natural Scale*;) the Lengths for the Extremes *la, la,* an *Octave,* are as 2 to 1, or 12 to 6. Thoſe for *la, la,* (in *la, fa, ſol, la,*) or *mi, la,* (in *mi, fa, ſol, la,*) a *Fourth,* as 4 to 3, or 12 to 9, or 8 to 6. Thoſe for *la, mi,* (in *la, fa, ſol, la, mi,*) or *la, la,* (in *la, mi, fa, ſol, la,*) a *Fifth* as 3 to 2, or 12 to 8, or 9 to 6. Thoſe for *la, mi,* the *Diazeutick-Tone* (or Difference of a *Fourth* and *Fifth,*) as 9 to 8. So have we for theſe four Notes *la, la, mi, la,* their Proportionate Length in the Numbers 12, 9, 8, 6.

Then, if we proceed in like manner to Divide a *Fifth* (or *Dia-pente,*) *la, fa, ſol, la, mi,* or *la, mi, fa, ſol, la,* or the Proportion of 3 to 2, into near Equals, (taking double Numbers in the ſame Proportion, 6, 4; and interpoſing the middle Number 5;) of theſe three Numbers, 6, 5, 4; that of 6 to 5, is the Proportion of a *Leſſer Third,* (called a *Tri-hemitone,* or *Tone* and half,) as *la, fa,* (in *la, mi, fa.*) And that of 5 to 4, is the Proportion of the *Greater Third,* (commonly called a *Ditone,* or two *Tones,* (as *fa, la,* (in *fa, ſol, la,*) which two put together make a *Fifth,* as 3 to 2; that is $\frac{6}{5}\times\frac{5}{4}=\frac{6}{4}=\frac{3}{2}$; and their Difference is, as 25 to 24: That is $\frac{6}{5})\,\frac{5}{4}\,(\frac{25}{24}$. So have we for theſe 3 Notes, *la, fa, la,* their Proportionate Lengths in Numbers, as 6, 5, 4.

In like manner, if we divide a *Ditone,* (or *Greater Third,*) as *fa, la,* (in *fa, ſol, la,*) whoſe Proportion is as 5 to 4 (or 10 to 8.) into two near Equals) by help of a middle Number 9;) then have we (in theſe three Numbers 10, 9, 8,) that of 10 to 9, for (what they call) the *Leſſer Tone*: And that of 9 to 8, for (what they call the *Greater Tone.*

But, whether *fa, ſol,* ſhall be made the *Leſſer* (as 10 to 9,) and *ſol, la,* the *Greater,* (as 9 to 8;) or, This the *Leſſer,* as 10 to 9,) and That the *Greater,* (as 9 to 8,) or ſome time This, ſome time That, as there is Occaſion, (to avoid what they call a *Schiſm*;) is ſomewhat indifferent. For, either way, the Compound will be as 5 to 4; and the Difference (which they call a *Comma,*) as 81 to 80. That is $\frac{9}{8}\times\frac{10}{9}=\frac{10}{9}\times\frac{9}{8}=\frac{10}{8}=\frac{5}{4}$. And $\frac{10}{9})\,\frac{9}{8}\,(\frac{81}{80}$.

Laſtly, if from that of the *Tri-hemi-tone* (or *Leſſer Third*) *la, mi, fa*; whoſe Proportion is as 6 to 5; we take that of the *Tone, la, mi,* (which is the Difference of a *Fourth* and *Fifth*) as 9 to 8; there remains for the *Hemi-tone, mi, fa,* (or *la, fa,*) that of 16 to 15. That is $\frac{9}{8})\frac{6}{5}(\frac{48}{45} = \frac{16}{15}$.

Or, the *Tri-hemi-tone* (or *Leſſer Third*) whoſe Proportion is as 6 to 5; may be divided into three near Equals, (by taking Triple Numbers, in the ſame Proportion 18, 15; and interpoſing the two Intermediates 17, 16;) which will therefore be as 18 to 17, and as 17 to 16, and as 16 to 15; That is, $\frac{18}{17} \times \frac{17}{16} \times \frac{16}{15} = \frac{18}{15} = \frac{6}{5}$.

Where alſo the *Greater Tone*, whoſe proportion is as 9 to 8 or 18 to 16, is divided into its two near Equals (commonly called *Hemi-tones*,) that of 18 to 17, and that of 17 to 16: That is, $\frac{18}{17} \times \frac{17}{16} = \frac{18}{16} = \frac{9}{8}$.

And the *Leſſer Tone*, that of 10 to 9, or 20 to 18, may be in like manner Divided into that of 20 to 19, and that of 19 to 18: That is, $\frac{20}{19} \times \frac{19}{18} = \frac{20}{18} = \frac{10}{9}$

Which Diviſions of the *Greater* and *Leſſer Tone*, anſwer to what is wont to be deſigned by *Flats* and *Sharps*.

So that (by this Compoſition,) of theſe Eight Notes, *la, fa, ſol, la, mi, fa, ſol, la*; their Proportions ſtand thus; that of *la, fa*, (or *mi, fa*) is as 16 to 15. That of *fa, ſol*, as 10 to 9, and that of *ſol, la*, as 9 to 8; (or elſe that of *fa, ſol*, as 9 to 8, and that of *ſol, la*, as 10 to 9,) That of *la, mi*, as 9 to 8. And, if either of the *Tones* (*Greater* or *Leſſer*) chance to be divided (by *Flats* or *Sharps*) into (what they call) *Hemi-tones*, their Proportions are to be ſuch as is already mentioned.

There may be a like Diviſion of a *Fourth*, (or *Dia-teſſeron*) into two Near Equals: And of ſome others of theſe, into three Near Equals. Which might be of uſe for (what they were wont to call) the *Chromatick* and *Enarmonick Muſick*. But, thoſe ſorts of *Muſick* having been long ſince laid aſide, there is now no need of theſe Diviſions, as to the *Muſick* now in Uſe.

The Imperfection of an Organ; by Dr. J. Wallis. n. 242. p. 249. July An. 1698.

IV. I think 'tis Evident that the *Pipe* in the *Organ* is intended to Expreſs a Diſtinct Sound at ſuch a *Pitch*; that is, in ſuch a Determinate Degree of *Gravity* or *Acuteneſs*; or (as it is now called) *Flatneſs* or *Sharpneſs*; And the Relative or Comparative Conſideration of Two (or more) ſuch Sounds or Degrees of *Flatneſs* or *Sharpneſs*, is the Ground of (what we call) *Concord*, and *Diſcord*; that is, a Soft, or Harſh Coincidence.

Now, concerning this, there were amongſt the Ancient *Greeks*, Two (the moſt conſiderable) Sects of *Muſicians*: the *Ariſtoxenians*, and *Pythagoreans*.

They both agreed thus far; that *Dia-teſſeron*, and *Dia-pente*, do together make up *Dia-paſon*; That is (as we now ſpeak, a *Fourth* and *Fifth*) do together make an *Eighth* or *Octave*: And, the Difference of thoſe two, of a *Fourth* and *Fifth*, they agreed to call a *Tone*, which we now call a *Whole Note*.

Such is that, (in our preſent *Muſick*,) of *la*, *mi*, (or, as it was wont to be called, *re*, *mi*.) For *la*, *fa*, *ſol*, *la*, or *mi*, *fa*, *ſol*, *la*, is a perfect *Fourth*: And *la*, *fa*, *ſol*, *la*, *mi*, or *la*, *mi*, *fa*, *ſol*, *la*, is a perfect *Fifth*: The Difference of which is *la*, *mi*, which is what the *Greeks* call the *Dia-zeutick-Tone*; which doth Disjoin two *Fourths* (on each ſide of it;) and being added to either of them doth make a *Fifth*; Which was, in their *Muſick*, that from *Meſe* to *Parameſe*; that is in our *Muſick*, from A to B: Suppoſing *mi* to ſtand in *B-fa-bi-mi*, which is accounted its Natural Poſition.

Now in order to this, *Ariſtoxenus* and his Followers did take that of a *Fourth*, as a Known *Interval*, by the Judgment of the Ear; and that of a *Fifth* likewiſe; And conſequently that of an *Octave*, as the Aggregate of both; and that of a *Tone*, as the Difference of thoſe Two.

And this of a *Tone* (as a Known *Interval*) they took as a common Meaſure, by which they did Eſtimate other *Intervals*: And accordingly they accounted a *Fourth* to contain two *Tones* and a Half; a *Fifth* to contain three *Tones* and a Half, and conſequently an *Eighth* to contain ſix *Tones*; or five *Tones* and two *Half Tones*.

And at this Rate our *Practical Muſicians* talk of *Notes* and *Half Notes* at this Day; ſuppoſing an *Octave* to conſiſt of Twelve *Hemi-tones*, or *Half Notes*.

But, *Pythagoras* and thoſe who follow him, not taking the Ear alone to be a Competent Judge in a Caſe ſo nice; choſe to diſtinguiſh theſe, not by *Equal Intervals*, but by *Due Proportions*: And this is followed by *Zarline*, *Kepler*, *Cartes*, and others, who treat of *Speculative Muſick* in this and the laſt Age. Accordingly they accounted that of an *Octave*, to be, when the Degree of *Gravity*, or *Acuteneſs* of the one Sound to that of the other, is Double, or as 2 to 1; that of a *Fifth*, when it is *Seſqui-alter*, or as 3 to 2; that of a *Fourth* when *Seſqui-tertian*, or as 4 to 3. Accounting that the Sweeteſt Proportion, which is expreſs'd in the ſmalleſt Numbers, and therefore (next to the *Uniſon*) that of an *Octave*, 2 to 1, then that of a *Fifth*, 3 to 2. and then that of a *Fourth*, 4 to 3.

And thus that of a *Fourth* and *Fifth* do together make an *Eighth*; For $\frac{4}{3} \times \frac{2}{3} = \frac{4}{2} = \frac{2}{1} = 2$, or the Proportion of 4 to 3, compounded with that of 3 to 2, is the ſame with that of 4 to 2, or 2 to 1. And conſequently, the Difference of thoſe two, which is that of a *Tone*, or Full Note, is that of 9 to 8. For $\frac{4}{3})\ \frac{3}{2}\ (\frac{9}{8}$; or, if out of the Proportion of 3 to 2, we take that of 4 to 3; the Reſult is that of 9 to 8.

Now, according to this Computation, it is manifeſt, That an *Octave* is ſomewhat leſs than Six *Full Notes*. For (as was firſt demonſtrated by *Euclid*, and ſince by others) the Proportion of 9 to 8, being ſix Times compounded, is ſomewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Caſe; they allowed (indiſputably) to that of the *Diazeutick Tone* (*la, mi,*) the full Proportion of 9 to 8, as a thing not to be altered; being the Difference of the *Dia-pente* and *Dia-teſſeron*, or the *Fifth* and *Fourth*.

All the Difficulty, was, How the remaining *Fourth* (*mi, fa, ſol, la,*) ſhould be divided into three parts, ſo as to anſwer (pretty near) the *Ariſtoxenians* Two *Tones* and a half: And might, altogether make up the Proportion of 4 to 3, which is that of a *Fourth* or *Dia-teſſeron*.

Many Attempts were made to this purpoſe: And according to thoſe they gave Names to the Different *Genera* or Kinds of *Muſick*, (the *Diatonick*, *Chromatick*, and *Enarmonick* Kinds,) with the ſeveral *Species*, or leſſer Diſtinctions under thoſe Generals.

The firſt was that of *Euclid* (which did moſt generally obtain for many Ages:) Which allows to *fa, ſol*, and to *ſol, la*, the full Proportion of 9 to 8; And therefore to *fa, ſol, la*, (which we call the *Greater Third*) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$. And, conſequently, to that of *Mi, fa*, which is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64}) \frac{4}{3} (\frac{256}{243}$; that is, if out of the Proportion of 4 to 3 we take that of 81 to 64, the Reſult is that of 256 to 243. To this they gave the Name of *Limma* (λεῖμμα), that is, the Remainder (to wit, over and above two *Tones*.) But, in common Diſcourſe (when we do not pretend to ſpeak nicely, nor intend to be ſo underſtood) it is uſual to call it an *Hemi-tone*, or *Half-Note*, (as being very near it) and the other, Two *Whole Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with *Two Full Tones*.)

Againſt this it is Objected (as not the moſt convenient Diviſion) that the Numbers of 81 to 64, are too great for that of a *Ditone*, or *Greater Third*; which is not Harſh to the Ear; but is rather Sweeter than that of a Single *Tone*, whoſe Proportion is 9 to 8. And, in that of 256 to 243, the Numbers are yet much greater. Whereas there are many Proportions (as $\frac{5}{4}, \frac{6}{5}, \frac{7}{6}, \frac{8}{7}$, in ſmaller Numbers than that of 9 to 8; of which, in this Diviſion, there is no Notice taken.

To Rectifie this, there is another Diviſion thought more convenient ; which is *Ptolemy*'s *Diatonum Intenſum* (of the *Diatonick* Kind, more *Intenſe* or *Acute* than that other.) Which inſtead of two *Full Tones* for *fa*, *ſol*, *la*, aſſigns (what we now call) a *Greater* and a *Leſſer Tone* ; (which by the more Nice *Muſicians* of this and the laſt Age ſeems to be more embraced;) Aſſigning to *fa*, *ſol*, that of 9 to 8, (which they call the *Greater Tone* ;) and to *ſol*, *la*, that of 10 to 9, (which they call the *Leſſer Tone* :) And therefore to *fa*, *la*, (the *Ditone* or *Greater Third*) that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And conſequently to *mi*, *fa*, (which is remaining of the *Fourth*) that of 16 to 15. For $\frac{5}{4}\,)\,\frac{4}{3}\,(\,\frac{16}{15}$. That, if out of that of 4 to 3 we take that of 5 to 4, there remains that of 16 to 15.

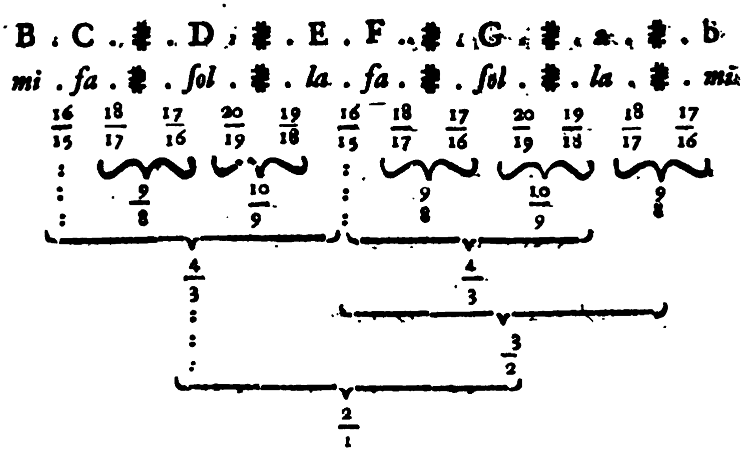
Many other ways there are (with which I ſhall not trouble you at preſent) of dividing the *Fourth* or *Dia-teſſeron*, or the proportion of 4 to 3, into three parts, anſwering to what (in a looſer way of Expreſſion) we call an *Half Note*, and *two Whole Notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as the moſt proper.

To which therefore I ſhall apply my Diſcourſe ; Where $\frac{16}{15}$ is (what we call) the *Hemi-Tone*, or *Half-Note*, in *mi*, *fa* ; $\frac{9}{8}$ that of the *Greater Tone*, in *fa*, *ſol*, and $\frac{10}{9}$ the *Leſſer Tone*, in *ſol*, *la*.

Only with this Addition; That each of thoſe *Tones*, is (upon Occaſion by *Flats* and *Sharps* (as we now ſpeak) divied into two *Hemi-tones*, or *Half Notes* : Which anſwers to what by the *Greeks* was called *Mutatio quoad Modos* (the Change of *Mood* ;) and what is now done by removing *mi* to another *Key*. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus by the help of *Flats* and *Sharps* (dividing each *whole Note*, be it the *Greater* or *Leſſer*, into two *Half Notes*, or what we call ſo,) the whole *Octave* is divided into Twelve parts or *Intervals* (contained beween Thirteen *Pipes*) which are commonly called *Hemi-tones* or *Half Notes*, not that each is preciſely *Half* a *Note*, but ſome what near it, and ſo called. And I ſay, by *Flats* and *Sharps*; For ſometime the one, ſometime the other is uſed. As for Inſtance, a *Flat* in D, or a *Sharp* in C, do either of them denote a Middling Sound (though not preciſely in the Midſt) between C and D ; *Sharper* than C, and *Flatter* than D.

Accordingly, ſuppoſing *mi* to ſtand in *B-fa-bi-mi*) which is accounted its *Natural* Seat) the Sounds of each *Pipe* are to bear theſe Proportions to each other, *viz.*


B . C . ♯ . D . ♯ . E . F . ♯ . G . ♯ . a . ♯ . b
mi . fa . ♯ . ſol . ♯ . la . fa . ♯ . ſol . ♯ . la . ♯ . mi
16/15 18/17 17/16 20/19 19/18 16/15 18/17 17/16 20/19 19/18 18/17 17/16
9/8 10/9 9/8 10/9 9/8
4/3 4/3
3/2
2/1

And ſo in each *Octave* ſucceſſively following. And, if the *Pipes* in each *Octave* be fitted to Sounds in theſe Proportions of *Gravity* and *Acuteneſs*, it will be ſuppoſed (according to this *Hypotheſis*) to be perfectly Proportioned.

But, inſtead of theſe ſucceſſive Proportions of each *Hemi-tone*; it is found neceſſary (if I do not miſtake the Practice) ſo to order the 13 *Pipes* (containing 12 *Intervals* which they call *Hemi-tones*) as that their Sounds (as to *Gravity* and *Acuteneſs*) be in Continual Proportion, (each to its next following, in one and the ſame Proportion ;) which all together ſhall compleat that of an *Octave* or *Dia-paſon*, as 2 to 1. Whereby it comes to paſs that each *Pipe* doth not expreſs its proper Sound, but very near it, yet ſomewhat varying from it ; Which they call *Bearing* ; which is ſomewhat of Imperfection in this *Noble Inſtrument* the Top of all.

It may be asked, ' Why may not the *Pipes* be ſo ordered, as to have their Sounds in juſt Proportion, as well as thus *Bearing* ?

I anſwer, It might very well be ſo, if all *Muſick* were *Compoſed* to the ſame *Key*, or (as the *Greeks* call it) the ſame *Mode* ; As for Inſtance, if, in all Compoſitions, *mi*, were always placed in *B-fa-bi-mi*. For then the *Pipes* might be ordered in ſuch Proportions as I have now deſigned.

But *Muſical Compoſitions* are made in a great Variety of *Modes*, or with great Diverſity in the *Pitch*. *Mi* is not always placed in *B-fa bi-mi* ; but ſometimes in *E-la-mi*, ſometimes in *A-la-mi-re*, &c. And (in Summ) there is none of thoſe 12 or 13 *Pipes* but may be made the *Seat* of *mi*. And, if they were exactly fitted to any one of theſe Caſes, they would be quite out of Order for all the reſt.

As for Inſtance, If *mi*, be removed from *B-fa-b-mi*, (by a *Flat* in B) to *E-la-mi*: Inſtead of the Proportions but now deſigned, they muſt be thus ordered,

B . ♯ . C . ♯ . D . ♯ . E . F . ♯ . G . ♯ . a . b
fa . ♯ . *ſol* . ♯ . *la* . ♯ . *mi* . *fa* . ♯ . *ſol* . ♯ . *la* . *fa*.
$\frac{18}{17}$ $\frac{17}{16}$ $\frac{20}{19}$ $\frac{19}{18}$ $\frac{18}{17}$ $\frac{17}{16}$ $\frac{16}{15}$ $\frac{18}{17}$ $\frac{17}{16}$ $\frac{20}{19}$ $\frac{19}{18}$ $\frac{16}{15}$

where 'tis manifeſt, that the Removal of *mi* doth quite diſorder the whole *Series* of Proportions. And the ſame would again happen, if *mi* be removed from E to A (by another *Flat* in E.) And again, if removed from to A to D. And ſo perpetually. But the *Hemitones* being made all equal, they do indiſferently anſwer all the Poſitions of *mi* (though not exactly to any:) Yet nearer to ſome than to others. Whence it is, that the ſame Tune ſounds better at one *Key* than at another.

It is asked, Whether this may not be remedy'd; by interpoſing more *Pipes*; and thereby dividing a *Note*, not only (as now) into *Half-Notes*, but into *Quarter-Notes*, or *Half-Quarter-Notes*, &c.

I anſwer; It may be thus remedy'd in Part; (that is, the Imperfection might thus be ſomewhat leſs, and the Sounds ſomewhat nearer to the Juſt Proportions:) but it can never be exactly true, ſo long as their Sounds (be they never ſo many) be in continual Proportion; that is, each to the next Subſequent in the ſame Proportion.

For it hath been long ſince demonſtrated, that there is no ſuch thing as a juſt *Hemitone* practicable in *Muſick* (and the like for the Diviſion of a *Tone* into any Number of equal Parts; three, four or more.) For, ſuppoſing the Proportion of a *Tone* or *Full Note*, to be $\frac{9}{8}$ (or, as 9 to 8) that of the *Half Note* muſt be as $\sqrt{9}$, to $\sqrt{8}$; that is as 3 to $\sqrt{8}$, or 3 to $2\sqrt{2}$.) which are *Incommenſurable Quantities*. And that of a *Quarter-Note*, as $\sqrt[4]{9}$ to $\sqrt[4]{8}$, which is yet more *Incommenſurate*. And the like for any other Number of equal Parts, which will therefore never fall in with the Proportions of Number to Number.

So that this can never be perfectly adjuſted for all Keys (without ſomewhat of *Bearing*) by multiplying Pipes; unleſs we would for every *Key*, or every different Seat of *mi*) have a different *Set* of *Pipes*, of which this or that is to be uſed, according as (in the Compoſition) *mi* is ſuppoſed to ſtand in this or that Seat. Which vaſt Number of *Pipes* (for every *Octave*) would vaſtly increaſe the Charge: And (when all is done) make the whole Impracticable.

A new Tuning of the Lyra-Viol; *by* S. Salvetti. n. 87. p. 5064. Aug. An. 1672.

V. *S. Salvetti*, about 4 Years ago invented a *New Tuning* of the *Ancient Lyra-Viol* with the uſual 13 *Strings*; by means of which *Tuning* it is render'd wholly perfect, ſo that you may expreſs upon it all *Concords*, *Diſcords*, and alſo the *Imperfect Concords*, as *Sevenths*, *Sixths*, *&c.* as well as upon any *Virginal* that hath the Quarters of Notes upon it. 'Tis true, 'tis only for melancholly and paſſionate Matter, and not for *Diviſion*, as is the proper Nature of the *Lyra*. I ſhall only add, that with the aboveſaid *Tuning*, it aſcends in *Alt* as high as *G-ſol-re-ut*; and deſcends as low as *Double C-fa-ut*; and can make every where the ſame Concords as above.

The ſtrange Effects reported of Muſick *in Former Times, examined*; *by Dr.* Wallis, n. 243. p. 297. Aug. An. 1698.

VI. 1. I take it for granted, that much of the Reports concerning the great Effects of *Muſick* in former Times, beyond what is to be found in latter Ages, is highly *Hyperbolical*, and next Door to Fabulous; and therefore great Abatements muſt be allowed to the Elogies of their *Muſick*.

2. We muſt conſider, That *Muſick* (to any tolerable Degree) was then (if not a new, at leaſt) a rare Thing, which the Ruſticks, on whom it is reported to have had ſuch Effects, had never heard before; and on ſuch a little Muſick will do great Feats; as we find at this Day, a *Fiddle* or a *Bag-pipe* at a Country Morice Dance.

3. We are to conſider, that their *Muſick* (even after it came to ſome good Degree of Perfection) was much more plain and ſimple than ours now-a-days, They had not *Conſorts* of two, three, four or more *Parts* or *Voices*: But one ſingle Voice or ſingle Inſtrument a-part; which to a rude Ear, is much more taking than more compounded Muſick. For that is at a Pitch not above their Capacity; whereas this other confounds it, with a great Noiſe, but nothing diſtinguiſhable to their Capacity.

4. We are to conſider, that *Muſick* with the *Ancients* was of a larger Extent than what we call *Muſick* now-a-days: For *Poetry* and *Dancing* (or comely Motion) were then accounted Parts of *Muſick*, when *Muſick* arriv'd to ſome Perfection. Now we know that *Verſe* of it ſelf, if in good Meaſures and affectionate Language, and this ſet to a *Muſical Tune*, and ſung by a decent Voice, and accompanied but with *ſoft Inſtrumental Muſick*, if any, ſuch as not to drown or obſcure the *emphatick* Expreſſions (like what we call *Recitative Muſick*) will work ſtrangely upon the Ear, and move all Affections ſuitable to the *Tune* and *Ditty*; (whether briſk and pleaſant, or ſoft and pitiful, or fierce and angry, or moderate and ſedate) eſpecially if attended with a *Geſture* and *Action* ſuitable. For, 'tis well known, that ſuitable *Acting* on a *Stage* gives great Life to the Words. Now all this together (which were all Ingredients in what they called *Muſick*) muſt needs operate ſtrongly on the Fancies and Affections of ordinary People, unacquainted with ſuch kind of Treatments. For, if the deliberate reading of a Romance (when well penn'd) will produce Mirth, Tears, Joy, Grief, Pity, Wrath, or Indignation, ſuitable to the reſpective Intents of it, much more would it ſo do, if accompanied with all thoſe Attendants.

5. You will aſk, perhaps, why may not all this be now done, as well as then? I anſwer, no doubt it may, and with like Effect, if an an Addreſs be made in proper Words, with moving Arguments, in juſt Meaſures, (*Poetical* or *Rhetorical)* with the *emphatick Words*, Words ſet in ſignal Places, pronounced with a good Voice, and a true Accent, and attended with a decent Geſture; and all theſe ſuitably adjuſted to the Paſſion, Affection, or Temper of Mind, particularly deſigned to be produced, (be it Joy, Love, Grief, Pity, Courage or Indignation) will certainly *now* as well as *then*, produce great Effects upon the Mind, eſpecially upon a Surprize, and where Perſons are not otherwiſe pre-engaged: And if ſo managed, as that you be, (or ſeem to be) in earneſt; and if not over-acted by apparent Affectation.

6. We are to conſider, that the uſual Deſign of what we now call *Muſick*, is very different from that of the *Antients*. What we now call *Muſick*, is but what they called *Harmonick*; which was but one Part of their *Muſick*, (conſiſting of Words, Verſe, Voice, Tune, Inſtrument, and Acting) and we are not to expect the ſame Effect of one Piece as of the whole.

7. When *Muſick* arrived to great Perfection, it was applied to particular Deſigns of exciting this or that particular Affection, Paſſion, or Temper of Mind; the *Tunes* and *Meaſures* being ſuitably adapted to ſuch Deſigns. But ſuch Deſigns ſeem almoſt quite neglected in our *preſent Muſick*. The chief Deſign now, in our moſt accompliſhed *Muſick*, being to pleaſe the Ear; when by a ſweet Mixture of different *Parts* and *Voices*, with *Cadencies* and *Concord* intermixed, a grateful Sound is produced, which only the Judicious *Muſician* can diſcern and diſtinguiſh.

8. 'Tis true, that even this *Compound Muſick* admits of different *Characters*, ſome are more briſk and airy; others more ſedate and grave; others more languid; as the different Subjects do require. But that which is moſt proper to excite particular *Paſſions* or *Diſpoſitions*, is ſuch as is more *Simple*, and *Uncompounded*: ſuch as a Nurſe's *languid Tune*, lulling her Babe to ſleep; or a continual Reading in an *Even Tone*; or even the ſoft Murmur of a little Rivulet, running upon Gravel or Pebbles; inducing a quiet Repoſe of the Spirits. And contrariwiſe, the Briſkneſs of a *Jig*, on a *Kit* or *Violin*, exciting to Dance. Which are more Operative to ſuch particular Ends, than an *Elaborate Compoſition* of *Full Muſick*.

9. To conclude; If we aim only at pleaſing the *Ear*, by a *ſweet Conſort*, I doubt not but our *Modern Compoſitions* may be equal, if not exceed thoſe of the *Ancients*: Amongſt whom I do not find any Footſteps of what we call ſeveral Parts or Voices, (as *Baſe*, *Treble*, *Mean*, &c. ſung in Conſort) anſwering each other, to compleat the Muſick. But if we would have our Muſick ſo adjuſted, as to excite particular Paſſions, Affections, or Temper of Mind, (as that of the *Ancients* is ſuppoſed to have done) we muſt apply more ſimple Ingredients, fitted to the Temper we would produce. And this, I doubt not, but a Judicious Compoſer may ſo effect, that (with the Help of ſuch *Hyperboles*, as with which the *Ancient Muſick* is wont to be ſet off) our *Muſick* may be ſaid to do as great Feats as any of theirs.

VII. *Account of Books*, Omitted.

n. 143. p. 20. 1. *Claudii Ptolemæi Harmonicorum* Libri tres, Ex Cod. MSS. undecim, nunc primum *Græce*, Editi. *Jo. Wallis*, S. S. Th. D. Reſenſuit, Edidit, Verſione & Notis Illuſtravit, & Auctuarium adjecit. *Oxon* 1682. in *4to.*

n. 231. p. 668. 2. *Porphyrii* Commentarius in Librum Primum *Harmonicorum Claudii Ptolemæi:* atque *Manuelis Bryennii* Commentarius in tres Libros *Harmonicos* ejuſdem *Ptolemæi.* (Qui ſoli reſtant ex *Græcis Muſicæ* Scriptoribus nondum Editi) *Græce* ac *Latine*, Cura *Jo. Walliſii* S. Th. D. *Fol.*

n. 80. p. 3095. 3. An Eſſay to the Advancement of *Muſick*, by *Tho. Salmon*, M. A. *Lond.* 1672. in 8*vo*.

n. 90. p. 5153. n. 100. p. 7000. 4. *Syntagma Muſicæ*; Treating of *Muſick Philoſophically, Mathematically*, and *Practically*; by *J. Birchenſha*, Eſq; This Book was preparing for the Preſs, 1674.

n. 100. p. 6194. 5. *Muſica* Speculativa del *Mengoli*. in *Bologna* 1670. *in 4to.*

n. 133. p. 835. 6. A *Philoſophical Eſſay* of *Muſick*. *Lond.* 1677. *in 4to.*

n. 208. p. 67. 7. A Treatiſe of the *Natural Grounds* and *Principles* of *Harmony*; by *Will. Holder*, D. D. *Lond.* 1694. *in* 8*vo.*

FINIS.
